# Impurity effects on the $A_1$-$A_2$ splitting of superfluid $^3$He in aerogel

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When liquid $^3$He is impregnated into silica aerogel a solid-like layer of $^3$He atoms coats the silica structure. The surface $^3$He is in fast exchange with the liquid on NMR time scales. The exchange coupling of liquid $^3$He quasiparticles with the localized $^3$He spins modifies the scattering of $^3$He quasiparticles by the aerogel structure. In a magnetic field the polarization of the solid spins gives rise to a splitting of the scattering cross section of for ''up'' vs ''down'' spin quasiparticles, relative to the polarization of the solid $^3$He. We discuss this effect, as well as the effects of nonmagnetic scattering, in the context of a possible splitting of the superfluid transition for $\uparrow\uparrow$ vs $\downarrow\downarrow$ Cooper pairs for superfluid $^3$He in aerogel, analogous to the $A_1$-$A_2$ splitting in bulk $^3$He. Comparison with the existing measurements of $T_c$ for $B<5$ kG, which shows no evidence of the $A_1$-$A_2$ splitting, suggests a liquid-solid exchange coupling of order $J\simeq 0.1$ mK. Measurements at higher fields, $B \gtrsim 20$ kG, should saturate the polarization of the solid $^3$He and reveal the $A_1$-$A_2$ splitting.

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One focus of experimental investigations of $^3$He in aerogel has been the determination of the phase diagram. Torsional oscillator, NMR, vibrating wire, and sound attenuation experiments on $^3$He in $\approx 98\%$ porosity aerogels suggest that there is just one superfluid phase for pressures above the critical pressure of 6 atm in zero magnetic field.[1–3] At lower pressures there appears to be only normal $^3$He down to zero temperature.[4] In short, a *B*-like phase, with reduced susceptibility, is the only stable superfluid phase observed in zero field. An *A*-like phase, if it exists in zero field, may be stable only in a very small region $\Delta T \leqslant 20\ \mu$K near $T_c$. As in bulk $^3$He, an *A*-like phase can be stabilized in a magnetic field; the region of stability, $T_{\rm AB} \leqslant T < T_c$, increases quadratically with field, $T_c - T_{\rm AB} = -g_{\rm AB} B^2$. However, a splitting of the transition for $\uparrow\uparrow$ and $\downarrow\downarrow$ pairs, analogous to the $A_1$-$A_2$ splitting in bulk $^3$He, has so far not been observed for fields up to $B \approx 5$ kG.[3,5]

In pure $^3$He, on application of a magnetic field, the $A_1$ phase, characterized by spin-polarized Cooper pairs composed of only $\uparrow\uparrow$ spins, nucleates at a temperature slightly higher than the zero-field transition, $T_c^{A_1} = T_c + \lambda^{A_1} B$.[29] This transition is followed by a second transition, shifted below the zero-field transition, at $T_c^{A_2} = T_c - \lambda^{A_2} B$, in which the $\downarrow\downarrow$ pairs nucleate. The region of stability of pure $\uparrow\uparrow$ pairs increases linearly with field, $\Delta T_{A_1-A_2} \equiv (T_c^{A_1} - T_c^{A_2}) = (\lambda^{A_1} + \lambda^{A_2}) B$, at a rate of $\lambda^{A_1} + \lambda^{A_2} \simeq 6.1\ \mu$K/kG at $p = 33.4$ bars.[6,7]

This splitting of the zero-field transition originates from the combined effect of the nuclear Zeeman coupling to the $^3$He spin, and particle-hole asymmetry in the normal-state density of states and pairing interaction. The original estimate of $\lambda^{A_1}$ by Ambegaokar and Mermin[8] was based on the asymmetry of the density of states for $\uparrow$ vs $\downarrow$ quasiparticles at the Fermi level. More involved calculations include the effects of spin polarization of the Fermi liquid on the pairing interaction.[9,10] Estimates of the splitting of the $A_1$ transition are of order

$$\lambda^{A_1} = \Lambda \left| \frac{\gamma\hbar}{2} \right| \left( \frac{k_B T_c}{E_f} \right), \tag{1}$$

where $\gamma$ is the gyromagnetic ratio for $^3$He, $k_B T_c / E_f \sim 10^{-3}$, and $\Lambda \sim \mathcal{O}(1-10)$ is a ''high-energy'' vertex. A first principles calculation of $\Lambda$ requires a solution of the many-body problem for the pairing interaction in $^3$He. Alternatively, we treat $\Lambda$ on the same level as other high-energy vertices in Fermi-liquid theory; $\Lambda$ is a Fermi-liquid parameter, which can be determined by comparing physical predictions of the Fermi-liquid theory with experiment—in this case the $A_1$-$A$ splitting. Once $\Lambda$ is determined, its effects on other low-energy properties of $^3$He can be calculated. Thus, the $A_2$ transition can be calculated in terms of $\Lambda$, and corrections to $T_c^{A_2}$ from the $\uparrow\uparrow$ pair condensate.[11]

The disorder introduced by the aerogel structure into liquid $^3$He is, on average, weak on the high-energy scale, $\hbar/\tau E_f \ll 1$. Thus, the Fermi-liquid interactions are essentially unaffected by the aerogel, and we can calculate the effects of aerogel on the low-energy excitations and superfluid properties within Fermi-liquid theory. The main effect of the aerogel structure is to scatter $^3$He quasiparticles moving with the Fermi velocity. At temperatures below $T^* \approx 10$ mK elastic scattering by the aerogel dominates inelastic quasiparticle-quasiparticle collisions.[12] This limits the mean free path of normal $^3$He quasiparticles to $\ell \simeq 130$–$180$ nm. In *p*-wave superfluids quasiparticle scattering is intrinsically pairbreaking and leads to renormalization of nearly all properties of the superfluid phases, including the $A_1$ and $A_2$ transition temperatures. The suppression of $T_c$, as well as pairbreaking effects on observable properties of the superfluid phases, have been analyzed theoretically for nonmagnetic scattering.[13–18] Here we analyze the effects of scattering by the aerogel on the $A_1$ and $A_2$ transitions.

