

# Time-dependent spintronic transport and current-induced spin transfer torque in magnetic tunnel junctions

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The responses of the electrical current and the current-induced spin transfer torque (CISTT) to an ac bias in addition to a dc bias in a magnetic tunnel junction are investigated by means of the time-dependent nonequilibrium Green function technique. The time-averaged current (time-averaged CISTT) is formulated in the form of a summation of dc current (dc CISTT) multiplied by the product of Bessel functions with the energy levels shifted by  $m\hbar\omega_0$ . The tunneling current can be viewed to occur between the photonic sidebands of the two ferromagnets. The electrons can pass through the barrier easily under high frequencies but difficultly under low frequencies. The tunnel magnetoresistance almost does not vary with an ac field. It is found that the spin transfer torque, still being proportional to the electrical current under an ac bias, can be changed by varying frequency. Low frequencies could yield a rapid decrease of the spin transfer torque, while a large ac signal leads to a decrease of both the electrical current and the spin torque. If only an ac bias is present, the spin transfer torque is sharply enhanced at the particular amplitude and frequency of the ac bias. A nearly linear relation between such an amplitude and frequency is observed.

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## I. INTRODUCTION

The spin-polarized electrical transport in magnetic multilayer structures has received much attention both experimentally and theoretically in the past several years.<sup>1</sup> The yield of this extensive investigation promises fascinating implications for applications in information technology. For instance, people may make use of the different resistive states corresponding to parallel and antiparallel magnetizations of different layers as memory elements in magnetic random-access memories (MRAM). It is now known that the antiparallel alignments of the moments in magnetic layers lead to a higher electrical resistance than the parallel alignments, giving rise to a so-called giant magnetoresistance (GMR) or tunnel magnetoresistance (TMR) depending on different magnetic junction systems. The origin of these phenomena is widely believed to be mainly caused by the spin-dependent scatterings of the conduction electrons. As is pointed out,<sup>2</sup> when the junction bias is increased, the phenomenon is significantly reduced. Generally speaking, the resistive state can be changed by applying an external magnetic field, because the latter could lead to variations of the traversing paths of the conduction electrons. However, it is now accepted that not only the electrical current is strongly affected by a magnetic state, but also the electrical current can conversely control the magnetic state of the magnetic junctions. This effect is predicted by Slonczewski<sup>3</sup> and Berger<sup>4</sup> in magnetic multilayer systems by noting that a spin-polarized electrical current can transfer local spin angular momenta of incident electrons to the scattering ferromagnet, thereby exerting a torque on the magnetic moments and therefore changing the magnetic state. When the current is large enough, the magnetization of the ferromagnetic layer can be switched. As a result, the spin transfer effect may provide a mechanism for a current-controlled magnetic memory element. Experiments

have given evidence of this effect in the Cu/Co multilayers,<sup>5</sup> nickel nanowires,<sup>6</sup> manganite junctions,<sup>7</sup> point contact magnetic multilayer devices,<sup>8</sup> bulk manganese oxides,<sup>9</sup> tunnel junctions,<sup>2</sup> Co/Cu/Co spin valve devices,<sup>10</sup> etc. A thermally activated switching of magnetic domains in a Co/Cu/Co spin valve is also confirmed.<sup>11</sup>

To deal with the spin transfer effect, it is useful to introduce concepts such as the spin current and the spin torque to describe the coupling between the conduction electrons and the magnetic moments of ferromagnetic materials. Those were first proposed by Slonczewski<sup>12</sup> based on a quantum-mechanical model. Then, this interaction is derived from the *s-d* coupling and manifests itself in magnetic multilayers<sup>3</sup> or bulk ferromagnets.<sup>13</sup> It also gives a contribution as a current-induced force on a domain wall or an interaction between spin waves and itinerant electrons.<sup>4</sup> The concepts are also used to deal with a variety of structures such as ferromagnet-normal-metal-ferromagnet (FM-NM-FM) junctions,<sup>14,15</sup> ferromagnet-superconductor-ferromagnet junctions,<sup>16</sup> and a FM-NM-FM trilayer contacting a normal-metal lead or a superconductor lead,<sup>17,18</sup> and so on. Apart from the quantum-mechanical method, the scattering matrix method is improved to cover this issue.<sup>15,17,18</sup> Heide *et al.* derived a set of coupled Landau-Lifshitz equations for the ferromagnetic layers by considering the nonequilibrium exchange interaction between layers.<sup>19</sup> Zhang *et al.* dealt with the dynamic spin transfer torque and thermally assisted magnetization reversal by means of the Landau-Lifshitz-Gilbert equation<sup>20,21</sup> in which a term describing the spin transfer torque is introduced. The nonequilibrium Green function is also applied to this question to consider the spin-flip scattering effect on the current-induced spin transfer torque.<sup>22</sup>

So far, the electrical current, the spin transfer effect, and the spin current in magnetic multilayer systems are extensively studied under a dc bias voltage, and the investigation under an ac bias is still sparse. Whether an ac bias voltage

applied to the magnetic tunnel junctions could induce some unusual properties (see, e.g., Ref. 23) is still not clear. In this paper, we shall show the effect of an ac bias on the electrical current and the spin transfer torque in a magnetic tunnel junction. Since the switching of magnetic domains depends on the magnitude and the directions of the spin transfer torque, while the direction of the spin transfer torque is related to the direction of the electrical current, it is of interest to anticipate that the response of the system under an ac bias is different from that under a dc bias. We have found that the spin transfer torque is sharply enhanced by applying an ac electrical field with a particular amplitude and frequency.

The rest of this paper is organized as follows. In Sec. II, a model is proposed, and the necessary formalism under an ac bias is established. In Sec. III, the tunneling current under an ac electrical field is derived and studied. In Sec. IV, the current-induced spin transfer torque under an ac electrical field is investigated. Finally, a brief summary is given in Sec. V.

