Induced nonequilibrium currents in the magnetization of mesoscopic dots in the quantum Hall regime

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We have measured the magnetization of a two-dimensional electron system that has been laterally patterned to an array of mesoscopic dots. In our experimental data we observe sawtooth-like de Haas–van Alphen (dHvA) oscillations of the equilibrium magnetization and induced nonequilibrium current signals which are superimposed on the dHvA signal at a small integer value of the filling factor. The nonequilibrium magnetization is found to be independent of the rate of magnetic-flux change which allows us to determine the critical current of nondissipative edge transport. We find that the magnitude of the critical current is directly related to the characteristic microscopic energy scales of the interacting many-body system.

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A two-dimensional electron system (2DES) subjected to a strong perpendicular magnetic field B shows a unique behavior at integer and fractional filling factors $\nu = n_s / N_L$ (n_s is the sheet carrier density and $N_L = eB/h$). In particular, such systems exhibit the quantum Hall effect, which is characterized by a Hall resistance which is quantized in units of h/e^2.[1] A remarkable property of the quantum Hall systems is the nondissipative nature of the current which causes the longitudinal resistance to vanish in the limit of zero temperature.[2] An elegant picture of the quantum Hall effect(QHE) is based on quasi-one-dimensional channels which are formed at the edge of the sample.[3,4] In these theoretical treatments closed sample geometries without any contacts have been considered. Experiments on the magnetization of a 2DES in the quantum Hall regime have shown that at integer value of the filling factor de Haas–van Alphen (dHvA) oscillations occur.[5,6] At low temperature and small value of the filling factor these were found to be accompanied by nonequilibrium magnetization signals which have been attributed to induced currents in the quantum Hall system. Both types of signals have also been observed in the fractional quantum Hall regime.[7,8] It has been proposed that the nonequilibrium current (NEC) signals are closely related to the nondissipative current transport which is characteristic to the QHE and the fractional QHE.[8,9] In this picture their magnitude is limited by the critical current I_c or critical current density at which the nondissipative transport in the QHE breaks down.[10] It is known that current contacts as well as voltage probes have strong impact on the dissipation in the quantum Hall systems (see, for example, Refs. 11,12). The investigation of the nonequilibrium signals in the magnetization of a 2DES thus provides interesting information about elementary dissipation processes without connecting the system to external reservoirs such as contacts. Furthermore, at the same time one directly obtains relevant information on the characteristic energetic structure of the system through the equilibrium magnetization, i.e., the dHvA effect, which at low temperature is determined by the changes in the ground-state energy of the many-body system. We have performed magnetization measurements on 2DES's, which have been laterally patterned on a micrometer scale. In particular we report on data which have been obtained on an array of 3-μm-diameter dots. We find that in these small systems the nonequilibrium magnetization signal is caused by a critical current, which at different even and odd filling factors is directly related to the Landau quantization and many-body enhanced spin-splitting energy, respectively.

The electron system was formed in an AlGaAs/GaAs heterojunction which was grown by molecular-beam epitaxy. The samples were patterned through standard optical lithography and wet chemical etching. For comparison, a sample with an unpatterned 2DES has been prepared from the same wafer. The total area of the investigated structures was $A_{\mathrm{tot}} = 1.28\ \mathrm{mm}^2$. The "active area" A_{act} was smaller by a factor of 0.3 in the dot geometry. The samples have been monolithically integrated to micromechanical cantilever magnetometers. These quasistatically detect the torque $\tau = \mathbf{M} \times \mathbf{B}$ acting on the magnetic-moment $\mathbf{M}$ of the 2DES in an external magnetic-field $\mathbf{B}$. This requires a finite tilt angle between the 2DES normal and the magnetic field, which in our experiments was adjusted to values below 20°. For clarity in the following, we only refer to the component of the magnetic field which is perpendicular to the 2DES, denoted by B. Further details of the technique are described in Refs. 13,14. All data discussed in this paper have been obtained after a brief illumination with a red light-emitting diode. The carrier density and mobility from magnetotransport under similar conditions was $n_s = 2.5 \times 10^{11}\ \mathrm{cm}^{-2}$ and $\mu = 1.3 \times 10^6\ \mathrm{cm}^2/\mathrm{V\,s}$. This means that in terms of transport characteristics the 3-μm-diameter dots may be considered as ballistic. The samples were cooled by placing them on the cold finger of a vacuum loading ^{3}He refrigerator. On the mesoscopic dot array additional low-temperature experiments were performed by placing the sample directly in the mixing chamber of a ^{3}He-^{4}He dilution refrigerator.