The aerogel has a strong effect on the *short-distance*, high-energy properties of the liquid locally near the silica

 

strands. The first few atomic layers of $^3$He are adsorbed on the silica structure and form a highly polarizable solidlike phase, observable as a Curie-like component of the magnetization of $^3$He aerogel.[19] The surface $^3$He is in fast exchange with the liquid on typical NMR time scales, implying a liquid-solid exchange interaction, $|J|/h \gtrsim 0.66$ MHz ($|J| \gtrsim 0.03$ mK).[19,20] The exchange coupling of liquid $^3$He quasiparticles with the localized $^3$He spins $J$ may modify the scattering of $^3$He quasiparticles by the aerogel structure.[21] Here we include the effect of magnetic scattering of $^3$He quasiparticles by polarizable $^3$He spins coating the aerogel strands. The differential scattering of $\uparrow$ vs $\downarrow$ spin quasiparticles by the polarized surface leads to an additional contribution to the splitting of the $\uparrow\uparrow$ vs $\downarrow\downarrow$ transitions, $\lambda_J \propto J$, which is determined by the nonmagnetic $u_0$ and exchange $J$ interactions and the density of $^3$He coating the aerogel.[22] Below we extend the analysis of Ref. 22 and examine the role of the exchange coupling on the possible $A_1$-$A_2$ splitting of the superfluid phases of $^3$He in aerogel.

The suppression of the AB transition in both pure and disordered $^3$He is quadratic in field on a scale set by $g_{\rm AB} \sim$ mK/kG$^2$.[3] Thus, for fields $B \gg B^* = \lambda^{A_1}/g_{\rm AB} \sim 1$ G and temperatures $T \approx T_c$, the $\uparrow\downarrow$ pairs are suppressed. In the field and temperature range of interest we can restrict the full $p$-wave, spin-triplet order parameter to two components, $\mathsf{d}_+$ for $\uparrow\uparrow$ pairs and $\mathsf{d}_-$ for $\downarrow\downarrow$ pairs. We assume that the orbital state is the same for both spin components and the axial (ABM) form, $\chi(\hat{\mathbf{p}}) = \hat{\mathbf{p}}\cdot(\hat{\mathbf{m}} + i\hat{\mathbf{n}})/\sqrt{2}$. Thus, in pure $^3$He the full Ginzburg-Landau (GL) free-energy functional (c.f. Ref. 23) reduces to[11]

$$\Omega[\mathsf{d}_+,\mathsf{d}_-] = \alpha(|\mathsf{d}_+|^2 + |\mathsf{d}_-|^2) - \eta B(|\mathsf{d}_+|^2 - |\mathsf{d}_-|^2) + \beta_{24}(|\mathsf{d}_+|^2 + |\mathsf{d}_-|^2)^2 + 4\beta_5|\mathsf{d}_+|^2|\mathsf{d}_-|^2, \tag{2}$$

where $\alpha$, $\eta$, and $\beta_{\rm i}$ are the known material parameters for superfluid $^3$He; $\alpha(T) = (1/3)N_f \ln(T/T_c)$ determines the zero-field transition and $\eta = (1/3)N_f \lambda^{A_1}/T_c$ determines the $A_1$ ($\uparrow\uparrow$) transition, where $N_f$ is the single-spin density of states at the Fermi level. The fourth-order coefficients determine the relative stability of the possible phases. In particular, $\beta_{24} > 0$ and $\beta_5 < 0$ favor an equal-spin-pairing (ESP) phase with $|\mathsf{d}_+| = |\mathsf{d}_-|$. The linear field term is symmetry breaking and competes with the fourth-order terms. The latter wins at lower temperatures and gives rise to the $A_2$ transition where $\downarrow\downarrow$ spins condense with $T_c^{A_2} = T_c - \lambda^{A_2} B$ and

$$\lambda^{A_2} = \left(\frac{\beta_{245}}{-\beta_5}\right)\lambda^{A_1}. \tag{3}$$

Within the homogeneous, isotropic scattering model (HSM) the rotational symmetry of $^3$He in aerogel is preserved on the coherence length scale, and the GL free energy has the same form as in pure $^3$He, but with material parameters, $\overline{\alpha}$, $\overline{\eta}$, etc., that are modified by the effects of scattering by the aerogel (we use a "bar" to denote the material parameters in the presence of aerogel scattering). These effects were calculated within the quasiclassical theory to leading order in $T_c/E_f$ (weak coupling), and one finds[13]

$$\overline{\alpha} = \tfrac{1}{3}N_f[\ln(T/T_{c0}) - 2\,S_1(x)], \tag{4}$$

where $x = v_f/2\pi T\ell$ and $\ell$ is the mean free path of quasiparticles scattering off the aerogel. In Eq. (4), and hereafter, we denote the transition temperature for pure $^3$He by $T_{c0}$. The superfluid transition in aerogel is determined by the condition $\overline{\alpha}(T_c) = 0$ and

$$S_1(x) = \sum_{n=0}^{\infty}\left(\frac{1}{2n+1+x} - \frac{1}{2n+1}\right). \tag{5}$$

The parameter $\eta$ is directly proportional to the high-energy vertex $\Lambda$ and so is unrenormalized by impurities to leading order in $\hbar/p_f\ell$. Thus, $\overline{\eta} = \eta = (1/3)N_f\lambda^{A_1}/T_{c0}$, where $\lambda^{A_1}$ is the rate for the splitting of the $A_1$-$A$ transition in pure $^3$He. Although $\eta$ is unrenormalized, the splitting parameters for the $A_1$ and $A_2$ transitions are renormalized by the impurity corrections to transition temperature and, in general, by the $\beta$ parameters.

