Electric field effect on the second-order nonlinear optical properties of parabolic and semiparabolic quantum wells

Li Zhang*
Department of Mechanism and Electron, Panyu Polytechnic, Panyu 511483, People's Republic of China

Hong-Jing Xie
Department of Physics, Guihuagang Campus, Guangzhou University, Guangzhou 510405, People's Republic of China
(Received 12 May 2003; revised manuscript received 7 October 2003; published 11 December 2003)

By using the compact-density-matrix approach and iterative procedure, a detailed procedure for the calculation of the second-harmonic generation (SHG) susceptibility tensor is given in the electric-field-biased parabolic and semiparabolic quantum wells (QW's). The simple analytical formula for the SHG susceptibility in the systems is also deduced. By adopting the methods of envelope wave function and displacement harmonic oscillation, the electronic states in parabolic and semi parabolic QW's with applied electric fields are exactly solved. Numerical results on typical $Al_xGa_{1-x}Al$/GaAs materials show that, for the same effective widths, the SHG susceptibility in semiparabolic QW is larger than that in parabolic QW due to the self-asymmetry of the semiparabolic QW, and the applied electric field can make the SHG susceptibilities in both systems enhance remarkably. Moreover, the SHG susceptibility also sensitively depends on the relaxation rate of the systems.

DOI: 10.1103/PhysRevB.68.235315 PACS number(s): 42.65.Ky, 42.79.Nv, 73.21.Fg, 78.66.Fd

I. INTRODUCTION

Recently nonlinear optical properties in semiconductor quantum wells (QW's) systems, superlattices, and nanostructures[1–21] are of considerable interest because of their relevance in studying practical applications and as a probe for the electronic structure of mesoscopic media. Much special attentions had been paid to the second-order nonlinear optical properties,[4–17] such as optical rectification (OR), second-harmonic generation (SHG), electro-optic effect (EOE), and so on, because the second-order nonlinear procedures are the simplest and the lowest-order nonlinear procedures, and the magnitudes of the second-order nonlinear are usually stronger than those of the higher-order nonlinears if the quantum systems have significant asymmetry.[13,14,18]

In a symmetric QW structure, the second-order nonlinear susceptibility is usually small except for the contribution of the bulk susceptibility.[4] Therefore, in order to obtain a strong second-order optical nonlinearity, the inversion symmetry of the quantum systems should be broken.[4–7] In general, people get these asymmetries through two ways, one is by using advanced material growing technology such as molecular-beam epitaxy and metal-organic chemical vapor deposition to obtain the systems with asymmetric confining potential,[7–10] the other is through applying an electric field to a symmetric system to get an asymmetric quantum system.[4,11,12] Gurnick and Detemple[8] have suggested obtaining this asymmetry by growing $Al_xGa_{1-x}As$ multiple QW's with asymmetric composition gradients of Al in the growing direction, and the authors have calculated the second-order nonlinearities for a Morse potential; the results reveal 10–100 times larger than in bulk materials. Khurgin[5] and Yuh and Wang[10] later suggested using an asymmetric couple QW and a step-QW structure, respectively. Rosencher and Bois[7] have shown that the step QW's could be designed so that the absorption could be doubly resonant, leading to SHG susceptibilities more than 10^3 times of magnitude higher than that in bulk GaAs. Ahn and co-worker[11] proposed to bias a symmetric QW electrically to obtain this asymmetry. This has been realized by Fejer *et al.*,[12] who obtained a SHG coefficient more than 70 times higher than in bulk GaAs. Recently, Guo and co-workers[13,14] investigated the OR and EOE in a symmetric parabolic QW with an applied electric field; results reveal that both effects were enhanced significantly with the increase of the magnitude of the electric field, and they reach nearly one and six orders higher than those in bulk GaAs, respectively. The huge nonlinear optical properties have the potential for device applications in far-infrared laser amplifies,[1] photodetectors,[2] and high-speed electro-optical modulators.[3]

In this paper, except for the symmetrical parabolic QW modes, a semiparabolic QW model has been brought forward. It is obvious that the semiparabolic QW system is an asymmetrical quantum system, and the applied electric field can adjust the asymmetry of the potential. Therefore, comparing with that in the symmetrical parabolic QW system, the second-order nonlinear effect in semiparabolic QW can be looked forward to having large enhancement. Bearing this idea, we will study the influences of electric field on the SHG susceptibility in parabolic and semiparabolic QW's.

