Statistics of current fluctuations in mesoscopic coherent conductors at nonzero frequencies

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We formulate a general approach which describes statistics of current fluctuations in mesoscopic coherent conductors at arbitrary frequencies and in the presence of interactions. Applying this approach to the noninteracting case, we analyze frequency dispersion of the third cumulant of the current operator $\mathcal{S}_3$ at frequencies well below both the inverse charge relaxation time and the inverse electron dwell time. This dispersion turns out to be important in the frequency range comparable to applied voltages. For comparatively transparent conductors it may lead to the sign change of $\mathcal{S}_3$.

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Investigations of current fluctuations in mesoscopic conductors can provide a great deal of information about properties of such systems. During past years much attention has been devoted to shot noise.[1] Experimental and theoretical studies of the second moment of the current operator describing shot noise revealed a rich variety of properties caused by an interplay between scattering, quantum coherence and charge discreteness. Furthermore, such studies can substantially deepen our understanding of the role of electron-electron interactions in mesoscopic transport because shot noise and interaction effects are known to be closely related.[2,3]

One can also go beyond the second moment and study higher-order correlators of the current operator thereby extending the amount of information already obtained from investigations of electron transport and shot noise. Recently the first experimental study of the third current cumulant in mesoscopic tunnel junctions was reported.[4]

A theoretical framework which enables one to analyze statistics of charge transfer in mesoscopic conductors was developed in Ref. 5. This theory of full counting statistics (FCS) allows to evaluate any cumulant of the current operator in the absence of interactions and in the zero-frequency limit. Under these conditions higher-order current cumulants were investigated by a number of authors.[6–9] In order to include interactions and to analyze frequency dispersion of current fluctuations it is necessary to go beyond the FCS theory and to develop a more general real time path-integral technique.[2,10,11]

The goal of the present paper is to address statistics of current fluctuations at nonzero frequencies. We first present a general and formally exact expression for the real time effective action of a coherent conductor described by an arbitrary energy independent scattering matrix. This expression enables one to fully describe interaction effects in such type of conductors. We will then demonstrate that in a noninteracting case this effective action provides a direct generalization of the FCS generating function[5] to nonzero frequencies. With the aid of our technique we will analyze the frequency dispersion of the third cumulant of the current operator in mesoscopic coherent conductors.

It is worthwhile to point out that the frequency dependence of current correlators can be caused by various reasons. One of them is the effect of an external electromagnetic environment which is important for quantitative interpretation of the experimentally detected behavior of higher cumulants.[12] Another source of the frequency dispersion is the internal dynamics of a quantum scatterer. Here the important time scales are the corresponding RC time τ_{RC} and the electron dwell time τ_D inside the conductor. The latter scale was recently taken into consideration in the analysis of the second[13] and the third[14] current cumulants for chaotic quantum dots.

In this paper we will address current fluctuations at frequencies not directly related to any of such scales. We will demonstrate that apart from the above mechanisms there exists an additional—intrinsic—frequency dispersion of the current correlators at the scale set by the voltage drop V across the conductor. Since V can vary in a wide range, this dependence is in general important and should be taken into account while interpreting the experimental results. In particular, in the absence of interactions the third cumulant of the current operator $\mathcal{S}_3$ is fully determined by the two parameters

$$\beta=\frac{\sum_n T_n(1-T_n)}{\sum_n T_n}, \quad \gamma=\frac{\sum_n T_n^2(1-T_n)}{\sum_n T_n}, \tag{1}$$

where T_n represents the transmission of the nth conducting channel of our system. The cumulant $\mathcal{S}_3$ can be expressed in the following general form:

$$\mathcal{S}_3=(\beta-2\gamma F)e^2\overline{I}. \tag{2}$$

Here $\overline{I}$ is the average current through the conductor, $-e$ stands for the electron charge, and F is a universal function of frequencies, voltage V and temperature T to be evaluated below. According to Eq. (2) the frequency dispersion of $\mathcal{S}_3$ originates only from the term proportional to the parameter γ, while the β term is dispersionless.