The weak-coupling results for the fourth-order coefficients are,[13]

$$\overline{\beta}_{24} = -2\overline{\beta}_5 = 4(\overline{\beta}^{\rm wc} + \overline{\beta}^{\overline{\sigma}}), \tag{6}$$

$$\overline{\beta}^{\rm wc} = \frac{N_f}{30\pi^2T^2}S_3(x),$$

$$\overline{\beta}^{\overline{\sigma}} = \frac{N_f}{9\pi^2T^2}(\overline{\sigma}_0 - \tfrac{1}{2})x\,S_4(x), \tag{7}$$

where $\overline{\sigma}_0$ is the dimensionless, nonmagnetic, $s$-wave scattering cross section, $0 < \overline{\sigma}_0 < 1$ (see below), and

$$S_p(x) = \sum_{n=0}^{\infty}\left(\frac{1}{2n+1+x}\right)^p, \quad p > 1\,. \tag{8}$$

Note that the ratio $\overline{r}_\beta = \overline{\beta}_{245}/(-\overline{\beta}_5) \equiv 1$ in the weak-coupling limit, even with (isotropic) impurity scattering. However, this ratio deviates substantially from 1 in pure $^3$He, particularly at high pressures, e.g., $r_\beta \approx 0.47$ at $p = 33.4$ bars. Thus, the asymmetry of the $A_1$-$A$ vs the $A_2$-$A$ transitions is a measure of the strong-coupling corrections to the $\beta$ parameters, $\delta\beta^{\rm sc} = \beta - \beta^{\rm wc}$, which are of order

$$\frac{\delta\beta^{\rm sc}}{\beta^{\rm wc}} \sim \frac{T_c}{E_f}\langle|\Gamma_{\rm N}|^2\rangle_{\rm FS}, \tag{9}$$

compared to the weak-coupling values, where $\langle\cdots\rangle_{\rm FS}$ is a Fermi-surface average of the normal-state quasiparticle-quasiparticle collision rate $\propto|\Gamma_{\rm N}|^2$.

Corrections to the weak-coupling $\beta$ parameters from quasiparticle scattering off the aerogel strands are of the order $\delta\overline{\beta}^{\rm wc}/\beta^{\rm wc} \sim x_c = v_f/2\pi\ell T_c$, which is small for high-porosity aerogels, but comparable to the strong-coupling corrections

from quasiparticle-quasiparticle collisions. Based on the suppression of $T_c$ and the aerogel mean free path we estimate $x_c \simeq 0.12$ at high pressures.

If we neglect aerogel scattering corrections to the intermediate quasiparticle states that enter the strong-coupling self-energies,[24] then the relative strong-coupling corrections for $^3$He in aerogel are scaled relative to their bulk ratios by the ratio of transition temperatures,

$$\frac{\delta\overline{\beta}^{\rm sc}}{\overline{\beta}^{\rm wc}} = \frac{\delta\beta^{\rm sc}}{\beta^{\rm wc}}\left(\frac{T_c}{T_{c0}}\right). \tag{10}$$

This approximation gives a good qualitative description of the suppression of strong-coupling parameters for $^3$He in aerogel as measured by the field dependence of the AB transition.[3]

A theoretical calculation of the $\beta$ parameters that includes some of the effects of aerogel scattering on the intermediate states of the strong-coupling self-energies was carried out by Baramidze and Kharadze[16] within the spin-fluctuation feedback theory of Brinkman and Anderson. This calculation predicts a suppression of strong-coupling effects with increased disorder, but at a rate that is slower than that predicted just on the basis of the suppression of $T_c$. We can use the results of Ref. 16 to estimate the strong-coupling correction to the predicted $A_2$ transition for $^3$He in aerogel. The results of Ref. 16 depend on a high-energy vertex, which we determine by comparison with the magnitude of strong-coupling ratio $r_\beta = \beta_{245}/(-\beta_5)$ for pure $^3$He. In particular, we fix the ratio $\delta\beta^{\rm sc}/\beta^{\rm wc}$ for pure $^3$He, using the measured value of $r_\beta$: $(1/2)\,\delta\beta^{\rm sc}/\beta^{\rm wc} = (1-r_\beta)/(1+r_\beta)$. Then, the impurity corrections to the strong-coupling $\beta$ parameters calculated in Ref. 16 give $\overline{r}_\beta = [1-(1/2)\,\delta\overline{\beta}^{\rm sc}/\overline{\beta}^{\rm wc}]/[1+(1/2)\,\delta\overline{\beta}^{\rm sc}/\overline{\beta}^{\rm wc}]$ where[30]

$$\frac{\delta\overline{\beta}^{\rm sc}}{\overline{\beta}^{\rm wc}} = \frac{\delta\beta^{\rm sc}}{\beta^{\rm wc}}\left(\frac{T_c}{T_{c0}}\right)\left(\frac{S_2(x_c)/S_2(0)}{S_3(x_c)/S_3(0)}\right). \tag{11}$$

To leading order in the pair-breaking parameter, $\delta\overline{\beta}^{\rm sc}/\overline{\beta}^{\rm wc} \approx (\delta\beta^{\rm sc}/\beta^{\rm wc})(1-a\,x_c)$. Based on Eq. (10), $a \simeq 2.47$. The rate of suppression is reduced to $a \simeq 1.28$ based on Eq. (11). In what follows we use Eq. (11) to estimate the suppression of the strong-coupling correction for $T_c^{A_2}$. This correction turns out to be small and relatively unimportant on the scale of corrections that are required to explain the lack of the $A_1$-$A_2$ splitting for $B \leqslant 5$ kG.

Finally, before discussing the effects of the liquid-solid exchange coupling, we consider a simplified version of an *inhomogeneous* scattering model discussed in Ref. 13 that incorporates correlations of the aerogel. The length scale at which aerogel reveals inhomogeneity, $\xi_a \sim 30$–$100$ nm, is typically comparable to the pair correlation length, $\xi$, and has a substantial effect on the transition temperature of $^3$He in aerogel, particularly at high pressures. The inhomogeneity of the aerogel on scales $\xi_a \sim \xi$ leads to higher superfluid transition temperatures than predicted by the HSM with the same quasiparticle mean free path. Regions of lower aerogel density, of size of order $\xi_a$, are available for formation of the