The paper is organized as follows: In Sec. II, by adopting the methods of envelope wavefunction and displacement harmonic oscillation, the electronic states in parabolic and semiparabolic QW's with applied electric fields are exactly solved first, then under the compact-density-matrix approach, the simple analytical formula for the SHG susceptibility in the systems is deduced. In Sec. III, numerical calculations on typical GaAs material are performed. The influences of the electric field on the energy level of the bound states in the semiparabolic QW systems are analyzed and discussed first, then the SHG susceptibility as functions of the change in the energy eigenvalues due to an applied electric field, the incident photon energy, and the relaxation rate of the systems are plotted, and the characteristics for these curves are analyzed and compared. The results reveal that the SHG susceptibilities are related to the strength of applied electric field and the relaxation rate of the system. As the energy eigenvalues decrease due to the applied electric field becoming strong, the SHG susceptibility in both systems increase monotonely. Furthermore, when the effective widths of the semiparabolic and parabolic QW's are same, the SHG susceptibility in semiparabolic QW is larger than that in parabolic QW, which means that the semiparabolic QW is a model of very promising candidates for second-order nonlinear optical properties.

II. THEORY

Under the effective-mass approximation, the electron Hamiltonian of a QW system with an applied electric field is described by

$$H=-\frac{\hbar^2}{2m^*}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]+V(z)+qFz, \quad (1)$$

with

$$V(z)=\tfrac{1}{2}m^*\omega_0^2 z^2, \quad -\infty<z<\infty \quad (2)$$

for parabolic quantum wells, and

$$V(z)=\begin{cases}\tfrac{1}{2}m^*\omega_0^2 z^2 & z\geqslant 0\\ \infty, & z<0\end{cases} \quad (3)$$

for semiparabolic quantum wells. Here z represents the QW's growth direction, F is the strength of the applied electric field parallel to z direction, q is the electron charge, and ω_0 is the frequency of the parabolic confining potential of the QW. Under the envelope wave-function approximation, the eigenfunctions $\psi_{n,\mathbf{k}}(\mathbf{r})$ and eigenenergies $\varepsilon_{n,\mathbf{k}}$ are the solutions of the Schrödinger equation for H and are given by[13]

$$\psi_{n,\mathbf{k}}(\mathbf{r})=\phi_n(z)U_c(\mathbf{r})\exp(i\mathbf{k}_{//}\cdot\mathbf{r}_{//}), \quad (4)$$

and

$$\varepsilon_{n,\mathbf{k}}=E_n+\frac{\hbar^2}{2m^*}|k_{//}|^2. \quad (5)$$

Here, $\mathbf{k}_{//}$ and $\mathbf{r}_{//}$ are the wave vector and coordinate in the xy plane and $U_c(\mathbf{r})$ is the periodic part of the Bloch function in the conduction band at $\mathbf{k}=0$. $\phi_n(z)$ and E_n are the solutions of one-dimensional Schrödinger equation

$$H_z\phi(z)=E\phi(z), \quad (6)$$

where H_z is the z part of the Hamiltonian H, and it is given by

$$H_z=-\frac{\hbar^2}{2m^*}\frac{d^2}{dz^2}+V(z)+qFz. \quad (7)$$

Using the analogous steps as in Refs. 16 and 17, we obtain the energy levels and corresponding wave functions in the parabolic QW, and they take the following forms:

$$E_n=\hbar\omega_0\left(n+\frac{1}{2}\right)-\frac{q^2F^2}{2m^*\omega_0^2}, \quad n=0,1,2,3\ \ldots, \quad (8)$$

$$\phi_n(z)=N_n\exp\left(-\frac{1}{2}[\alpha(z+\beta)]^2\right)H_n[\alpha(z+\beta)], \quad (9)$$

with

$$\alpha=\sqrt{\frac{m^*\omega_0}{\hbar}}, \quad \beta=\frac{qF}{m^*\omega_0^2}, \quad (10)$$

where $H_t(z)$ is the Hermite functions[22] [when t is integer, $H_t(z)$ becomes Hermite polynomial], and $N_n=[\alpha^{-1}\sqrt{\pi}2^n n!]^{-1/2}$ is the normalization constant.

