Coulomb drag effect between Luttinger liquids

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The exact solution of the one-dimensional Hubbard model, the supersymmetric t-J model, and the gas of fermions interacting via a δ-function potential is used to calculate the drag current between two parallel quantum wires. The critical exponent for the drag current at low temperatures is obtained by means of the mesoscopic energy spectrum and conformal field theory. For a repulsive interaction between the carriers the drag current is opposite to the driving current, while for attractive potentials the two currents are parallel.

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I. INTRODUCTION

Charge carriers moving in one conductor may interact via the Coulomb interaction with carriers in another conductor located nearby. Via momentum conservation the charges in conductor 1 can exert a force on the carriers in the conductor 2 and induce a drag current. This drag mechanism was proposed by Pogrebinskii[1] for a semiconductor-insulator-semiconductor layer structure. At very low temperatures the drag effect in a two-dimensional system is dominated by phonons. The theoretical and experimental developments of the electron drag effect in a coupled electron system have recently been reviewed by Rojo.[2]

A Coulomb drag is also present between two parallel quantum wires. While in two-dimensional layers the Fermi-liquid picture is expected to remain valid, in one-dimensional systems the correlations between electrons lead to exotic properties generically referred to as Luttinger liquids.[3] The Luttinger liquid properties change the Coulomb drag response for ballistic electrons. Characteristics of 1D systems are the charge and spin separation, i.e., the charge and spin contents of the wave functions propagate with different velocities, and the disappearance of the Fermi-liquid quasiparticle pole in the excitation spectrum, which is replaced by incoherent structures. Hence, the Fermi-liquid picture breaks down for interacting electrons in one dimension. Experimental realizations are quasi one-dimensional organic conductors, e.g., the "Bechgaard salts," which have strongly anisotropic electronic properties,[4] and quantum wires, such as single-wall carbon nanotubes.[5]

Electron transport in a wire is ballistic if the scattering length of phonons and impurities is sufficiently long. For simplicity we assume here that only the lowest one-dimensional subband of the conductors is occupied. The Coulomb-drag between quantum wires has previously been studied within both, the Fermi-liquid and Luttinger liquid, pictures and is extensively reviewed in Ref. 6. We limit ourselves to investigate the drag current in linear response to the voltage applied in the driving wire. While within the Fermi liquid approach this leads to a drag current proportional to temperature,[7] the Luttinger liquid picture gives rise to non-universal power laws with critical exponents that depend on the interaction strength.[8–11] In the diffusive regime the drag rate also strongly depends on the effects of disorder.[12]

The Coulomb drag is typically discussed for equal wires with the carriers being spinless interacting fermions.[6] This model can be mapped onto the traditional Luttinger liquid description for a one-dimensional conductor with the "spin" components playing the role of the driving and drag wires. The bosonization of the fermions in conjunction with the renormalization-group flow then yield the drag current, which is given by the renormalized backward-scattering amplitude (sine-Gordon term) of the traditional Luttinger liquid. The experimental results[13–16] on the Coulomb drag between parallel wires remain sparse, probably because (1) the drag voltage usually has a very small amplitude and (2) it is difficult to create parallel electrically isolated quantum wires that are sufficiently long and close enough to yield a measurable drag voltage.[6]

In short wires the low-T conductance is affected by the discrete energy-level spectrum and the current increases in steps as the voltage is increased.[5] We consider here sufficiently long wires so that the frequency can be treated as a continuous variable. At very low T and small ω, of course, there will be a crossover to the quantum limit, where the discreteness of the energy spectrum plays a dominant role.[17] On the other hand, the wires have to be sufficiently short, so that the transport can be considered ballistic, i.e., scattering by phonons and defects can be neglected.

In this paper we calculate the critical exponents of the drag-current response function to a driving current in a parallel wire for a Luttinger liquid. The critical behavior is determined by the low-energy excitation spectrum (mesoscopic corrections to the ground-state energy) via conformal field theory. To model the Luttinger liquid we use the exact Bethe *Ansatz* solution of integrable one-dimensional systems.[18] The situations of repulsive and attractive interactions have to be distinguished. In the former case the drag current is opposite to the driving current, while in the latter the two currents are parallel due to the formation of bound states of carriers between the two wires. For a repulsive interaction the results include those obtained perturbatively, e.g., via bosonization and renormalization group. In the attractive case a spin-gap opens, which cannot be obtained perturbatively, i.e., from the bosonization of fermions.

The remainder of the paper is organized as follows. In Sec. II we discuss the standard situation of spinless carriers interacting via a repulsive potential in terms of the Bethe *Ansatz* solution of (i) the Hubbard model, (ii) the gas of particles with parabolic dispersion and a δ-function potential, and (iii) the SU(3)-invariant *t-J* model. In Sec. III we investigate the situation of spinless carriers interacting via an attractive potential using the exact solutions for (i) the Hubbard model, (ii) the gas of particles with a δ-function repulsion, and (iii) the supersymmetric *t-J* model. In Secs. IV and V some results of Secs. II and III, respectively, are extended to the situation of carriers with spin. This can only be accomplished for the δ-function potential gas and the *t-J* models, but not for the Hubbard model. Conclusions follow in Sec. VI.

II. SPINLESS CARRIERS WITH REPULSIVE INTERACTION

The standard approach[6] considers two parallel quantum wires with spinless carriers interacting via a repulsive potential. Each conductor has a Fermi point for forward (backward) moving charges. When an electric field is applied to the driving wire, some carriers are transferred from one Fermi point (e.g., for the backward movers) to the other Fermi point (i.e., for forward movers), thus generating a current. The momentum conservation of the interaction for carriers between wires yields then a backward momentum transfer in the drag wire, inducing this way a drag current opposite to the driving current. The response function for this current is proportional to a power of the temperature or frequency, with the critical exponent being nonuniversal, depending on the model, the interaction strength and the bandfilling. The critical exponents are determined by the low-energy excitations of the model, i.e., the conformal towers of the Luttinger liquid under consideration. Below we briefly review the excitation spectra and discuss the critical exponents for three models.

A. The Hubbard model

We assume that each wire consists of spinless fermions on a nearest-neighbor tight-binding Hamiltonian and that carriers on different wires interact with a local Coulomb interaction U. This corresponds to the Hubbard model,

$$H_U=-\sum_{i\sigma}\,(c^{\dagger}_{i\sigma}c_{i+1\sigma}+c^{\dagger}_{i+1\sigma}c_{i\sigma})+U\sum_i\, n_{i1}n_{i2}, \quad (1)$$

where $\sigma=1,2$ labels the conductors, the hopping matrix element is set equal to 1, and $n_{i\sigma}$ is the number operator. We consider N_e itinerant carriers on a chain of N_a sites with periodic boundary conditions. Model (1) has been exactly diagonalized by means of two nested Bethe *Ansätze* by Lieb and Wu[19] in terms of two sets of rapidities, $\{k_j\}$ for $j=1,\dots,N_e$ representing the charges, and $\{\lambda_\alpha\}$ for $\alpha=1,\dots,M$, with N_e-2M corresponding to the population difference between the two wires (spinons for the standard Hubbard chain). These rapidities are self-consistently determined by the Bethe *Ansatz* equations and the energy is given by $E=-\sum_{j=1}^{N_e}2\cos(k_j)$.