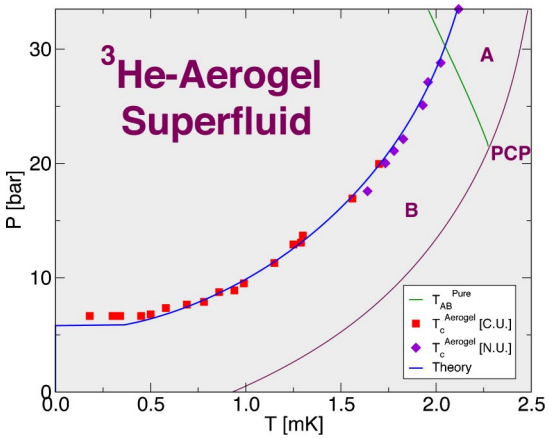


FIG. 1. (Color online) The phase diagram for $^3$He in 98% aerogel. The data are from Refs. 4 and 25. The theoretical curve is calculated from $\overline{\alpha}(T_c) = 0$ using Eq. (4) in zero field with the effective pair-breaking parameter $\tilde{x}$ evaluated with $\xi_a = 502$ Å and $\ell = 1400$ Å. The phase boundaries for pure $^3$He are shown for comparison.

condensate. Thus, the qualitative picture is that of a random distribution of low density regions, ''voids,'' with a typical length scale $\xi_a$ in an aerogel with a quasiparticle mean free path, $\ell$. When $\xi \sim \xi_a \ll \ell$, the superfluid transition is determined by the pairbreaking effects of dense regions surrounding the voids, and scales as $\delta T_c/T_{c0} \propto -(\xi/\xi_a)^2$. However, when the pair size is much larger than $\xi_a$ the aerogel is effectively homogeneous on the scale of the pairs and pair-breaking results from homogeneous scattering defined by the transport mean free path, which scales as $\delta T_c/T_{c0} \propto -(\xi/\ell)$. This latter limit is achieved at low pressures. We incorporate the correlation effect by introducing an effective pairbreaking parameter in Eq. (4) that interpolates between these two limits, $x \to \tilde{x} = x/(1+\zeta_a^2/x)$, where $\zeta_a \equiv \xi_a/\ell$. This heuristic treatment of aerogel correlations provides a good description of the pressure dependence of $T_c$ in zero field for $^3$He in aerogel over the whole pressure range, as shown in Fig. 1 for $\ell = 1400$ Å and $\xi_a = 502$ Å. Alternatively, we can adjust the mean free path $\ell$ with pressure in order to simulate the correlation effect on $T_c$. However, we prefer to identify $\ell$ with the pressure-independent geometric mean free path and introduce aerogel correlation effects via the effective pair-breaking parameter $\tilde{x}$. In either scenario, the GL theory for $^3$He in aerogel predicts transitions for the $A_1$ and $A_2$ phases, which correspond to the condensation of $\uparrow\uparrow$ and $\downarrow\downarrow$ Cooper pairs as in pure $^3$He; the transition temperatures are of the same form,

$$T_c^{A_1} = T_c + \overline{\lambda}^{A_1} B, \quad T_c^{A_2} = T_c - \overline{\lambda}^{A_2} B, \tag{12}$$

but with renormalized parameters,

$$\overline{\lambda}^{A_1} = \lambda^{A_1}\left(\frac{T_c}{T_{c0}}\right)[1 + 2\tilde{x}_c'\,S_2(\tilde{x}_c)], \quad \overline{\lambda}^{A_2} = \overline{r}_\beta \overline{\lambda}^{A_1}, \tag{13}$$

where $\tilde{x}_c' \equiv T_c d\tilde{x}_c/dT_c$, and $\overline{r}_\beta = \overline{\beta}_{245}/(-\overline{\beta}_5)$ is calculated including both impurity scattering and strong-coupling corrections as described above. These results predict the $A_1$-$A_2$

splitting of $\Delta T_c^{A_1-A_2}/B=(\overline{\lambda}^{A_1}+\overline{\lambda}^{A_2})\simeq 6.3\ \mu$K/kG at $p=33.4$ bars, comparable to that of pure $^3$He. There is currently no experimental evidence of the $A_1$-$A_2$ splitting in $^3$He aerogel. Since the width of the transition is less than 20 $\mu$K, inhomogeneities within the aerogel cannot account for the absence of the $A_1$-$A_2$ splitting.

For $^3$He in aerogel an additional mechanism contributing to the splitting of the $\uparrow\uparrow$ and $\downarrow\downarrow$ transitions is possible.[22] It originates from an exchange coupling between liquid $^3$He quasiparticles and the surface $^3$He spins adsorbed on the silica structure. Such a surface solid of $^3$He has been observed for $^3$He impregnated into silica aerogel. The signature is a Curie-like susceptibility $\chi_S=C/(T-\Theta_S)$ with a Curie temperature, $\Theta_S\approx 0.4$ mK.[21]

Thus, the model for scattering of quasiparticles by aerogel that we adopt is a modified version of the scattering model described above which includes an exchange coupling between $^3$He quasiparticles in the liquid and localized $^3$He atoms bound to the silica aerogel structure. This coupling is described by an exchange term in the quasiparticle-impurity potential,

$$u=u_0+J\mathbf{S}\cdot\boldsymbol{\sigma}, \tag{14}$$

where $J$ is the liquid-solid exchange coupling, $\mathbf{S}$ is the localized $^3$He spin operator, and $\boldsymbol{\sigma}$ is the Pauli spin operator for the $^3$He quasiparticles. There are no direct measurements of $J$ for $^3$He on aerogel, and theoretical calculations for $^3$He on planar substrates give indirect exchange interactions that vary over a wide range of values, $J_{\rm ind}\sim 0.1\ \mu$K–1.0 mK, and may be either ferromagnetic or antiferromagnetic depending on the specific mechanism and details of the theoretical model (cf. Ref. 26).

