Linear temperature dependence of conductivity in the apparent insulating regime of dilute two-dimensional holes in GaAs

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(Received 22 August 2003; revised manuscript received 6 October 2003; published 23 December 2003)

The conductivity of extremely high mobility dilute two-dimensional holes in GaAs changes linearly with temperature in the insulating side of the metal-insulator transition. Hopping conduction, characterized by an exponentially decreasing conductivity with decreasing temperature, is not observed when the conductivity is smaller than e^2/h . We suggest that strong interactions in a regime close to the Wigner crystallization must be playing a role in the unusual transport.

DOI: 10.1103/PhysRevB.68.241308

PACS number(s): 73.40.Kp, 71.30.+h

In recent years, there has been great interest in the transport of low-density two-dimensional (2D) charge carrier systems. In contradiction to the scaling theory of localization¹ which predicted that all states in 2D in the absence of electron-electron interactions are localized, experimental observations of "metallic" behavior showing increasing conductivity (σ) with decreasing temperature (*T*) ($d\sigma/dT < 0$) and an apparent metal-insulator transition (MIT) as the carrier density is lowered have been reported on many low-density, low disorder 2D systems.² Although there have been many experimental and theoretical studies trying to understand this metallic behavior, there is still no consensus on its origin to date.

On the other hand, the "insulating" behavior $(d\sigma/dT)$ >0) for lower densities in these low disorder systems has not been studied as extensively as the metallic behavior. In highly disordered systems, the transport in the insulating regime is described by variable range hopping^{3,4} (VRH) among the localized states, and the T dependence of σ is expressed as $\sigma(T) = \sigma_0 \exp[-(T_0/T)^x]$, where σ_0 is a prefactor and T_0 is a characteristic temperature. The exponent x depends on the density of states (DOS) at the Fermi energy. For a constant DOS in the absence of interactions, x = 1/3 corresponding to the Mott VRH,³ and for a DOS which has a Coulomb gap at the Fermi energy due to interactions, x = 1/2 corresponding to the Efros-Shklovskii VRH.⁴ In clean systems with low disorder, however, the origin of the insulating behavior $(d\sigma/dT > 0)$ in low-density regime, where the interactions between carriers are strong, could be significantly different from that described by simple localization. In the ideal case, it was predicted that the ground state of the system in the low-density limit is a Wigner crystal,⁵ which can be pinned by even a tiny amount of impurities in the system.

We have recently studied the MIT of extremely high mobility dilute 2D holes in GaAs and presented a detailed analysis for the *T* dependence of σ on the metallic side of the transition.⁶ The metallic behavior persisted up to $r_s = 57$, much higher than 37 predicted for Wigner crystallization [r_s is the interaction energy to Fermi energy ratio, given by r_s $= (p\pi)^{-1/2}m^*e^2/4\pi\hbar^2\epsilon$, where *p* is the hole density, m^* effective mass, and ϵ the dielectric constant]. With the persistence of metallic behavior up to such a large r_s value, the question arises regarding the nature of the insulating behavior observed in this 2D system for r_s reaching 80. To address this question, we present the *T* dependence of σ on the insulating side of the MIT in this paper. We show that the conductivity exhibits a linear dependence on temperature and does not follow the VRH behavior, and discuss its implications with regard to strong interaction effects close to or in the Wigner crystallization regime.

The sample used in this study is a heterojunction insulated-gate field-effect transistor (HIGFET) made on a (100) surface of GaAs.^{6,7} A metallic gate, separated by an insulating AlGaAs from the semiconducting GaAs, is used to induce the 2D holes at the interface between the GaAs and AlGaAs. The mobility μ of our sample measured at T =65 mK reaches 1.8×10^6 cm²/Vs for $p = 3.2 \times 10^{10}$ cm⁻², which is the highest achieved for 2D holes in this low density regime. This high mobility allows us to measure the temperature dependence of conductivity down to very low densities reaching $p = 1.5 \times 10^9$ cm⁻², with r_s near 80. The MIT, determined from the sign of $d\sigma/dT$, is observed at a critical density of $p_c = 3 \times 10^9$ cm⁻², i.e., $r_s = 57$. The experiment was done in a dilution refrigerator with a base temperature around 65 mK. The resistivity of the 2D holes was measured by a low-frequency lock-in technique with frequency as low as 0.1 Hz to maintain the out-of-phase signal less than 5%. Excitation current of as small as 50 pA was used to ensure that the effect of Joule heating is negligible $(\leq 10 \text{ fW/cm}^2)$. In addition, the drive current was varied at base temperature to make sure that the linear response was measured.

