On-site interaction effects on localization: Dominance of nonuniversal contributions

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The influence of on-site (Hubbard) electron-electron interactions on disorder-induced localization is studied in order to clarify the role of electronic spin. The motivation is based on the recent experimental indications of a ''metal-insulator'' transition in two-dimensional systems. We use both analytical and numerical techniques, addressing the limit of weak short-range interactions. The analytical calculation is based on random matrix theory (RMT). We demonstrate that, at least in the diffusive regime, delocalization can indeed be induced by electron-electron interactions and that an in-plane magnetic field has a strong influenece on this effect. It is found that although RMT gives a qualitative explanation of the numerical results, it is quantitatively incorrect. This is due to an exact cancellation of short-range and long-range correlations in RMT, which does not occur in the nonuniversal corrections to RMT. An estimate for these contributions is given.

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I. INTRODUCTION

The question considered in this paper is whether electron-electron interactions can reduce disorder-induced localization, thus enabling metallic behavior in two-dimensional disordered systems.

The common view about the subject in the last 20 years has been based on the well-known scaling theory of localization,[1] according to which two-dimensional systems will always be localized (i.e., insulating), no matter how weak the disorder is. Although the original scaling theory did not take interactions into account, it was shown that when the interaction is weak (i.e., when the electronic density is high), it does not affect its results.[2] On the other hand, in the limit of very strong interactions (i.e., very dilute systems) it is known that the electron liquid freezes into a Wigner lattice, which is pinned by disorder and is therefore insulating.[3] All these results have led to the opinion that the repulsion between electrons can only further decrease the conductance, so that all two-dimensional systems will show insulating behavior, regardless of the strength of the interaction between the electrons.

A series of experiments performed in the last few years showed that even though in the limits of both very dense and very dilute systems we get the expected insulating behavior, for intermediate values of density (corresponding to r_s between 8 and 40, where r_s is the average interelectron distance measured in the units of the Bohr radius) metalliclike temperature dependence is found.[4] The transition from an insulating behavior to a metallic one as the density decreases is called a two-dimensional metal-insulator transition (2DMIT). An important feature of these systems is that the application of an in-plane magnetic field (which cannot affect the electrons' orbital motion but can direct their spins) reduces the conductance in the metallic regime, until for high enough magnetic fields the conductance saturates as a function of the field and the systems show the expected insulating behavior. This saturation field was estimated to be the field of full alignment of all spins.

These results arouse much interest and many ideas were suggested for their explanation. A debate started on the question of whether there is really a metallic behavior and a phase transition, probably caused by electron-electron interaction,[5] or the system is really insulating, but the experimentally accessible temperatures are high enough to exhibit temperature-dependent scattering, thus causing the apparent metallic behavior.[6]

Analytical[5] and numerical[7] calculations have shown that, as expected, for spinless electrons repulsion can only further localize the electrons and does not lead to a metal-insulator transition. However, when taking spin into account, the situation is still unclear.[5] In a recent numerical exact-diagonalization study,[8] an Anderson model with both long-range Coulomb interactions and short-range Hubbard interactions was considered. It was shown that the Coulomb interaction, existing between any two electrons regardless of their spin, can only increase localization. On the other hand, the not-too-strong Hubbard interaction was seen to cause delocalization (a strong Hubbard interaction will lead to a Mott-Hubbard insulator). Since this interaction exists only between electrons with opposite spins, its effect is decreased by an in-plane magnetic field and disappears when all the spins are aligned. This dependence of localization on interaction strength and in-plane magnetic field thus mimics, at least qualitatively, the experimentally observed phenomena. Similar results were obtained recently using quantum Monte Carlo methods.[9]

In this paper we wish to study further the weak short-range interaction regime, in which interaction-induced delocalization was observed. We will first address the problem analytically, using a random matrix theory (RMT) approach,[10] and then compare it to numerical simulations on an Anderson model. It will be shown that RMT can give only a qualitative but not a quantitative explanation for the numerical results, since RMT does not take into account nonuniversal correlations existing between wave functions in the diffusive regime. An estimate for the effect's order of magnitude and its dependence on the parameters of the system will be given.

