Role of spinon in the presence of spinon singlet pair excitations on phase transitions in *d*-wave superconductors

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We examine the roles of massless Dirac spinon and spin singlet pair excitations on both quantum and classical phase transitions in extreme type-II *d*-wave superconductors. We discuss that the massless Dirac fermion (spinon) excitations in the presence of the spin singlet pair excitations do not alter the nature of the quantum phase transition at $T=0$, that is, the XY universality class, while at finite temperature they are seen to induce an additional logarithmic interaction potential between vortices, further stabilizing vortex-antivortex pairs at low temperature for KT transition for lightly doped high-T_c samples.

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Recent thermal Hall conductivity measurements[1] suggest the existence of vortices in the pseudogap (PG) phase. This implies that preformed pairs are present in the PG phase.[2] In the PG phase, vortex-antivortex pairs remain broken to cause a state of globally incoherent but locally coherent Cooper pairs. Vortex-induced phase transitions in underdoped region have been an issue of great interest.[3–8] In this paper, by applying a duality transformation we extend the U(1) gauge Lagrangian[9] obtained from the slave-boson mean-field theory[10,11] in order to examine how vortex-induced phase transitions in *d*-wave superconductors at low energy are affected by the presence of both the massless spinon and spinon singlet pair excitations. In other studies,[7,8] the flavor number of the massless Dirac fermions in the absence of the spinon singlet pair excitations is shown to alter the nature of the phase transition. According to this study,[7] as the flavor number increases, the type-*II* superconductivity is preferred showing the second-order phase transition which deviates from the XY universality class. In the case of small flavor number it leads to the type-I superconductivity and the transition becomes first order owing to the strong fluctuations of the massless gauge field. Our present study differs from other previous studies,[7,8] in that in our case the U(1) effective gauge field of interest becomes massive as a result of the spinon (spin) singlet pair excitations. Particle-hole excitations of the massless Dirac spinons lead to the renormalized kinetic energy of the U(1) gauge field[4,12,13] [the second term in Eq. (3)]. Based on the effective Lagrangian involved with the massive gauge field caused by the presence of the spinon singlet pair excitations, Eq. (3), we discuss zero- and finite-temperature phase transitions in the extreme type-II limit.[14] First, we find that the $(2+1)D$ XY universality class at $T=0$ is not altered despite the presence of the massless Dirac fermions (spinons), as long as the spinon singlet pair excitations exist in the underdoped high T_c cuprates and thus the effective gauge particle [Eqs. (5) and (12)] remains sufficiently massive.[15] Second, it is shown from the present study that at finite temperature the interaction potentials between vortices are modified to bring an additional logarithmic interaction [Eqs. (13) and (23)] as a consequence of the massive gauge field, thus maintaining the KT transition.

Our primary focus is to examine how low-energy excitations, specifically the Dirac spinon excitations in the presence of spinon singlet pair excitations affect the vortex-induced phase transition. Here the low-energy excitations refer to the phase fluctuations of both the spinon singlet pair and the single holon order parameter and the massless Dirac spinon excitations near the *d*-wave nodes of the spinon singlet pair. Gauge-field fluctuations[16] are introduced to allow the presence of internal flux responsible for the stabilization of the system by energy lowering. Thus considering gauge fluctuations and proper phase fluctuations (involved with $\phi_{sp}=e^{i\theta_{sp}}$ and $\phi_b=e^{i\theta_b}$) for the spinon pairing order field, and the single holon order field, respectively, we rewrite the low-energy effective Lagrangian of Lee[9] in compact form

$$\mathcal{L}=\frac{K_{b,\mu}}{2}|\partial_\mu\theta_b+a_\mu-A_\mu|^2+\frac{K_{sp,\mu}}{2}|\partial_\mu\theta_{sp}+2a_\mu|^2$$
$$+\psi_1^\dagger[\partial_\tau+v_F\tau^3 i\partial_x+v_\Delta\tau^1 i\partial_y]\psi_1+(1\rightarrow 2,x\rightarrow y)+iJ_{f\mu}$$
$$\times(\partial_\mu\theta_{sp}+2a_\mu)+i\overline{\rho}_{sp}(\partial_\tau\theta_{sp}-2\partial_\tau\theta_b+2A_0), \tag{1}$$