## II. MODEL AND FORMALISM

### A. Model

Let us consider a magnetic tunnel junction in which two ferromagnets (FM) which are stretched to infinity are separated by a thin insulator ( $I$ ). The molecular field in the left ferromagnet is assumed to align along the  $z$  axis, which is in the junction plane, while the orientation of the molecular field in the right ferromagnet deviates the  $z$  axis by an angle  $\theta$ , which is along the  $z'$  axis (such that the frame  $x'o'z'$  deviates  $xoz$  by an angle  $\theta$ ). The tunnel current flows along the  $x$  axis, which is perpendicular to the junction plane. The Hamiltonian reads

$$H = H_L + H_R + H_T, \quad (1)$$

with

$$H_L = \sum_{k\sigma} \varepsilon_{k\sigma}^L(t) a_{k\sigma}^\dagger a_{k\sigma},$$

$$H_R = \sum_{q\sigma} [(\varepsilon_{qR}(t) - \sigma M_2 \cos \theta) c_{q\sigma}^\dagger c_{q\sigma} - M_2 \sin \theta c_{q\sigma}^\dagger c_{q\bar{\sigma}}], \quad (2)$$

$$H_T = \sum_{kq\sigma\sigma'} [T_{kq}^{\sigma\sigma'} a_{k\sigma}^\dagger c_{q\sigma'} + T_{kq}^{\sigma\sigma'} * c_{q\sigma'}^\dagger a_{k\sigma}],$$

where  $a_{k\sigma}$  and  $c_{k\sigma}$  are annihilation operators of conduction electrons with momentum  $k$  and spin  $\sigma$  ( $= \pm 1$ ) in the left and right ferromagnets, respectively. When the time-dependent bias voltage is applied, the single-particle energies in the left and right ferromagnets,  $\varepsilon_{k\sigma}^{L,R}(t)$ , become time-dependent:<sup>24,25</sup>  $\varepsilon_{k\sigma}^L(t) = \varepsilon_{kL}(t) - \sigma M_1$ ,  $\varepsilon_{kL}(t) = \varepsilon_{kL}^0 + \Delta_L(t) - eV_0$ ,  $\Delta_L(t) = eV_L \cos \omega_0 t$ ,  $M_1 = g \mu_B h_L / 2$ ,  $\varepsilon_{qR}(t) = \varepsilon_{qR}^0 + \Delta_R(t)$ ,  $\Delta_R(t) = eV_R \cos \omega_0 t$ ,  $M_2 = g \mu_B h_R / 2$ , where  $V_0$  is the applied dc bias,  $\Delta_L(t)$  and  $\Delta_R(t)$  are from the applied ac bias,  $g$  is the Landé factor,  $\mu_B$  is the Bohr magneton,  $h_{L(R)}$  is the magnitude of the molecular field of the left (right) ferromagnet,  $\varepsilon_{kL(R)}^0$  is the single-particle dispersion of the left (right) FM electrode,  $T_{kq}^{\sigma\sigma'}$  denotes the spin and momentum dependent tunneling amplitude through the insulating barrier. Note that the spin-flip scattering is included in  $H_T$  when  $\sigma' = \bar{\sigma} = -\sigma$ . It is this term that violates the spin conservation in the tunneling process.

### B. Green functions of uncoupled leads

First let us write down the lesser Green function and the retarded (advanced) Green function for the isolated leads which will be used subsequently. The single-particle energies of the isolated leads for spin up and down are splitting, i.e.,  $\varepsilon_{k\sigma}^{L,0} = \varepsilon_{kL} - \sigma M_1$  ( $\sigma = \pm 1$ , corresponding to  $\uparrow, \downarrow$ ). Following the standard procedure, it is not difficult to obtain the lesser Green function for the left FM lead

$$g_{kL}^<(t, t') = \begin{pmatrix} if_L(\varepsilon_{k\uparrow}^{L,0}) e^{-i(\varepsilon_{k\uparrow}^{L,0} - eV_0)(t-t')} & 0 \\ 0 & if_L(\varepsilon_{k\downarrow}^{L,0}) e^{-i(\varepsilon_{k\downarrow}^{L,0} - eV_0)(t-t')} \end{pmatrix} e^{-i \int_{t'}^t dt_1 \Delta_L(t_1)} \\ = \sum_{m,n} J_m \left( \frac{eV_L}{\omega_0} \right) J_n \left( \frac{eV_L}{\omega_0} \right) \begin{pmatrix} if_L(\varepsilon_{k\uparrow}^{L,0}) e^{-i(\varepsilon_{k\uparrow}^{L,0} - eV_0)(t-t')} & 0 \\ 0 & if_L(\varepsilon_{k\downarrow}^{L,0}) e^{-i(\varepsilon_{k\downarrow}^{L,0} - eV_0)(t-t')} \end{pmatrix} e^{-i\omega_0(mt - nt')}, \quad (3)$$

where  $f_L(x)$  is the Fermi function of the left uncoupled lead,  $J_m(eV_L/\omega_0)$  is the  $m$ th order of the Bessel function. Throughout this paper,  $\hbar = 1$  is assumed. Similarly, we can write down the lesser Green function for the right lead. The retarded and advanced Green functions have the form of

$$g_{k\alpha}^{r(a)}(t, t') = \mp i \theta(\pm t \mp t') \sum_{mn} J_m \left( \frac{eV_\alpha}{\omega_0} \right) J_n \left( \frac{eV_\alpha}{\omega_0} \right) e^{\pm i\omega_0(mt - nt')} \begin{pmatrix} e^{\pm i(\varepsilon_{k\uparrow}^{\alpha,0} - eV_0)(t-t')} & 0 \\ 0 & e^{\pm i(\varepsilon_{k\downarrow}^{\alpha,0} - eV_0)(t-t')} \end{pmatrix}, \quad (4)$$

where  $\alpha=L,R$ . A further evaluation will make use of the double time Fourier transform as follows:<sup>26,27</sup>

$$F(E_1, E_2) = \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 F(t_1, t_2) \exp[i(E_1 t_1 - E_2 t_2)]. \quad (5)$$

The above equations will be invoked in the following analysis.

### III. TUNNELING ELECTRICAL CURRENT UNDER AN ac BIAS

In this section, we shall investigate the tunneling electrical current in FM-*I*-FM junctions under an ac electrical field. Following the method in Refs. 24 and 25, the tunneling current can be expressed as

$$I(t) = \frac{2e}{\hbar} \text{Re} \sum_{kq} \text{Tr}_\sigma [\Omega \mathbf{G}_{kq}^<(t, t)], \quad (6)$$

where  $\Omega = \mathbf{TR}$ ,

$$\mathbf{T} = \begin{pmatrix} T_1 & T_2 \\ T_3 & T_4 \end{pmatrix}$$

with the elements  $T_i$  ( $i=1, \dots, 4$ ) of the tunneling matrix which are assumed to be independent of  $k$  and  $q$  for the sake of simplicity,<sup>28</sup> which is reasonable in the assumption of a wide-band limit (WBL),<sup>24,25,29</sup> and the elements  $T_2$  and  $T_3$  describe the effect of spin-flip scatterings,<sup>30</sup>

$$\mathbf{R} = \begin{pmatrix} \cos \frac{\theta}{2} & -\sin \frac{\theta}{2} \\ \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix},$$