For semiparabolic QW, the electronic energy levels and corresponding wave functions are given as follows:

$$E_n=\hbar\omega_0\left(t_n+\frac{1}{2}\right)-\frac{q^2F^2}{2m^*\omega_0^2}, \quad (11)$$

$$\phi_n(z)=N_n\exp\left(-\frac{1}{2}[\alpha(z+\beta)]^2\right)H_{t_n}[\alpha(z+\beta)], \quad (12)$$

where t_n is determined by

$$H_{t_n}(\alpha\beta)\equiv 0, \quad n=1,2,3\ \ldots, \quad (13)$$

and the normalization constant N_n is determined by

$$N_n=\left\{\int_0^\infty \exp\{-[\alpha(z+\beta)]^2\}\{H_{t_n}[\alpha(z+\beta)]\}^2 dz\right\}^{-1/2}. \quad (14)$$

Next, the formulas of the SHG susceptibility in the two models will be deduced. Assuming a monochromatic incident field $E(t)=\tilde{E}\exp(-i\omega t)+\tilde{E}^*\exp(i\omega t)$ is applied to the system. The evolution of the density matrix is given by the time-dependent Schrödinger equation

$$\frac{\partial\rho_{ij}}{\partial t}=\frac{1}{i\hbar}[H_0-qzE(t),\rho]_{ij}-\Gamma_{ij}(\rho-\rho^{(0)})_{ij}. \quad (15)$$

For simplicity, only one value of the relaxation rate is assumed $\Gamma_{ij}=\Gamma_0=1/T$. Equation (15) is solved using the usual iterative method,[7,11]

$$\rho(t)=\sum_n \rho^{(n)}(t), \quad (16)$$

with

$$\frac{\partial\rho_{ij}^{(n+1)}}{\partial t}=\frac{1}{i\hbar}\{[H_0,\rho^{(n+1)}]_{ij}-i\hbar\Gamma_{ij}\rho_{ij}^{(n+1)}\} -\frac{1}{i\hbar}[qz,\rho^{(n)}]_{ij}E(t). \quad (17)$$

TABLE I. The calculated energy eigenvalues and their intervals in eV.

	Semiparabolic QW							Parabolic QW				
$F\times10^7$ V/m	E_1	E_2	E_3	E_4	ΔE_1	ΔE_2	ΔE_3	E_1	E_2	E_3	E_4	$\Delta E_i(i=1,2,3)$
0.0	0.3554	0.8293	1.3032	1.7770	0.4739	0.4739	0.4739	0.2369	0.7108	1.1847	1.6586	0.4739
0.5	0.3432	0.8110	1.2802	1.7502	0.4676	0.4692	0.4700	0.2369	0.7108	1.1846	1.6585	0.4739
1.0	0.3315	0.7930	1.2576	1.7238	0.4615	0.4646	0.4662	0.2367	0.7106	1.1844	1.6583	0.4739
1.5	0.3201	0.7755	1.2355	1.6978	0.4554	0.4600	0.4623	0.2364	0.7102	1.1841	1.6580	0.4739
2.0	0.3091	0.7583	1.2137	1.6721	0.4492	0.4554	0.4584	0.2359	0.7098	1.1837	1.6576	0.4739
3.0	0.2883	0.7253	1.1714	1.6222	0.4371	0.4461	0.4507	0.2347	0.7085	1.1824	1.6563	0.4739
4.0	0.2689	0.6939	1.1308	1.5738	0.4250	0.4369	0.4430	0.2329	0.7068	1.1806	1.6545	0.4739
5.0	0.2510	0.6640	1.0917	1.5270	0.4130	0.4277	0.4353	0.2306	0.7045	1.1784	1.6522	0.4739
8.0	0.2058	0.5842	0.9850	1.3979	0.3784	0.4009	0.4129	0.2207	0.6946	1.1685	1.6424	0.4739
10.0	0.1822	0.5383	0.9210	1.3187	0.3561	0.3828	0.3977	0.2116	0.6855	1.1594	1.6333	0.4739