II. ANALYTICAL RESULTS: RANDOM MATRIX THEORY

We will consider an Anderson Hamiltonian with on-site Hubbard interaction:

TABLE I. Values of the average of the numerator in Eq. (5) for all the possible combinations of level numbers l, m, and n and sites s and s'.

	$s=s'$	$s\neq s'$
$l\neq m\neq n$	$\frac{3}{N(N+2)(N+4)(N+6)}$	$-\frac{3(N+3)}{(N-1)N(N+1)(N+2)(N+4)(N+6)}$
$l=m\neq n$	$\frac{9}{N(N+2)(N+4)(N+6)}$	$-\frac{9(N+3)}{(N-1)N(N+1)(N+2)(N+4)(N+6)}$
$m=n\neq l$	$\frac{15}{N(N+2)(N+4)(N+6)}$	$-\frac{9}{(N-1)N(N+2)(N+4)(N+6)}$

$$\hat{H}=\sum_{s;\sigma}\epsilon_s\hat{n}_{s;\sigma}-t\sum_{\langle s,s'\rangle;\sigma}\hat{a}^{\dagger}_{s;\sigma}\hat{a}_{s';\sigma}+U_H\sum_{s}\hat{n}_{s;\uparrow}\hat{n}_{s;\downarrow}, \quad (1)$$

where $\hat{a}^{\dagger}_{s;\sigma}$, $\hat{a}_{s';\sigma}$, and $\hat{n}_{s;\sigma}$ denote electron creation, annihilation, and number operators, respectively, for a state on site s with spin projection σ on some axis. The first term is a random on-site potential, where ϵ_s is chosen randomly from the range $[-W/2,W/2]$; the second is the hopping or kinetic term, where the sum is over nearest-neighbor sites s, s'; and t is an overlap integral; the third is the Hubbard term, the electrostatic interaction between two electrons in the same site (which must have opposite spins), whose strength is determined by the parameter U_H. In the following we will concentrate on the situation at zero temperature and work in the canonical ensemble, taking the number of electrons as fixed, realization independent. We will also neglect any mechanism of dephasing.

To quantify localization, we will calculate the inverse participation ratio (IPR), defined by $P^{-1}=\Sigma_s|\psi(s)|^4$. This quantity is of order 1 for localized states and of order N^{-1} for delocalized states, where N is the number of lattice sites (or, equivalently, the Hamiltonian matrix size). The IPR thus decreases when the single-particle wave function ψ becomes less localized and gives us an estimate for the changes in the conductance of the system.

We now turn to use RMT. We note that relying on RMT limits the validity of our considerations to the nonlocalized regime (i.e., to the case when the system's size is smaller than the localization length). Nevertheless, we can still gain some insight about the interplay of disorder-induced localization and interaction in mesoscopic systems. We thus assume that without interaction the single-electron energy and state vector distributions for the ensemble of Anderson Hamiltonians are described by the corresponding distributions for an ensemble of Gaussian real symmetric matrices—i.e., the Gaussian orthogonal ensemble (GOE). This ensemble is defined by the well-known distribution[10]

$$P(\hat{H})\mu(\hat{H})=\exp\left(-\frac{\beta}{4\lambda^2}\mathrm{Tr}(\hat{H}^2)\right)\mu(\hat{H}), \quad (2)$$

where $\beta=1$, λ is a constant determining the energy scale, and $\mu(\hat{H})$ is a suitable measure. The eigenvectors are then a set of random orthogonal real normalized vectors. The average IPR without interaction for an electron in the nth level with spin σ is thus[11]

$$P_n^{-1}=\sum_{s}\langle|\psi^{(0)}_{n;\sigma}(s)|^4\rangle=\frac{3}{N+2}, \quad (3)$$

where the superscript (0) denotes the state without interaction and angular brackets denote ensemble average.