$\text{Tr}_\sigma$  stands for the trace of the matrix taking over the spin space, and  $\mathbf{G}_{kq}^<(t, t')$  is the lesser Green function in spin space defined as

$$\mathbf{G}_{kq}^<(t, t') = \begin{pmatrix} G_{kq}^{\uparrow\uparrow, <}(t, t') & G_{kq}^{\downarrow\uparrow, <}(t, t') \\ G_{kq}^{\uparrow\downarrow, <}(t, t') & G_{kq}^{\downarrow\downarrow, <}(t, t') \end{pmatrix}, \quad (7)$$

where  $G_{kq}^{\sigma\sigma', <}(t, t') \equiv i \langle c_{q\sigma'}^\dagger(t') a_{k\sigma}(t) \rangle$ . By using the non-equilibrium Green function technique,<sup>24,25</sup> we can get  $\mathbf{G}_{kq}^<(t, t')$  to the first order:

$$\mathbf{G}_{kq}^<(t, t') = \int dt_1 [g_{qR}^r(t, t_1) \Omega^\dagger g_{kL}^<(t_1, t') + g_{qR}^<(t, t_1) \Omega^\dagger g_{kL}^a(t_1, t')], \quad (8)$$

where  $g_{kL}^r$ ,  $g_{qR}^a$ ,  $g_{kL}^<$ , and  $g_{qR}^<$  are retarded, advanced, lesser Green function of the left and the right ferromagnets, respectively. Then, substituting Eq. (8) into Eq. (6), and making the double-time Fourier transform of Green functions, we have

$$I(t) = \frac{\pi e}{\hbar} \sum_{mnm'n'} \int dE J_{(mn)'}^{(m'n')} [f_L(E'') - f_R(E')] \Lambda(E', E'', \theta) \cos(l\omega_0 t), \quad (9)$$

where

$$J_{(mn)'}^{(m'n')} = J_m \left( \frac{eV_L}{\omega_0} \right) J_n \left( \frac{eV_L}{\omega_0} \right) J_{m'} \left( \frac{eV_R}{\omega_0} \right) J_{n'} \left( \frac{eV_R}{\omega_0} \right),$$

$$E'' = E - m\omega_0 + eV_0, \quad E' = E - n'\omega_0,$$

$$l = m + m' - n - n',$$

$$\Lambda(E', E'', \theta) = \text{Tr}_\sigma [\Omega \mathbf{D}_R(E') \Omega^\dagger \mathbf{D}_L(E'')],$$

and

$$\mathbf{D}_{L(R)} = \begin{pmatrix} D_{L(R)\uparrow} & 0 \\ 0 & D_{L(R)\downarrow} \end{pmatrix}$$

with  $D_{L(R)\uparrow(\downarrow)}(\varepsilon) = D_{L(R)}(\varepsilon \pm M_{1(2)})$  the density of states (DOS) of the conduction electrons with spin up and down in the left (right) ferromagnet. In Eq. (9), the real part of the current is taken. The time average of the electrical current  $I(t)$  can be defined as  $\langle I(t) \rangle = (1/T_0) \int_{-T_0/2}^{T_0/2} dt I_L(t)$  with  $T_0$  the time interval between which the physical quantity is measured. Thus, the time-averaged tunneling current is given by

$$I_{ac}^{aver} = \langle I(t) \rangle = \begin{cases} \sum_{mnm'n'} I_{(m'n')}^{(mn)} = \sum_{mnm'n'} J_{(mn)'}^{(m'n')} I_{dc}(V_0 - m\omega_0/e, n'\omega_0/e), & l=0 \\ 0, & l \neq 0 \end{cases} \quad (10)$$

where

$$I_{(m'n')}^{(mn)} = J_{(mn)'}^{(m'n')} I_{dc}(V_0 - m\omega_0/e, n'\omega_0/e),$$

$$I_{dc}(eV_0 - m\omega_0, n'\omega_0) = (\pi e/\hbar) \int dE [f_L(E'') - f_R(E')] \Gamma_I(E', E'') \{1 + P_2(E') [P_1(E'') \cos \theta + P_3(E'') \sin \theta]\},$$

$$P_1 = \frac{D_{L\uparrow}(T_1^2 - T_2^2) - D_{L\downarrow}(T_4^2 - T_3^2)}{D_{L\uparrow}(T_1^2 + T_2^2) + D_{L\downarrow}(T_3^2 + T_4^2)},$$

$$P_2 = \frac{D_{R\uparrow} - D_{R\downarrow}}{D_{R\uparrow} + D_{R\downarrow}},$$

$$P_3 = \frac{2(D_{L\uparrow}T_1T_2 + D_{L\downarrow}T_3T_4)}{D_{L\uparrow}(T_1^2 + T_2^2) + D_{L\downarrow}(T_3^2 + T_4^2)},$$

and

$$\Gamma_I(E', E'') = [D_{R\uparrow}(E') + D_{R\downarrow}(E')][D_{L\uparrow}(E'')(T_1^2 + T_2^2) + D_{L\downarrow}(E'')(T_3^2 + T_4^2)].$$

It can be seen that the average ac tunneling current in a magnetic tunnel junction is modulated via Bessel functions. This result is quite similar to Eq. (3.3) in Ref. 31 in which the superconductor tunnel junctions are investigated. By analogy, the time dependence of the wave function for every electron state is modulated by  $J_n(eV_{L(R)}/\omega_0)e^{\pm in\omega_0 t}$ . Each single-particle level is modulated in terms of a probability  $J_n(eV_{L(R)}/\omega_0)$  and is displaced in energy by  $n\hbar\omega_0$ . These displacements in energy contributing to the amplitude of the average tunneling current are equivalent to that dc voltages  $(V_0 - m\omega_0/e)$  applied across the junction with a probability  $J_{mn}^{(m'n')}$ . In Eq. (10), an explicit relation between the time-averaged ac tunneling current and the dc current is given, where  $l=0$  gives a nontrivial result. This result implies that only the elastic transmission through the tunnel barrier contributes to the average current, and the net number of photons absorbed from the ac field must be zero.<sup>32</sup> So, every term in the summation of Eq. (10) describes the tunneling process that the electrons tunnel from the excited states  $eV_0 - m\omega$  absorbed  $m$  photons of the left ferromagnet to the excited states  $n'\omega_0$  absorbed  $n'$  photons of the right one. In this way, the summation is taken over all the excited states of the left and the right ferromagnets. The ac field gives a correction to the transition rate  $\Gamma_I$  by adding the product of Bessel functions, i.e.,  $J_{mn}^{(m'n')}$ , which describes the probability of electron population in the excited states. The tunneling current can be viewed as what happens between the photonic sidebands of the two ferromagnets.