The electronic polarization of the QW will also be a series expansion as in Eq. (16), and be limited to the first two orders, i.e.,

$$P(t)=(\varepsilon_0\chi_{\omega}^{(1)}\tilde{E}e^{i\omega t}+\varepsilon_0\chi_{2\omega}^{(2)}\tilde{E}^2e^{2i\omega t})+\text{c.c.}+\varepsilon_0\chi_0^{(2)}\tilde{E}^2, \tag{18}$$

where $\chi_{\omega}^{(1)}$, $\chi_{2\omega}^{(2)}$, and $\chi_0^{(2)}$ are the linear susceptibility, SHG, and OR coefficients, respectively. ε_0 is the vacuum permittivity. The electronic polarization of the nth order is given by

$$P^{(n)}(t)=\frac{1}{S}\text{Tr}(\rho^{(n)}qz), \tag{19}$$

where S is the area of the interaction and the symbol ''Tr'' denotes the summation over the diagonal elements of the matrix.

In this paper, we lay emphasis on the calculations of the SHG susceptibility. By using the same compact-density-matrix approach and iterative procedure as Refs. 7, 11, 13 and 16–18, and under the condition of two-photon resonance, i.e., $\hbar\omega\approx E_{21}\approx E_{32}\approx\hbar\Omega$ (From the above discussion on electron states, it can be seen that the resonance condition in parabolic QW with applied field can obtain rigorous satisfaction, and the resonance condition in semiparabolic QW with applied field can also obtain approximate satisfaction when the field is not too strong), the SHG susceptibility per unit volume is derived, and it is given by

$$\chi_{2\omega}^{(2)}=\frac{q^3\mu_{12}\mu_{23}\mu_{31}\rho_s}{\varepsilon_0}\times\frac{1}{(E_{31}-2\hbar\omega+i\hbar\Gamma_0)(E_{21}-\hbar\omega+i\hbar\Gamma_0)}. \tag{20}$$

The volume SHG susceptibility has a resonant peak value for $\hbar\omega=\hbar\Omega$ given by

$$\chi_{2\omega,Max}^{(2)}=\frac{q^3\rho_s\mu_{12}\mu_{23}\mu_{31}}{\varepsilon_0(\hbar\Gamma_0)^2}, \tag{21}$$

where ρ_s is the surface density of electrons in the QW, $E_{ij}=E_i-E_j$ is the energy interval of two different electronic states, and μ_{ij} is the off-diagonal matrix element which is given by $\mu_{ij}=|\langle\phi_i|z|\phi_j\rangle|$ $(i,\ j=1,2,3)$.

III. NUMERICAL RESULTS AND DISCUSSION

It is well known that parabolic QW is a symmetrical quantum system, while semiparabolic QW is an asymmetrical quantum system. So the effective width for the parabolic QW can be defined as $2|z_0|$, but that for the semiparabolic QW as $|z_0|$ (z_0 is the maximum size of the quantum well in z direction). In order to compare the SHG susceptibility and the influences of electric field on the SHG susceptibility in the two systems, the same effective widths and the same barrier heights of the two QW's are assumed, which implies that $\frac{1}{2}m^*\omega_S^2z_0^2=\frac{1}{2}m^*\omega_A^2(2z_0)^2$. Via this relation, it is easy to get $\omega_A=\omega_S/2$, where ω_S and ω_A denote the confined potential frequencies of the symmetrical parabolic QW and the asymmetrical semiparabolic QW, respectively. In the following discussion, we mark $\omega_A=\omega_S/2=\omega_0$ for convenience.