For $U>0$ all the rapidities are real in the ground state and densely distributed (without holes) between the respective Fermi points at $\pm Q$ for the charges and $\pm B$ for the "spinons."[19] If both wires have the same number of carriers, then the spinon band is completely filled, so that $B=\infty$. The critical properties of correlation functions at low temperatures and small frequencies are determined by the low-energy excitation spectrum, which is given by the finite size corrections of mesoscopic order to the ground-state energy in terms of a set of quantum numbers,[20]

$$E=N_a\epsilon_\infty+\sum_l\,\frac{\pi v_l}{2N_a}\left[\sum_q\,(\hat{z}^{-1})_{lq}\Delta N_q\right]^2+\sum_l\,\frac{2\pi v_l}{N_a}\left\{\left[\sum_q\,z_{ql}D_q\right]^2+n_l^{+}+n_l^{-}-\frac{1}{12}\right\}, \quad (2)$$

where ϵ_∞ is the ground-state energy density in the thermodynamic limit, l and q label the two bands and take values c and s (for charges and spinons), and v_l denote the group velocities of the two bands. Here ΔN_q is the departure of the number of particles in the band q from the equilibrium value, i.e., ΔN_e and ΔM, respectively. Note that each band has two Fermi points corresponding to forward and backward moving states. D_q is the backward-scattering quantum number, i.e., $2D_q$ represents the difference of forward to backward moving states in each band. These quantities are sensitive to the parity in each set of rapidities. Finally, $n_q^{\pm}$ define the low-lying particle-hole excitations about each of the Fermi points. Here ΔN_q, $n_q^{\pm}$, and $2D_q$ take always integer values; hence D_q can either be an integer or half-integer depending on the initial conditions.

The quantities z_{lq} in Eq. (2) are the dressed generalized charges of the excitations, which describe the interplay of the different Fermi points when particles (charges or spinons) are added or removed, and $\hat{z}^{-1}$ denotes the inverse of the matrix. The integral equations determining $z_{lc}=\xi_{l,c}(Q)$ and $z_{ls}=\xi_{l,s}(B)$ simplify considerably for $B=\infty$, where $z_{cs}=0$, $z_{ss}=1/\sqrt{2}$, $z_{sc}=\frac{1}{2}z_{cc}$, and z_{cc} is determined from[20,21]

$$\xi_{c,c}(k)=1+\int_{-Q}^{Q}dk'\cos(k')G[\sin(k)-\sin(k')]\xi_{c,c}(k'),$$

$$G(x)=\mathrm{Re}\left[\Psi(1+ix/U)-\Psi\left(\frac{1}{2}+ix/U\right)\right]\Big/(\pi U), \quad (3)$$

where Ψ is the digamma function and Re denotes real part. $\theta=2z_{cc}^2$ is related to the charge stiffness. The Fermi momentum for the charges is $p_{Fc}=\pi n$, with $n=N_e/N_a$, and the one for the spinons $p_{Fs}=\pi n/2$, so that for the Fermi momentum of the wires we have $p_{F1}=p_{F2}=\pi n/2$.

In terms of the quantum numbers defined above, the total momentum of the system is given by[20,21]

$$P=\frac{2\pi}{N_a}\sum_l\,[N_lD_l+n_l^{+}-n_l^{-}]. \quad (4)$$

From Eqs. (3) and (4) we obtain the conformal dimensions of primary fields characterized by the above quantum numbers[22]

$$\Delta_c^{\pm}=n_c^{\pm}+\frac{1}{8}[z_{cc}(2D_c+D_s)\pm\Delta N_e/z_{cc}]^2,$$

$$\Delta_s^{\pm}=n_s^{\pm}+\frac{1}{16}[2D_s\pm(2\Delta M-\Delta N_e)]^2. \tag{5}$$

We can now compute the simplest case for the drag-current response function. In wire 1 (driving wire) a carrier with momentum $-p_{F1}$ is transferred to the other Fermi point with p_{F1}, and simultaneously in the drag wire (wire 2) a charge undergoes the transformation $p_{F2}\rightarrow -p_{F2}$. If $p_{F1}=p_{F2}$ this process conserves momentum and energy. The quantum numbers for this process are $\Delta N_e=\Delta M=0$, since each of the wires conserves its charges, and $n_c^{\pm}=n_s^{\pm}=0$ because particle-hole excitations about the Fermi points only contribute to higher order. The process involving both, the driving and the drag, wires corresponds to $D_c=\pm 1$ and $D_s=\mp 2$, so that $\Delta_c^{\pm}=0$ and $\Delta_s^{\pm}=1$. The critical exponent for this correlation function does then neither depend on the interaction strength nor on the band filling, because the overall process is a conserving one, independent of U. If, on the other hand, we consider only the drag current, then $D_c=0$ and $D_s=\mp 1$, so that $\Delta_c^{\pm}=\theta/16$ and $\Delta_s^{\pm}=1/4$.

At $T=0$ the drag-current correlation function for long times and large distances is then proportional to

$$\frac{\exp(-2ip_{Fs}x)}{[x^2+(v_ct)^2]^{\theta/8}[x^2+(v_st)^2]^{1/2}}, \tag{6}$$

where the exponential represents the momentum transfer. The response function falls off with a power law for long times and distances, where the critical exponent, $\theta/4+1$, is nonuniversal and depends on the interaction strength and bandfilling. The extension to finite temperature and chain length yields[18,21,22]

$$\left\{\frac{\pi T/N_a}{\sinh[\pi T(x-iv_ct)/v_c]}\,\frac{\pi T/N_a}{\sinh[\pi T(x+iv_ct)/v_c]}\right\}^{\theta/8}$$
$$\times\left\{\frac{\pi T/N_a}{\sinh[\pi T(x-iv_st)/v_s]}\,\frac{\pi T/N_a}{\sinh[\pi T(x+iv_st)/v_s]}\right\}^{1/2}, \tag{7}$$

where the exponential representing the momentum transfer has now been dropped. We now consider the equal-time correlation function $(t=0)$ and integrate with respect to x to obtain the temperature dependence of the drag current, which is proportional to $(\pi T/N_a)^{\theta/4}$. The exponent θ is shown in Fig. 1(a) as a function of the band filling $n=N_e/N_a$ for several U.[18,22] For $U=0$ and $U=\infty$ we have that $\theta=4$ and 2, respectively. θ is a decreasing function of U. Note the strong dependence of θ as $n\rightarrow 1$ as a consequence of the metal-insulator transition.

It is also of interest to calculate the temperature dependence of the primary current, i.e., the driving current in wire 1. To transfer a charge from one Fermi point to the other in wire 1, it requires that $D_c=1$ and $D_s=-1$ with all other quantum numbers being zero. The conformal dimensions are