where $K_{b,\mu}\equiv(1/u_b,K_b,K_b)$ with $1/u_b$ ($\sim 1/t$), the compressibility and K_b ($\sim 2t\chi_0\delta$), the phase stiffness of the single holon field and $K_{sp,\mu}\equiv(1/u_{sp},K_{sp},K_{sp})$ with $1/u_{sp}$ ($\sim 1/J$), the compressibility and K_{sp} ($\sim J\Delta_0^2$), the phase stiffness of the spinon pair order field. $\psi_{n\sigma}=\begin{pmatrix} e^{-i\theta_{sp}/2}f_{n\sigma} \\ e^{i\theta_{sp}/2}\epsilon_{\sigma\sigma'}f^\dagger_{n\sigma'}\end{pmatrix}$ is the renormalized Nambu spinor associated with the *d*-wave gap nodes n. v_F ($\sim J\chi_0$) and v_Δ ($\sim J\Delta_0$) are the Fermi and gap velocities of the Dirac spinons, respectively. $J_{f\mu}=\frac{1}{2}(\Sigma_n\psi^\dagger_{n\sigma}\tau^3\psi_{n\sigma}, iv_F\psi^\dagger_{1\sigma}\psi_{1\sigma}, iv_F\psi^\dagger_{2\sigma}\psi_{2\sigma})$ is the three-current of the spinon quasiparticles and $\overline{\rho}_{sp}$ is the average density of spinon pairs.

By introducing the gauge shift (unitary gauge) $\tilde{a}_\mu=2a_\mu+\partial_\mu\theta_{sp}$, we rewrite Eq. (1) as

$$\mathcal{L}=\frac{K_{b,\mu}}{8}|\partial_\mu\theta_p-\tilde{a}_\mu+2A_\mu|^2+\frac{K_{sp,\mu}}{2}\tilde{a}_\mu^2+iJ_{f\mu}\tilde{a}_\mu$$
$$+\psi_1^\dagger[\partial_\tau+v_F\tau^3 i\partial_x+v_\Delta\tau^1 i\partial_y]\psi_1+(1\rightarrow 2,x\rightarrow y)$$
$$+i\overline{\rho}_{sp}(\partial_\tau\theta_p+2A_0), \tag{2}$$

where $\theta_p=\theta_{sp}-2\theta_b$. θ_p is the phase of the Cooper pair order parameter $\Delta_{Cooper}(k)=|\Delta^0_{Cooper}(k)|\phi_p$, where $\Delta_{Cooper}(k)=\langle c_{k\uparrow}c_{-k\downarrow}\rangle=\langle b^* f_{k\uparrow} b^* f_{-k\downarrow}\rangle\sim\langle f_{k\uparrow}f_{-k\downarrow}\rangle\langle b^*\rangle^2=|\Delta_0(k)||\langle b^*\rangle|^2\phi_{sp}\phi_b^{*2}$ and thus $\phi_p=e^{i\theta_p}=\phi_{sp}\phi_b^{*2}=e^{i\theta_{sp}}e^{-i2\theta_b}$. $\tilde{a}_\mu$ is the massive effective gauge field associated with the phase fluctuations of spinon singlet pair order parameter and the original internal U(1) gauge field a_μ. The mass $m_{a,\mu}$ of the effective gauge field $\tilde{a}_\mu$ is defined by the phase stiffness $K_{sp,\mu}$ of the spinon pairing order parameter involved with the PG phase of the doped Mott insulator. In the above equation, the fluctuating fields of θ_p, $\tilde{a}_\mu$ and ψ_n, are U(1) gauge invariant,[16] thus satisfying Elitzur's theorem.[17]

Integrating over the Nambu spinor fields and expanding the resulting logarithmic term up to second order in $\tilde{a}_\mu$,[4,12,13] we get an effective U(1) Lagrangian involved with the massive gauge field $\tilde{a}_\mu$,

$$Z=\int D\theta_p D\tilde{a}_\mu \exp\left(-\int_0^\beta d\tau\int dx^2\mathcal{L}\right),$$

$$\mathcal{L}=\frac{\tilde{K}_{b,\mu}}{2}|\partial_\mu\theta_p-\tilde{a}_\mu+2A_\mu|^2+\frac{N}{16}(\partial\times\tilde{a})\frac{1}{\sqrt{-\partial^2}}(\partial\times\tilde{a})+\frac{1}{2}m^2_{a,\mu}\tilde{a}^2_\mu+i\overline{\rho_{sp}}(\partial_\tau\theta_p+2A_0), \tag{3}$$

where $\tilde{K}_{b,\mu}=K_{b,\mu}/4$ and $m^2_{a,\mu}=K_{sp,\mu}$. N is the number of flavors (i.e., the number of nodal points) of the Dirac fermions. The kinetic-energy term (the second term) of the effective gauge field $\tilde{a}_\mu$ originates from the massless excitations of the spinon quasiparticles (Dirac fermions).[4,12,13] The Berry phase contribution $i\overline{\rho_{sp}}\partial_\tau\theta_p$ is related to the Cooper pair boson density.[18,19] In $(2+1)D$ the chiral-symmetry breaking is expected to occur and the fermions get a dynamically generated mass.[6,12,20,21] This will not alter the XY universality class at $T=0$ owing to the presence of the spin singlet pair excitations but will change the interaction potential between vortices. This will be discussed later.