Next, let us present the numerical results of the time-dependent electric current. Before going on, we shall first give some presumptions. A parabolic dispersion for band electrons is assumed, on which is based that the DOS of conduction electrons are calculated. The Fermi energy and the molecular field will be taken as  $E_f = 1.295$  eV and  $|\mathbf{h}_1| = |\mathbf{h}_2| = 0.90$  eV, which are given in Ref. 33 for Fe. We note that the elements of the coupling matrix  $T_2$  and  $T_3$  mean the strength of spin-flip scatterings which were discussed in Refs. 22 and 30. In order to focus our attention on the effect of an ac bias, we shall not consider the effect of spin-flip scatterings here for brevity and simplicity. So we take  $T_2 = T_3 = 0$ , and  $T_1 = T_4 = 0.01$  eV. In addition,  $V_R = 0$  is assumed, then  $V_L = V_{ac}$ . Under this assumption,  $I_{ac}^{aver} = \sum_m J_m^2(eV_{ac}/\omega_0) I_{dc}(V_0 - m\omega_0/e, \theta)$ , where  $J_m(eV_{ac}/\omega_0)$  is the  $m$ th Bessel function. When the ac bias is absent, we get  $G_{dc}(V_0 = 0, T = 0, \theta) = \partial I_{dc} / \partial V_0 = G_0 [1 + P_2 \sqrt{P_1^2 + P_3^2} \cos(\theta - \theta_f)]$  (Ref. 30) at zero dc bias, zero

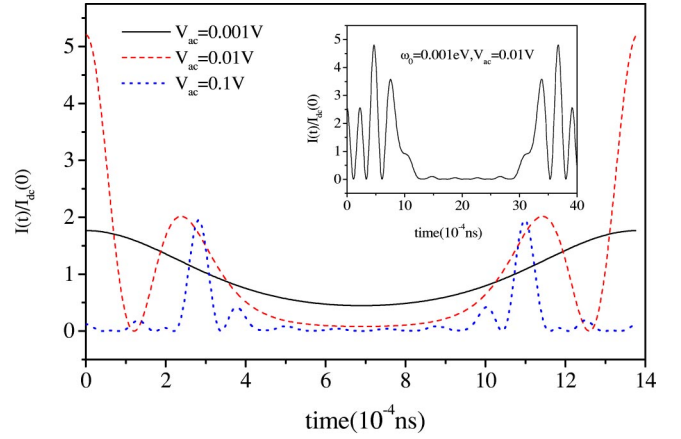


FIG. 1. (Color online) Time dependence of the tunneling current, where the effective masses of the left and the right ferromagnets are taken as unity, the molecular fields are assumed to be 0.9 eV, the Fermi energy is taken as 1.295 eV which are taken from Ref. 33 for Fe, and  $T_1 = T_4 = 0.01$  eV,  $T_2 = T_3 = 0$ , the other parameters are assumed as  $\hbar = 1$ ,  $V_0 = -0.1$  V,  $\omega_0 = 0.003$  eV,  $\theta = \pi/3$  and the temperature is at 100 K.

temperature ( $T=0$ ) and  $\theta$ , where  $P_1, P_2$ , and  $P_3$  are similar to the formulas as given before,  $\tan \theta_f = P_3/P_1$  and

$$G_0 = (\pi e^2 / 2\hbar) [(T_1^2 + T_2^2) D_{L\uparrow} + (T_3^2 + T_4^2) D_{L\downarrow}] (D_{R\uparrow} + D_{R\downarrow}).$$

As a consequence,  $I_{dc}(T=0, \theta) = G_{dc}(0, 0, \theta) V_0$  can be used as a scale to measure the ac current.

The time evolution of the tunnel electrical current  $I(t)$  in response to an ac field is depicted in Fig. 1, where  $V_0 = -0.1$  V and  $\omega_0 = 0.003$  eV. As can be seen, when the dc bias is positive, the current can be negative. When the ac signal is small (e.g.,  $V_{ac} = 0.001$  V), the current varies with time in a cosine manner, which appears to be proportional to the ac bias. In this case, the current response is similar to the dc case, i.e., the larger the bias, the larger the current. However, when the ac signal is stronger, things become different. There appear some resonant peaks, which can be regarded as being result from the photon-assisted tunneling, and the tunnel current is still periodic with time. The inset of Fig. 1 shows the case at  $V_{ac} = 0.01$  V but with a lower frequency for a comparison. It is found that the peaks split into several peaks in one period of time, suggesting that the external frequency can change the oscillating frequency of the tunnel current.

The frequency dependence of the averaged tunneling current is shown in Fig. 2. Small oscillations of the current with frequency can be seen, which are caused by the summation of the  $m$ th current  $I_{dc}^{(mn)}$  with different  $m, n, m'$ , and  $n'$  owing to  $I_{dc}^{(mn)}$  oscillating with the frequency and having many peaks. The peaks of the  $m$ th current correspond to the troughs of the  $(m+1)$ th current. Furthermore, when the ac signal is small (e.g.,  $V_{ac} = 0.01$  V), the current first increases with small oscillations, and then the current almost does not vary with the frequency like a pure resistance because the pure resistance should not vary with the frequency in the common sense. However, it can be observed that a large ac

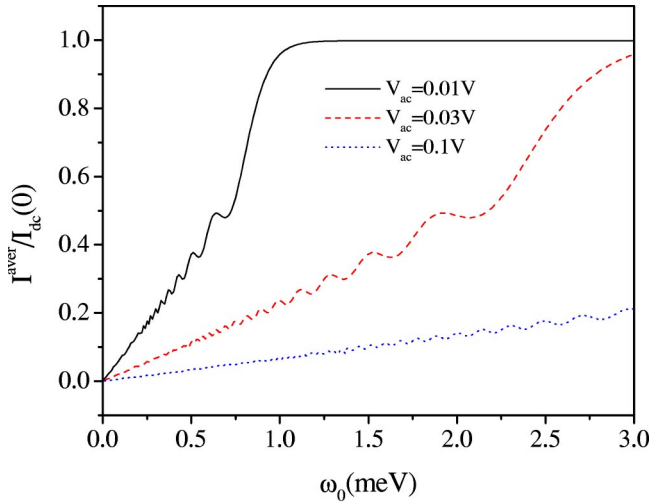


FIG. 2. (Color online) The frequency dependence of the time-averaged tunneling current. The remaining parameters are assumed to be the same as those in Fig. 1.