We will now add a weak Hubbard interaction, taking it to first-order in perturbation theory. Thus, the effect of spin-down electrons on the electrons with spin up will be the following effective potential (since only electrons with different spins interact, we have no exchange term):

$$\hat{V}=U_H\sum_{s}|\psi^{(0)}_{m;\downarrow}(s)|^2\hat{n}_{s;\uparrow}. \quad (4)$$

According to the familiar first-order perturbation theory, the first-order change in the IPR of a spin-up electron in the nth state due to its interaction with a spin-down electron in the mth state is

$$\Delta_m P_n^{-1}\sim 4U_H\sum_{\substack{l\neq n\\ s,s'}}\left\langle\frac{[\psi^{(0)}_{m;\downarrow}(s')]^2\psi^{(0)}_{n;\uparrow}(s')\psi^{(0)}_{l;\uparrow}(s')[\psi^{(0)}_{n;\uparrow}(s)]^3\psi^{(0)}_{l;\uparrow}(s)}{E^{(0)}_n-E^{(0)}_l}\right\rangle. \quad (5)$$

Since the wave functions can be chosen as real due to time reversal symmetry, we omitted the absolute value and complex conjugate notations in this and the following expressions.

According to RMT, the eigenvectors distribution is independent of the eigenvalues distribution, so we can separate the averages of the numerator and denominator in the above expression.

As for the average of the numerator, its value can be found in the literature,[11,12] and the results are summarized in

Table I. We note that when $s=s'$ we have an average of even powers of wave functions at different sites, which is expected to be positive and vary as N^{-4}, since we have eight wave function values in the expression, each of which going as $N^{-1/2}$. On the other hand, when $s\neq s'$, it may appear at first glance that since we have an average of odd powers of values of wave functions at different sites, which are uncorrelated, we should get zero. However, we get in this case a nonzero negative value, going as N^{-5}. This result is due to correlations resulting from the orthogonality requirement on the eigenvectors.

To understand this, we may note that squaring the orthogonality relation $\Sigma_s \psi_j(s)\psi_k(s)=0$ for $j\neq k$ and averaging, using the known result[11]

$$\langle (\psi_j(s))^2 (\psi_k(s))^2 \rangle = \frac{1}{N(N+2)}, \tag{6}$$

we find that

$$\langle \psi_j(s)\psi_j(s')\psi_k(s)\psi_k(s') \rangle = -\frac{1}{(N-1)N(N+2)}, \tag{7}$$

for $s\neq s'$; i.e., if two different wave functions have the same sign on one site, from orthogonality they will tend to have opposite signs on another site and vice versa, hence the above nonzero negative average.

As for the average value of the energy denominator in Eq. (5), in principle it might be possible to calculate its value using RMT. However, to estimate the leading order we will assume that the spectrum is composed of equidistant levels, with mean level spacing Δ.

Combining all these results together, we get, to leading order in N, the following result for the change in the IPR of a spin-up electron in the nth level due to its interaction with a spin-down electron in the mth level:

$$\Delta_m P_n^{-1} = \begin{cases} -\dfrac{24}{N^3}\dfrac{U_H}{\Delta}[\Phi(N-n)-\Phi(n-1)], & m=n, \\ \dfrac{24}{N^4}\dfrac{U_H}{\Delta}\left(\Phi(N-n)-\Phi(n-1)+\dfrac{2}{m-n}\right), & m\neq n, \end{cases} \tag{8}$$

where $\Phi(n)$ is defined by

$$\Phi(n)=\sum_{k=1}^{n}\frac{1}{k}.$$

We observe that for $m=n$ the correction is always negative (for n in the lower half of the band); i.e., the interaction between two electrons in the same state tends to delocalize them, which is the only way to reduce their mutual interaction energy. For $m\neq n$ the correction will usually be positive; i.e., electrons in different levels repulse each other, resulting in further localization. As can be expected, the former effect is larger than the latter, due to the identity of the two interacting electrons' wave functions in the former case. However, the order-N difference between the case $m=n$ and the case $m\neq n$ is caused by an excat cancellation of the leading-order dependence on N between the single short-range ($s=s'$) term and all the $N-1$ long-range ($s\neq s'$) terms in the latter case, which does not occur in the former. We will see below that this cancellation, together with the positive sign of the result for $m\neq n$, is correct only in RMT.