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Though negligible for tunnel junctions $\gamma\to 0$, the frequency dispersion of $\mathcal{S}_3$ may become important in other situations. For instance, at $T\to 0$ one finds $F\to 1$ at frequencies much smaller than eV, while in the opposite high frequency limit one gets $F=0$ and, hence, $\mathcal{S}_3=\beta e^2\overline{I}$ in the latter limit. To give some numbers, for an important case of diffusive conductors one has $\beta=1/3$ and $\beta-2\gamma=1/15$, i.e., in this case the quantity $\mathcal{S}_3$ changes by the factor of 5 depending on whether relevant frequencies are below or above eV. For conductors with $\beta<2\gamma$ even the sign of $\mathcal{S}_3$ will differ in these two limits.

In our analysis we will use the real time path-integral formalism developed for the systems of interacting fermions.[15] After the standard Hubbard-Stratonovich decoupling of the interaction term in the Hamiltonian one can exactly integrate out fermions and arrive at the effective action S which depends on the fluctuating fields $V_{1,2}(t,\boldsymbol{r})$. Let us define

$$e^{iS_0}=\mathrm{Tr}[\mathcal{T}e^{-i\int_0^t dt' \boldsymbol{H}_1(t')}\hat{\boldsymbol{\rho}}_0\tilde{\mathcal{T}}e^{i\int_0^t dt' \boldsymbol{H}_2(t')}], \tag{3}$$

with the trace taken over the fermionic variables. Here $\hat{\boldsymbol{\rho}}_0$ is the initial N-particle density matrix of electrons,

$$\boldsymbol{H}_{1,2}=\sum_\sigma \int d^3\boldsymbol{r}\,\hat{\Psi}^\dagger_\sigma(\boldsymbol{r})\hat{H}_{1,2}(t)\hat{\Psi}_\sigma(\boldsymbol{r}),$$

$$\hat{H}_{1,2}(t)=-\frac{\nabla^2}{2m}+U(\boldsymbol{r})-eV_{1,2}(t,\boldsymbol{r}) \tag{4}$$

are the effective Hamiltonians on the forward and backward parts of the Keldysh contour, $U(\boldsymbol{r})$ describes the static potential, $\mathcal{T}$ and $\tilde{\mathcal{T}}$ are, respectively, the forward and backward time ordering operators. Integrating out fermions in Eq. (3), we obtain $iS=iS_0+iS_{\mathrm{em}}$, where

$$iS_0=2\,\mathrm{Tr}\ln[1+(\hat{u}_2^{-1}\hat{u}_1-1)\hat{\rho}_0], \tag{5}$$

$\hat{u}_{1,2}(t)=\mathcal{T}\exp(-i\int_0^t dt'\hat{H}_{1,2}(t'))$ are the evolution operators pertaining to the Hamiltonians (4), $\hat{\rho}_0$ stands for the initial single-particle density matrix and the term S_{em} accounts for the electromagnetic contribution which may also include the effect of an external circuit.

In order to evaluate the evolution operators $\hat{u}_{1,2}$ it is necessary to specify the model of a mesoscopic conductor. Here we will adopt the standard model of a (comparatively short) coherent conductor placed in between two bulk metallic reservoirs. The electron dwell time τ_D is supposed to be shorter than any relevant time scale in our problem. Energy and phase relaxation times are, on the contrary, assumed to be long, i.e., inelastic relaxation is allowed in the reservoirs but not inside the conductor. Under these assumptions electron transport through the conductor can be described by the energy independent scattering matrix

$$\hat{S}=\begin{pmatrix}\hat{r} & \hat{t}'\\ \hat{t} & \hat{r}'\end{pmatrix}, \tag{6}$$

and the effective action Eq. (5) can be expressed via the fluctuating phase fields $\varphi_{1,2}$ which are in turn related to the jumps of the fields $V_{1,2}$ across the scatterer as $\dot{\varphi}_{1,2}=e(V_{L1,2}-V_{R1,2})$, where $V_{L,R}$ are fluctuating in time but constant in space fields in the left and right reservoirs. We note that in this case the right hand side of Eq. (3) differs from the FCS generating functional introduced in Ref. 16 only by a gauge transformation.