signal leads to a small averaged current, also as those expressed in Fig. 3. The reason for this feature is that an ac bias is imposed on an dc bias, while the summation current decreases with increasing ac bias. A large  $V_{ac}$  means a large argument of Bessel function, thus leading to a strong modulation to the current, which makes the current approach zero. If  $V_{ac}$  is fixed, a larger  $\omega_0$  gives a larger averaged current, suggesting that the current flows easily in this system under a higher frequency. This character is consistent with the classical feature of systems that metallic leads are separated by an insulator. Here, we can consider two limiting cases. When  $\omega_0 \rightarrow 0$ , the argument of the Bessel function  $x_B = eV_{ac}/\omega_0 \rightarrow \infty$ , then  $J_m^2(x_B) \sim (2/\pi x_B) \cos^2[x_B - (\pi/2)m - \pi/4] \rightarrow 0$ , which suggests that the current cannot pass easily through the system. When  $V_{ac} \rightarrow 0$  and the frequency becomes larger, the argument of the Bessel function  $x_B \rightarrow 0$ , then  $J_{m \neq 0}(x_B \rightarrow 0) \sim 0$  and  $J_0(0) \sim 1$ , which suggests that the  $m=0$  term

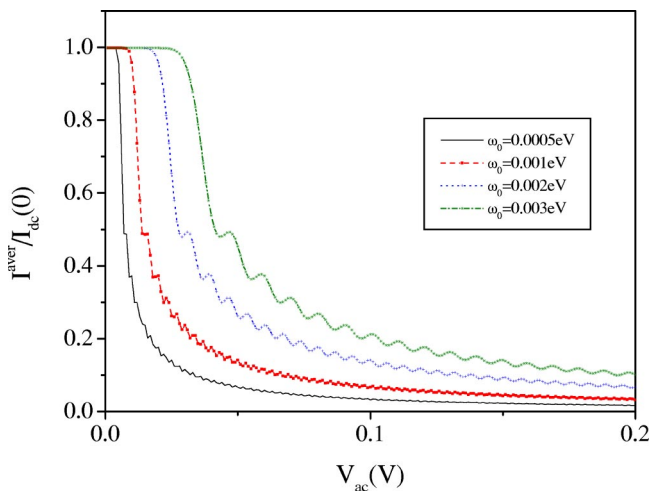


FIG. 3. (Color online) The time-averaged tunneling current as a function of the amplitude of the ac bias,  $V_{ac}$ . The remaining parameters are assumed to be the same as those in Fig. 1.

is dominant. This case shows that the current can pass easily through the barrier under higher frequencies. It is the character of a capacitance in the usual sense.

Let us define  $TMR = (J_P - J_{AP})/J_{AP}$  by using the real part of the current. We have investigated the response of TMR to an ac bias. It is found that TMR almost does not alter with  $V_{ac}$  and frequency  $\omega_0$ , with the varying range about in 0.01%–0.1%. To understand this result, we would like to remark that TMR is mainly contributed by the spin-dependent scatterings. When electrons from one ferromagnet enter into another whose magnetization deviates an angle to the first one, spin up and down electrons bear different potentials. While the ac field provides the same modulation of quasiparticle levels of the spin up and spin down electrons shifted by  $n\hbar\omega_0$ , leading to a change of the magnitude of the current, it affects less spin-dependent scatterings. If we impose a time-dependent magnetic field on both electrodes, it is conceivable that TMR would be remarkably influenced by such an ac magnetic field. Work towards this direction is now in progress.

#### IV. CURRENT-INDUCED SPIN TRANSFER TORQUE UNDER AN ac BIAS

As is known, the relative orientation of magnetizations on both electrodes can affect considerably the magnitude of the electrical current flowing through the magnetic tunnel junctions, and meanwhile, the spin-polarized electrical current can also switch the direction of magnetization of electrodes. This latter effect comes from an indirect interaction between ferromagnets, which is caused by the so-called current-induced spin-transfer torque (CISTT).<sup>3,4</sup> The physical meaning of the CISTT can be understood as follows. When the electrons in the first ferromagnet tunnel through the barrier and enter into the right ferromagnet, the incoming polarized electrons will precess, eventually to align with the magnetization direction at an angle  $\theta$  in the second ferromagnet. In this process, there should be a difference of spin angular momenta between the incoming and outgoing spins in the second ferromagnet. The missing spin angular momentum must be absorbed by the local moments, thereby generating a torque which exerts on the moments of the second ferromagnetic layer. This kind of torque reflects actually the spin angular momentum transfer from the first ferromagnet to the second by the polarized current. In Ref. 22, we have discussed the CISTT by means of the nonequilibrium Green function technique under a dc bias. By generalizing our treatment, we now consider the case under a simultaneous application of both ac and dc biases. The CISTT is related not only to the available energy levels of both ferromagnets, but also to the direction of the current. In the ac case, the available energy levels of ferromagnets will be shifted by absorbing or emitting photons. The tunneling can be viewed as occurring between the photonic sidebands of the left and right ferromagnets, and the magnitude of the current can be tuned. The ac field can change the current directions by frequency  $2\omega_0$ . It may be expected that the ac field may impose considerable effect on the CISTT. The total spin of the right ferromagnet can be expressed as<sup>22</sup>

$$\mathbf{s}_R(t) = \frac{\hbar}{2} \sum_{k\mu\nu} c_{k\mu}^\dagger c_{k\nu} (\mathbf{R}^{-1} \chi_\mu)^\dagger \hat{\sigma} (\mathbf{R}^{-1} \chi_\nu), \quad (11)$$

where  $\hat{\sigma}$  is the Pauli matrices and  $\chi_{\mu(\nu)}$  is spin states, which is written down in the  $xyz$  coordinate frame while the spins  $\mathbf{s}_R$  are quantized in the  $x'y'z'$  frame, and the rotation matrix  $\mathbf{R}$  is the same as before. Since

$$\hat{\mathbf{s}}_{L,R} \sim I_e \hat{s}_{L,R} \times (\hat{s}_L \times \hat{s}_R), \quad (12)$$

where  $\hat{s}_{L,R}$  are unit vectors  $\hat{s}_i = \vec{s}_i/s_i$  and  $\hbar \vec{s}_i$  represents the respective total spin momenta per unit area of the ferromagnets,<sup>3</sup> we know that the direction of the spin transfer torque  $\hat{\mathbf{s}}_R$  is just along the  $x'$  direction in the  $x'y'z'$  coordinate frame. From Eq. (11) we obtain