Numerical calculations are carried out on typical $Al_xGa_{1-x}As/GaAs$ parabolic and semiparabolic QW's. The material parameters adopted in the present work are from Refs. 13 and 18: $m^*=0.067m_0$ (m_0 is the bald electron mass), $\rho_s=5\times10^{24}$ m^{-3}, and $T=0.14$ ps.

In order to better visualize the QW systems under consideration, when ω_0 is kept at 3.6×10^{14} s^{-1}, the first four energy eigenvalues and their intervals for semiparabolic QW and parabolic QW were listed in Table I, and the first three eigenfunctions for the two systems were plotted in Fig. 1. From Table I, it can be observed that, with the increase of F, all the energy eigenvalues E_i ($i=1,2,3,4$) in the two systems decrease monotonously, but the variations of the energy intervals ΔE_i between adjoining energy levels in the two systems are obviously different. ΔE_i in semiparabolic QW decreases monotonely as F increases, while ΔE_i in parabolic QW is kept unchanged with the increase of F. It also can be noted that, when electric field is absent $(F=0)$, the energy intervals ΔE_i in both QW systems are the same, which is not occasional. It is well known that, when $F=0$, the energy levels in parabolic QW can be expressed by $E_n=\hbar\omega_S(n+\frac{1}{2})$ via Eq. (8) (Ref. 23), and the expression of the energy

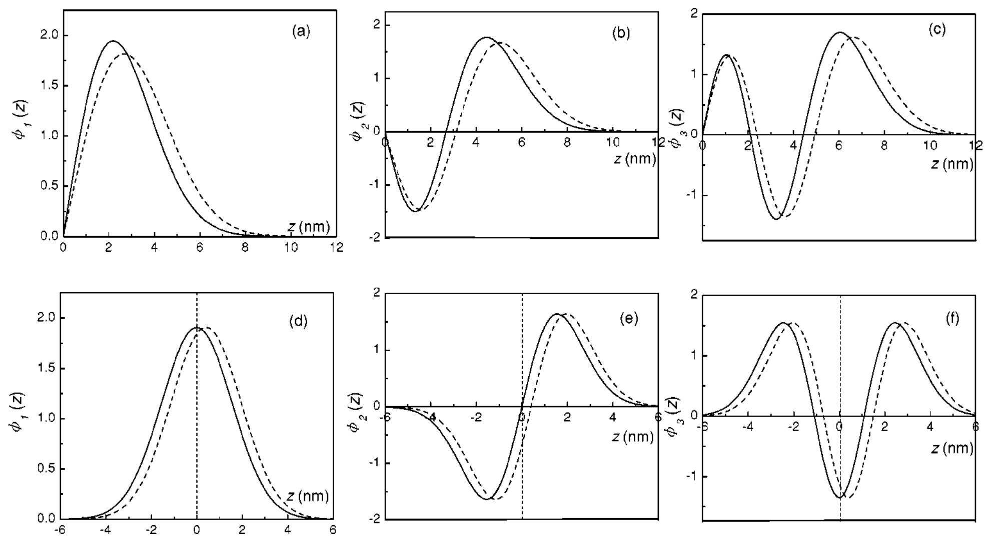

FIG. 1. The first three wave functions in semiparabolic QW and parabolic QW along with the confining potential. Panels (a), (b), and (c) correspond to the first three states in semiparabolic QW, respectively. Panels (d), (e), and (f) correspond to the first three states in parabolic QW, respectively. The solid lines denote the wave functions without electric field and the dashed lines denote the wave functions with the electric field of $F=8\times10^7$ V/m.

