Microcavity effect on the electron-hole relative motion in semiconductor quantum wells

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(Received 13 May 2003; published 24 November 2003)

We rigorously find that for a quantum well embedded within a semiconductor microcavity near resonance with the 1*s*-exciton transition, the upper (lower) polariton is associated with an electron-hole relative-motion probability-density distribution larger (smaller) than the two-dimisional exciton Bohr radius if half the normal-mode splitting exceeds the 1*s*-exciton binding energy. In this case, the exciton continuum comprises an *essential* part of the description of the upper mode, leading to a strong asymmetry of the microcavity lines observed in transmission.

DOI: 10.1103/PhysRevB.68.205325 PACS number(s): 73.50.Mx, 42.65.Ky, 72.30.+q

Recently, quantum wells (QWs) embedded within semiconductor microcavities (MCs) have attracted considerable interest due to the formation of spectroscopically pronounced hybrid exciton-photon modes known as *cavity polaritons* (CPs).[1] The two CPs are usually described as arising from the coupling of the crystal-ground-state (CGS)–1*s*-exciton transition dipole moment to the optical resonance of the MC. This picture, however, assumes that once the Schrödinger equation for the exciton's internal motion is solved (yielding the quasi-two-dimensional exciton envelope functions), all states but the energetically lowest (1*s*) may be discarded since the CGS–1*s*-exciton transition has the dominant dipole moment. This transition dipole moment is then coupled to the confined mode of the MC.

This approach, however, neglects the coherent recycling of photons between the various dipole-allowed interband transitions.[2] For a bare QW, indeed, its neglect is seen below to be justified; however, in a high-*Q* MC, most of the photons arising from the interband polarization associated with one exciton internal state are fed back into the system, leading to a photon-mediated radiative renormalization of the optically allowed (i.e., *s*-like) exciton internal states. Thus, the new resonances of the MC system may possess electron-hole (*e*-*h*) relative-motion probability density (henceforth *spatial correlation*) whose spatial extents may differ strongly from the otherwise dominant 1*s* states, i.e., from the two-dimensional (2D) exciton Bohr radius $a_0/2$ (a_0 is the 3D value).[3] In particular, for a near-resonant MC in which half the CP splitting exceeds the binding energy of the 1*s* exciton, the upper (lower) CP may have a spatial extent larger (smaller) than $a_0/2$. This is the *very strong coupling* regime.[2] For typical parameters, the effect on the upper CP is more pronounced.

Below, we present calculations demonstrating this effect. The exactly solvable model captures all the essential features we wish to explore. We begin with a rigorous Green-function (GF) treatment of the *e*-*h* spatial correlations in the MC. Our interest is in the linear optical spectra associated with dipole-allowed transitions from the CGS to exciton states near the bandgap at the Γ point; hence, we begin with the polarization equation of the semiconductor Bloch equations (SBEs) in the low-density limit [see Eq. (12.25) of Ref. 4] coupled to the Maxwell equations, neglecting the carrier densities. Scattering (e.g., carrier-phonon) is accounted for by a single dephasing rate.

We aim to find the interband polarization, the field transmitted through the MC, and the self-consistent electric field due to the interaction of the applied field with the induced polarization. We consider light normally incident on the MC so that the excitons have no in-plane center-of-mass momentum. Also, our treatment applies to N identical QWs in the MC provided that the electromagnetic phase varies negligibly over the N QWs; i.e., the total thickness of the QWs must be much less than the relevant optical wavelength in the medium. We treat a two-band model in the 2D limit, in which all quantities may be evaluated in a closed form. The inclusion of more subbands or a nonzero QW width may necessitate a numerical assault, but does not alter our conclusions.

We denote the weak optical-frequency ω electric field at the QW location $z=0$ as $\mathcal{E}_\omega(z=0)\exp(-i\omega t)$. The induced interband polarization in response to $\mathcal{E}_\omega(0)$ is given by the polarization equation of the SBE in the low-density limit,[4]

$$(\omega-\hat{\mathcal{H}}_{eh})P(\mathbf{r})=-d_{cv}U\mathcal{E}_\omega(0)\,\delta^2(\mathbf{r}), \tag{1}$$