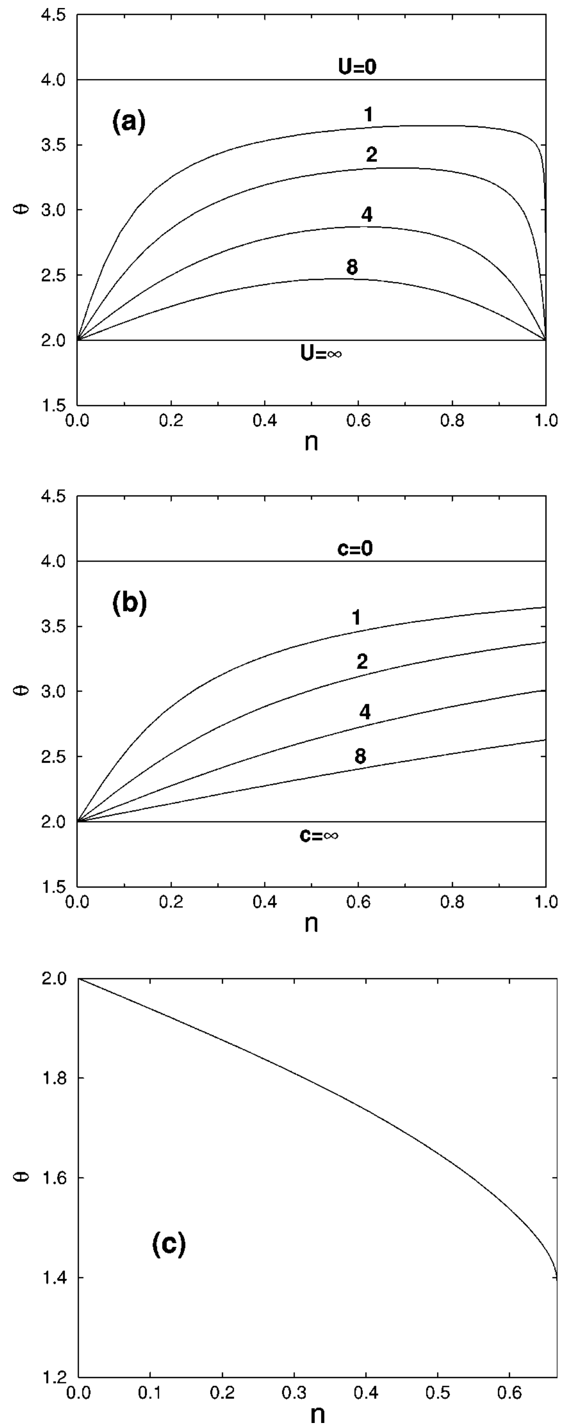

FIG. 1. Critical exponent θ as a function of the band filling n for (a) the Hubbard model for various values of the repulsion U, (b) the gas of charges with δ-function repulsion for various coupling strengths c, and (c) the supersymmetric t-J model with SU(3) invariance $(2t=J)$. For $n\rightarrow 0$, $\theta=2$ for all three models, while the behavior is model dependent for larger n. For the Hubbard model $\theta\rightarrow 2$ as $n\rightarrow 1$ because of the metal-insulator transition.

then $\Delta_c^{\pm}=\theta/8$ and $\Delta_s^{\pm}=1/4$, i.e., the same values as for the drag current. Hence, as expected, the two correlation functions are identical.

For a drag current to take place we required that p_{F1}

$=p_{F2}$. Otherwise the momentum is not conserved and it will require a phonon to make up for the difference in momentum. Hence, at $T=0$, we obtain a δ function if $p_{F1}=p_{F2}$, while for $T\neq 0$ a sharp peak of width proportional to T is expected, due to the lifetime of the excitations in a Luttinger liquid.

There are of course other possibilities for a momentum transfer between the two wires. The general condition for n_1 particles being backscattered in wire 1 and n_2 backscattered in wire 2 is $n_1 p_{F1}=n_2 p_{F2}$. In terms of the Fermi momenta of the two Bethe *Ansatz* bands, i.e., charges and spinons, this condition reads $n_1 p_{Fc}=(n_1+n_2)p_{Fs}$. Small deviations from this condition can still be compensated by temperature broadening. The quantum numbers for the drag current (n_2 carriers are pushed backward) are all zero except $D_s=n_2$. The conformal dimensions for this case are $2\Delta_c^{\pm}=(z_{sc}n_2)^2$ and $2\Delta_s^{\pm}=(z_{ss}n_2)^2$. Here z_{sc} and z_{ss} have to be determined by solving the coupled integral equations for $\xi_{i,j}$, since now $B\neq\infty$. The equal-time correlation function is now (dropping the phase factor $e^{-2in_2 p_{Fs}x}$)

$$\left[\frac{\pi T/N_a}{\sinh(\pi T x/v_c)}\right]^{2(z_{sc}n_2)^2}\left[\frac{\pi T/N_a}{\sinh(\pi T x/v_s)}\right]^{2(z_{ss}n_2)^2}, \tag{8}$$

and after integrating over x we obtain for the temperature dependence of the drag current, $(\pi T/N_a)^{2(z_{sc}^2+z_{ss}^2)n_2^2-1}$. Note that, since $p_{F1}\neq p_{F2}$, the exponent of the temperature dependence of the driving current response function is now different from that of the drag-current correlation function. Other mechanisms to transfer momentum between the wires involve an umklapp process, i.e., a vector of the reciprocal lattice is absorbed, so that

$$2n_1 p_{F1}=2n_2 p_{F2}+G, \quad G=2\pi n, \tag{9}$$

where n is another integer.

B. δ-function potential

An alternative model with similar characteristics is the gas of fermions interacting via a δ-function potential.[23] The two conductors are labeled 1 and 2, and as for the Hubbard model this label plays the role of the spin. The model can be written as

$$H_c=-\sum_{i=1}^{N_e}\left(\frac{\partial}{\partial x_i}\right)^2+2c\sum_{i<j}\delta(x_i-x_j), \tag{10}$$

where c is the interaction strength and $\{x_i\}$ are the positions of the carriers. The dispersion in the wires here is parabolic with the effective mass equated to 1/2. For periodic boundary conditions the Bethe *Ansatz* solution of this model is similar to that of the Hubbard model. Two sets of rapidities, corresponding to charges and spinons, are required to diagonalize the model. For $c>0$ all the rapidities are real in the ground-state and if both wires have the same number of carriers, then the spinon band is completely filled.[23] The finite-size corrections of mesoscopic order to the ground-state energy, the momentum, and the conformal dimensions are also given by Eqs. (2), (4), and (5), respectively. The same set of quantum numbers as for the Hubbard model describes the low-energy excitations.[18,20] The group velocities and the dressed generalized charges are, however, different. The simplest case for momentum transfer between wires is again $p_{F1}=p_{F2}$, for which $z_{cs}=0$, $z_{ss}=1/\sqrt{2}$, $z_{sc}=\frac{1}{2}z_{cc}$, and $z_{cc}=\xi_{c,c}(Q)$ is determined from[18]

$$\xi_{c,c}(k)=1+\int_{-Q}^{Q}dk' H_N(k-k')\xi_{c,c}(k'),$$

$$H_N(x)=\frac{1}{\pi Nc}\mathrm{Re}\left[\Psi\left(1+\frac{ix}{Nc}\right)-\Psi\left(\frac{1}{N}+\frac{ix}{Nc}\right)\right] \tag{11}$$

for $N=2$. Here Ψ is again the digamma function. The drag current is proportional to $(\pi T/N_a)^{\theta/4}$. The exponent $\theta=2z_{cc}^2$ for the δ-function potential is shown in Fig. 1(b) as a function of the band filling $n=N_e/N_a$ for several c. The dependence of θ on c is similar to the Hubbard model, except for $n\to 1$, where the continuum limit has no metal-insulator transition. All other conclusions are the same as for the Hubbard model, except that for a continuum model there are no umklapp processes.

C. SU(3)-invariant *t-J* model

The *t-J* model involves three possible states per site, namely, a hole, an up-spin electron and a down-spin electron. A permutation symmetry of these three states between neighboring sites leads to a supersymmetric algebra[24,25] and hence to integrability. Here we assume that the number of holes is larger than both the number of up-spin and down-spin electrons. The Hamiltonian is then given by

$$H_{tJ}=-\sum_{i\sigma}P(c_{i\sigma}^{\dagger}c_{i+1\sigma}+c_{i+1\sigma}^{\dagger}c_{i\sigma})P$$

$$+\sum_{i\sigma\sigma'}c_{i+1\sigma}^{\dagger}c_{i\sigma'}^{\dagger}c_{i\sigma}c_{i+1\sigma'}+\sum_i n_i n_{i+1}, \tag{12}$$

where P is a projector that excludes the multiple occupancy at each site and $n_i=\Sigma_\sigma c_{i\sigma}^{\dagger}c_{i\sigma}$ is the number operator for site i. This variant of the *t-J* model corresponds to an effective repulsion of particles on nearest-neighbor sites.