Within the above model the evolution operators $\hat{u}_{1,2}$ were evaluated in Refs. 2 and 10. Combining these expressions with Eq. (5) after some algebra we find

$$iS_0=2\,\mathrm{Tr}\ln\Bigg\{\hat{a}\,\delta(x-y)+\theta(t-x)\theta(x)\times\begin{bmatrix}\hat{t}^\dagger\hat{t}(e^{i\varphi^-(x)}-1) & 2i\hat{t}^\dagger\hat{r}'\sin\frac{\varphi^-(x)}{2}\\ 2i\hat{r}'^\dagger\hat{t}\sin\frac{\varphi^-(x)}{2} & \hat{t}'^\dagger\hat{t}'(e^{-i\varphi^-(x)}-1)\end{bmatrix}\times\begin{bmatrix}\rho_0(y-x)e^{i[\varphi^+(x)-\varphi^+(y)]/2} & 0\\ 0 & \rho_0(y-x)e^{i[\varphi^+(y)-\varphi^+(x)]/2}\end{bmatrix}\Bigg\}. \tag{7}$$

Here we introduced $\varphi^+=(\varphi_1+\varphi_2)/2$, $\varphi^-=\varphi_1-\varphi_2$, and

$$\rho_0(x)=\int\frac{dE}{2\pi}\frac{e^{iEx}}{1+e^{E/T}}. \tag{8}$$

Taking the trace in Eq. (7) implies convolution with respect to internal time variables (x and y). The total time span is denoted by t.

Equation (7) defines a formally exact effective action for a coherent conductor described by an arbitrary energy independent scattering matrix (6). This expression allows to fully determine statistics of current fluctuations at arbitrary frequencies and in the presence of interactions. In the case of equilibrium fluctuations the formula Eq. (7) represents a real time analog of the effective action derived in Refs. 17 and 18 within the Matsubara technique. A formula similar to Eq. (7) was also presented recently in Ref. 11. In addition we note that, provided the field φ^- does not depend on time, Eq. (7) coincides with the generating function considered, e.g., in the problem of adiabatic pumping through mesoscopic conductors.[19]

Let us illustrate the relation between the effective action Eq. (7) and the FCS generating function.[5] For this purpose we (i) disregard interactions and set $\varphi^+(t)=eVt$ and (ii) suppress fluctuations of φ^- and set $\varphi^-=\mathrm{const}$. After these simplifications from Eq. (7) we obtain

$$iS_{FCS}=\frac{t}{\pi}\mathrm{Tr}\int dE\,\ln[1+\hat{t}^{\dagger}\hat{t}\,n_L(E)((1-n_R(E))(e^{i\varphi^-}-1)+\hat{t}^{\dagger}\hat{t}\,n_R(E)(1-n_L(E))(e^{-i\varphi^-}-1)], \tag{9}$$

where $n_{L,R}(E)=1/[1+\exp((E\pm eV/2)/T])$. Equation (9) is just the FCS generating function[5] which can be used to recover all cumulants of the current operator in the zero-frequency limit and in the absence of interactions.

The expression (7) is more general since it includes all possible fluctuations of the phase fields $\varphi^{\pm}$. For instance, if one allows for temporal variations of φ^-, with the aid of Eq. (7) one can easily describe the frequency dispersion of the current correlators. Below we will illustrate this point by directly evaluating the third cumulant of the current operator.