$$\begin{aligned} \mathbf{s}_R(t) &= \frac{\hbar}{2} \sum_{k\sigma} (c_{k\sigma}^\dagger c_{k\bar{\sigma}} \cos \theta - \sigma c_{k\sigma}^\dagger c_{k\sigma} \sin \theta) \\ &= s_{Rx'0} \cos \theta - s_{Rz'0} \sin \theta, \end{aligned}$$

where  $s_{Rx'0}$  and  $s_{Rz'0}$  are  $x'$  and  $z'$  components of the total spins in the  $x'y'z'$  coordinate frame in which the spins  $\mathbf{s}_R$  are quantized. So the CISTT can be obtained<sup>22</sup>

$$\begin{aligned} \tau^{Rx}(t) &= -\cos \theta \operatorname{Re} \sum_{kq} \operatorname{Tr}_\sigma [\mathbf{G}_{kq}^<(t,t) \hat{\sigma}_1 \mathbf{T}^\dagger] \\ &\quad + \sin \theta \operatorname{Re} \sum_{kq} \operatorname{Tr}_\sigma [\mathbf{G}_{kq}^<(t,t) \hat{\sigma}_3 \mathbf{T}^\dagger], \end{aligned} \quad (13)$$

where

$$\hat{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

are Pauli matrices, and  $\mathbf{G}_{kq}^<(t,t')$  is the lesser Green function in spin space defined as above. By using the nonequilibrium Green function technique<sup>24</sup> (a similar framework employed in Ref. 22), we can get the torque to the first order of the Green function

$$\begin{aligned} \mathbf{G}_{kq}^<(t,t') &= \int dt_1 [\mathbf{g}_{kL}^r(t,t_1) \mathbf{TR} \mathbf{g}_{qR}^<(t_1,t') \mathbf{R}^\dagger \\ &\quad + \mathbf{g}_{kL}^<(t,t_1) \mathbf{TR} \mathbf{g}_{qR}^a(t_1,t') \mathbf{R}^\dagger], \end{aligned} \quad (14)$$

where  $\mathbf{g}_{kL}^r$ ,  $\mathbf{g}_{qR}^a$ ,  $\mathbf{g}_{kL}^<$ , and  $\mathbf{g}_{qR}^<$  are retarded, advanced, lesser Green function of the isolated left and the right ferromagnets, respectively. After some algebra, we obtain the Fourier transform of the CISTT

$$\begin{aligned} \tau^{Rx}(\omega) &= 2\pi^2 \sum_{mnm'n'} \int dE J_{(mn}^{(m'n')} \\ &\quad \times [f_L(E'') - f_R(E')] \Gamma(E', E'') \\ &\quad \times [P_1(E'') \sin \theta - P_3(E'') \cos \theta] \delta(\omega + l\omega_0), \end{aligned}$$

where

$$\begin{aligned} \Gamma(E', E'') &= [D_{R\uparrow}(E') + D_{R\downarrow}(E')] [D_{L\uparrow}(E'') (T_1^2 + T_2^2) \\ &\quad + D_{L\downarrow}(E'') (T_3^2 + T_4^2)]. \end{aligned}$$

Here we may note that the direction of the spin torque is related not only to whether the applied dc bias is positive or negative<sup>3,5,7,8,10</sup> (namely, the direction of the electrical current in a steady state), but also to the frequency of the external ac bias and the Bessel function's order. The time average of CISTT may be defined as  $\langle \tau^{Rx} \rangle = (1/T_0) \int_{-T_0/2}^{T_0/2} dt \tau^{Rx}(t)$ . Then, we get

$$\tau_{aver}^{Rx} = \langle \tau^{Rx} \rangle = \begin{cases} \pi \sum_{mnm'n'} \int dE J_{(mn}^{(m'n')} [f_L(E'') - f_R(E')] \Gamma(E', E'') P(E'', \theta), & l=0 \\ 0, & l \neq 0 \end{cases} \quad (15)$$

where  $l = m + m' - n - n'$ ,  $E'$ ,  $E''$ ,  $J_{(mn}^{(m'n')}$ ,  $P(E'', \theta) = [P_1(E'') \sin \theta - P_3(E'') \cos \theta]$ , and  $\Gamma(E', E'')$  are defined as above.

The  $\theta$  dependence of the spin-transfer torque is similar to that in Refs. 3, 15, 17 and 18. In a colinear case ( $\theta=0$  or  $\pi$ ) and without spin-flip scatterings, the CISTT disappears, even though an ac bias is present. At  $\theta=\pi/2$ , the CISTT tends to a maximum in the presence of no-flip of spins.<sup>22</sup> In this case,  $\omega_0$ ,  $V_{ac}$ , and  $V_0$  can affect the maximum of the torques. In the following discussions, we suppose that  $T_1=T_4$ ,  $T_2=T_3=0$ ,  $V_R=0$  and  $V_L=V_{ac}$ . We shall use  $\tau_{dc}(T=0, \theta) = G_{dc}^\tau(0,0, \theta) \cdot V_0$  as a scale, where  $G_{dc}^\tau(0,0, \theta) = e\pi T_1^2 [D_{R\uparrow}(E_f)$

$+ D_{R\downarrow}(E_f)] [D_{L\uparrow}(E_f) + D_{L\downarrow}(E_f)] P_1(E_f) \sin \theta$  with  $P_1(E_f) = [D_{L\uparrow}(E_f) - D_{L\downarrow}(E_f)] / [D_{L\uparrow}(E_f) + D_{L\downarrow}(E_f)]$ .

First, in order to observe the effect of an ac bias on the CISTT, we present  $V_{ac}$  dependence of the time-dependent spin-transfer torque at different frequency  $\omega_0$  in Fig. 4. It is seen that the amplitude and oscillating frequency of the torque are modulated. As the direction of the spin-transfer torque is related to the direction of the electrical current, it is changed continuously with the ac current. Note that in Fig. 4, a dc voltage is applied simultaneously with an ac bias. When  $V_{ac}$  is small, the oscillating amplitude becomes large, and when  $V_{ac}$  becomes larger, the torque is strongly suppressed. Besides, one may see that a larger  $\omega_0$  gives a small oscillat-