levels in semiparabolic QW becomes $E_n=\hbar\omega_A(2n+\frac{3}{2})$ (Ref. 24) ($n=0,1,2\ldots$), thus the energy intervals in the two systems have the same values $2\hbar\omega_A$ or $\hbar\omega_S$ (note $\omega_S=2\omega_A$). Comparing the corresponding energy levels in these two systems, we found that the energy level in semiparabolic QW is higher than the corresponding energy level in parabolic QW, which is because the confinement for the electron in the semiparabolic QW is stronger than that in parabolic QW. Detailed calculation denotes that, when $F\rightarrow\infty$, ΔE_i ($i=1,2,3$) of semiparabolic QW with applied electric field approaches $\hbar\omega_0$. This characteristic for ΔE_n in semiparabolic QW can be explained reasonably as follows. It is well known that, in a symmetric parabolic QW with or without applied electric field, all the intervals of the adjoining energy levels are $\hbar\omega_0$.[23] Therefore, it can be deduced that, when F approaches infinity, the potential function $V(z)+qFz$ of semiparabolic QW more and more approaches that in symmetric parabolic QW, and only the symmetrical axis of semiparabolic QW with electric field moves from the position $z=0$ to $|qF/m^*\omega_0^2|$, and the potential bottom of the semiparabolic QW decreases from 0 to $-q^2F^2/2m^*\omega_0^2$. Thus, the intervals ΔE_i in semiparabolic QW with strong electric fields approach those in symmetrical parabolic QW, namely, the intervals approach $\hbar\omega_0$.

Figures 1(a)–1(c) are for the wave functions of the ground state, the first excited state, and the second excited state in semiparabolic QW, while Figs. 1(d)–1(f) are for the wave functions of the first three states in parabolic QW, respectively. In the six figures, the solid lines denote the wave functions without electric field and the dashed lines denote the wave functions with the electric field of $F=8\times10^7$ V/m. Due to the infinite barrier at $z=0$ for semiparabolic QW, their wave functions are zero at the origin. The applied electric field make each wave function an obvious right-shift, which is a reasonable result.

The SHG susceptibility peak value $\chi^{(2)}_{2\omega,max}$ as functions of the eigenenergy of ground states E_1 for semiparabolic QW and parabolic QW are depicted in Figs. 2(a) and 2(b), respectively. In fact, in Fig. 2, the confined potential frequency ω_0 is kept at $3.6\times10^{14}\ \mathrm{s}^{-1}$, and F varies from 0 to 1×10^8 V/m. From Fig. 2 it can be seen that both curves decrease monotonously with the increase of the ground-state energy, which means that the SHG susceptibility increases with the increase of the applied electric field. It can also be seen that, in the energy ranges of the two systems under consideration, the SHG susceptibility in semiparabolic QW is much larger than that in parabolic QW. This feature is due to the self-asymmetry for the semiparabolic QW, and it is just the prospective result.

Figures 3 and 4 depict the SHG susceptibility $|\chi^{(2)}_{2\omega}|$ as

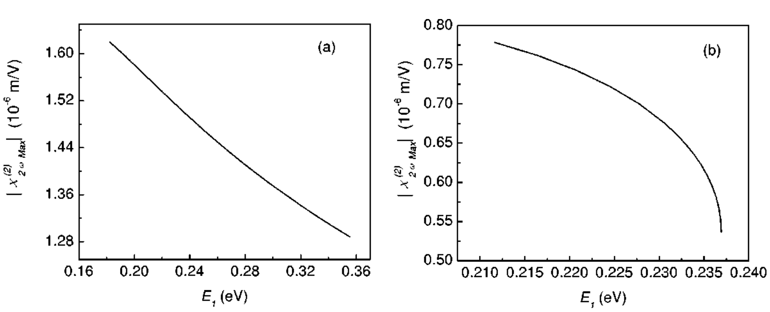

FIG. 2. The SHG susceptibility peak value $\chi^{(2)}_{2\omega,Max}$ as functions of the eigenenergy of ground states E_1 for semiparabolic QW (a) and parabolic QW (b). The confined-potential frequency ω_0 is kept at $3.6\times10^{14}\ \mathrm{s}^{-1}$ and F varies from 0 to 1×10^8 V/m.