Within our model the current operator can be defined as $\hat{I}(t)=(e/2)d(\hat{N}_L(t)-\hat{N}_R(t))/dt$, where $\hat{N}_{L(R)}$ is the total number of electrons in the left (right) lead. Combining this definition with Eq. (3), one finds

$$\langle\hat{I}(t)\rangle=-e\int\mathcal{D}\varphi^{\pm}\,\delta S[\varphi^{\pm}]/\delta\varphi^-(t)e^{iS[\varphi^{\pm}]}.$$

Similarly one can define higher moments of the current operator. Here we consider the following correlation functions: $\widetilde{\mathcal{S}}_2=\langle\hat{I}(t_1)\hat{I}(t_2)+\hat{I}(t_2)\hat{I}(t_1)\rangle/2$ and

$$\begin{aligned}\widetilde{\mathcal{S}}_3=\frac{1}{8}\{&\langle\hat{I}(t_1)(\mathcal{T}\hat{I}(t_2)\hat{I}(t_3))\rangle+\langle[\widetilde{\mathcal{T}}\hat{I}(t_2)\hat{I}(t_3)]\hat{I}(t_1)\rangle\\&+\langle\hat{I}(t_2)[\mathcal{T}\hat{I}(t_1)\hat{I}(t_3)]\rangle+\langle[\widetilde{\mathcal{T}}\hat{I}(t_1)\hat{I}(t_3)]I(t_2)\rangle\\&+\langle\hat{I}(t_3)[\mathcal{T}\hat{I}(t_1)\hat{I}(t_2)]\rangle+\langle[\widetilde{\mathcal{T}}\hat{I}(t_1)\hat{I}(t_2)]\hat{I}(t_3)\rangle\\&+\langle\mathcal{T}\hat{I}(t_1)\hat{I}(t_2)\hat{I}(t_3)\rangle+\langle\widetilde{\mathcal{T}}\hat{I}(t_1)\hat{I}(t_2)\hat{I}(t_3)\rangle\}.\end{aligned}\tag{10}$$

The correlation function $\widetilde{\mathcal{S}}_2$ is important because the symmetric combination of voltages V^+ can be viewed as a classical, measurable, voltage.[16] The noise is deduced from the measurable product $V^+(t_1)V^+(t_2)$, which is related to the symmetric correlator $\widetilde{\mathcal{S}}_2$. Similarly, the measured product $V^+(t_1)V^+(t_2)V^+(t_3)$ is related to the correlation function of the current operators $\widetilde{\mathcal{S}}_3$ defined in Eq. (10), see also Refs. 9,16, and 20.

Let us now disregard interaction effects which remain small provided either the scatterer conductance $1/R=(2e^2/h)\Sigma_n T_n$ or that of attached external leads strongly exceeds the quantum conductance unit e^2/h. In this case fluctuations of $\varphi^{\pm}$ are effectively suppressed and, hence, one should set $\varphi^+(\tau)=eV\tau$ and $\varphi^-(\tau)\to 0$ in the end of the calculation. Then for the noise correlator one gets

$$\widetilde{\mathcal{S}}_2=(ie)^2\frac{\delta^2}{\delta\varphi^-(t_1)\,\delta\varphi^-(t_2)}e^{iS_0}\bigg|_{\varphi^-=0}, \tag{11}$$

and a similar expression is obtained for $\widetilde{\mathcal{S}}_3$. Of interest is the irreducible part of the correlator $\widetilde{\mathcal{S}}_3$ which reads

$$\begin{aligned}\mathcal{S}_3=e^3\frac{\delta^3 S_0}{\delta\varphi^-(t_1)\,\delta\varphi^-(t_2)\,\delta\varphi^-(t_3)}\bigg|_{\varphi^-=0}&=\widetilde{\mathcal{S}}_3(t_1,t_2,t_3)\\&-\langle\hat{I}(t_1)\rangle\widetilde{\mathcal{S}}(t_2,t_3)-\langle\hat{I}(t_2)\rangle\widetilde{\mathcal{S}}(t_1,t_3)-\langle\hat{I}(t_3)\rangle\widetilde{\mathcal{S}}(t_1,t_2)\\&+2\langle\hat{I}(t_1)\rangle\langle\hat{I}(t_2)\rangle\langle\hat{I}(t_3)\rangle.\end{aligned}\tag{12}$$