In Fig. 1, the resistivity ρ of the 2D holes as a function of T is shown for five densities varying from 5×10^9 cm⁻² ($r_s = 44$) to 1.5×10^9 cm⁻² ($r_s = 80$). Data for higher densities deep inside the metallic region have been presented in Ref. 6. For $p = 5 \times 10^9$ cm⁻² and $p = 4 \times 10^9$ cm⁻², ρ shows a nonmonotonic dependence on T. It increases with decreasing T at high T and decreases with decreasing T at low T. This decrease of ρ with decreasing $T (d\rho/dT > 0)$ is typical of the metallic behavior, the physical origin of which has been the focus of much recent research on 2D MIT. When



FIG. 1. ρ vs T for p = 5, 4, 3, 2.3, and 1.5×10^9 cm⁻² from the bottom. Inset: ρ vs p at various T's.

the density is lowered further, below 3×10^9 cm⁻², ρ only increases with decreasing *T*, with an apparent transition to an insulating behavior. In the inset, we show ρ as a function of *p* at different *T*'s. The crossing point seen at p=3 $\times 10^9$ cm⁻² is an indicative of the transition from a metallic behavior $(d\rho/dT>0)$ to an insulating behavior $(d\rho/dT$ <0). Following a widely adopted method in the literature, we identify this point as a critical point of the MIT. In this paper, we show that the insulating behavior determined from the sign of $d\rho/dT$ does not exhibit characteristics of a localized state even though ρ increases rapidly with decreasing *T*.

In Fig. 2, we show the same data as σ vs *T*. Now, the top two traces are on the metallic side. For $p=5\times10^9$ cm⁻², σ decreases with increasing *T* ($d\sigma/dT<0$) from 0.06 K to 0.15 K. At *T*>0.15 K, σ increases with increasing *T* ($d\sigma/dT>0$) and the increase becomes linear for *T* >0.25 K. This linear dependence of σ on *T* continues for *T* exceeding T_F , the Fermi temperature of the 2D hole gas (marked by the vertical bar in Fig. 2), which is 0.36 K for this density. For $p=4\times10^9$ cm⁻² ($T_F=0.29$ K), similar *T* dependence of σ is observed, although the metallic behavior is much weaker, and the crossover from $d\sigma/dT<0$ to



FIG. 2. σ vs T for p=5, 4, 3, 2.3, and 1.5×10^9 cm⁻² from the top. The Fermi temperature T_F is marked by the vertical bar.



FIG. 3. σ vs *T* for $p = 1.5 \times 10^9$ cm⁻². Insets: Trial fitting using the VRH model with (a) x = 1/3 and (b) x = 1/2.

 $d\sigma/dT > 0$ occurs at lower T (~0.1 K). The linear dependence sets in at T=0.22 K.

However, for $p=3\times 10^9$ cm⁻² ($r_s=57$), the third trace from the top, $d\sigma/dT > 0$ is observed in all our T ranges, implying that this density corresponds to p_c of the MIT in our sample. The metallic behavior $(d\sigma/dT < 0)$ is not seen in the data down to the lowest T. We note that for T<0.13 K σ becomes smaller than e^2/h , suggestive of an insulator, although the value of σ does not solely determine the MIT. The bottom two traces are for $p = 2.3 \times 10^9$ cm⁻² and 1.5 $\times 10^9$ cm⁻², corresponding to the insulating behavior. For these densities, ρ in Fig. 1 showed a strong increase with decreasing T. When the same data are plotted as σ vs T in Fig. 2, however, σ decreases with decreasing T only linearly. For $p=2.3 \times 10^9 \text{ cm}^{-2}$ ($r_s=65$), $\sigma < e^2/h$ for T < 0.22 Kand the linear dependence extends down to the base temperature. This linear dependence appears to be very similar to that seen in the high-density metallic side $(5 \times 10^9 \text{ cm}^{-2} \text{ and}$ 4×10^9 cm⁻²), and the same linear dependence is seen for $\sigma < e^2/h$ as well as for $\sigma > e^2/h$. Further reducing the density down to 1.5×10^9 cm⁻² ($r_s = 80$) does not change the T dependence very much, but simply decreases the magnitude of σ , making the T range where $\sigma < e^2/h$ wider.