In passing we would like to briefly discuss a nodeless case. That is, we ignore the Dirac spinon quasiparticles and thus consider only the $N=0$ flavor limit (which corresponds to the isotropic *s*-wave superconductivity). It is then obvious that the kinetic-energy term of the gauge field in Eq. (3) disappears. Integrating over the effective gauge field $\tilde{a}_\mu$ in Eq. (3), we obtain

$$\mathcal{L}=\frac{K_{p,\mu}}{2}|\partial_\mu\theta_p+2A_\mu|^2+i\overline{\rho_{sp}}(\partial_\tau\theta_p+2A_0), \tag{4}$$

where $K_{p,\mu}=\tilde{K}_{b,\mu}K_{sp,\mu}/(\tilde{K}_{b,\mu}+K_{sp,\mu})$. This reveals that the phase stiffness $K_{p,\mu}$ of the Cooper pair field is in a "reduced mass" form[22] between the holon and the spinon pair, and the usual logarithmic type of interaction between vortices arises under a duality transformation.

Now for the case of $N\neq 0$ flavor, the duality transformation[3,5,18,19,23] of Eq. (3) with the introduction of vortex mass and self-interaction terms leads to an effective Lagrangian for the vortex field,

$$Z=\int D\psi_{pV}Dc_\mu D\tilde{a}_\mu\exp\left(-\int dx^3\mathcal{L}\right),$$

$$\mathcal{L}=|(\partial_\mu+ic_\mu)\psi_{pV}|^2+m^2_{pV}|\psi_{pV}|^2+\frac{u_{pV}}{2}|\psi_{pV}|^4+\frac{1}{2\tilde{K}_{b,\mu}}|\partial\times\mathbf{c}|^2+i(\partial\times\mathbf{c})_\mu(\tilde{a}_\mu-2A_\mu)-\mu(\partial\times\mathbf{c})_\tau+\frac{N}{16}(\partial\times\tilde{a})\frac{1}{\sqrt{-\partial^2}}(\partial\times\tilde{a})+\frac{1}{2}m^2_{a,\mu}\tilde{a}^2_\mu, \tag{5}$$

where $m^2_{pV}\sim\tilde{K}_b-\tilde{K}_{bc}\sim\delta-\delta_c$, with $\tilde{K}_{bc}$ as the critical phase stiffness of the holon (boson) field, δ the hole doping concentration, and δ_c the critical hole doping concentration and $\mu=-\tilde{u}_b\overline{\rho_{sp}}$ with $\tilde{u}_b\sim 4t$. ψ_{pV} represents the Cooper pair vortex field and c_μ the dual gauge field to mediate interactions between vortices. Noting that μ is analogous to an applied "magnetic field" H_z and $(\partial\times\mathbf{c})_\tau$ to B_z,[18,19] the sixth term in Eq. (5), $-\mu(\partial\times\mathbf{c})_\tau$ (Refs. 18 and 19) which results from the Berry phase term $i\overline{\rho_{sp}}\partial_\tau\theta_p$ is analogous to interaction energy $-H_zB_z$ associated with the vortex field. We note that in case of $\delta<\delta_c$ vortex condensation occurs.

In association with the Lagrangian, Eq. (3) it is of interest to examine the instanton contribution resulting from the compactness of the original gauge field a_μ in the expression of the unitary gauge $\tilde{a}_\mu=2a_\mu+\partial_\mu\theta_{sp}$. The Cooper pair field is "charge" neutral and can not couple with the U(1) gauge field a_μ. In the following we discuss the conservation of vortex current to show that the contribution of instanton disappears. We rewrite our dual Lagrangian, Eq. (5) in the first quantized representation,

$$\mathcal{L}=\frac{1}{2\tilde{K}_{b,\mu}}|\partial\times\mathbf{c}|^2+ic_\mu J^V_{p,\mu}+i(\partial\times\mathbf{c})_\mu(\tilde{a}_\mu-2A_\mu)+\frac{N}{16}(\partial\times\tilde{a})\frac{1}{\sqrt{-\partial^2}}(\partial\times\tilde{a})+\frac{1}{2}m^2_{a,\mu}\tilde{a}^2_\mu-\mu(\partial\times\mathbf{c})_\tau, \tag{6}$$

where $\mathbf{J}^V_p=\partial\times\partial\theta_p$ is the Cooper pair vortex three current. From this we obtain the equation of motion for the dual gauge field c_μ,

$$-\frac{1}{\mathbf{K}_b}\partial\times\partial\times\mathbf{c}=\mathbf{J}^V_p-(\partial\times\tilde{a})+2(\partial\times\mathbf{A}). \tag{7}$$