Thus, if the lowest $n_\downarrow$ levels are occupied by spin-down electrons, the total change in the IPR of a spin-up electron in the nth level is

$$\Delta P_n^{-1} = \begin{cases} -\dfrac{24}{N^3}\dfrac{U_H}{\Delta}\left[\left(1-\dfrac{n_\downarrow-1}{N}\right)[\Phi(N-n)-\Phi(n-1)] \right. \\ \qquad \left. -\dfrac{2}{N}[\Phi(n_\downarrow-n)-\Phi(n-1)],\right] & n\leqslant n_\downarrow, \\ +\dfrac{24}{N^3}\dfrac{U_H}{\Delta}\left[\dfrac{n_\downarrow}{N}[\Phi(N-n)-\Phi(n-1)] \right. \\ \qquad \left. -\dfrac{2}{N}[\Phi(n-1)-\Phi(n-n_\downarrow-1)]\right], & n>n_\downarrow. \end{cases} \tag{9}$$

The main features in the behavior of ΔP_n^{-1} are as follows: For $n\leqslant n_\downarrow$ the negative contribution of the spin-down electron at the same level n as the affected spin-up electron dominates the usually positive contribution of the other spin-down electrons. Therefore, ΔP_n^{-1} is negative, but decreases in absolute value when $n_\downarrow$ increases. For $n>n_\downarrow$, there are spin-down electrons only in levels different from n; thus ΔP_n^{-1} is positive and increases when $n_\downarrow$ increases. At $n=n_\downarrow$ there is a discontinuous jump of ΔP_n^{-1}. In both cases, since $\Delta\sim N^{-1}$ in real systems (although not in RMT), the effect is of order N^{-2}, if we keep the concentration of spin-down electrons constant. [We neglect here the logarithmic

factor coming from the function $\Phi(n)$.] A plot of these formulas will be shown in the next section, where these expressions will be compared to numerical results.

III. NUMERICAL RESULTS

In this section we will examine results of numerical calculations and compare them to the analytical results discussed above. Two model Hamiltonians will be considered: an RMT Hamiltonian and an Anderson Hamiltonian. It will be shown that their results differ by an order of magnitude as well as in other characteristics. The theoretical predictions will be shown to agree with the former but not with the latter, and reasons for the discrepancy will be given.

A. Random matrix Hamiltonian

We will first consider the change in the IPR for a true RMT Hamiltonian. Since we consider here only the weak-interaction regime, instead of solving the many-body problem exactly, we simply first diagonalize the Hamiltonian without interaction and then use the wave functions to construct the effective potential, given in Eq. (4). This potential is then used to calculate the wave functions and the IPR with interaction. The applicability of this one-loop Hartree-Fock approximation is justified by the fact that the change in P_n^{-1} was found to be linear in U_H, as required.

The matrix size chosen was 408×408, and the elements were chosen according to the distribution law in Eq. (2). We have chosen $\lambda=0.1t$, so that the mean level spacing is $\Delta=0.0196t$, approximately equal to the spacing in the Anderson Hamiltonian, Eq. (1), used in the next section ($0.022t$–$0.025t$ for W between $2.0t$ and $4.0t$). The interaction strength U_H was taken as $1.0t$. The calculated quantities were averaged over an ensemble of 5×10^4 different realizations.

The numerical results for the change in the IPR versus the level number of the affected spin-up electron due to its interaction with different numbers of spin-down electrons are shown in Fig. 1, together with the theoretical formula, Eq. (9). The theoretical formula was corrected, taking into account that the mean level spacing is not constant across the spectrum, but varies according to the semicircle law[10]

$$\frac{1}{\Delta(E)}=\rho(E)=\frac{1}{2\pi\lambda^2\beta}\sqrt{4\lambda^2\beta N-E^2}, \qquad (10)$$

where $\rho(E)$ is the density of states.

As can be seen, there is good agreement between the numerical and theoretical results. All the main features discussed at the end of the previous section can clearly be seen in the numerical data.

B. Anderson Hamiltonian

We will now discuss the changes in the IPR for the Anderson Hamiltonian given in Eq. (1). The calculation was performed in the same method as was used for the random matrix Hamiltonian (i.e., one-loop Hartree-Fock approximation).