In a magnetic field $\mathbf{B}=-B\hat{\mathbf{z}}$ the solid $^3$He spins are polarized, $\mathbf{S}=S(T,B)\hat{\mathbf{z}}$, with $S(T,B)=\mathcal{P}(B,T)\mathsf{s}$, where $0\leqslant\mathcal{P}\leqslant 1$ is the fractional polarization and $\mathsf{s}=(1/2)$. For sufficiently low fields, and temperatures well above the ordering temperature for the solid $^3$He spins, the polarization is expected to be linear in field with $\mathcal{P}(B,T)\approx|\mu|B/k_BT$. In this limit the $A_1$-$A_2$ splitting is given by Eqs. (12) and (13), but with $\overline{\lambda}^{A_1}\to\overline{\lambda}^{A_1}+\lambda_J$, where $\lambda_J$ represents the effect of the surface polarization and exchange coupling on the transition temperatures for $\uparrow\uparrow$ vs $\downarrow\downarrow$ pairs. The polarization-induced splitting $\lambda_J\propto J$ depends on the sign of the exchange coupling. Thus, this term may either enhance or reduce the intrinsic splitting $\overline{\lambda}^{A_1}$. In what follows we calculate the exchange splitting $\lambda_J$ and discuss the result in relation to the existing data for $T_c$.

To calculate the liquid-solid exchange contribution to the $A_1$-$A_2$ splitting we use the quasiclassical theory of superfluid $^3$He,[27] with effects of scattering by the aerogel described by the HSM,[13] modified to include the exchange coupling in Eq. (14). The instability temperatures for $\uparrow\uparrow$ and $\downarrow\downarrow$ Cooper pairs are obtained by solving the weak-coupling gap equation for the spin-triplet components of the order parameter,

$$\frac{1}{3}\ln(T/T_{c0})\boldsymbol{\Delta}(\hat{\mathbf{p}})=T\sum_{\varepsilon_n}\int\frac{d\Omega_{\hat{\mathbf{p}}'}}{4\pi}(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')\times\left(\mathbf{f}(\hat{\mathbf{p}}';\varepsilon_n)-\pi\frac{\boldsymbol{\Delta}(\hat{\mathbf{p}}')}{|\varepsilon_n|}\right), \tag{15}$$

where $\mathbf{f}(\hat{\mathbf{p}};\varepsilon_n)=(f_1,f_2,f_3)$ are the "pair" amplitudes for the three spin-triplet states: $f_{\uparrow\downarrow}=f_3$, $f_{\uparrow\uparrow}=(-f_1+if_2)$, and $f_{\downarrow\downarrow}=(f_1+if_2)$. The pairing interaction and density of states at the Fermi level, as well as the cutoff, have already been adsorbed into $T_{c0}$.

The scattering of quasiparticles off the aerogel structure is described by a random distribution of scattering centers ("impurities"). The impurity self-energy, to leading order in $\hbar/\tau E_f$, is determined by a $t$ matrix for multiple scattering by a single impurity and the mean density of impurities,

$$\hat{\boldsymbol{\Sigma}}_{\rm imp}(\hat{\mathbf{p}};\varepsilon_n)=n_s\hat{t}(\hat{\mathbf{p}},\hat{\mathbf{p}};\varepsilon_n). \tag{16}$$

The model for scattering of quasiparticles by aerogel that we adopt is described by an isotropic, nonmagnetic scattering amplitude $u_0$ and an exchange term in the quasiparticle-impurity scattering potential; in $4\times 4$ Nambu representation $\hat{u}=u_0\hat{1}+JS\,\hat{\boldsymbol{\Sigma}}_z$, where $\hat{\boldsymbol{\Sigma}}=(\hat{1}+\hat{\tau}_3)\boldsymbol{\sigma}/2+(\hat{1}-\hat{\tau}_3)\boldsymbol{\sigma}^{\rm tr}/2$ is the Nambu representation for the quasiparticle spin. For simplicity we also assume $J$ to be isotropic. The $t$ matrix for repeated scattering of quasiparticles off a random distribution of these polarized scattering centers is

$$\hat{t}=\hat{u}+N_f\hat{u}\langle\hat{g}\rangle\hat{t}, \tag{17}$$

where $\langle\hat{g}\rangle$ is the Fermi-surface-averaged propagator. For normal $^3$He in aerogel and even in the presence of magnetic fields and magnetic scattering, the propagator reduces to $\hat{g}_N=-i\pi\,\mathrm{sgn}(\epsilon_n)\hat{\tau}_3$. Thus, the solution to the scattering $t$ matrix is given by

$$\hat{t}=\frac{1}{\pi N_f}(\hat{1}+is_\epsilon\hat{u}\hat{\tau}_3)^{-1}\hat{u}, \tag{18}$$

where $s_\epsilon=\mathrm{sgn}(\epsilon_n)$, and the dimensionless scattering potential is $\hat{u}=u\hat{1}+v\hat{\boldsymbol{\Sigma}}_z$, with $u=\pi N_f u_0$, $v=\pi N_f JS$.

For nonmagnetic scattering ($S=0$) the $t$ matrix is parametrized by the $s$-wave scattering phase shift, $\delta_0=\tan^{-1}(u)$,

$$\hat{t}=\frac{1}{\pi N_f}\sin\delta_0\,e^{-is_\epsilon\delta_0\hat{\tau}_3}. \tag{19}$$

In this minimal model for aerogel scattering, the mean density of impurities and scattering rate for normal quasiparticles are fixed by the mean free path $\ell$ and scattering cross section $\sigma$,

$$n_s=\frac{1}{\sigma\ell},\quad\text{with}\quad\sigma=\frac{4\pi\hbar^2}{p_f^2}\sigma_0, \tag{20}$$

where the normalized cross section is related to the scattering potential by

$$\overline{\sigma}_0 = \frac{u^2}{1+u^2}. \tag{21}$$

Note that $\overline{\sigma}_0 \to 0$ is the Born scattering limit, while $\overline{\sigma}_0 \to 1$ is the unitary limit.