Thus we obtain

$$\partial\cdot\mathbf{J}^V_p=\partial\cdot\partial\times\tilde{a}=\partial\cdot\partial\times(2a+\partial\theta_{sp})=2\rho_M+\mathbf{J}^V_{sp}, \tag{8}$$

where $\rho_M=\partial\cdot\partial\times a$ is the instanton ("magnetic" monopole) density[19,23] associated with the original gauge field a_μ and $\mathbf{J}^V_{sp}=\partial\times\partial\theta_{sp}$, the spinon pair vortex three current. Taking $\tilde{\rho}_M=\partial\cdot\partial\times\tilde{a}$ with $\tilde{\rho}_M$, the monopole density of the effective gauge field $\tilde{a}_\mu$, we write $\partial\cdot\mathbf{J}_p^V=\tilde{\rho}_M$ for the relation between the vortex current and the monopole density.[19] Since the spinon pair vortex current carries $hc/2\tilde{e}$ flux quantum, where $\tilde{e}$ is the internal (gauge) charge, $\partial\cdot\mathbf{J}_p^V=\tilde{\rho}_M=2\rho_M+\partial\cdot\mathbf{J}_{sp}^V=0$ must hold. This indicates that the instanton contribution disappears to conserve the vortex current of the Cooper pairs. Thus the compactness of a_μ and thus $\tilde{a}_\mu$ does not alter the $(2+1)D$ XY universality class.

We now prove that $2\rho_M+\partial\cdot\mathbf{J}_{sp}^V=0$. Performing the duality transformation of the starting Lagrangian, Eq. (1), we obtain

$$\mathcal{L}_{dual}=\frac{1}{2K_{sp,\mu}}|\partial\times\mathbf{c}_{sp}+\mathbf{J}_f|^2+ic_{sp,\mu}J_{sp,\mu}^V-i(\partial\times c_{sp})_\mu 2a_\mu -\mu_s(\partial\times\mathbf{c}_{sp})_\tau+\frac{1}{2K_{b,\mu}}|\partial\times\mathbf{c}_b|^2+ic_{b,\mu}J_{b,\mu}^V -i(\partial\times\mathbf{c}_b)_\mu(a_\mu-A_\mu)-\mu_b(\partial\times\mathbf{c}_b)_\tau+\mathcal{L}_\psi, \tag{9}$$

where $\mu_{sp}=-u_{sp}\overline{\rho_{sp}}$ with $u_{sp}\sim J$ and $\mu_b=u_b\overline{\rho_{sp}}$ with $u_b\sim t$ are the applied magnetic fields for the spinon pair and holon vortices, respectively, and $\mathcal{L}_\psi=\psi_1^\dagger[\partial_\tau+v_F\tau^3i\partial_x+v_\Delta\tau^1i\partial_y]\psi_1+(1\to2,x\to y)$ is the Dirac fermion Lagrangian. $\mathbf{J}_{sp(b)}^V$ is the vortex three-current of the spinon singlet pair (holon) and $c_{sp(b),\mu}$ the dual gauge field to mediate interactions between the spinon pair (holon) vortices. From the above dual Lagrangian we obtain the equations of motion for $c_{sp,\mu}$ and $c_{b,\mu}$, respectively,

$$-\frac{1}{\mathbf{K}}{}^{sp}\partial\times(\partial\times\mathbf{c}_{sp}+\mathbf{J}_f)=\mathbf{J}_{sp}^V+2\partial\times a,$$

$$-\frac{1}{\mathbf{K}}{}^{b}\partial\times\partial\times\mathbf{c}_b=\mathbf{J}_b^V+\partial\times a-\partial\times\mathbf{A}. \tag{10}$$

Thus we obtain

$$\partial\cdot\mathbf{J}_{sp}^V+2\partial\cdot\partial\times a=\partial\cdot\mathbf{J}_{sp}^V+2\partial\cdot\mathbf{b}=\partial\cdot\mathbf{J}_{sp}^V+2\rho_M=0,$$

$$\partial\cdot\mathbf{J}_b^V+\rho_M=0. \tag{11}$$

The above result proves that the instanton contribution $\tilde{\rho}_M=2\rho_M+\partial\cdot\mathbf{J}_{sp}^V$ for the effective gauge field $\tilde{a}_\mu$ is zero. Instantons associated with a_μ and thus $\tilde{a}_\mu$ do not affect dynamics of the Cooper pair field and thus the vortex field since it is a gauge neutral particle. On the other hand, instantons are expected to affect dynamics of the gauge noninvariant objects (i.e., spinon pairs and holons) since instantons act as the source of the spinon pair vortex and holon vortex current, respectively [see Eq. (11)]. If we consider the dynamics of spinon pairs and holons to examine the confinement physics involved with the opposite charges for these particles, we must consider the instanton effect. The issue of the instanton effect on the confinement physics and the superconducting phase transition will be discussed in a later study.