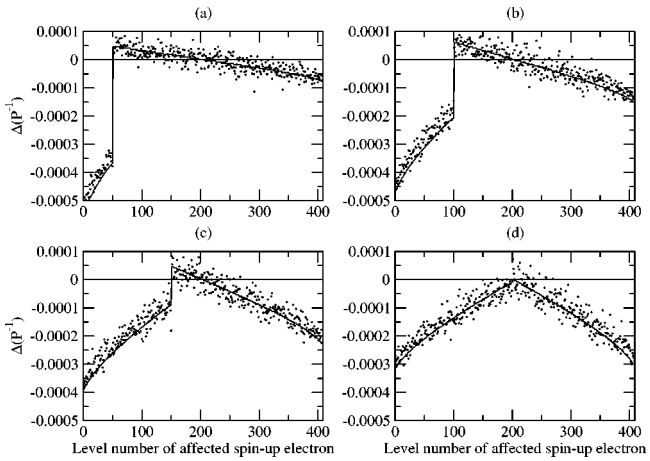

FIG. 1. Change in the IPR of a spin-up electron due to its interaction with spin-down electrons, according to RMT. The change is plotted as a function of the level number of the affected spin-up electron for different numbers of spin-down electrons: (a) $n_\downarrow=50$, (b) $n_\downarrow=100$, (c) $n_\downarrow=150$, and (d) $n_\downarrow=200$. In all the graphs the line indicates the theoretical formula, while the dots indicate the numerical results. The numerical results are averages over an ensemble of 5×10^4 realizations of 408×408 RMT Hamiltonians. The estimated error approximately equals the roughness of the numerical results. Further parameters are given in the text.

We have chosen a 17×24 lattice, corresponding to a 408×408 matrix. As for the RMT calculations, we took $U_H=1.0t$, while four values of disorder were used: $W=2.0t$, $W=2.5t$, $W=3.0t$, and $W=4.0t$. The results were averaged over 10^4 realizations of disorder.

First, in Fig. 2, the value of the IPR without interaction is shown for the four values of disorder, as well as the RMT value, Eq. (3). We can see a difference here, as the Anderson model gives higher (i.e., more localized) values of the IPR than RMT. The effect is caused by nonuniversal (i.e., beyond

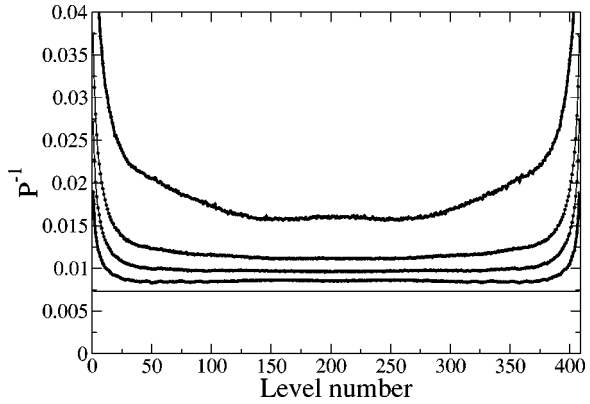

FIG. 2. The IPR for noninteracting electrons in the Anderson model. The IPR is plotted as a function of the level number. The lowest curve shows the RMT value, while the other ones are the Anderson model results for $W=2.0t$, $W=2.5t$, $W=3.0t$, and $W=4.0t$, from lower to upper, respectively. The results are averages over an ensemble of 10^4 realizations of systems on a 17×24 sites lattice. The estimated error approximately equals the roughness of the numerical results. Further parameters are given in the text.