where $\hat{\mathcal{H}}_{eh}=E_g-\nabla^2/(2\mu)-e^2/(\epsilon_0 r)$ is the *e*-*h* Hamiltonian (with E_g the QW band gap, μ the reduced *e*-*h* mass, e the electron charge, ϵ_0 the static dielectric constant, and $\mathbf{r}$ the in-plane *e*-*h* separation), $P(\mathbf{r})$ is related to the spatial-frequency components of the 2D interband polarization $P_{\mathbf{k}_\parallel}$ via $P(\mathbf{r})=(2\pi)^{-2}S\int d^2k_\parallel\, P_{\mathbf{k}_\parallel}\exp(i\mathbf{k}_\parallel\cdot\mathbf{r})$, with S the normalization area, and d_{cv} is the interband dipole matrix element. The 2D interband polarization is $P(\mathbf{r}=0)$; the actual interband polarization that couples to the optical field is $UP(0)$. Note that $P(\mathbf{r})$ is the spatial correlation of the internal motion of the *e*-*h* excitation. The *e*-*h* overlap integral is $U=\int dz f_e(z)f_h(z)$ where $f_e(f_h)$ is the electron (hole) single-particle envelope function of the subband of interest; for QWs that are neither too narrow nor too wide, $U\sim 1$. We emphasize that $\mathcal{E}_\omega(0)$ is the amplitude of the optical-frequency part of the *Maxwell* field at the QW location. In particular, $\mathcal{E}_\omega(0)$ contains contributions from the applied field as well as from the self-field due to reradiation by the interband polarization.

Equation (1) holds both for the case in which a cavity is present and the case in which the cavity is absent. The trick is to express the total optical amplitude $\mathcal{E}_\omega(0)$ at the QW location in terms of the externally applied amplitude. Before doing so, a few remarks concerning the spatial degrees of freedom associated with the e-h motion are in order. Consider the retarded GF $D^{(0)}(\mathbf{r},\mathbf{r}';\omega)$ of Eq. (1). It is defined by

$$(\omega+i0^+-\hat{\mathcal{H}}_{eh})D^{(0)}(\mathbf{r},\mathbf{r}';\omega)=\delta^2(\mathbf{r}-\mathbf{r}'). \tag{2}$$

Thus, we can express the excitonic wave packet and the optical polarization excited by $\mathcal{E}_\omega(0)$ in terms of the Maxwell field as $P(\mathbf{r})=-d_{cv}U\mathcal{E}_\omega(0)D^{(0)}(\mathbf{r},0;\omega)$. Now $D^{(0)}(\mathbf{r},\mathbf{r}';\omega)$ contains all the information about the spectrum of $\hat{\mathcal{H}}_{eh}$:

$$D^{(0)}(\mathbf{r},\mathbf{r}';\omega)=\sum_{nm}\frac{\varphi_{nm}(\mathbf{r})\varphi^*_{nm}(\mathbf{r}')}{\omega-\omega_{nm}+i0^+}. \tag{3}$$

Here $\varphi_{nm}(\mathbf{r})$ and ω_{nm} are the solutions of $\omega_{nm}\varphi_{nm}(\mathbf{r})=\hat{\mathcal{H}}_{eh}\varphi_{nm}(\mathbf{r})$; i.e., $\varphi_{nm}(\mathbf{r})$ is a quasi-2D exciton envelope function[4] of energy ω_{nm}, with n the principal quantum number and m the axial quantum number, including bound and continuum states. We are only considering a pair of subbands; the inclusion of more subbands requires adding subband indices to the wavefunctions φ and to the overlap integral U. Because only s states play a role in the linear optical properties [only $D^{(0)}(\mathbf{r},0;\omega)$ enters into the expressions for $P(\mathbf{r})$], the summation over m collapses to the single value $m=0$. Thus, we require the GF $D_s^{(0)}(r,r';\omega)$ restricted to s states. The desired exact expression has been obtained by Zimmermann.[5] In d dimensions,

$$D_s^{(0)}(r,r';\omega)=\frac{1}{E_0a_0^2}\frac{(2\kappa)^{d-2}}{S_d}\frac{\Gamma(a)}{\Gamma(b)}e^{-\kappa(r+r')/a_0}\times M(a,b,2\kappa r_</a_0)U(a,b,2\kappa r_>/a_0), \tag{4a}$$

$$a=\frac{d-1}{2}-\frac{1}{\kappa},\quad b=d-1,\quad S_d=\frac{2\pi^{d/2}}{\Gamma(d/2)},\quad \kappa=\left(\frac{E_g-\omega-i\gamma}{E_0}\right)^{1/2} \tag{4b}$$

with M and U Kummer functions, $r_<=\min(r,r')$, $r_>=\max(r,r')$, $a_0=\hbar^2\epsilon_0/(e^2\mu)$ the three-dimensional exciton Bohr radius, and $E_0=\hbar^2/(2\mu a_0^2)$ the 3D exciton Rydberg. For the problem at hand, note that $D^{(0)}(\mathbf{r},0;\omega)=D_s^{(0)}(r,0;\omega)$ and $D^{(0)}(0,\mathbf{r}';\omega)=D_s^{(0)}(0,r';\omega)$. Thus, $D_s^{(0)}(r,r',\omega)$ contains precisely the information we desire regarding the *e-h* spatial correlation; it will play a key rôle in the sequel.Finally, we replace the 0^+ in the denominator of Eq. (3) by the nonradiative contribution γ to the homogeneous linewidth, which will be assumed to be state independent.