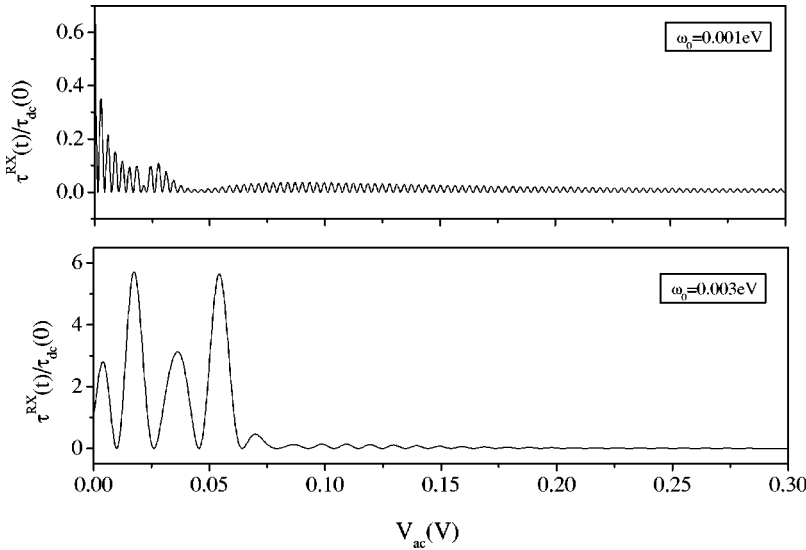


FIG. 4. Time-dependent spin-transfer torque as a function of  $V_{ac}$  under different frequencies, where  $t = 15 \times 10^{-4}$  ns,  $V_0 = -0.05$  V, and the other parameters are assumed to be the same as those in Fig. 1.

ing frequency of the torque with  $V_{ac}$ . The frequency ( $\omega_0$ ) dependence of the time-dependent CISTT at different amplitudes of the ac bias is shown in Fig. 5. It is seen that the torque oscillates with the external frequency, and it appears that the signal of the CISTT is strong in some regime of  $\omega_0$ , and is almost vanishing in other regimes. This feature becomes more evident at a larger  $V_{ac}$ .

The time-averaged spin torque  $\langle \tau^{Rx} \rangle$  is more interesting. In Fig. 6, the  $V_{ac}$  dependence of the time-averaged CISTT is plotted under different frequencies of the ac bias. One may find that the time-averaged CISTT increases slowly with small  $V_{ac}$ , and then decreases rapidly with small oscillations. The CISTT approaches zero when  $V_{ac}$  becomes larger. As shown above, it seems that a large amplitude of the ac bias field may suppress the spin-transfer torque. When  $V_{ac}$  is decreasing,  $\langle \tau^{Rx} \rangle / \tau_{dc}(0)$  approaches asymptotically to a single curve for different frequency  $\omega_0$ . At a given  $\omega_0$ ,  $\langle \tau^{Rx} \rangle / \tau_{dc}(0)$  has a maximum at a specific  $V_{ac}$ . In this case, the time-averaged CISTT,  $\langle \tau^{Rx} \rangle$  (note that we use  $\tau_{aver}^{Rx}$  to denote it in figures hereafter), in the simultaneous presence of an ac bias and a dc bias, can be expressed by that under only a dc bias:

$$\langle \tau^{Rx} \rangle = \sum_{mnm'n'} J_{mn}^{m'n'} \tau_{dc}^{Rx}(V_0 - n' \omega_0 / e, m \omega_0 / e), \quad (16)$$

where the subscript  $dc$  represents the quantity under only a dc bias. It is clear that the Bessel function modulates the amplitude of the torque. From Eq. (12) one may see that a large dc current generates a large torque. In the case under an ac bias, the electrical current might have the similar character. Since the tunneling is viewed as taking place between the modulated levels of ferromagnets, the modulations lead to different transmissions, thus enabling the spin-transfer torque to exhibit various features. This character also manifests itself in Fig. 7, in which the time-averaged spin-transfer torque as a function of the external frequency  $\omega_0$  for different  $V_{ac}$  is shown. It can be found that small frequencies affect the time-averaged CISTT dramatically, manifested explicitly by an approximately linear relation between the CISTT and small  $\omega_0$ . In comparison to Fig. 2, it may be concluded that  $\langle \tau^{Rx}(t) \rangle$  is proportional to the time-averaged electrical current, say  $\langle I_e(t) \rangle$ , and the CISTT can be still induced by the ac electrical current. Note that Eq. (12) was proposed under

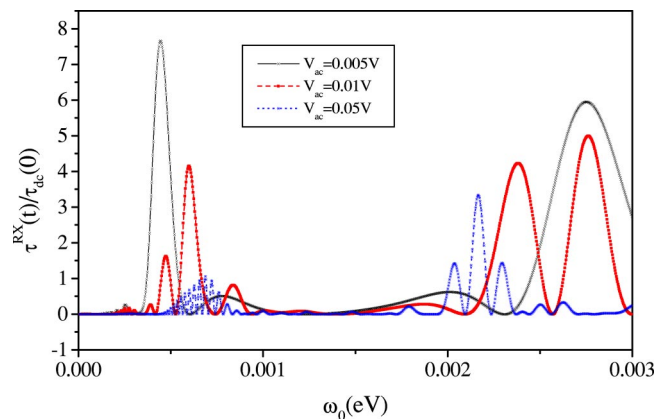


FIG. 5. (Color online) Time-dependent spin-transfer torque as a function of frequency  $\omega_0$  under different  $V_{ac}$ , where  $V_0 = -0.1$  V, and the other parameters are the same as those in Fig. 4.

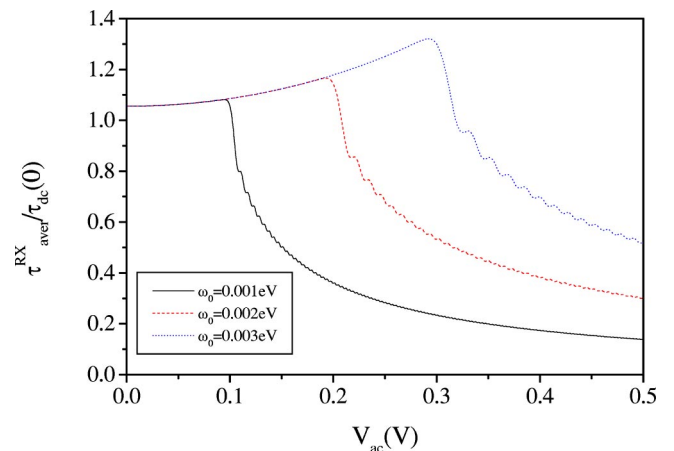


FIG. 6. (Color online)  $V_{ac}$  dependence of the time-averaged spin transfer torque under different frequencies  $\omega_0$ , where the other parameters are the same as those in Fig. 1.