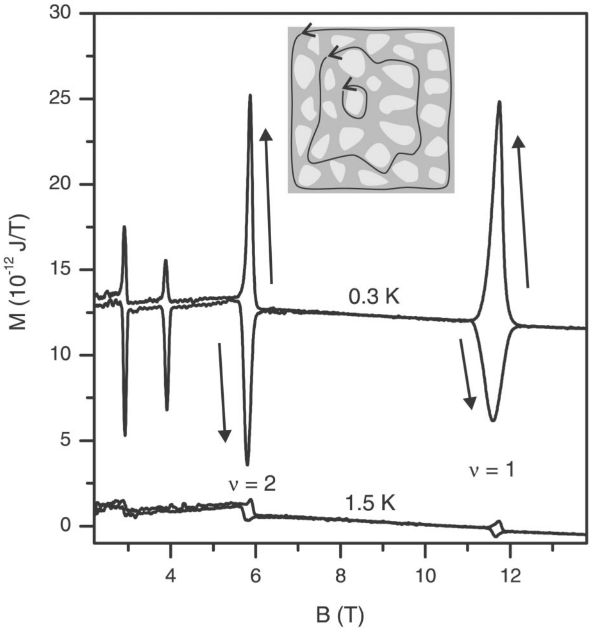

FIG. 1. Experimental magnetization of the reference 2DES. The curves taken at $T=0.3$ K and $T=1.5$ K have been offset for clarity. The arrows indicate the sweep direction of the magnetic field. The inset illustrates the contribution of multiple current paths to the total induced current in an inhomogeneous 2DES.

Experimental magnetization data of the unpatterned 2DES are shown in Fig. 1. Two features have to be distinguished: The sawtoothlike dHvA oscillations of the equilibrium magnetization and the peaked signals at integer filling factors, which are due to the induced NEC's. The magnetization of the mesoscopic dot array is shown in Fig. 2. Here the strong

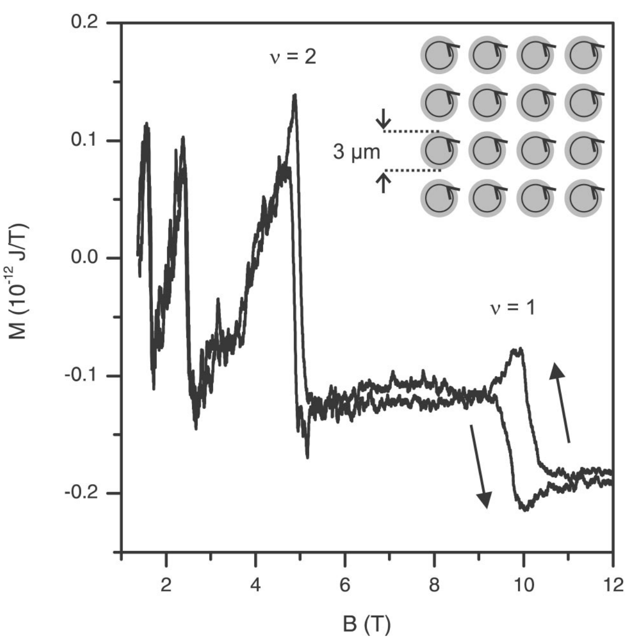

FIG. 2. Experimental magnetization of the mesoscopic dot-array sample at $T=0.02$ K. The arrows indicate the sweep direction of the magnetic field. The inset illustrates the geometry of the dot array and the path of the nonequilibrium current.

TABLE I. Characteristics of the dHvA oscillation at $\nu=2$: carrier density, and oscillation amplitude in absolute units and normalized to the carrier number $N=n_s A_{act}$ in units of the effective Bohr magneton $\mu_B^*=e\hbar/2m^*$.

	n_s (10^{11} cm^{-2})	ΔM (10^{-12} J/T)	$\Delta M/N$ (μ_B^*)
Unpatterned	2.82	0.66	1.33
Dot	2.39	0.20	1.57

signals due to the induced NEC are significantly smaller even at the lowest experimental temperature of $T=0.02$ K and the equilibrium dHvA effect predominates. Nevertheless, the nonequilibrium signals can be clearly resolved.