When $S \neq 0$ there are different phase shifts for the scattering of $\uparrow$ $(+)$ and $\downarrow$ $(-)$ spin quasiparticles, which we parametrize as

$$\delta^{\pm} = \delta_0 \pm \Delta\delta. \tag{22}$$

The $t$ matrix can now be expressed as

$$\hat{t} = \frac{1}{\pi N_f}\{\sin\delta_0 \cos(\Delta\delta)\hat{1} + \cos\delta_0 \sin(\Delta\delta)\hat{\Sigma}_z\} \times e^{-is_\epsilon \delta_0 \hat{\tau}_3} e^{-is_\epsilon \Delta\delta \hat{\Sigma}_z \hat{\tau}_3}. \tag{23}$$

The quasiparticle-impurity scattering rates for $\uparrow$ and $\downarrow$ quasiparticles are calculated from the retarded self-energy $\hat{\Sigma}^{\mathrm{R}}_{\mathrm{imp}} = n_s \hat{t}^{\mathrm{R}}$, obtained from Eq. (23) by setting $s_\epsilon = +$. Thus, for quasiparticles the self-energy for spin $\sigma_z = \uparrow$ and $\sigma_z = \downarrow$ becomes

$$\Sigma^{\mathrm{R}}_{\uparrow,\downarrow} = \Gamma_{\mathrm{N}} \sin\delta_{\uparrow,\downarrow}(\cos\delta_{\uparrow,\downarrow} - i\sin\delta_{\uparrow,\downarrow}), \tag{24}$$

where $\Gamma_{\mathrm{N}} = n_s/\pi N_f$. The scattering rates for $\uparrow$ and $\downarrow$ spin quasiparticles are then

$$\frac{1}{2\tau_\pm} = \Gamma_{\mathrm{N}} \sin^2\delta^{\pm} = \Gamma_{\mathrm{N}} \overline{\sigma}_{\pm} = \Gamma_{\mathrm{N}} \frac{(u \pm v)^2}{1+(u \pm v)^2}, \tag{25}$$

where $\overline{\sigma}_{\pm}$ is the dimensionless cross section for scattering of $\uparrow$ vs $\downarrow$ spin quasiparticles. In both the unitary ($\delta_0 \to \pi/2$) and the Born ($\delta_0 \to 0$) limits, the $\uparrow$ and $\downarrow$ spin scattering rates are equivalent,

$$\frac{1}{2\tau_\pm} \to \begin{cases} \Gamma_{\mathrm{N}} \cos^2(\Delta\delta), & \delta_0 = \pi/2 \\ \Gamma_{\mathrm{N}} \sin^2(\Delta\delta), & \delta_0 = 0. \end{cases} \tag{26}$$

Only when $\delta_0 \neq 0, \pi$ is the scattering rate for $\uparrow$ and $\downarrow$ spin quasiparticles differ. In general, we can parametrize the scattering rates as

$$\frac{1}{\tau^{\pm}} = \frac{1}{\overline{\tau}} \pm \frac{1}{\tau_{\mathrm{S}}}, \tag{27}$$

or equivalently,

$$\frac{1}{\tau_{\mathrm{S}}} = \frac{1}{\overline{\tau}}\left(\frac{\overline{\sigma}_+ - \overline{\sigma}_-}{\overline{\sigma}_+ + \overline{\sigma}_-}\right), \tag{28}$$

where $1/\overline{\tau}$ is the polarization-independent scattering rate. It is convenient to express the normal-state self-energy in terms of base particle-hole matrices,

$$\hat{\Sigma}_{\mathrm{N}} = \Sigma_{11}\hat{1} + \Sigma_{13}\hat{\Sigma}_z + \Sigma_{31}\hat{\tau}_3 + \Sigma_{33}\hat{\tau}_3\hat{\Sigma}_z, \tag{29}$$

with components

$$\Sigma_{11} = +\tfrac{1}{2}\Gamma_{\mathrm{N}} \sin(2\delta_0)\cos(\Delta\delta), \tag{30}$$

$$\Sigma_{13} = +\tfrac{1}{2}\Gamma_{\mathrm{N}} \cos(2\delta_0)\sin(\Delta\delta), \tag{31}$$

$$\Sigma_{31} = -\frac{i}{2}\Gamma_{\mathrm{N}} s_\epsilon [1 - \cos(2\delta_0)\cos(2\Delta\delta)], \tag{32}$$

$$\Sigma_{33} = -\frac{i}{2}\Gamma_{\mathrm{N}} s_\epsilon \sin(2\delta_0)\sin(2\Delta\delta). \tag{33}$$

To calculate the instability temperatures for $\uparrow\uparrow$ and $\downarrow\downarrow$ pairs we need the off-diagonal propagator to linear order in the pairing self-energy. Thus, we expand the transport equation, self-energies, and normalization condition in powers of $\hat{\Delta}$. The zeroth-order terms are the normal-state propagator and self-energy [Eq. (29)]. To first order we obtain[27]

$$[i\varepsilon_n \hat{\tau}_3 - \hat{\Sigma}_{\mathrm{N}}, \hat{g}^{(1)}] = [\hat{\Delta}, \hat{g}_{\mathrm{N}}], \tag{34}$$

and $\hat{\tau}_3 \hat{g}^{(1)} + \hat{g}^{(1)} \hat{\tau}_3 = 0$ from the normalization condition. We reduce the equations to $2\times 2$ spin space by writing $\hat{H}_{\mathrm{N}} = i\epsilon_n \hat{\tau}_3 - \hat{\Sigma}_{\mathrm{N}} = (1/2)(\hat{1} + \hat{\tau}_3)H_{\mathrm{N}} + (1/2)(\hat{1} - \hat{\tau}_3)\overline{H}_{\mathrm{N}}$ with $H_{\mathrm{N}} = i\epsilon_n - \Sigma_{\mathrm{N}}$ and $\overline{H}_{\mathrm{N}} = -i\epsilon_n - \overline{\Sigma}_{\mathrm{N}}$. Note that $\hat{g}^{(1)}$ is purely off-diagonal with the upper-right pair amplitude satisfying the equation in spin space,

$$H_{\mathrm{N}} f^{(1)} - f^{(1)} \overline{H}_{\mathrm{N}} = 2i\pi\,\mathrm{sgn}(\epsilon_n)\Delta. \tag{35}$$