Integrating over the effective gauge field $\tilde{a}_\mu$ in Eq. (5), we get

$$Z=Z_0^a\int D\psi_{pV}Dc_\mu\exp\left(-\int d^3x\mathcal{L}_{eff}\right)\mathcal{L}_{eff} =|(\partial_\mu+ic_\mu)\psi_{pV}|^2+m_{pV}^2|\psi_{pV}|^2+\frac{u_{pV}}{2}|\psi_{pV}|^4 +\frac{1}{2\tilde{K}_{b,\mu}}|\partial\times\mathbf{c}|^2+\frac{1}{2}(\partial\times\mathbf{c})\frac{1}{\frac{N}{8}\sqrt{-\partial^2}+m_{a,\mu}^2}(\partial\times\mathbf{c}) -i(\partial\times\mathbf{c})_\mu 2A_\mu-\mu(\partial\times\mathbf{c})_\tau,$$

$$Z_0^a=\int D\tilde{a}_\mu\exp\left(-\int d^3x\frac{N}{16}(\partial\times\tilde{a})\frac{1}{\sqrt{-\partial^2}}(\partial\times\tilde{a}) +\frac{1}{2}m_{a,\mu}^2\tilde{a}_\mu^2\right). \tag{12}$$

The fifth term represents an additional kinetic energy of the dual gauge field resulting from the presence of the massless Dirac spinon field and the spinon singlet pair field which results in the massive effective gauge field $\tilde{a}_\mu$. In the case of weak external field μ, the nature of the XY universality class at $T=0$ will not be affected in the extreme type-II limit despite the contribution of the massless Dirac fermions (as shown in the fifth term) as long as the mass of the gauge field, that is, the phase stiffness K_{sp} of the spinon singlet pair order parameter is substantially large; the lower the temperature, the larger is the K_{sp} in the PG phase.

Thus far we discussed the zero-temperature phase transition which maintains the $(2+1)D$ XY universality class in the presence of negligible magnetic fluctuations A_μ. Now we examine the finite-temperature phase transition by considering dimensional reduction. We calculate the dual gauge-field propagator to obtain the interaction potential between vortices. The dual field propagator involved with the fourth and fifth terms in Eq. (12) is obtained:

$$P_{ij}(q)=P(q)\left(\delta_{ij}-\frac{q_iq_j}{q^2}\right)$$

$$P(q)=\frac{\tilde{K}_b\left(q+\frac{8}{N}m_a^2\right)}{q^2\left(q+\frac{8}{N}m_a^2+\frac{8}{N}\tilde{K}_b\right)}=K_p\frac{1}{q^2}+\frac{K_b}{4}z_J\frac{1}{q(q+M^2)}, \tag{13}$$

where $K_p=K_bK_{sp}/(K_b+4K_{sp})$, $M=\sqrt{8/N(K_{sp}+K_b/4)}$, and $z_J=K_b/(K_b+4K_{sp})$. $i,j=1,2$ denotes the space component index (x,y). In the real space, the above equation leads to the total interaction potential between a vortex and an antivortex:

$$V(x-x')=2\pi K_p\ln|x-x'|-\pi^2\tilde{K}_bz_J(\mathrm{Struve}H_0(M^2|x-x'|) -Y_0(M^2|x-x'|))\sim2\pi K_p\ln|x-x'| +\pi^2\tilde{K}_bz_J\ln|x-x'|\quad\text{for}\quad M^2|x-x'|\ll1, \tag{14}$$

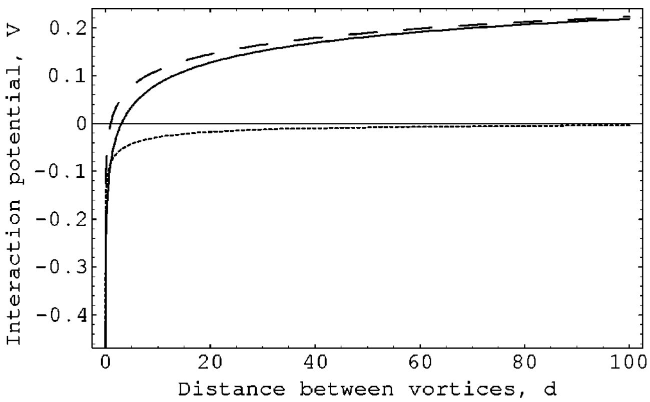

FIG. 1. The total interaction energy (solid line), logarithmic interaction energy (dashed line), and additional interaction energy (dotted line) (in the unit of t) at underdoping $\delta \sim 0.035$ as a function of vortex distance d (in the unit of t^{-1}).

$$\sim 2\pi K_p \ln|x-x'| - \frac{2\pi \tilde{K}_b z_J}{M^2}\frac{1}{|x-x'|} \quad \text{for} \quad M^2|x-x'| \gg 1, \tag{15}$$