In Fig. 3, we show the T dependence of σ for p=1.5 $\times 10^9$ cm⁻² in more detail. Clearly, σ depends linearly on T (as shown by the solid line) in the entire T range, and σ $\langle e^2/h$ in almost the same range. To see if an exponential dependence could be possibly consistent with our data, we show in the inset a fit of the data with the VRH model⁸ with exponent x equal to either 1/3 or 1/2. Clearly, none of them produced a good fit. If we extract σ_0 and T_0 from these relatively poor fits, $\sigma_0 \approx 8e^2/h$ and $T_0 \approx 2.54$ K for x = 1/3, and $\sigma_0 \approx 3e^2/h$ and $T_0 \approx 0.4$ K for x = 1/2. $\sigma_0 \approx 8e^2/h$ for x = 1/3 is too large for phonon-assisted hopping, where σ_0 $\ll e^2/h$. It is also significantly larger than that $(\sigma_0 \approx e^2/h)$ theoretically expected⁹ and that $(\sigma_0 \approx 2e^2/h)$ experimentally observed¹⁰ for an electron-assisted hopping. For x = 1/2, σ_0 $\approx 3e^2/h$ is also larger than that $(\sigma_0 \approx e^2/h)$ observed in other experiments.^{10,11} Furthermore, $T_0 \approx 0.4$ K gives a localization length $\xi \approx 20 \ \mu \text{m}$ from $T_0 = C e^2 / k_B \epsilon \epsilon_0 \xi$ with C = 6.2and $\epsilon = 13$. This localization length, in turn, gives a hopping length $r_h \approx 13 \ \mu m$ at $T = 60 \ mK$. This hopping length is so much larger than the distance from the 2D holes to the gate in our sample, 0.46 μ m, that the Efros-Shklovskii VRH is not expected.¹² All these rule out the possibility that the data could be described by the VRH model. This suggests that the transition to an insulating behavior, determined from the sign of $d\sigma/dT$, as we lower the 2D hole density within the T range of our experiment is not a transition to a localized state but a crossover to an apparent insulating regime. In this context, we note that Altshuler *et al.*² argued that the critical density for the MIT must be determined from the disappearance of the exponential behavior in resistivity rather than the sign change in $d\rho/dT$. While one could, therefore, expect that further reducing the density will eventually lead to an exponentially localized regime, the origin of the apparent insulating behavior and linear T dependence observed for this density is of great interest and needs further exploration.

Experiment on low-density 2D electrons in HIGFET (Ref. 13) showed that σ varies as $T^{0.75}$ and $T^{0.5}$ for $n=1.6 \times 10^9$ cm⁻² and $n=2.0\times 10^9$ cm⁻², respectively, in the insulating side of the transition. Data for 2D holes in modulation-doped (311)A GaAs quantum well¹⁴ are also consistent with such a power-law behavior, $\sigma \approx T^{0.82}$ and $T^{0.38}$ for $p=3.8\times 10^9$ cm⁻² and $p=4.0\times 10^9$ cm⁻², respectively, even when the measurements were made down to much lower temperatures. We note that the exponent of the power-law behavior increases with decreasing carrier density and appears to approach 1.

Das Sarma and Hwang¹⁵ calculated the T dependence of σ by taking into account the charged impurity scattering. In their predicted T dependence, the nonmonotonic dependence $(d\sigma/dT < 0$ at low T and $d\sigma/dT > 0$ at high T) seen in the high-density metallic side is explained as a quantum-toclassical crossover. In the classical regime, when $T \gg T_F$, the scattering of the 2D carriers by charged impurities would give rise to linear increase of σ with increasing T. They argued¹⁶ that the insulating behavior at low densities just below the critical density results from the same classical origin (high-T behavior), exhibiting a power-law dependence of σ on T. The linear dependence observed in the insulating regime of our sample seems consistent with their prediction. However, strong interactions (with $r_s = 80$) which must be present for this density $(p=1.5\times10^9 \text{ cm}^{-2})$ make it unlikely that their predicted T dependence is the adequate explanation even though it is remarkably similar to that observed in our experiment.