In order to map the Coulomb drag problem onto the supersymmetric *t-J* model we consider spinless carriers traveling along two quantum wires with $\sigma=1,2$ labeling the two conductors. The Bethe *Ansatz* solution of the *t-J* model in terms of two sets of rapidities (for the charges and spinons, respectively) can be found in Ref. 26. For the ground-state all rapidities are real and densely distributed. The simplest condition for the Coulomb drag is again that $p_{Fc}=2p_{Fs}=\pi n$, where n is the fermion density, N_e/N_a, and p_{Fc} and p_{Fs} are the Fermi momenta of the charge and spinon bands, respectively. In terms of the Fermi momenta for the two conductors this condition is $p_{F1}=p_{F2}=\pi n/2$, i.e., both wires have the same number of carriers. As a consequence of the momentum conservation, the drag current is then again in the opposite direction to the driving current.

As for the Hubbard model, the critical behavior at long times and large distances is determined by the finite-size corrections to the ground-state energy. The general expression of the conformal towers has the same form as in Eq. (2), where the quantum numbers also have the same meaning here. The ground-state energy in the thermodynamic limit, the group velocities, and the generalized dressed charges are, however, model dependent and, hence, different. The conformal dimensions only involve the generalized dressed charges, which for equal density of carriers in both conductors are again $z_{cs}=0$, $z_{ss}=1/\sqrt{2}$, $z_{sc}=\frac{1}{2}z_{cc}$, and $z_{cc}=\xi_{c,c}(Q)$, where now $\xi_{c,c}(p)$ satisfies the linear integral equation

$$\xi_{c,c}(p)=1-\int_{-Q}^{Q}dp' K_N(p-p')\xi_{c,c}(p'),$$

$$K_N(x)=\frac{1}{\pi N}\mathrm{Re}\left[\Psi\left(1+\frac{1}{N}+\frac{ix}{N}\right)-\Psi\left(1+\frac{ix}{N}\right)\right] \quad (13)$$

for $N=2$. Here Ψ is the digamma function.

The quantum numbers for the drag-current response function are the same as for the previous cases, i.e., $D_s=\pm 1$ and all others are zero. Here we again assume the driving wire has the label 1 and wire 2 hosts the drag current. The conformal dimensions for the drag current are then $\Delta_c^{\pm}=\theta/16$ and $\Delta_s^{\pm}=1/4$, and the temperature dependence of the correlation function is $(\pi T/N_a)^{\theta/4}$. The exponent θ is shown in Fig. 1(c) as a function of the band filling $n=N_e/N_a$. Note that for this model n cannot exceed the value 2/3. Note that θ is now less than 2 and decreases with n. The current correlation function for the driving wire has the same temperature dependence. All other conclusions discussed for the Hubbard model remain valid here.

III. SPINLESS CARRIERS WITH ATTRACTIVE INTERACTION

If the interaction between carriers in different wires is attractive, the dominant drag current is parallel to the driving current. Below we discuss this effect using three integrable models, namely, the Hubbard model, the gas with δ-function interaction, and the standard supersymmetric t-J model.

A. The Hubbard model

We consider model (1) but for $U<0$. The Bethe *Ansatz* solution (i.e., the discrete Bethe equations) is the same as for repulsive interaction.[19] The attractive interaction pairs the electrons into Cooper-like singlet states without off-diagonal long-range order even at $T=0$.[27] Here the two wires play the role of the spin. For *equal number of carriers* in each wire, the singlet pairs introduce a "spin-gap" (binding energy) in the excitation spectrum of unpaired electrons. The pairs act like hard-core bosons and are characterized by a set of real rapidities Λ with Fermi points at $\pm Q$. Since excitations involving only one conductor (spin excitations) are suppressed by the gap, the finite-size corrections to the ground-state energy are those of a one-component Luttinger liquid,[28]

$$E=N_a\epsilon_\infty+(\pi v_F/2N_a)(\Delta M/z)^2 +(2\pi v_F/N_a)[z^2D^2+n^++n^--1/12], \quad (14)$$

where again ϵ_∞ is the ground-state energy density and v_F is the group velocity of the paired charges. The quantum numbers have similar meanings as before, ΔM is the departure of the number of pairs from the equilibrium value, $2D$ is the difference of forward to backward moving pairs, and $n^\pm$ represents the low-lying particle-hole excitations for paired electron states about each of the Fermi points. Again ΔM, $n^\pm$, and $2D$ take always integer values; hence D can either be an integer or half-integer depending on the initial conditions.

The quantity z is the generalized dressed charge, related to the stiffness of the Cooper pairs, given by $z=\xi(Q)$, where $\xi(\Lambda)$ satisfies the integral equation[28]

$$\xi(\Lambda)+\int_{-Q}^{Q}d\Lambda' a_2(\Lambda-\Lambda')\xi(\Lambda')=1, \quad (15)$$

where $a_2(x)=(|U|/2\pi)/(x^2+U^2/4)$. The Fermi momentum of the Cooper pairs is $p_F=\pi n/2$, and the total momentum of the system in terms of the quantum numbers is $P=(2\pi/N_a)[MD+n^+-n^-]$. The conformal dimensions of primary fields are obtained from Eq. (14) and P (Ref. 28),

$$\Delta^\pm=n^\pm+\frac{1}{2}[\Delta M/(2z)\pm zD]^2. \quad (16)$$

Since the electrons are all paired in bound states, the drag and driving currents are parallel. The voltage applied to the driving wire can then only transfer charges from one Fermi point to the other without creating spin excitations, which are gapped. Consequently, there is a simultaneous charge transfer in both wires. The quantum numbers corresponding to this process are $D=1$ and all others are zero. In terms of carrier field operators the current operator for simultaneous conductivity in both wires is $\psi_{1-}^\dagger(x,t)\psi_{2-}^\dagger(x,t)\psi_{2+}(x,t)\psi_{1+}(x,t)$ and the corresponding equal-time response function is proportional to

$$e^{-4ip_Fx}\left(\frac{\pi T/N_a}{\sinh(\pi Tx/v_F)}\right)^\theta, \quad (17)$$

where $\theta=2z^2$ is related to the stiffness of the pairs. Integrating with respect to x we obtain $(\pi T/N_a)^{\theta-1}$ for the simultaneous conduction through both channels. θ as a function of the electron density $n=N_e/N_a$ is displayed in Fig. 2(a) for several values of U. Note that for $U\to-\infty$ we obtain $\theta=2$ and for $U\to 0$ we have $\theta=1$. Note the strong dependence on n as $n\to 1$ due to the Van Hove singularity of the filled band of pairs.