We first briefly address the equilibrium magnetization, i.e., the dHvA effect. The carrier densities of the samples and the oscillation amplitudes at $\nu=2$ are summarized in Table I. The absolute amplitude is found to scale with the total electron number $N=n_s A_{act}$. In Ref. 15 it was demonstrated that the amplitude ΔM of the dHvA oscillation at integer filling factor is a measure of the jump in the chemical potential, i.e., the "energy gap" ΔE, through the relation

$$\Delta E=\Delta M B/N. \tag{1}$$

At $\nu=2$ this yields a gap which is about 70% of the cyclotron energy $\hbar\omega_c$, the value expected for a noninteracting ideal 2DES. According to the previous studies such a reduction is explained by the presence of disorder. At $\nu=1$, where the 2DES becomes spin polarized, we find that the gap corresponds to a value which is enlarged by a factor of 16 with respect to the bare Zeeman energy. Like the enhanced spin splitting which is deduced from magnetotransport studies,[16,17] it is due to the Coulomb-exchange interaction. In our case we would infer an enhanced g factor of $g^*=7$. Later we will use these important results for the interpretation of the nonequilibrium magnetization signals.

We now turn to the signals due to the NEC's which, in contrast to the equilibrium dHvA effect, reverse their polarity when the direction of the magnetic-field sweep is reversed, i.e., they are negative for $dB/dt>0$ and vice versa.

In a macroscopic 2DES one can expect that the main contribution to the magnetic signal is from current paths with large enclosed area, i.e., from NEC's that flow in close vicinity to the boundary of the 2DES. In the simplest model, the NEC is driven by the change of flux $d\phi/dt$ through the 2DES area A and $d\phi/dt \propto dB/dt$. Surprisingly, one finds in the experiments that the magnitude of the signals depends only very weakly and in a nonlinear way on $|dB/dt|$. If the magnetic-field sweep is stopped at a value close to the maximum of such a peak, a remarkably slow decay is observed, which has been shown to be approximated by a two-component exponential decay.[18] For the slow component at $\nu=2$ we find a time constant which at $T=0.3$ K is of the order of 20 min. This behavior underlines that the system is in a nonequilibrium state and approaches thermal equilibrium in a very slow characteristic equilibration process.[19] Our results on the unpatterned 2DES are in agreement with

TABLE II. Nonequilibrium magnetization signal $M_{\rm NEC}$ in 10^{-14} J/T averaged between upward and downward sweeps of the magnetic field, critical currents I_c in μA, and ratio of $M_{\rm NEC}$ in the dot array and in the unpatterned sample.

ν	$M_{\rm NEC}$ Unpatterned	I_c Unpatterned	$M_{\rm NEC}$ Dot	I_c Dot	$M_{\rm dot}/M_{\rm unp}$
1	930	7.27	6.2	0.17	0.004
2	1080	8.44	12.0	0.33	0.006
3	440	3.44	1.3	0.05	0.001
4	610	4.77	9.1	0.28	0.007

the earlier works concerning the nonequilibrium magnetization of a large-area 2DES.[8,9,18]

Following these works and assuming that the nonequilibrium magnetization $M_{\rm NEC}$ is due to a critical current $I_{\rm c} = M_{\rm NEC}/A$ flowing at the edge of the mesa, we evaluate I_c directly from our experimental data. For our unpatterned sample this yields values in the order of several 10^{-6} A at $T=0.3$ K. We attribute the small asymmetry between upward and downward spikes in Fig. 1 to a different microscopic current distribution when the direction of the current is changed in a magnetic field. In the mesoscopic dot sample the currents are smaller by more than one order of magnitude. The values of the nonequilibrium magnetization $M_{\rm NEC}$ at the lowest experimental temperature, averaged between both sweep directions, and the calculated critical currents are summarized in Table II. For the unpatterned sample the minimal temperature was $T_{\rm min}=0.3$ K and for the dot array sample $T_{\rm min}=0.02$ K. The ratio of $M_{\rm NEC}$ in the dot and unpatterned geometries, $M_{\rm dot}/M_{\rm unp}$, takes values between 10^{-3} and 10^{-2}. This is significantly smaller than the geometric factor of 0.3 introduced through the patterning. For unpatterned 2DES samples we find that the temperature dependence of the NEC signal at $\nu=2$ is approximately linear between 0.3 and 2 K. We do not observe a saturation of the NEC signal below $T=1.5$ K like in Ref. 9. If the linear temperature dependence is extrapolated to $T=0.02$ K a further increase of $M_{\rm NEC}$ of about 20% can be expected. This would correspond to an even smaller value of $M_{\rm dot}/M_{\rm unp}$. The main result here is that even at the lowest temperature the nonequilibrium magnetization of the dot-array sample remains smaller by more than two orders of magnitude if compared to the unpatterned 2DES.