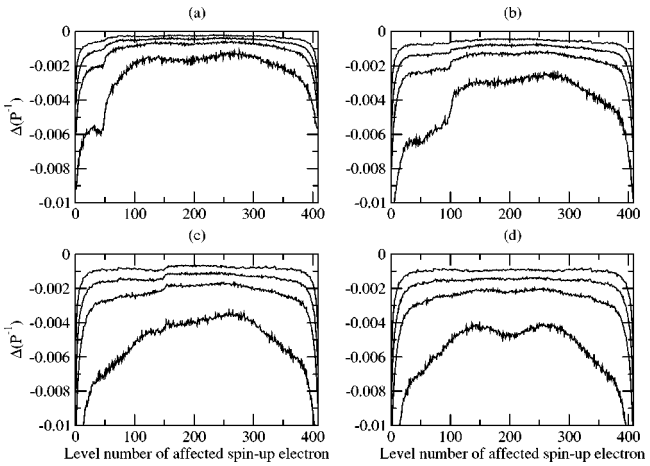

FIG. 3. Change in the IPR of a spin-up electron due to its interaction with spin-down electrons in the Anderson model. The change is plotted as a function of the level number of the affected spin-up electron for different numbers of spin-down electrons: (a) $n_\downarrow = 50$, (b) $n_\downarrow = 100$, (c) $n_\downarrow = 150$, and (d) $n_\downarrow = 200$. In all the graphs the curves correspond to $W = 4.0t$, $W = 3.0t$, $W = 2.5t$, and $W = 2.0t$, from lower to upper, respectively. The results are averages over an ensemble of 10^4 realizations of systems on a 17×24 sites lattice. The estimated error approximately equals the roughness of the numerical results. Further parameters are given in the text.

RMT) corrections to the IPR and is more pronounced for higher disorder. The corrections for the IPR were calculated using the supersymmetry technique,[13] resulting in $P^{-1} - P_{RMT}^{-1} \sim g^{-1} N^{-1}$ (g being the dimensionless conductance). We can also see, as can be expected, that the levels near the band edge have higher IPR, and are thus more localized, than levels near the center of the band.

Now we move to interaction effects in the Anderson model. The results are shown in Fig. 3, with the same occupation numbers as those chosen in the previous RMT calculations, for the four values of the disorder. As in RMT, ΔP_n^{-1} is negative for $n \leqslant n_\downarrow$ and changes sharply (though not discontinuously) at $n = n_\downarrow$. Nevertheless, it does not change its sign there. Moreover, ΔP_n^{-1} is larger by about an order of magnitude than the one found from RMT. Also, even in the range $n \leqslant n_\downarrow$, it increases in absolute value, rather than decreases, when $n_\downarrow$ increases. All this is in contrast with Eq. (9) and the discussion following it.

Another point is that the effect increases with disorder. This is seen by comparing ΔP_n^{-1} for the same level n, but different values of W, or by observing that, for the same value of W, levels near the band edge, which are more localized, show larger ΔP_n^{-1}.

The reason for these differences is the above-mentioned cancellation between long-range and short-range wave function correlations in RMT. As has been seen in our RMT calculations (Table I), the average of the wave function products appearing in the numerator of Eq. (5) is of order N^{-4} and positive when the two sites considered coincide, but is only of order N^{-5} and negative when the sites are different. Since there are $N-1$ terms of the latter type for each term of the former type, their total contributions are of the same order but their signs are opposite. Due to the equality of the numerical coefficients of the two types of terms when the interacting electrons are in different levels, they cancel out exactly to the leading order in N, leaving behind a small negative term of order N^{-5}. Therefore, in RMT the interaction between electrons in different levels increases their localization, opposite to the situation for electrons in the same level. From this followed the decrease in the absolute value of ΔP_n^{-1} as $n_\downarrow$ increases in the range $n \leqslant n_\downarrow$, its positive value for $n > n_\downarrow$, and the overall N^{-2} dependence of the effect for constant density of spin-down electrons.

All this is correct when g is infinite. For finite g there exist nonuniversal corrections to the wave function averages. Those corrections were not calculated before for the averages required here, but their behavior can be conjectured from known corrections for simpler averages [like those in Eqs. (6) and (7) (Ref. 14)]. We may expect them to have the same N dependence and sign as the RMT value, but to be smaller by a factor of g. The corrections for the short-range ($s = s'$) terms and long-range ($s \neq s'$) terms will not, in general, have equal numerical coefficients, even when the interacting electrons are in different levels. In fact, since hopping in the RMT Hamiltonian and, thus, correlations in RMT do not depend on distance, the occurrence of the same leading-order cancellations between correlations in realistic models like the Anderson Hamiltonian is highly improbable. Hence, after summation over s' we are left with an order-$g^{-1}N^{-4}$ contribution instead of the order-N^{-5} contribution in RMT. For this reason, although the nonuniversal corrections are of order g^{-1}, for most of the averaged terms they are about N times larger, so they will determine both the magnitude and sign of the interaction-induced change in the IPR. Since the corrections for $s = s'$ will, in general, have a long-range part, persisting for $s \neq s'$ and having the same sign for neighboring sites (although for larger distances we may expect some oscillations), their sign will dominate the overall sign of the results. We will thus get a negative change in the IPR not only from the interaction between electrons in the same level but also when the interacting electrons are in different levels. Hence, ΔP_n^{-1} will always be negative, as can be seen in the numerical results.