where $Y_0(x)$ is the zeroth-order Bessel function of the second kind and Struve$H_0(x)$ represents the zeroth-order Struve function.[24] The additional interaction in Eq. (15) shows a power-law behavior ($|x-x'|^{-1}$) showing its decrease at large vortex separation distances while it is logarithmic in nature at short separations as shown in Eq. (14). In Fig. 1, we plot this interaction potential as a function of distance in the underdoped region.[25] As shown in Fig. 1, the additional attraction between vortices does not introduce a significant change in the net attractive interaction and shows virtually no change particularly at large separations. It is noted that the additional attractive interaction combined with the original logarithmic term results in the renormalized form of a net logarithmic interaction, $2\pi K' \ln|x-x'|$, where $K' = K_p + (\pi/2)\tilde{K}_b z_J$, with $K' \sim K_p$ particulary at low doping (in the limit of $z_J \to 0$) in the underdoped region. The vortex-antivortex unbinding transition temperature $T_{KT}(\delta)$ as a function of hole doping δ is expected to linearly scale with the phase stiffness K_p of the Cooper pair field. For $K_{sp} \gg K_b$, K_p is seen to be linearly dependent on δ particularly in the lightly doped region. On the other hand, the strength of the additional interaction [the second term in Eq. (15)] shows a nonlinear (quadratic) dependence of δ owing to the linear doping dependence of $K_b(\sim\delta)$ and $z_J(\sim\delta)$ in the lightly underdoped region.[25]

Now we discuss the case of the chiral-symmetry breaking in the $(2+1)D$ systems and show that this will not alter the XY universality class only in the presence of negligible magnetic fluctuations A_μ,[14] but change the strength of interaction between the vortices. Introducing the massive Dirac fermions for the case of the chiral-symmetry-broken phase,[6,12,20,21] we rewrite the effective Lagrangian, Eq. (2),

$$\mathcal{L} = \frac{\tilde{K}_{b,\mu}}{2}|\partial_\mu \theta_p - \tilde{a}_\mu + 2A_\mu|^2 + \frac{1}{2}m_{a,\mu}^2 \tilde{a}_\mu^2 + i\overline{\rho}_{sp}(\partial_\tau \theta_p + 2A_0) + \overline{\psi}_l \gamma_\mu(\partial_\mu - i\tilde{a}_\mu)\psi_l + m\overline{\psi}_l\psi_l, \tag{16}$$

where m is the mass of the Dirac fermion. Integrating over the massive Dirac fermions[6,12,21] and performing the duality transformation, we obtain the low-energy effective Lagrangian

$$Z = \int D\psi_{pV} Dc_\mu D\tilde{a}_\mu \exp\left(-\int dx^3 \mathcal{L}\right),$$

$$\mathcal{L} = |(\partial_\mu + ic_\mu)\psi_{pV}|^2 + m_{pV}^2|\psi_{pV}|^2 + \frac{u_{pV}}{2}|\psi_{pV}|^4 + \frac{1}{2\tilde{K}_{b,\mu}}|\partial\times\mathbf{c}|^2 + i(\partial\times\mathbf{c})_\mu(\tilde{a}_\mu - 2A_\mu) - \mu(\partial\times\mathbf{c})_\tau + \frac{N}{12\pi m}|\partial\times\tilde{a}|^2 + \frac{1}{2}m_{a,\mu}^2\tilde{a}_\mu^2. \tag{17}$$

Note that the kinetic energy of the gauge field is proportional to q^2, but not to q as in the case of the chiral-symmetric phase. After integrating over the massive effective gauge field, we obtain

$$Z = \int D\psi_{pV} Dc_\mu \exp\left(-\int dx^3 \mathcal{L}\right),$$

$$\mathcal{L} = |(\partial_\mu + ic_\mu)\psi_{pV}|^2 + m_{pV}^2|\psi_{pV}|^2 + \frac{u_{pV}}{2}|\psi_{pV}|^4 + \frac{1}{2\tilde{K}_{b,\mu}}|\partial\times\mathbf{c}|^2 + \frac{1}{2}(\partial\times\mathbf{c})\frac{1}{\frac{N}{6\pi m}(-\partial^2) + m_{a,\mu}^2} \times(\partial\times\mathbf{c}) - i(\partial\times\mathbf{c})_\mu 2A_\mu - \mu(\partial\times\mathbf{c})_\tau. \tag{18}$$

In the long-wavelength limit $\frac{1}{2}(\partial\times\mathbf{c})[1/(N/6\pi m(-\partial^2) + m_{a,\mu}^2)](\partial\times\mathbf{c})$ is reduced to $(1/2K_{sp,\mu})|\partial\times\mathbf{c}|^2$ as in the case of the chiral-symmetric phase. Thus the superconducting phase transition at $T=0$ falls into the XY universality class in both cases of chiral-symmetry and chiral symmetry breaking in the extreme type-II limit.[14] Only the interaction between vortices is affected as a result of chiral-symmetry breaking. In the chiral-symmetry broken phase the interaction potential is obtained to be, from Eq. (18),

$$P_{ij}(q) = P(q)\left(\delta_{ij} - \frac{q_i q_j}{q^2}\right),$$

$$P(q) = \frac{\tilde{K}_b\left(q^2 + \frac{6\pi m}{N}m_a^2\right)}{q^2\left(q^2 + \frac{6\pi m}{N}(m_a^2 + \tilde{K}_b)\right)} = K_p\frac{1}{q^2} + \frac{K_b}{4}z_J\frac{1}{q^2+M^2}, \tag{19}$$

with m the mass of the Dirac fermion and M the effective mass of the effective gauge field defined by $M^2 = (6\pi m/N)(m_a^2 + \tilde{K}_b)$. In real space this is expressed as