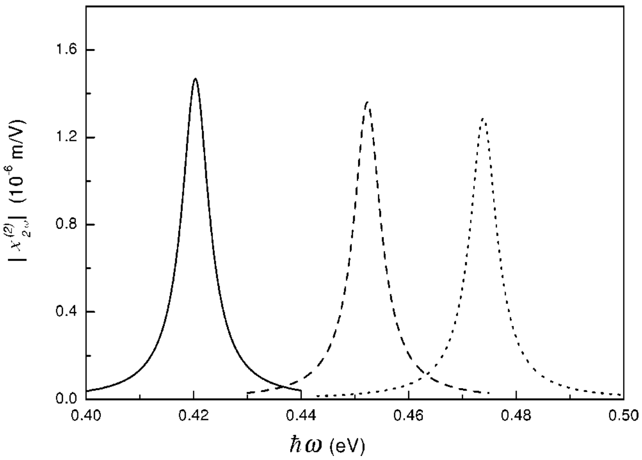

FIG. 3. The SHG susceptibility $|\chi^{(2)}_{2\omega}|$ as functions of the photon energy $\hbar\omega$ for semiparabolic QW when the confined-potential frequency ω_0 kept at 3.6×10^{14} s^{-1}. The solid line, dashed line, and dotted line correspond to the electric field strengths $F=5\times10^7$ V/m, $F=2\times10^7$ V/m, and $F=0$, respectively.

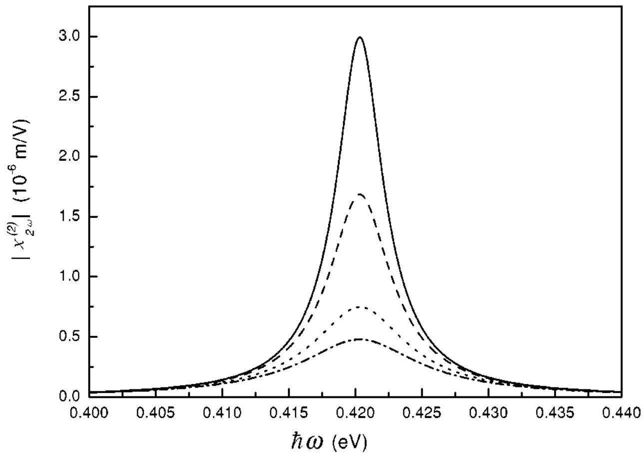

FIG. 5. The SHG susceptibility $|\chi^{(2)}_{2\omega}|$ as functions of the photon energy $\hbar\omega$ for four different relaxation times $T=0.2$ ps (solid line), 0.15 ps (dashed line), 0.1 ps (dotted line), and 0.08 ps (dashed-dotted line) with $\omega_0=3.6\times10^{14}$ s^{-1} and $F=1.5\times10^7$ V/m in semiparabolic QW.

functions of the photon energy $\hbar\omega$ for semiparabolic QW and parabolic QW, respectively. The confined potential frequency ω_0 is kept at 3.6×10^{14} s^{-1}. In the two figures, solid line, dashed line, and dotted line correspond to the electric field strengths $F=5\times10^7$ V/m, $F=2\times10^7$ V/m, and $F=0$, respectively. It is observed that each curve of the SHG susceptibility dependent on the photon energy reveals a single resonant peak. The stronger the electric fields are, the sharper the resonant peak will be and the bigger the peak intensity will be. The most obvious difference between Figs. 3 and 4 is that, corresponding to the three different electric field strengths 5×10^7 V/m, 2×10^7 V/m, and $F=0$, the resonant peaks for semiparabolic QW appear at three different values of photon energy, $\hbar\omega=0.4204$ eV, 0.4523 eV, and 0.4739 eV, while the three resonant peaks for parabolic QW appear at the same photon energy 0.4739 eV. This feature is because, as F increases, for the semiparabolic QW, the intervals of adjoining energy levels become narrower and narrower, which results in the redshifts of resonant peak with the creasing of F; but for the parabolic QW, the intervals of adjoining energy levels are kept unchanged. These results can also be deduced directly from the discussions of the electronic states, or be observed directly from the Table I.