Moreover, repeating the calculations with the nonuniversal correction to the averages of the wave functions product, we can estimate the dependence of the effect on the system parameters. We expect the total change in the IPR of a spin-up electron due to its interaction with $n_\downarrow$ spin-down electrons to vary as

$$\Delta P_n^{-1} \sim -\frac{1}{g} \frac{U_H}{\Delta} \frac{n_\downarrow}{N^3}. \tag{11}$$

This expression does not include a factor coming from the sum over energy denominators, which has only a weak dependence on N and $n_\downarrow$ (logarithmic for equidistant levels, a weak power law for a nonconstant density of states). Because wave functions corresponding to neighboring levels are more correlated than wave functions corresponding to faraway levels, there is also a factor which changes sharply (though not discontinuously) when we pass from $n \leqslant n_\downarrow$ to $n > n_\downarrow$, as

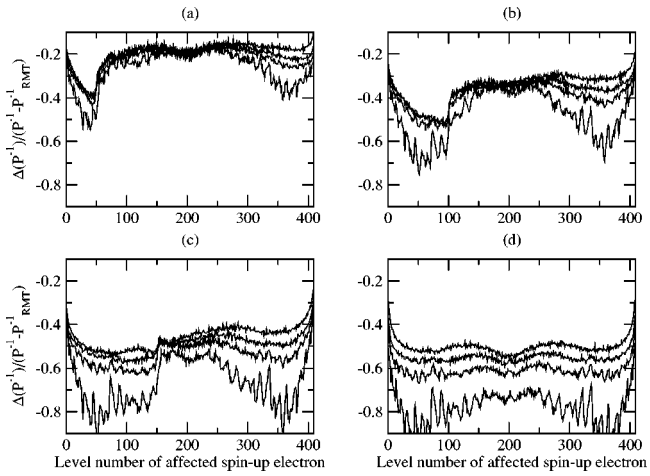

FIG. 4. Ratio between the change in the IPR of a spin-up electron due to its interaction with spin-down electrons in the Anderson model and the nonuniversal part of the IPR without interaction. The ratio is plotted as a function of the level number of the affected spin-up electron for different numbers of spin-down electrons: (a) $n_\downarrow=50$, (b) $n_\downarrow=100$, (c) $n_\downarrow=150$, and (d) $n_\downarrow=200$. In all the graphs the curves correspond to $W=2.0t$, $W=2.5t$, $W=3.0t$, and $W=4.0t$, from lower to upper, respectively. The results are averages over an ensemble of 10^4 realizations of systems on a 17×24 sites lattice. The estimated error approximately equals the roughness of the numerical results. Further parameters are given in the text.