TABLE I. Comparison of the interaction potential between vortices in *s*-wave and *d*-wave superconductors.

s-wave superconductors	*d*-wave superconductors
$\mathcal{L}=\frac{K_p}{2}\vert\partial\theta_p\vert^2$	$\mathcal{L}=\frac{K_p}{2}\vert\partial\theta_p\vert^2-iz_J J_f\partial\theta_p+\overline{\psi_l}\gamma\partial\psi_l$
$\mathcal{L}_{dual}=\frac{1}{2K_p}\vert\partial\times c\vert^2+icJ_V\ (J_V\equiv\partial\times\partial\theta_p)$	$\mathcal{L}_{dual}=\frac{1}{2K_p}\vert\partial\times c\vert^2+icJ_V+\frac{N}{16}z_J^2 J_V\frac{1}{\sqrt{-\partial^2}}J_V$
$V(q)=\frac{K_p}{q^2}$	$V(q)=\frac{K_p}{q^2}+\frac{Nz_J^2}{8q}$
$V(x)=K_p\ln\vert x\vert$	$V(x)=K_p\ln\vert x\vert-\frac{N}{8}z_J^2\frac{1}{\vert x\vert}$

$$V(r)=2\pi\left(K_p\ln r-\frac{K_b}{4}z_J K_0[Mr]\right), \tag{20}$$

where r is the separation distance between a vortex and an antivortex and $K_0[x]$ is the modified Bessel function, which is $\int_0^\infty dt\cos(xt)/\sqrt{t^2+1}$, where $x>0$.[24] In the limit of large separation we obtain[24]

$$V(r)=2\pi\left(K_p\ln r-\frac{K_b}{4}z_J\sqrt{\frac{\pi}{2M}}\frac{e^{-Mr}}{\sqrt{r}}\right). \tag{21}$$

At a large separation distance the correction term is ineffective to alter the nature of phase transition. Thus owing to the main contribution of the logarithmic term particulary at low temperature, the classical transition is KT type as in the case of the chiral-symmetric phase.

To grasp the origin of the additional attractive interaction between vortices in a different angle, we now take a different procedure. Integrating over the effective gauge field $\tilde{a}_\mu$ first in Eq. (2), we obtain the effective Lagrangian[9] involved with the U(1) gauge-invariant particles and the Doppler energy shift term,[26] $z_\mu J_{f\mu}\partial_\mu\theta_p$,

$$Z=\int D\psi D\theta_p\exp\left(-\int_0^\beta d\tau\int dx^2\mathcal{L}\right),$$

$$\begin{aligned}\mathcal{L}=&\frac{K_{p,\mu}}{2}\vert\partial_\mu\theta_p-2A_\mu\vert^2-iz_\mu J_{f\mu}(\partial_\mu\theta_p-2A_\mu)\\&+\psi_1^\dagger[\partial_\tau+v_F\tau^3 i\partial_x+v_\Delta\tau^1 i\partial_y]\psi_1+(1\to2,x\to y)\\&+\frac{1}{2}\frac{J_{f\mu}^2}{\tilde{K}_{b,\mu}+K_{sp,\mu}},\end{aligned} \tag{22}$$

with $K_{p,\mu}=K_{b,\mu}K_{sp,\mu}/(K_{b,\mu}+4K_{sp,\mu})\equiv(1/u_p,K_p,K_p)$ the phase stiffness of the Cooper pair order parameters and $z_\mu=K_{b,\mu}/(K_{b,\mu}+4K_{sp,\mu})\equiv(z_\rho,z_J,z_J)$ the effective charge of the spinon quasiparticles,[9] as mentioned earlier. The above Lagrangian is the low-energy effective Lagrangian of the *d*-wave BCS theory with the doping dependent phase stiffness K_p ($\sim\delta$) and effective charge z_J ($\sim\delta$).[9]

To see the effects of the Dirac fermions on the phase fluctuations of the Cooper pair fields, we integrate over the Dirac fermion fields ignoring the local interactions of the Dirac fermions and the temporal fluctuations in Eq. (22) to find

$$\mathcal{L}=\frac{K_p}{2}\vert\nabla\theta_p-2A\vert^2+\frac{N}{16}z_J^2(\nabla\times\nabla\theta_p-2\nabla\times A)\times\frac{1}{\sqrt{-\nabla^2}}(\nabla\times\nabla\theta_p-2\nabla\times A). \tag{23}$$