It follows immediately that in order to evaluate the third current cumulant in the absence of interactions it suffices to formally expand the exact effective action Eq. (7) up to the third order in φ^-,

$$iS_0[\varphi^{\pm}]=iS^{(1)}[\varphi^{\pm}]+iS^{(2)}[\varphi^{\pm}]+iS^{(3)}[\varphi^{\pm}], \tag{13}$$

keeping the full nonlinearity in φ^+ in each of these terms. The first two terms of this expansion were evaluated in Ref. 2. Being combined with Eq. (11), the term $S^{(2)}[\varphi^{\pm}]$ allows to recover the well-known expression for the shot-noise spectrum.[1] Proceeding further with the expansion in φ^- for the term $iS^{(3)}$ one finds[10]

$$\begin{aligned}iS^{(3)}[\varphi^{\pm}]=&\frac{i\beta}{6e^2R}\int_0^t d\tau(\varphi^-(\tau))^3\dot{\varphi}^+(\tau)\\&-\frac{2\pi i\gamma}{3e^2R}\int_0^t d\tau_1\int_0^t d\tau_2\int_0^t d\tau_3\varphi^-(\tau_1)\varphi^-(\tau_2)\\&\times\varphi^-(\tau_3)f(\tau_2,\tau_1)f(\tau_3,\tau_2)f(\tau_1,\tau_3),\end{aligned}\tag{14}$$

where

$$f(\tau_2,\tau_1)=\frac{T\sin[(\varphi^+(\tau_2)-\varphi^+(\tau_1))/2]}{\sinh[\pi T(\tau_2-\tau_1)]}. \tag{15}$$

Let us substitute the above expressions into Eq. (12) and, after taking derivatives over φ^-, set $\varphi^+(\tau)=eV\tau$ and $\varphi^-\to 0$. This is sufficient provided the time differences $|t_1-t_2|$, $|t_1-t_3|$ exceed the charge relaxation time τ_{RC} and provided $eV\ll 1/\tau_{RC}$. Equation (12) then yields

$$\begin{aligned}\mathcal{S}_3=\beta e^2\overline{I}\delta(t_1-t_2)\,\delta(t_1-t_3)-\frac{4\pi e\gamma}{R}f(t_2-t_1)\\\times f(t_3-t_2)f(t_1-t_3),\end{aligned}\tag{16}$$

where $f(\tau)=T\sin(eV\tau/2)/\sinh(\pi T\tau)$. Performing the Fourier transformation

$$\mathcal{S}_3(\omega_1,\omega_2)=\int d\tau_1 d\tau_2\,e^{i\omega_1\tau_1+i\omega_2\tau_2}\mathcal{S}_3(t_1,t_1-\tau_1,t_1-\tau_2)$$

we arrive at the final result

$$\mathcal{S}_3=\beta e^2\overline{I}-2\gamma e^2\overline{I}F(v,w_1,w_2), \tag{17}$$

$$F=\frac{\sinh^3(v/2)}{4v}\int_{-\infty}^{\infty}\frac{d\omega}{\chi(\omega)\chi(\omega-w_1)\chi(\omega+w_2)}. \tag{18}$$

Here we defined $v=eV/T$, $w_{1,2}=\omega_{1,2}/2T$, and

$$\chi(\omega)=\cosh^2\omega+\sinh^2(v/4). \tag{19}$$

Equations (17)–(19) represent the main result of this paper. They fully describe the third cumulant of the current operator at voltages, temperatures, and frequencies smaller than both $1/\tau_{RC}$ and $1/\tau_D$.

Let us briefly analyze Eqs. (17)–(19) in various limits. For $\omega_{1,2}=0$ we recover the well-known result[7]

$$F(v,0,0)=1+3\frac{1-(\sinh v/v)}{\cosh v-1}, \tag{20}$$

which in turn yields $F\to 1$ in the limit of large voltages $v \gg 1$. Equation (20) also holds for $w_{1,2}\ll v$.