If the *number of carriers* in the two wires is *different*, not all the charges can be paired, and a second branch of states (corresponding to unpaired charges) appears in the ground-state. This band of rapidities is then no longer gapped, but has a Fermi surface. The finite-size corrections to the ground-state energy are then again of the general form of Eq. (2), where l and q label the paired (p) and unpaired (u)

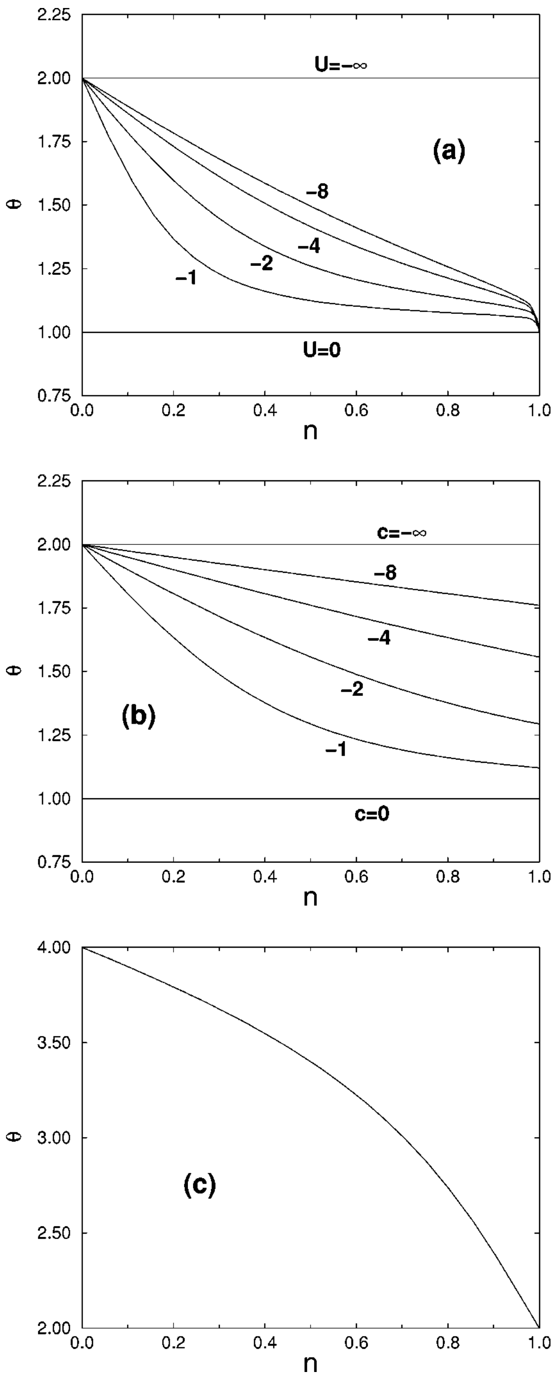

FIG. 2. Critical exponent θ as a function of the band filling n for (a) the Hubbard model with attractive U for various interaction strengths, (b) the gas of charges with attractive δ-function potential for various coupling strengths c, and (c) the standard supersymmetric t-J model. For $n \to 0$, $\theta = 2$ for the first two models, while $\theta = 4$ for the t-J model.

states. The conformal dimensions are similar to Eq. (5), but with indices p and u. Since there are now two Dirac seas, it is possible to consider the driving and drag currents separately.

In order to discuss the drag current we have to distinguish two cases: either (i) the drag wire or (ii) the drive wire has no unpaired carriers. In case (i) the driving voltage accelerates paired and unpaired charges. The paired carriers give rise to a drag current parallel to the driving current and the drag current is determined by the quantum numbers $D_p = -1$ and $D_u = 1$, while all other quantum numbers are zero, i.e., it is proportional to $(\pi T/N_a)^{2(z_{pp}-z_{up})^2+2(z_{pu}-z_{uu})^2-1}$. The unpaired charges can induce a backward current of pairs if $p_{Fu} = 2p_{Fp}$ to satisfy momentum conservation, which has the same temperature dependence as the pair-induced current. In case (ii) the driving voltage accelerates only the pairs, but the drag current has two components, namely, from paired and unpaired charges. The drag current arising from paired charges, which is parallel to the driving current, is characterized by the nonzero quantum number $D_p = 1$ and has the exponent $2(z_{pp}^2 + z_{pu}^2) - 1$. The drag current arising from the unpaired states, on the other hand, is opposite to the driving current (characterized by $D_u = 1$ and all other quantum numbers being zero) and has the critical exponent $2(z_{up}^2 + z_{uu}^2) - 1$. As a function of the unpaired charges, the exponents change discontinuously from case (i) to (ii).

B. δ-function potential

We consider model (10) but for $c < 0$. The discrete Bethe *Ansatz* equations are the same as for a repulsive interaction.[23] As for the Hubbard model the attractive potential forms bound states of carriers between the two wires.[27] We again have to distinguish the cases of equal and different number of carriers in each wire. In the former case all charges are paired in bound states, while in the latter case there are unpaired carriers left over.

For *equal number of carriers* in each wire, the excitations of unpaired electrons are gapped, and the finite-size excitation spectrum is again given by Eq. (14).[28] The group velocity for the pairs and the dressed generalized charge are, however, different from those of the Hubbard model. The drag and driving currents are parallel, since the carriers cannot be depaired. For simultaneous conductivity in both wires the quantum numbers are $D = 1$ and all others zero, and the critical exponent for the current is $\theta - 1$, where $\theta = 2z^2$. θ as a function of $n = N_e/N_a$ is displayed in Fig. 2(b) for several values of c. Since in the continuum there is no upper band edge, there is no strong dependence of θ as $n \to 1$.

For wires with *different carrier densities* not all carriers are paired, so that the rapidity band of unpaired carriers is partially filled and the system behaves like a two-component Luttinger liquid. As for the Hubbard model with attractive U, the corresponding conformal towers are given by Eq. (2), where l and q label the paired (p) and unpaired (u) states. The dressed generalized charges entering the conformal towers and dimensions are given by $z_{pq} = \xi_{pq}(Q)$ and $z_{uq} = \xi_{uq}(B)$,

$$\xi_{pl}(\Lambda) + \int_{-Q}^{Q} d\Lambda' a_2(\Lambda - \Lambda')\xi_{pl}(\Lambda') + \int_{-B}^{B} d\Lambda' a_1(\Lambda - \Lambda')\xi_{ul}(\Lambda') = \delta_{pl},$$

$$\xi_{ul}(\Lambda) + \int_{-Q}^{Q} d\Lambda' a_1(\Lambda - \Lambda')\xi_{pl}(\Lambda') = \delta_{ul}, \qquad (18)$$

where $a_n(x)=(n|c|/2\pi)/[x^2+(nc/2)^2]$, and Q and B determine the number of paired and unpaired carriers, respectively. The conformal dimensions are given by

$$2\Delta_p^{\pm}=(z_{pp}D_p+z_{up}D_u)^2,\quad 2\Delta_u^{\pm}=(z_{pu}D_p+z_{uu}D_u)^2. \tag{19}$$

With two Dirac seas, it is now possible to have separate driving and drag currents. The analysis of the drag current follows in complete analogy to the last paragraph in Sec. III A.

C. Supersymmetric *t-J* model

In this section we consider the standard *t-J* model, which involves three possible states per site (a hole, an up-spin electron and a down-spin electron). In contrast to the variant discussed in Sec. II C, which is SU(3)-invariant, the present model is also integrable for $2t=J$ and has an underlying supersymmetric algebra.[24,25] The Hamiltonian is given by

$$H_{tJ}=-\sum_{i\sigma} P(c_{i\sigma}^{\dagger}c_{i+1\sigma}+c_{i+1\sigma}^{\dagger}c_{i\sigma})P + \sum_{i\sigma\sigma'} c_{i+1\sigma}^{\dagger}c_{i\sigma'}^{\dagger}c_{i\sigma}c_{i+1\sigma'}-\sum_i n_i n_{i+1}, \tag{20}$$

where P is a projector that excludes the multiple occupancy at each site and $n_i=\Sigma_\sigma c_{i\sigma}^{\dagger}c_{i\sigma}$ is the number operator for site *i*. This model corresponds to the large *U*-limit of the Hubbard model[29] and the Zhang-Rice singlet state.[30] Again the spin index labels the two wires.