Projecting out the spin-triplet components, we obtain $(i\epsilon_n - \Sigma_\pm) f_\pm = i\pi\,\mathrm{sgn}(\epsilon_n)\Delta_\pm$ with $\Sigma_\pm = \Sigma_{31} \pm \Sigma_{33} = -i\,\mathrm{sgn}(\epsilon_n)/2\tau_\pm$, and for $f_{\uparrow\uparrow(\downarrow\downarrow)} \equiv f_{+(-)} = \mp f_1 + i f_2$,

$$f_\pm = \frac{\pi\Delta_\pm}{|\epsilon_n| + 1/2\tau_\pm}. \tag{36}$$

The linearized gap equations for $\Delta_\pm$ are given by Eq. (15) with $\mathbf{f} \to f^{\pm}$, $\mathbf{\Delta} \to \Delta^{\pm}$, and $T \to T_c^{\pm}$. For nonunitary, axial states

$$\Delta_\pm(\hat{\mathbf{p}}) = d_\pm(\hat{p}_x + i\hat{p}_y)/\sqrt{2}, \tag{37}$$

the eigenvalue equation for $d_\pm$ yields the weak-coupling equation for the instability temperatures $T_c^{\pm}$. In the absence of the polarization, the aerogel transition temperature is given by

$$\ln(T_c/T_{c0}) = 2S_1(x_c), \tag{38}$$

where $x_c = 1/2\pi\tau T_c$, and the spin-independent rate for quasiparticles scattering off the aerogel is given by

$$\frac{1}{2\tau} = \Gamma_{\mathrm{N}} \sin^2\delta_0 \equiv \Gamma_{\mathrm{N}} \overline{\sigma}_0. \tag{39}$$

In the presence of a liquid-solid exchange coupling, and polarization of the solid $^3$He, the instability temperatures for the $\uparrow\uparrow$ and $\downarrow\downarrow$ condensates are given by

$$\ln(T_c^{\pm}/T_{c0}) = 2\,S_1(x^{\pm}), \tag{40}$$

where $x^{\pm} \equiv 1/2\pi\tau_\pm T_c^{\pm}$. For $u_0 \neq 0$, the leading-order polarization correction to the scattering cross sections gives

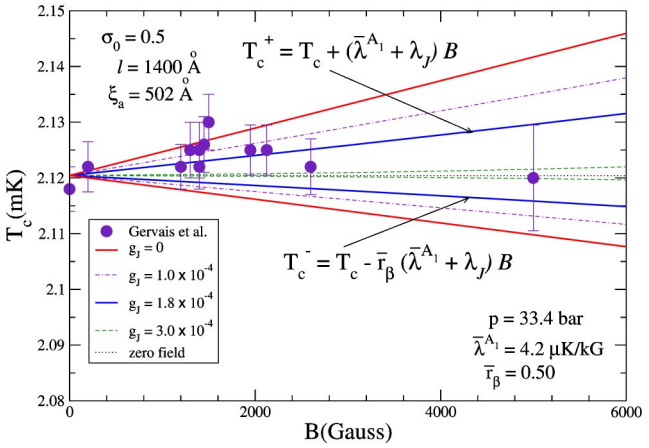


FIG. 2. (Color online) The low-field (linear) splitting of $T_c$ with magnetic field for $^3$He in aerogel with a mean free path of $\ell = 140$ nm, a correlation length of $\xi_a = 50$ nm and a typical cross section $\bar{\sigma}_0 = 1/2$. The splitting for $^3$He aerogel without liquid-solid exchange is indicated by the solid (red) lines. The data points are taken from Ref. 25.

$$\frac{1}{\tilde{\tau}} = \frac{1}{\tau}, \quad \frac{1}{2\tau_S} = 2n_s JS(T,B)\sqrt{\bar{\sigma}_0}(1-\bar{\sigma}_0)^{3/2}. \tag{41}$$

In the low-field region, and above the magnetic ordering temperature, $\mathcal{P} = |\mu|B/k_B T_c$, and we obtain

$$\lambda_J = g_J\left(\frac{|\mu|}{k_B}\right)\left(\frac{(1-\bar{\sigma}_0)^{3/2}}{\sqrt{\bar{\sigma}_0}}\right)\left(\frac{-2x_c S_2(x_c)}{1-2x_c S_2(x_c)}\right), \tag{42}$$

where $x_c = (\xi_0/\ell)(T_{c0}/T_c)$, and the dimensionless exchange coupling is

$$g_J = 2\pi N_f Js. \tag{43}$$

Note the impurity-induced exchange splitting vanishes in the unitary limit.[31] Equation (42) is easily generalized to include aerogel correlations within the heuristic "random void" model described above; the result for $\lambda_J$ has the same form as Eq. (42), but with $-x_c S_2(x_c) \to \tilde{x}_c' S_2(\tilde{x}_c)$, where $\tilde{x}_c = x_c^2/(x_c + \zeta_a^2)$, $x_c = 1/2\pi\tau T_c$, and $\tilde{x}_c' \equiv T_c\, d\tilde{x}_c/dT_c$.

The effects of the liquid-solid exchange coupling $g_J$ and the polarization of the solid $^3$He coating the aerogel strands on the $A_1$-$A_2$ splitting are shown in Fig. 2, and compared with measurements of the superfluid transition in 98% aerogel reported in Ref. 25; these authors found no evidence of the $A_1$-$A_2$ splitting for fields up to $B = 5$ kG. The data for the superfluid transition of $^3$He in 98% aerogel for fields up to $B = 5$ kG are shown in Fig. 2. The error bars are conservative estimates of the uncertainty in defining $T_c$; the experiment shows no evidence of a splitting to within the error of determining $T_c$, and consequently we can assume that the splitting to be less than the error bars for $T_c$.[32]

The calculation of the $A_1$-$A_2$ splitting includes aerogel correlations, which are most important at high pressures. Both the mean free path $\ell$ and the aerogel correlation length $\xi_a$ contribute. The values of $\ell = 1400$ Å and $\xi_a = 502$ Å correspond to $T_c = 2.12$ mK at $p = 33.4$ bars and yield close agreement with $T_c(p)$ over the full pressure range. The dimensionless cross section $\bar{\sigma}_0$ is not known with any certainty; there is likely a distribution of cross-sections provided by the aerogel. In the absence of detailed knowledge we assume an average value of $\bar{\sigma}_0 = 1/2$. The values of $\lambda^{A_1}$ and $\lambda^{A_2}$ for pure $^3$He, and thus the strong-coupling parameter $r_\beta$ are taken from Ref. 6. The effects of nonmagnetic scattering by aerogel lead to small corrections for $\bar{\lambda}^{A_1}$ and $\bar{\lambda}^{A_2}$; these terms alone (shown in Fig. 2 as $g_J = 0$) generate the $A_1$-$A_2$ splitting that is substantially larger ($\approx \times 2$) than the error reported for the superfluid $^3$He transition in Ref. 5. An antiferromagnetic exchange coupling ($g_J > 0$) decreases the $A_1$-$A_2$ splitting. The magnitude of the predicted splitting is reduced to lie within the error bars for $T_c$ for $g_J = 1.8 \times 10^{-4}$, which corresponds to an exchange coupling of $J \simeq 0.1$ mK per liquid $^3$He spin.