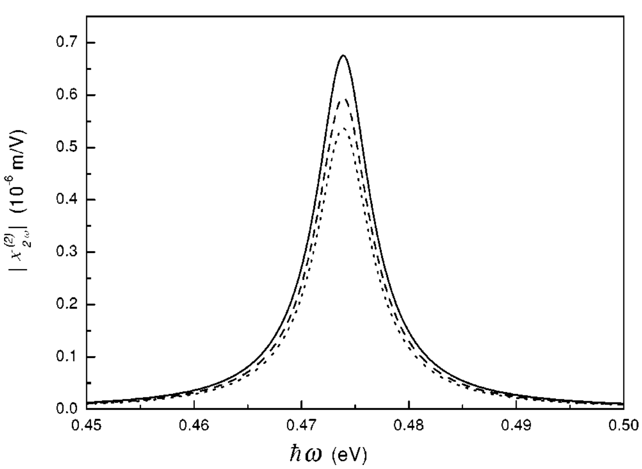

FIG. 4. The SHG susceptibility $|\chi^{(2)}_{2\omega}|$ as functions of the photon energy $\hbar\omega$ for parabolic QW when the confined-potential frequency ω_0 kept at 3.6×10^{14} s^{-1}. The solid line, dashed line and dotted line are corresponding to the electric field strengths $F=5\times10^7$ V/m, $F=2\times10^7$ V/m, and $F=0$, respectively.

In Fig. 5, we show the SHG susceptibility $|\chi^{(2)}_{2\omega}|$ as functions of the photon energy $\hbar\omega$ for four different relaxation times $T=0.2$ ps, 0.15 ps, 0.1 ps, and 0.08 ps, with $\omega_0=3.6\times10^{14}$ s^{-1} and $F=1.5\times10^7$ V/m in semiparabolic QW, which are shown by the solid line, dashed line, dotted line, and dash-dotted line, respectively. The relaxation rate Γ_0 is the inverse of relaxation time T. From the figure, we observed that the relaxation time T has a great influence on the SHG susceptibility $|\chi^{(2)}_{2\omega}|$, namely, with the increase of relaxation rate Γ_0, or the decrease of the relaxation time T, the SHG susceptibilities $|\chi^{(2)}_{2\omega}|$ decrease obviously. On the other hand, the relaxation rate is related not only to the materials constituting the QW, but also to some other factors, such as the temperature of the system, boundary conditions, and the electron-impurity and electron-phonon scattering interactions, etc. Hence, in order to obtain a large SHG susceptibility, one should reduce the influences of these factors on the systems.

IV. SUMMARY

In conclusion, by using the compact-density-matrix approach, the SHG susceptibility in a parabolic and semiparabolic QW with applied fields have been deduced and investigated in detail. Before studying the nonlinearity, the exact and analytical electronic states in both QW systems with applied electric fields have been deduced by the methods of displacement harmonic oscillation. Numerical calculation on the typical $Al_xGa_{1-x}As/GaAs$ QW's are performed; results show that the applied electric fields have an important influence on the electronic energy levels, for example, with the increase of electric field from 0 to ∞, the energy intervals of the adjoining energy levels in semiparabolic QW's decrease from $2\hbar\omega_0$ to $\hbar\omega_0$, and the reason for these characters has been explained. Calculation found that, for the same effective widths of the parabolic and semiparabolic QW, the SHG susceptibility in semiparabolic QW is larger than that in parabolic QW due to the self-asymmetry of the semiparabolic QW, which means that the semiparabolic QW is a model of very promising candidates for second-order nonlinear optical properties. In addition, the applied electric field can make the SHG susceptibility enhance remarkably. For example, when F varies from 0 to 6×10^7 V/m, the SHG susceptibility in semiparabolic QW have 25.7% enhancement for the effect of the electric field. Moreover, the result calculated reveals that the SHG susceptibility is also related to the relaxation rate. As we know, with the recent advances in nanofabrication technology, the manufactures of such semiconductor parabolic or semiparabolic QW's become possible. According to our calculations, if we choose an optimized parabolic confinement frequency and an appropriate electric field, we can obtain a large SHG susceptibility. Therefore, theoretical study may make a great contribution to experimental studies. We hope that this paper being helpful in the study of the influence of the QW's potential shape and electric field on the second-order nonlinear optical properties will stimulate more experimental work.

ACKNOWLEDGMENT

The project was financially supported by Guangdong Provincial Natural Science Foundation of China.

*Corresponding author. Email address: zhangli-gz@263.net

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