The Bethe *Ansatz* solution of this model involves two sets of rapidities, one for the charges and one for the spins. For equal number of carriers in each wire, in the ground-state all the carriers are paired in analogy to the Hubbard and δ-function models. The key difference of the *t-J* model with the other two models is that the binding energy is zero, i.e., the unpaired carrier excitations are not gapped in this case. Hence, we do not have to distinguish between the gapped and gapless situations. The system is a two-component [paired (*p*) and unpaired (*u*) carriers] Luttinger liquid and the finite-size corrections to the ground-state energy is of the form of Eq. (2).

For simplicity we limit ourselves to the case of equal carrier density in the driving and drag wires. As for the Hubbard model with repulsive U the matrix of dressed generalized charges has dimension 2×2. [Here we follow the notation of Refs. 18 and 31, which is actually in terms of holons and up-spin holes, rather than electrons.] For the dressed charges we obtain that $z_{pu}=0$, $z_{uu}=2^{-1/2}$, $z_{up}=z_{pp}/2$, and $z_{pp}=\xi(Q)$, where

$$\xi(\Lambda)=1+\int_{-Q}^{Q}d\Lambda' H_N(\Lambda-\Lambda')\xi(\Lambda'), \tag{21}$$

with H_N given by Eq. (11) for $N=2$ and $c=1$. The parameter $\theta=2z_{pp}^2$ appearing in the critical exponents is shown in Fig. 2(c) as a function of $n=N_e/N_a$. For all three models in this section, θ is a decreasing function of *n*.

For the drag current we have to consider two contributions. On the one hand, the driving potential pulls on the paired electrons, giving rise to a drag current parallel to the driving current. The quantum numbers corresponding to the simultaneous conductivity in both wires are $D_p=1$ and all others zero, and the critical exponent for the current is $2(z_{pp}^2+z_{pu}^2)-1=\theta-1$. On the other hand, if either the driving current or the drag current have a component of unpaired charges, then if $p_{Fu}=2p_{Fp}$ there is the possibility of transferring momentum between the two wires. In this case the driving and drag currents flow in opposite directions. The quantum numbers for this process are $D_p=1$ and $D_u=-1$, while all other quantum numbers are zero. The critical exponent for the drag current is $2(z_{pp}-z_{up})^2+2(z_{pu}-z_{uu})^2-1$. The situation $p_{Fu}=2p_{Fp}$ can only be satisfied if the charge density in the two wires is very different.

IV. REPULSIVE INTERACTION: THE ROLE OF THE SPIN

In this section we reconsider the situation of repulsive interaction between the carriers, but in contrast to Sec. II the carriers now have spin. The spin and the two wires give rise to four internal degrees of freedom. Unfortunately, the degenerate Hubbard model [of more than two internal variables $(N=2)$] is not integrable. We therefore limit ourselves to the δ-function interaction and the SU(5)-invariant *t-J* model.

A. δ-function potential

The gas of fermions with four colors interacting via a δ-function potential, defined by the Hamiltonian (10), is also integrable in terms of four nested Bethe *Ansätze*.[32] There are then four sets of rapidities, corresponding to the charges (total number of particles) and three spinons sets for the relative population of the internal degrees of freedom. For the ground-state and $c>0$ all the rapidities are real.[18,32] In the absence of an external magnetic field, the first and third spinon rapidity bands are completely filled.[33] If in addition the two wires have the same density of electrons, then the second spinon rapidity band is also completely filled. We limit our discussion to this case.

The finite-size corrections of mesoscopic order to the ground-state energy the momentum and the conformal dimensions are given by Eqs. (2), (4), and (5), respectively, but with l and q running over four indices.[18] The low-energy excitations are given in terms of four sets of quantum numbers, i.e., for each rapidity band, the change in the number of rapidities ΔN_q, the quantum number for backward scattering D_q and particle-hole excitations about each Fermi point, $n_q^{\pm}$. The matrix of generalized dressed charges has dimension 4×4 and has the form characteristic of SU(N) invariance, i.e., $z_{qc}=[(N-q)/N]z_{cc}$, $z_{cq}=\delta_{q,0}z_{cc}$ and the spin sector is given by the symmetric 3×3 matrix $\mathcal{Z}_N$ defined by[34]

$$(\mathcal{Z}_N^{-2})_{ij}=2\delta_{i,j}-\delta_{i,j-1}-\delta_{i,j+1},$$

$$(\mathcal{Z}_N^{-1})_{ij}=-\frac{2}{N}\frac{y_{1/2}y_iy_j}{x_i^2+x_j^2-2x_{1/2}x_ix_j-y_{1/2}^2}, \tag{22}$$

where $x_j = \cos(\pi j/N)$ and $y_j = \sin(\pi j/N)$. Since all components are equally populated, there are only two different conformal dimensions of fields, one associated with the charge sector (of central charge 1) and the other one with the SU(N) symmetry (of central charge $N-1=3$, Refs. 18 and 34)

$$2\Delta_c^{\pm} = 2n_c^{\pm} + \left[z_{cc} \sum_{j=0}^{N-1} \frac{N-j}{N} D_j \pm \frac{\Delta N_e}{2z_{cc}} \right]^2,$$

$$2\Delta_S^{\pm} = 2n_S^{\pm} + \frac{1}{4} \sum_{i,j=1}^{N-1} \Delta M_i (\mathcal{Z}_N^{-2})_{ij} \Delta M_j + \sum_{i,j=1}^{N-1} D_i (\mathcal{Z}_N^2)_{ij} D_j \pm \sum_{i=1}^{N-1} \Delta M_i D_i, \quad (23)$$

where $n_S^{\pm}$ is the sum over the quantum numbers for the excitations at the forward and backward Fermi points for the $q=1,2,3$, and $q=0$ refers to the charges. Note that $\Delta_S^{\pm}$ is independent of the interaction strength and band filling; it is the same conformal dimension characterizing the SU(N) Heisenberg chain and corresponding critical vertex models.[35] If the electron density is the same in both wires and per spin component [SU(4) invariance], then the Fermi momenta of the rapidity bands are $p_{F0}=4p_{F3}$, $p_{F1}=3p_{F3}$, and $p_{F2}=2p_{F3}$, where $p_{F3}=\pi n/4$ and $n=N_e/N_a$. The dressed generalized charge $z_{cc}=\xi_{c,c}(Q)$ is given by Eq. (11) for $N=4$.

To study the drag effect we use the representation $N_{1\uparrow} \geqslant N_{1\downarrow} \geqslant N_{2\uparrow} \geqslant N_{2\downarrow}$, where wire 1 carries the driving current and wire 2 hosts the drag current. Momentum transfer between the wires is optimal when all $N_{j\sigma}$ are equal. Under these circumstances $p_{Fj\sigma}=p_{F3}$ for all j and σ. The drag current has now an up-spin and a down-spin channel. The corresponding currents are expected to be equal. For the up-spin channel $D_2=1$ and $D_3=-1$, while all other quantum numbers are zero. For the down-spin channel, on the other hand, $D_3=1$ and all other quantum numbers are zero. Both cases yield $2\Delta_c^{\pm}=z_{cc}^2/16$ and $2\Delta_S^{\pm}=3/4$, so that the critical exponent for the drag current is $\theta/16-1/2$. θ as a function of the total electron density is displayed in Fig. 3(a) for various coupling strengths.