We should also mention that localization corrections to the conductivity in the weakly interacting classical regime do not explain our data. Experimental study on weak localization in the classical regime has been performed for electrons on solid hydrogen surface.¹⁷ In this case, the correction to the conductivity is proportional to T_F/T , and our data are not consistent with this expectation. The interaction correction is proportional to $(T_F/T)^2$. This term is negligible compared with that from the weak localization, and does not explain our data either.

The linear dependence of σ on *T* observed in the insulating regime of our data must result from strong interaction

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effects. For instance, in a 2D electronic system with such a large r_s value ($r_s \approx 80$), formation of a Wigner crystal should be considered. In the quantum regime, Wigner crystallization is predicted⁵ to occur for $r_s > 37$. In our sample, we should also consider the crystallization in the classical regime. We note that decreasing the carrier density not only increases r_s , but also decreases T_F , thus decreases the quantum-toclassical crossover temperature. In our data, for hole densities $p=5\times10^9$ cm⁻² and $p=4\times10^9$ cm⁻², the linear dependence of σ on T predicted for the $T \gg T_F$ classical regime actually extends down to a T even below T_F , suggesting that the crossover from classical to the quantum regime seems to occur much below T_F . In any case, for a classical 2D electronic system, the relevant interaction parameter is Γ , which is given by $\Gamma = e^2 \sqrt{\pi n} / 4\pi \epsilon k_B T$, with *n* being the carrier density. This parameter is equivalent to r_s in the degenerate regime, with the kinetic energy replaced by k_BT . Studies of electrons on liquid Helium^{18–21} have shown that transition to the Wigner crystal is around $\Gamma = 127$. In our sample, for the lowest hole density, $p = 1.5 \times 10^9$ cm⁻², $T_F = 0.11$ K, and Γ is between 146 and 27 for T from 0.06 K to 0.32 K.

Either from the quantum or the classical picture, the 2D holes in our sample appear to be in the crystallization regime. When the Wigner crystal is pinned by disorder, the transport is expected through thermal activation of the crystal over the pinning potential. In this case, σ should decrease exponentially with decreasing T. Our data, on the contrary, show a linear T dependence in the insulating regime. Here, we should consider the nonzero measurement temperatures, which lead to a higher r_s value for the crystallization, and we should also consider the finite r_s values, which lead to a higher Γ value for the crystallization, as studied by Chui and Esfarjani.²² In their calculation, the phase boundary between the liquid and the crystal shifts to a higher r_s as the reduced temperature $t = 127/\Gamma$ increases, or shifts to a lower t (higher Γ) as r_s decreases. For $r_s = 80$ ($p = 1.5 \times 10^9$ cm⁻² in our sample), Γ must be larger than 230 for the 2D holes to be crystallized. Therefore, Wigner crystallization is not likely to occur in the temperature and density range of our measurements, even though we should expect the 2D holes to be very close to the Wigner crystallization and strongly correlated. We note that a more recent electron-on-helium experiment²³ has reported residual signature of the Wigner crystal up to a much higher temperature with Γ down to 46.

In this respect, we think that the recent theory by Spivak²⁴ may be relevant to our data. In his theory, the metallic state is a state in which a small concentration of Wigner crystal droplets is embedded in the Fermi liquid, and the insulating state corresponds to one in which a small fraction of Fermi liquid is present in a mostly crystalline phase. In the metallic side, the resistivity increases linearly in T in the low-temperature degenerate regime as the size of the Wigner crystal droplets grows. In the insulating side, he argues that the crystal forms a supersolid, which is not pinned by disorder, and σ remains nonzero as T goes to zero. At high temperatures, Wigner crystal droplets melt and the system behaves classically. In this case, the resistivity is inversely proportional to T and σ is linear in T. We note that this

predicted *T* dependence, especially the linear *T* dependence of σ in the classical regime, is identical to that by Das Sarma and Hwang, but it is based on strong correlation effects close to the Wigner crystallization.

In summary, we have studied the temperature dependence of the conductivity in the insulating side of the MIT for extremely high mobility dilute 2D hole system with r_s reach-

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ing 80. A linear dependence of conductivity on temperature has been observed in the insulating side. We suggest that strong interaction effects in both degenerate and classical regimes close to the Wigner crystallization should be consid-

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ered to provide an understanding for this unusual transport. This work was supported by the NSF and the DOE at

Princeton University.

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