The resulting additional attractive interaction between vortices leads to the second term in Eq. (15) (Ref. 27) as a result of the supercurrent affected by the massless Dirac-fermions involved with the Doppler-shift term $z_\mu J_{f\mu}\partial_\mu\theta_p$. For a brief guidance we list differences in interaction potentials between vortices in the *s*-wave and the *d*-wave superconductors in Table I and a comparison of the *d*-wave BCS theory and the present theory in Table II. We note that in the limit of large M, $M^2\gg q$ in Eq. (13), the interaction energy between vortices obtained by the gauge theory is the same as that obtained by the *d*-wave BCS formalism [Eq. (23)]. This limit corresponds to the case in which the local interactions [the last term in Eq. (22)] of the massless Dirac fermions are ignored. The additional interactions may affect the vortex lattice structure.[28] Since our present theory is able to handle the doping dependence of vortex interactions, it is advantageous to study in the future how the vortex dynamics and lattice structure vary with hole concentration.

In this paper we examined the nature of both the quantum ($T=0$) and the classical ($T\neq0$ phase transitions at low temperature in the lightly doped region of high-T_c cuprates. First, we note that the quantum phase transition in underdoped cuprates[14] falls into the XY universality class or the inverted XY (IXY) universality class[29] depending on the strength of magnetic fluctuations A_μ. This situation of XY or IXY transition will not be altered even in the case of the massless spinon quasiparticle as long as there exists the massive gauge field $\tilde{a}_\mu$ which results from the spinon singlet pair excitations. It is known that high-T_c cuprates (such as $YBa_2Cu_3O_{7-\delta}$) of extreme type II obey a 3D XY scaling as they have high values of Ginzburg-Landau parameter κ in the range of 70–100 in which case the magnetic fluctuations play no significant role.[14] On the other hand, the inverted $(2+1)D$ XY transition[29] may occur for weakly type-II

TABLE II. Comparison of the interaction potential between vortices in the d-wave BCS formalism and in the present gauge theory formalism.

d-wave BCS theory	Our theory
$\mathcal{L}=\frac{K_p}{2}\lvert\partial\theta_p\rvert^2-iz_J J_f\partial\theta_p+\bar{\psi}_l\gamma\partial\psi_l$	$\mathcal{L}=\frac{\tilde{K}_b}{2}\lvert\partial\theta_p-\tilde{a}\rvert^2+\frac{N}{16}(\partial\times\tilde{a})\frac{1}{\sqrt{-\partial^2}}(\partial\times\tilde{a})+\frac{1}{2}m_a^2\tilde{a}^2$
$\mathcal{L}_{dual}=\frac{1}{2K_p}\lvert\partial\times c\rvert^2+icJ_V$	$\mathcal{L}_{dual}=\frac{1}{2\tilde{K}_b}\lvert\partial\times c\rvert^2+icJ_V$
$+\frac{N}{16}z_J^2 J_V\frac{1}{\sqrt{-\partial^2}}J_V$ $(J_V\equiv\partial\times\partial\theta_p)$	$+\frac{1}{2}(\partial\times c)\frac{1}{\frac{N}{8}\sqrt{-\partial^2}+m_a^2}(\partial\times c)$
$V(q)=\frac{K_p}{q^2}+\frac{Nz_J^2}{8q}$	$V(q)=\frac{K_p}{q^2}+\tilde{K}_b z_J\frac{1}{q(q+M^2)}$
$V(x)=K_p\ln\lvert x\rvert-\frac{N}{8}z_J^2\frac{1}{\lvert x\rvert}$	$V(x)=K_p\ln\lvert x\rvert-\frac{N}{8}z_J^2\frac{1}{\lvert x\rvert}$ for $q\ll M^2$
	$V(x)=\left(K_p+\frac{\pi}{2}\tilde{K}_b z_J\right)\ln\lvert x\rvert$ for $q\gg M^2$

superconductors.[14] Second, we show that at finite temperature the massless Dirac fermions in the presence of the spinon singlet pairs induces the additional attractive interaction of a logarithmic behavior $(\pi/2)\tilde{K}_b z_J\ln|x-x'|$ at short distances and of a power-law behavior $-(\tilde{K}_b z_J/M^2)|x-x'|^{-1}$ at large distances. As shown in Fig. 1 of our paper, the additional attraction between vortices did not introduce a significant change in the net attractive interaction and showed virtually no change particularly at large separations. It is noted that the additional attractive interaction combined with the original logarithmic term results in the renormalized form of a net logarithmic interaction $2\pi K'\ln|x-x'|$, where $K'=K_p+(\pi/2)\tilde{K}_b z_J$, with $K'\sim K_p$ particulary at low doping (in the limit of $z_J\rightarrow 0$) in the underdoped region. It is then expected that the nature of phase transition will not be altered since no marked change of vortex fugacity will occur particulary in the low doping spin gap phase region of present interest.[30] For a detailed analysis renormalization-group calculation is needed in the future. The present study has been made based on the U(1) slave-boson theory concerned with the single holon order parameter. Thus it will be of great interest to apply our recent SU(2) holon pair boson theory[31] in the future.

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