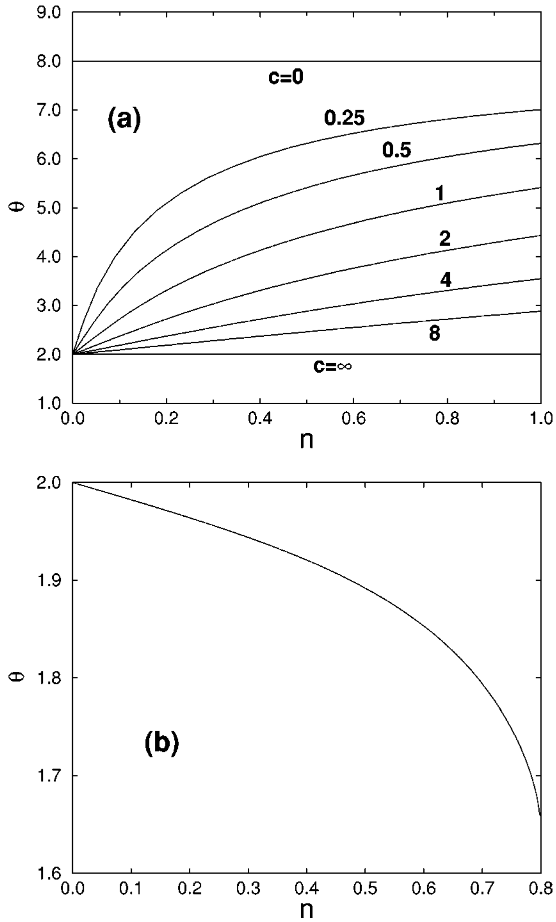

FIG. 3. Critical exponent θ as a function of the band filling n for (a) the gas of electrons with δ-function repulsion ($N=4$) for various coupling strengths c and (b) the supersymmetric t-J model with SU(5) invariance. For $n\to 0$, $\theta=2$ for both models, while the behavior is model dependent for larger n. For the t-J model the maximum electron density is $n=N/(N+1)$.

B. SU(5)-invariant *t-J* model

We now extend the model of Sec. II C to include spin degrees of freedom. The Hamiltonian is still given by Eq. (12) but with σ running from 1 through 4 (spin-1/2 times two wires). The model still satisfies a supersymmetric algebra equivalent to the permutation of the degrees of freedom between neighboring sites.[24,25] The degrees of freedom can be represented by a spin 2 and, hence, the invariance of the model is SU(5). The Bethe *Ansatz* solution of the model involves four sets of rapidities (one for the charges, two for the spin degrees of freedom of the wires, and one for the relative population of the wires).[26] For the ground-state all rapidities are real and densely distributed.

We again use the representation $N_{1\uparrow} \geqslant N_{1\downarrow} \geqslant N_{2\uparrow} \geqslant N_{2\downarrow}$, where wire 1 carries the driving current and wire 2 hosts the drag current, and consider the limit of equal electron density in each wire and for each spin-component. The general form of the finite-size corrections to the ground-state energy, the momentum, the dressed generalized charges and the conformal dimensions is the same as for the δ-function potential described in the preceding subsection, as a consequence of the SU(4) invariance of the internal degrees of freedom. The dressed charge for the charge sector is now given by $z_{cc}=\xi_{c,c}(Q)$ with $\xi_{c,c}(p)$ being the solution of Eq. (13) for $N=4$. The exponent θ as a function of the total electron density is displayed in Fig. 3(b). In contrast to the δ-function potential, for the present model θ decreases with n.

If the electron density is the same in both wires and per spin component, then the Fermi momenta of the rapidity bands are $p_{F0}=4p_{F3}$, $p_{F1}=3p_{F3}$, and $p_{F2}=2p_{F3}$, where $p_{F3}=\pi n/4$. Momentum transfer between the Fermi points in the driving wire 1, and global momentum conservation yields a backflow drag current. As in Sec. III A the drag current has an up-spin and a down-spin channel, which are both equal. The corresponding quantum numbers are all zero, except $D_2=1$ and $D_3=-1$ for the up-spin channel, and

$D_3=1$ for the down-spin channel. Both cases yield $2\Delta_c^{\pm}=z_{cc}^2/16$ and $2\Delta_S^{\pm}=3/4$, so that the critical exponent for the drag current is $\theta/16-1/2$.

V. ATTRACTIVE INTERACTION: THE ROLE OF THE SPIN

In this section we consider the situation of an attractive potential between the carriers with spin (electrons). As in Sec. IV the spin and the two wires give rise to four internal degrees of freedom. Since the degenerate Hubbard model is not integrable, we limit ourselves to the δ-function interaction and the supersymmetric t-J model.

A. δ-function potential

As discussed in Sec. IV A the gas of fermions with four colors interacting via a δ-function potential, Eq. (10), is also integrable in terms of four sets of rapidities.[18,32,33] For an attractive interaction ($c<0$) the electrons form bound states of up to four electrons (there are four degrees of freedom). If $N_{1\uparrow}=N_{1\downarrow}=N_{2\uparrow}=N_{2\downarrow}$, all electrons are bound in four clusters, and clusters of less than four electrons are gapped. There is then only one branch with Fermi surface and the finite-size contributions to the ground-state energy are of the form of Eq. (14) with the dressed generalized charge given by $z=\xi(Q)$, where

$$\xi(\Lambda)=1-\int_{-Q}^{Q}d\Lambda' J_N(\Lambda-\Lambda')\xi(\Lambda'),$$

$$J_N(x)=\frac{1}{\pi|c|}\mathrm{Re}\left[\Psi\left(N+\frac{ix}{|c|}\right)-\Psi\left(1+\frac{ix}{|c|}\right)\right] \qquad (24)$$

for $N=4$. For $N=2$ this expression reduces to Eq. (18) for equal carrier density in the two wires. The driving potential in this case has to pull on the bound states of four, so that the driving and drag currents are parallel. For simultaneous conductivity in both wires the quantum numbers are $D=1$ and all others zero, and the critical exponent for the current is $\theta-1$, where $\theta=2z^2$. θ as a function of $n=N_e/N_a$ is displayed in Fig. 4(a) for several values of c.

Another case that is of interest is when the conductors have very different densities, so that electrons are bound in clusters of four and the remainder of electrons in the majority band in Cooper pairs (spin singlets). In this case the system is a two-component Luttinger liquid. Now there is the possibility of transferring momentum between the two components of the Luttinger liquid (the Fermi momentum of the Cooper pairs has to be twice that of the four electron bound states), which gives rise to a backflow drag current, in addition to the forward drag current discussed above.

B. Supersymmetric t-J model

The extension of the standard t-J Hamiltonian, Eq. (20), to four internal degrees of freedom (spin times two conductors) can also be used to model the Coulomb drag problem. The Bethe *Ansatz* solution[36] involves four sets of rapidities, one for the charges and three for the internal degrees of freedom. For equal number of electrons in each wire and per spin component, in the ground-state all the electrons are clustered in units of four, in analogy to the case of the attractive δ-function potential. The key difference between the two models is that the binding energy is zero for the t-J model, i.e., the bound states of less than four electrons are not gapped.[36] The system is a four-component Luttinger liquid and the finite-size corrections to the ground-state energy is of the form of Eq. (2) with the summation index running over four bands. For $N_{1\uparrow}=N_{1\downarrow}=N_{2\uparrow}=N_{2\downarrow}$, the generalized dressed charge $z_{cc}=\xi(Q)$, where $\xi(\Lambda)$ is the solution of Eq. (21) with H_N given by Eq. (11) for $N=4$ and $c=1$. The remaining components of the matrix of dressed charges are $z_{qc}=[(N-q)/N]z_{cc}$, $z_{cq}=\delta_{q,0}z_{cc}$, and the "spin" sector is given by the symmetric 3×3 matrix $\mathcal{Z}_N$ defined by Eq. (22).[34] Here the charges "c" correspond to $q=0$.