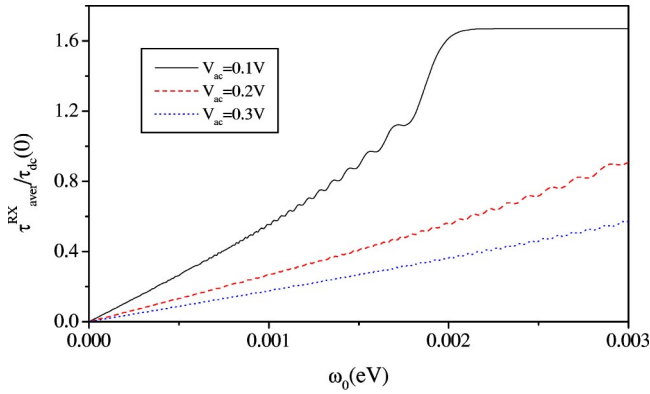


FIG. 7. (Color online) Time-averaged spin transfer torque as a function of  $\omega_0$  for different  $V_{ac}$ , where the other parameters are the same as those in Fig. 1.

an dc bias.<sup>3</sup> Under a small ac signal, the torque almost does not vary in higher frequencies, which is saturated with the magnitude larger than that under only a dc bias.

In above, we have applied simultaneously both the dc and ac biases to the tunnel junction. In the presence of only an ac bias, the time-averaged CISTT as a function of  $V_{ac}$  is given in Fig. 8. It is seen that the time-averaged spin transfer torque first increases sharply and then decreases rapidly with increasing  $V_{ac}$ . In other words, the CISTT takes its larger values in a narrow regime of  $V_{ac}$ . The higher the external frequency  $\omega_0$ , the larger the maximum of the torque. This is because the spin transfer torque is related to the subtraction of two Fermi functions, and the latter is very small when only an ac bias is present. The principal contribution to the result comes from a narrow regime of  $V_{ac}$ . The inset of Fig. 8 gives a plot of  $V_{ac}^p$  versus  $\omega_0$ , where  $V_{ac}^p$  is the specific value of the amplitude of the ac bias at which the peaks of the torque appear. It is to note that  $V_{ac}^p$  is almost proportional to the frequency  $\omega_0$ . Compared to Fig. 6, where the dc and ac biases are present simultaneously, the peaks in Fig. 8 are

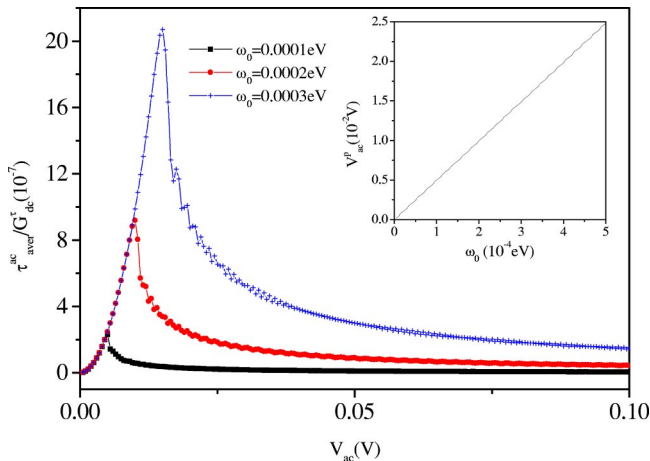


FIG. 8. (Color online) Time-averaged spin transfer torque versus  $V_{ac}$  for different frequencies when only the ac bias is present ( $V_0 = 0$ ). The inset is the particular amplitude,  $V_{ac}^p$ , at which the CISTT is peaked, versus frequency  $\omega_0$ . The parameters are the same as those in Fig. 1.

more sharp. These peaks can be regarded as the evidence of the photon-assisted enhancements of the CISTT. In Fig. 6, the averaged spin transfer torque increases slightly with increasing  $V_{ac}$ , and tends to a maximum, then decreases rapidly. If the dc bias disappears slowly, the spin transfer torque tends to zero when  $V_{ac}$  tends to zero, while the situation in Fig. 6 is not. This reveals that the CISTT exhibits different behaviors under different biases. Our results show that an ac bias could enhance the CISTT at particular values of the amplitude and frequency.

## V. DISCUSSION AND SUMMARY

We have presented the calculation of the response of the electrical tunnel current and the current-induced spin transfer torque to ac and dc biases in FM-I-FM tunnel junctions by means of the time-dependent nonequilibrium Green function technique. In general, when a spin-polarized current is injected into a ferromagnet, the polarized electrons may experience the fields such as the external field, the anisotropy field, the dipolar-dipolar interaction, and the demagnetization field,<sup>19,34</sup> leading to the precession of the polarized electrons around these fields. To conserve the spin angular momentum, the magnetization also precesses around the polarized direction of the injected electrons, giving rise to a spin transfer which is quite different from a magnetic field induced by the current. The ways to realize the switching of magnetization can be either by a magnetic field, or by a magnetic field induced by the current, or by the spin transfer effect.<sup>35</sup> The way through the mechanism of spin transfer offers a convenient and fast choice to switch magnetization, which could be observed from other works and our present study.

In summary, we have investigated the spin-dependent tunneling in the presence of an ac bias applied to a FM-I-FM system by considering the ac tunneling current and the ac CISTT. We have formulated the time-averaged current (time-averaged CISTT) in the form of a summation of dc current (dc CISTT) multiplied by products of Bessel functions with the energy levels shifted by  $m\hbar\omega_0$ . The tunneling current can be viewed as what happens between the photonic sidebands of the two ferromagnets. Our calculation shows that low-frequency ac field suppresses the current, and the electrons may tunnel more easily through the barrier under a high-frequency ac field, similar in character to the response for a capacitance to an ac field in a classical case. It is found that the TMR almost does not vary with an ac bias, which suggests that the ac electric field contributes less to the spin-dependent scatterings. The current-induced spin transfer torque under an ac bias has also been investigated. It has been shown that an ac bias may overall suppress the spin transfer torque, but in a narrow regime of the ac bias, the CISTT is greatly enhanced, characterized by a sharp peak which can be viewed as the photon-assisted enhancement. It has been found that the particular amplitude of the ac bias at which the CISTT shows a peak has a linear relation with the frequency of the ac bias approximately. As a consequence, people could adjust the proper values of the amplitude and



the frequency of the ac bias in accordance with such a linear relation to realize the enhancement of the CISTT, which gives another possible option to enhance the CISTT externally. In addition to using an dc current, our result might give an alternative clue for controlling the local moments of ferromagnets, and also to control the resistance states.

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