The existing data, while suggestive that the liquid-solid layer coupling may be playing an important role in suppressing the $A_1$-$A_2$ splitting are not conclusive. If scattering by polarized $^3$He is responsible for the suppressed $A_1$-$A_2$ splitting for pure $^3$He in aerogel, then heat capacity or acoustic attenuation measurements with $^4$He added to displace the solid $^3$He, should exhibit the $A_1$-$A_2$ splitting that is comparable to that of pure bulk $^3$He. Measurements of Sprague *et al.*[21] at $p = 18.7$ bars and at $B = 1.47$ kG do show an increase in $T_c$ from 1.69 mK without $^4$He coverage to 1.76 mK with the addition of one monolayer of $^4$He to remove the solid $^3$He; thus, $\Delta T_c \simeq 70\ \mu$K. By comparison, if we suppress the polarization component of the scattering rate in our theoretical calculation we obtain an increase in $T_c$ from the conventional component of the $A_1$-$A_2$ splitting of $\Delta T_c^{A_1\text{-}A_2} = 3.1\ \mu\text{K/kG}\, B \simeq 4.6\ \mu$K, which is more than an order of magnitude smaller than the change in $T_c$ observed by

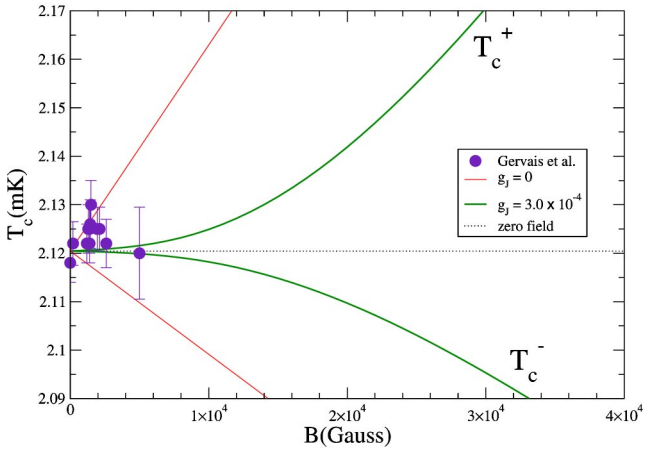


FIG. 3. (Color online) The field-evolution of the splitting of $T_c$ for $^3$He in aerogel with the same parameters as those used in Fig. 2. The linear field splitting expected in the absence of polarized solid $^3$He is indicated by the solid (red) lines. The nonlinear field evolution of the splitting is indicated by the thick (green) lines and corresponds to the value of $g_J = 3.0 \times 10^{-4}$. The Curie temperature is taken from Ref. 19, $\Theta_S \simeq 0.4$ mK, and the exchange field is $B_S = k_B\Theta_S/|\mu| \simeq 5.14$ kG. The data points are from Ref. 25.

adding $^4$He. Thus, the addition of $^4$He also modifies the nonmagnetic contribution to the pairbreaking, and this effect is dominant at these low fields.

Measurements on pure $^3$He in aerogel at higher fields should not suffer from this problem and should be able to resolve some or all of the uncertainty in the mechanism suppressing the $A_1$-$A_2$ splitting at low fields. In particular, if an exchange coupling, $J \approx 0.1$–$0.2$ mK is responsible for the suppressed $A_1$-$A_2$ splitting at $B \leqslant 5$ kG, then for higher fields, $B \gg B_S = k_B \Theta_S / |\mu| \approx 5$ kG the polarization of the solid $^3$He should saturate, producing a field-independent shift from scattering off the polarized $^3$He, and the $A_1$-$A_2$ splitting that increases with field, for $B \gg B_S$, at a rate comparable to that for pure $^3$He.

The Curie temperature for the solid $^3$He provides the temperature and field scale for the polarization, i.e., $\mathcal{P}(B/B_S, T/\Theta_S)$. In order to estimate the field dependence of the $A_1$-$A_2$ splitting at higher fields we use the mean-field theory for the $s = 1/2$, near-neighbor Heisenberg ferromagnet to calculate the polarization.[28] The result is shown in Fig. 3 for the same parameters used to obtain the low-field suppression of the $A_1$-$A_2$ splitting shown in Fig. 2. Thus, even if fully suppressed at low fields $B \lesssim B_S$ the $A_1$-$A_2$ splitting should emerge for fields above $B \approx 20$ kG.

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[29] The $A_1$ phase corresponds to pairs of $^3$He quasiparticles with their magnetic moments aligned along the field. We follow the notation of Ref. 23 and define the quantization axis for the spin to be $\hat{\mathbf{z}} || -\mathbf{B}$ to compensate for the negative gyromagnetic ratio of $^3$He.
[30] This is the result for $\overline{\sigma_0} = 1/2$. There is a correction to the third term on the right side of Eq. (11) for $\overline{\sigma_0} \neq 1/2$.
[31] The splitting also vanishes in the Born limit; although Eq. (42) is not valid in the Born limit since it is based on a expansion of $J/u_0$. Nevertheless, in the limit $u_0 = 0$, i.e., with only pure exchange coupling the cross sections for $\uparrow$ and $\downarrow$ spin scattering are equal and the splitting vanishes.
[32] W.P. Halperin (private communication).