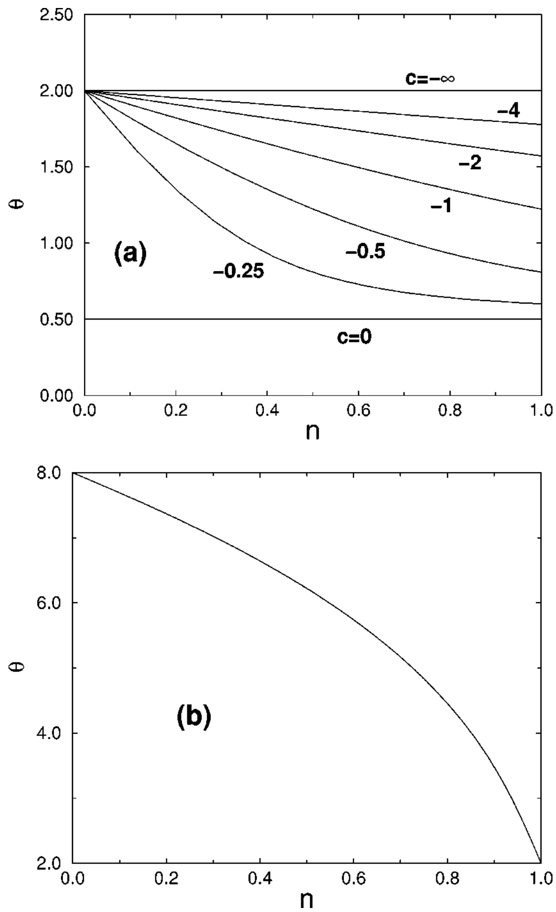

FIG. 4. Critical exponent θ as a function of the band filling n for (a) the gas of electrons ($N=4$) with attractive δ-function potential for various coupling strengths c and (b) the supersymmetric t-J model for $N=4$. For $n\to 0$, $\theta=2$ for the δ-function potential, while $\theta=8$ for the t-J model.

The Coulomb drag current has several components. Here we assume that the pull by the driving potential is on the clusters of four. We consider only two contributions, namely, (1) the drag current arising from the clusters of four electrons, which is parallel to the driving current and (2) the drag current on paired electron states in the drag wire, which is opposite to the driving current. For case (1) the quantum numbers corresponding to the simultaneous conductivity in both wires are all zero, except $D_c = -1$. The critical exponent for the current is then $2z_{cc}^2 - 1 = \theta - 1$. θ as a function of $n = N_e/N_a$ is displayed in Fig. 4(b). For case (2), on the other hand, momentum has to be transferred between the wires, which is only possible if $p_{F2} = 2p_{Fc}$. The situation $p_{F2} = 2p_{Fc}$ can only be satisfied if the charge density in the two wires is very different. The quantum numbers for this process are $D_c = 1$ and $D_2 = -1$, while all other quantum numbers are zero. The critical exponent for the drag current is then $2\Sigma_{j=0}^{3}(z_{cj} - z_{2j})^2 - 1$, where $j = 0$ refers to the charge c. The matrix of dressed generalized charges for this case has to be determined by solving the system of integral equations (see Ref. 34).

VI. CONCLUSIONS

We considered two nearby parallel quantum wires with carriers interacting via repulsive or attractive potentials.[6] Each conductor has at least one Fermi point for forward (backward) moving charges. When an electric field is applied to the driving wire, some carriers are transferred from one Fermi point (e.g., for the backward movers) to the other Fermi point (i.e., for forward movers), thus generating a current. This driving current may induce a drag current in the second wire via the interaction among the carriers. The spin degree of freedom of the electrons only plays a secondary role in this process, so that frequently spinless carriers are considered in this context.

For a repulsive interaction for carriers between wires, the momentum conservation of the interaction yields a backward momentum transfer in the drag wire, inducing in this way a drag current opposite to the driving current. The response function for this current is proportional to a power of the temperature, with a nonuniversal critical exponent. The exponent depends on the model, the interaction strength, and the band-filling.

For an attractive interaction among carriers between the wires, on the other hand, the charges form bound states. The potential applied to the driving wire pulls the bound electrons and hence the drag and driving currents are parallel. If the excitations into unbound carrier states are all gapped, then this is the only contribution to the drag current. However, if the band of unbound carriers has a Fermi surface, a backflow current may also arise, in the case where the Fermi momenta are matched so that momentum transfer between bands is possible. The current response functions follow power laws of the temperature, again with nonuniversal exponents that depend on the model, the band filling, and the interaction strength.

We considered the exact Bethe *Ansatz* solutions of several integrable models to obtain the critical exponents from the conformal towers. The exact solution yields all excitation branches, which either have Fermi points or are gapped. The former constitute a Luttinger liquid component. Only the low-energy excitations of the Luttinger liquid are needed to calculate the asymptotic dependence of correlation functions, which are described in terms of four quantum numbers (number of particles, backward-scattering quantum number, and particle-hole excitations at each of the Fermi points). The excitation spectrum is obtained from the mesoscopic finite-size corrections to the ground-state energy in terms of the group velocities and the matrix of dressed generalized charges, which describes the interplay of the different excitation branches as a consequence of the interaction.

If the carriers are spinless and the interaction is repulsive, the system can be described in terms of the Hubbard model, where the spin-index labels the wire (see Sec. II A). The simplest situation corresponds to equal carrier concentration in the two wires. In this case the critical exponent is determined by θ, which is related to the charge stiffness. The exponent decreases with increasing U and has a strong n dependence close to the metal-insulator transition (for $n \to 1$). The latter dependence is absent in the continuum variant (δ-function potential), since this model has no metal-insulator transition. Here θ increases monotonically with the band filling. Finally, for the SU(3)-invariant t-J model the value of θ is always smaller than 2 and a decreasing function of n. In Sec. IV these results were extended to the case of carriers with spin. As mentioned above the effect of the spin is quantitative but not qualitative.

For spinless carriers with an attractive interaction the system can be modeled by the Hubbard Hamiltonian with negative U. In this case all carriers are paired in bound states and states of unpaired carriers are gapped. The system is a one-component Luttinger liquid and the parameter θ decreases with n and is always less than one. The continuum limit of the model (δ-function potential) yields similar results except in the limit $n \to 1$. The case of the standard t-J model is physically different because the unpaired carrier excitations are not gapped (see Sec. III C). The exponent θ in this case also decreases monotonically with n. In Sec. V the extension of these results to carriers with spin is presented. Again the effect of the spin is only quantitative.

The usual drag current discussed in the literature is the one induced via back scattering of ballistic carriers and momentum conservation through a repulsive interaction. The linewidth of the excitations in the Luttinger liquid are proportional to T and smear the δ function for the momentum conservation. It then allows for a small mismatch (of order T) between the Fermi momenta of the two wires. There are many other possibilities to introduce a drag current, for both repulsive and attractive interactions. Some of these results involve backscattering of several carriers and the absorption of a vector of the reciprocal lattice (Umklapp process) if the interaction is repulsive (see Sec. II A), and backscattering for an attractive interaction if the carrier density in the two wires is very different (see Sec. III A).

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