To discuss possible mechanisms for this strongly different behavior, let us first assume that we can describe the induced current in a dissipative channel along the edge of a 2DES by $I_{\rm ind}=-R_c^{-1}d\phi/dt$, where R_c is the resistance of the edge channel. The corresponding magnetic moment is given by $M_{\rm NEC}=I_{\rm ind}A$, which in an array consisting of $N_{\rm D}$ dots sums up to a total magnetic moment of $N_{\rm D}M_{\rm NEC}$. In our case the total number of dots is $N_{\rm D}=51\,200$. In the diffusive limit one would expect the edge-channel resistance R_c to scale with the channel length, i.e., $R_c \propto A^{1/2}$. Comparing $M_{\rm NEC}$ for an array of dots, in which the single dot has an area $A_{\rm D}$, to a large-area 2DES which covers the same area $A_{\rm tot}$ as the dot array one would expect a reduction by a factor of $N_{\rm D}(A_{\rm D}/A_{\rm tot})^{3/2}$, which for our dot-array sample is a factor of 6.6×10^{-4}. This would be a very strong reduction of the nonequilibrium signal, even stronger than that observed in the experiment (see Table II). However, if one assumes that, independent on the 2DES size, the current reaches a universal critical value I_c one would expect a reduction by a factor of $N_{\rm D}A_{\rm D}/A_{\rm tot}$, which, for our samples, is a factor 0.3, i.e., the ratio of the active area in the dot array as compared to the reference 2DES. This effect would be too small to explain our experimental observations. Furthermore our data show that the NEC signal in the magnetization of the mesoscopic dot array is due to a *critical current*: In Fig. 3 the magnetization at $\nu=2$ is shown for different rates of the magnetic-field change. The hysteresis of the magnetization which is caused by the NEC signal is independent of $|dB/dt|$ within the experimental accuracy. In Fig. 3, for the slowest sweep rate we also show the averaged signal of up and down sweep directions $(M_{\rm down}+M_{\rm up})/2$ and the difference signal $(M_{\rm down}-M_{\rm up})/2$. The former is the dHvA oscillation of the equilibrium magnetization and the latter is the superimposed nonequilibrium signal due to the induced NEC. Both contributions are of the same strength. This is important because it nicely demonstrates that our observation of a strongly reduced NEC signal in the dot-array sample is not an effect of the smaller induced voltage combined with a small but finite resistivity. Given that in the smaller systems the basic mechanisms of dissipation remain the same as in the macroscopic system, one rather must conclude that this is an intrinsic effect due to a different distribution of the current.

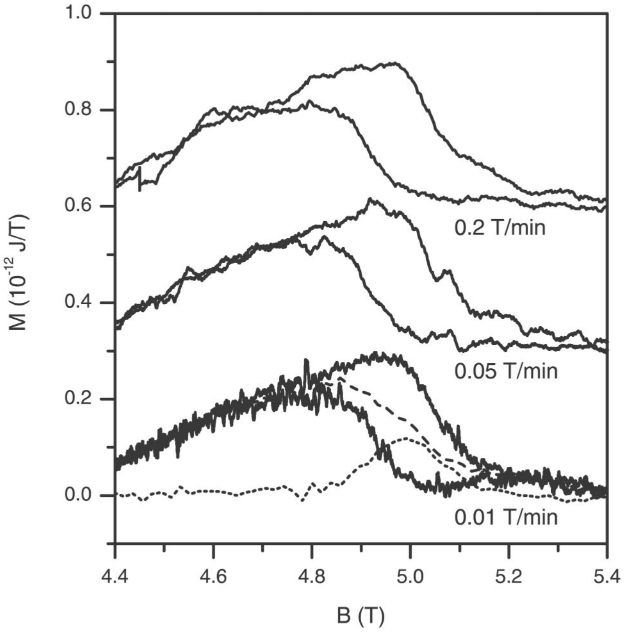

FIG. 3. Magnetization of the mesoscopic dot-array sample at $\nu=2$. Curves taken with different sweep rates of the magnetic field have been offset for clarity. The maximum and minimum sweep rates correspond to induced voltages of 2.36×10^{-14} V and 1.18×10^{-15} V, respectively. The dashed line in the lowest dataset is the average of both sweep directions, the dotted line the difference.