In the limit $v\ll 1$ one finds

$$F(v\ll 1,w,0)=F(v\ll 1,w,-w)$$
$$=\frac{9\sinh w+\sinh(3w)-12w\cosh w}{48\sinh^5 w}v^2 \tag{21}$$

and, similarly,

$$F(v\ll 1,w,w)$$
$$=\frac{\sinh(4w)+4\sinh(2w)-8w\cosh(2w)-4w}{128\sinh^5 w\cosh^3 w}v^2. \tag{22}$$

These equations demonstrate that at large frequencies $w\gg 1$ the function F decays exponentially with w. From Eqs. (21) and (22) we find $F\propto v^2 e^{-2w}/3$ and $F\propto v^2 e^{-4w}$, respectively.

Finally let us turn to the most interesting limit of low temperatures, in which case one always has $v,w_1,w_2\gg 1$. Neglecting small corrections $\sim 1/v, 1/w$ we obtain

$$F(v,w_1,w_2)=1-2\left|\frac{w_{12}}{v}\right| \quad \text{if } 2|w_{12}|<|v|, \tag{23}$$

$$F(v,w_1,w_2)=0 \quad \text{if } 2|w_{12}|>|v|. \tag{24}$$

Here the value w_{12} is defined differently depending on the sign of the product w_1w_2. For $w_1w_2>0$ we have $w_{12}=w_1+w_2$, while in the opposite case $w_1w_2<0$ we define $w_{12}=\max[|w_1|,|w_2|]$. We observe that in both cases the function F depends linearly on frequency and vanishes as soon as $|w_{12}|$ exceeds $|v|/2$.

At arbitrary values of v, w_1, and w_2 the integral Eq. (18) can be evaluated numerically. The corresponding result for the function $F(v,w,0)$ is depicted in Fig. 1. The overall form of the function $F(v,w,w)$ is similar but—as compared to $F(v,w,0)$—it demonstrates a somewhat faster decay with increasing frequency.

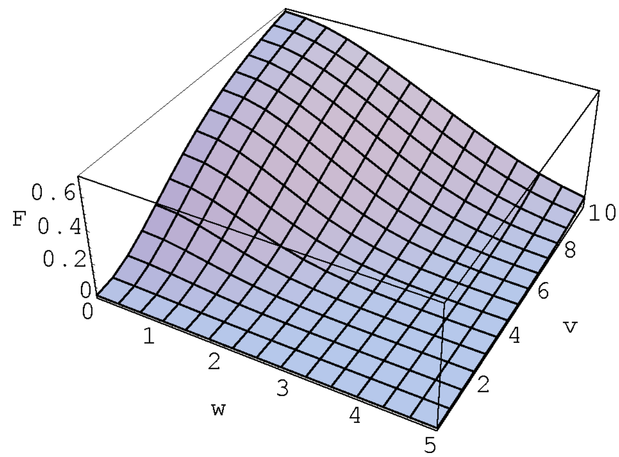

FIG. 1. (Color online) The function $F(v,w,0)=F(v,w,-w)$, see Eq. (18).

Finally let us point out that our predictions can be experimentally tested in various types of coherent mesoscopic conductors, such as, e.g., break junctions, quantum point contacts, or short diffusive metallic bridges/films.[21,22] In all these systems both τ_D and τ_{RC} can be small enough in order to satisfy all the assumptions adopted here. For instance, in diffusive samples[21,22] one finds $1/\tau_D$ of the order of few Kelvins and, hence, the condition $eV<1/\tau_D$ is obeyed in a wide range of voltages $V\lesssim 0.1$–0.5 mV. We also note that the work[22] reports the experimental analysis of the frequency dispersion of shot noise,[1] which is important in the same frequency range as that of the third cumulant studied here.

In conclusion, we have presented a general approach which allows to describe statistics of current fluctuations in mesoscopic coherent conductors at arbitrary frequencies and in the presence of interactions. Restricting ourselves to the noninteracting case, we have analyzed frequency dispersion of the third cumulant of the current operator. This dispersion was found negligible only in the case of tunnel junctions, while in a general case it turns out to be important in the frequency range comparable to eV. Similar results are also expected for higher-order cumulants of the current operator.

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