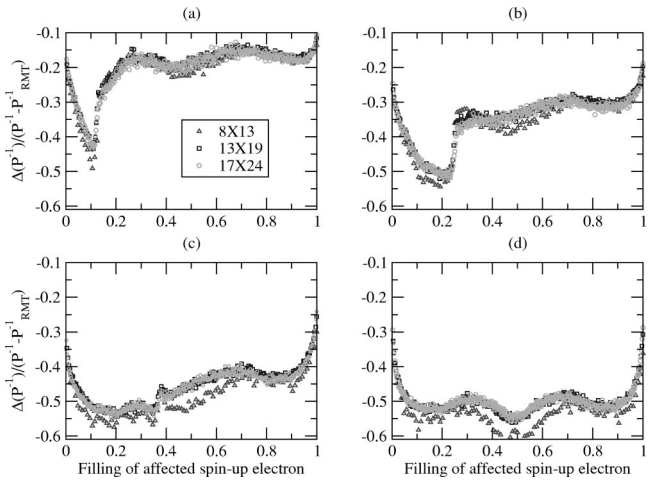

FIG. 5. Ratio between the change in the IPR of a spin-up electron due to its interaction with spin-down electrons in the Anderson model and the nonuniversal part of the IPR without interaction. The ratio is plotted as a function of the filling of the affected spin-up electron (i.e., the ratio of the number of spin-up electrons and the number of lattice sites) for different fillings of spin-down electrons: (a) $\nu_\downarrow\approx1/8$, (b) $\nu_\downarrow\approx1/4$, (c) $\nu_\downarrow\approx3/8$, and (d) $\nu_\downarrow\approx1/2$. In each graph we use three different lattice sizes 8×13, 13×19, and 17×24, but a constant value of disorder, $W=4.0t$. The results are averages over an ensemble of 10^4 realizations. The estimated error approximately equals the roughness of the numerical results. Further parameters are given in the text.

seen in the numerical results. Since $\Delta\sim N^{-1}$ in real systems (although not in RMT), the effect is of order $g^{-1}N^{-1}$ if we keep the concentration of spin-down electrons constant. This is in contrast to the N^{-2} dependence in RMT. Because N/g is much larger than unity in our numerical calculations, we can now understand the order-of-magnitude difference between RMT and Anderson model results. Thus, all the features of the numerical data can be explained by taking nonuniversal corrections into account.

As we have mentioned before, the nonuniversal part of the IPR without interaction—i.e., the difference between the value of the IPR without interaction in the Anderson model and its value in RMT—varies as $g^{-1}N^{-1}$. According to our estimate, the change in the IPR due to interaction in the Anderson model also goes as $g^{-1}N^{-1}$. Thus, their ratio $\Delta P_n^{-1}/(P^{-1}-P_{RMT}^{-1})$ should be independent of g—i.e. of the degree of disorder. It should also be independent of the number of lattice sites N if the densities of spin-up and spin-down electrons are kept constant. Thus, this ratio may be used to test our conjecture for the parametric form of ΔP_n^{-1}.

We first test the g independence of the ratio $\Delta P_n^{-1}/(P^{-1}-P_{RMT}^{-1})$ by plotting it in Fig. 4 for systems with identical lattice sizes (taken to be 17×24, as in the previous calculations), but different values of disorder. We can clearly see that the differences between curves corresponding to different W values are much smaller than the corresponding differences in Fig. 3. The only exception is the value $W=2.0t$ (the lowest curve), which shows a marked difference from the other W values. This is probably due to the fact that for $W=2.0t$ disorder is not high enough, so the electrons' motion is not fully diffusive and ballistic boundary effects may be important.

We now test N independence of the ratio $\Delta P_n^{-1}/(P^{-1}-P_{RMT}^{-1})$ by plotting it in Fig. 5 for systems with the same value of disorder (taken as $W=4.0t$) but different lattice sizes 8×13, 13×19, and 17×24. In all cases the densities of spin-up and spin-down electrons are approximately equal (the horizontal axis is not the level number of the affected spin-up electron as before, but the filling ν, defined as the ratio between the number of spin-up electrons n and the total number of lattice sites N). We can clearly see that the different curves are almost identical. The only exception is the small 8×13 lattice, whose slightly different behavior can again be attributed to ballistic boundary effects.

IV. CONCLUSIONS

In conclusion, we have shown how a spin-dependent interaction can cause delocalization, at least for weak short-range interactions. Localized electrons highly repulse each other, especially if they have the same orbital wave function and thus a different spin. This results in a tendency for interaction-induced delocalization. The effect on an electron with a given orbital level and spin direction is stronger if the same orbital level is occupied by an electron with an opposite spin and increases with the total number of electrons with opposite spin. The delocalization is thus reduced by an in-plane magnetic field. All this is in accordance, at least qualitatively, with recent experimental findings[4] and numeri-

cal simulations[8,9] regarding the in-plane magnetoresistance.

We have also seen that the main difference in the influence of the Hubbard interaction between realistic finite g systems and the RMT stems from exact cancellation of the leading-order long-range and short-range terms in the former. Thus, while in RMT a state is correlated only to the same state with an opposite spin (except for weak anticorrelations with all other states), for finite g correlations between different states lead to a stronger repulsion between these states, resulting in a stronger delocalization due to the on-site interactions. Nevertheless, the order of magnitude and parametric dependence of the IPR can be calculated using RMT, once the nonuniversal corrections are properly taken into account.

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