In the following we explain our observations within the edge-state picture. In a homogeneous dot, edge states develop at the outer boundary. Non-equilibrium current flow requires a redistribution of electrons between bulk and edge states, i.e., the Hall voltage U_{Hall} builds up between the edge and the bulk of the system. When eU_{Hall} exceeds the characteristic level spacing ΔE, dissipation via interlevel transitions becomes energetically possible. The critical current is then defined by

$$U_{\mathrm{Hall}}=\nu^{-1}\frac{h}{e^2}I_c=\frac{\Delta E}{e}. \tag{2}$$

In the scenario of one current path near the edge of the dot, this model provides an upper limit for the critical current. If we use Eq. (1) to determine ΔE at $\nu=1$ and $\nu=2$ from the equilibrium magnetization and estimate the maximum NEC according to Eq. (2), we find $I_c=0.17$ μA and $I_c=0.52$ μA, respectively. These values are in fair agreement with the currents deduced from our experimental data on the mesoscopic dots (Table II). Within the experimental accuracy, a similar consistency is also found at $\nu=3$ and $\nu=4$, where the model predicts a critical current of $I_c=0.10$ μA and $I_c=0.47$ μA, respectively. For $\nu\geqslant 2$ the measured values of I_c are systematically smaller than the theoretical ones. This is possibly due to the finite width of the edge states in a real sample, which would distribute the current over a broader region and shift the current distribution away from the edge of the dot. This would reduce the corresponding magnetic signal. We find that in the dot geometry at each of the filling factors the value of the critical current is determined by the characteristic microscopic energies of the many-body system. We conclude that our mesoscopic dots are smaller than the characteristic length scale of the inhomogeneities in the 2DES and only one peculiar path in the periphery of the dot contributes to the NEC, as sketched in the inset of Fig. 3.

In the unpatterned sample we find values of I_c which are larger by more than one order of magnitude (Table II). In Ref. 8 the filling-factor dependence of I_c was described by assuming a definite scattering mechanism involving quasielastic inter-Landau-level scattering processes. However, to explain the absolute signal strength the authors introduced a phenomenological factor accounting for "domain formation." Accordingly, the large signals in the unpatterned 2DES can be explained by the formation of multiple current paths, which arises from local inhomogeneities. This scenario is illustrated in the inset of Fig. 1. While it implies current paths in the interior of the 2DES, the main contribution to the nonequilibrium magnetization still comes from current paths which have a large enclosed area, i.e., which are located near the outer boundary of the 2DES. The importance of inhomogeneities for the formation of the NEC's is further stressed by the observations of Wiegers *et al.* who find that the magnitude of the NEC signal is typically reduced by illumination and depends in a nonmonotonous fashion on the sample mobility.[20] This means that the ratio of the critical currents determined from our magnetization experiments on the unpatterned 2DES and the dot-array sample may be identified with the number of different current paths which contribute to the nonequilibrium magnetization of the unpatterned sample. For example, at $\nu=2$ this gives a number of $8.44/0.33\approx 30$. The dot-array sample presented in this paper is the smallest feature size we have investigated which has clearly shown the NEC signals. In experiments on smaller dots with lateral dimension below 1 μm we observe that the equilibrium magnetization exhibits an oscillatory behavior which is drastically different from the dHvA effect of the unpatterned 2DES and of the 3-μm dots investigated here. In these small structures the NEC signal vanishes, showing that the small dots can no longer be considered as a quantum Hall system. In such structures the interplay of the quantum confinement and the Coulomb interaction gives rise to an oscillatory equilibrium magnetization which is drastically different from the dHvA effect of a 2DES.[21]

In conclusion, we have demonstrated that the magnitude of the NEC, which is a signature of nondissipative transport in a 2DES under conditions of the quantum Hall effect, strongly depends on the size of the system. This behavior is found to be in contrast to the equilibrium dHvA effect, which scales with the area of the system and can be considered as a bulk effect. We have shown that in a sufficiently small 2DES the critical current of nondissipative transport which is evaluated from the nonequilibrium magnetization can be understood in the edge-state picture. The strength of the critical current is determined by the characteristic microscopic energy scales of the system. In particular it reflects the properties of the interacting many-body system.

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