The electron-hole degrees of freedom are now fully described; it remains to relate these quantities to the electric field. Specifically, we must still relate $\mathcal{E}_\omega(0)$ to the incident field $\mathcal{E}^+_\omega(-z_0)$ impinging from the outside of the MC at $z=-z_0$ and traveling in the $+z$ direction. This brings the Maxwell equations into play. These may be solved via the transfer-matrix approach. The application of the transfer-matrix method to MC problems was introduced in Ref. 6; however, a comprehensive treatment free of several errors appearing in Ref. 6 is given in Ref. 7.

Again, our results are quite general; however, to obtain fully closed-form expressions, we consider a simple model for the MC. That is, the MC is formed from two identical dielectric multilayer mirrors, i.e., distributed Bragg reflectors (DBRs), separated by L. The DBR materials are assumed to be lossless and to have indices of refraction close to $\sqrt{\epsilon_b}$, with ϵ_b (Ref. 8) the background high-frequency dielectric constant in the absence of the excitonic resonances of interest. Near the DBR stop-band center, the mirror reflectivity R is real and the phase varies linearly with frequency, and hence will be neglected. In addition, the mirrors have a transmission coefficient T. Light of frequency ω is incident normally on the MC; the light wave vector is $k_z=\omega\sqrt{\epsilon_b}/c_0$ where c_0 is the *in vacuo* speed of light. The QW is assumed to lie in the center of the MC, though the present treatment applies with little modification to other MC designs.[9]

We define $\chi(\omega)=\Sigma_{\mathrm{QW}}(\omega)D^{(0)}(0,0;\omega)$ associated with exciton resonances in the QW. The radiative self-energy for a bare QW $\Sigma_{\mathrm{QW}}(\omega)$ describes the decay of the interband polarization associated with excitons. It is given by (at $\mathbf{k}_\parallel=0$) $\Sigma_{\mathrm{QW}}(\omega)=-2\pi iN\omega(c_0\sqrt{\epsilon_b})^{-1}d_{cv}^2|U|^2$.[10,11] If the incident electric field is of unit amplitude, $\mathcal{E}^+(z)=\exp(ik_zz)$, the reflected and transmitted field amplitudes R_c and T_c are given by[7]

$$\begin{bmatrix}T_c\\0\end{bmatrix}=\begin{bmatrix}\frac{1}{T^*} & -\frac{R}{T^*}\\ -\frac{R}{T} & \frac{1}{T}\end{bmatrix}\begin{bmatrix}e^{ik_zL/2} & 0\\ 0 & e^{-ik_zL/2}\end{bmatrix}\begin{bmatrix}1+\chi & \chi\\ -\chi & 1-\chi\end{bmatrix}\times\begin{bmatrix}e^{ik_zL/2} & 0\\ 0 & e^{-ik_zL/2}\end{bmatrix}\begin{bmatrix}\frac{1}{T^*} & \frac{R}{T}\\ \frac{R^*}{T^*} & \frac{1}{T}\end{bmatrix}\begin{bmatrix}1\\R_c\end{bmatrix}. \tag{5}$$

The inverse of the 22 component of the product of matrices in Eq. (5) is T_c (Refs. 6 and 7): $T_c=T^2[(1-\chi)e^{-ik_zL}-(1+\chi)R^2e^{ik_zL}-2\chi R]^{-1}$. We focus on a $L=\lambda$ MC, i.e., L is the wavelength of light in the medium corresponding to the bare MC photon mode of interest. In the limit $R\to1$, we have

$$T_c=i\frac{T^2}{4\pi}\frac{\omega_c}{\omega-\omega_c+i\gamma_c}\frac{D_s(0,0;\omega)}{D_s^{(0)}(0,0;\omega)}, \tag{6}$$

where the MC resonant frequency is $\omega_c=2\pi c_0/(\sqrt{\epsilon_b}L)$ and $\gamma_c\approx\omega_c(1-R)/(2\pi)$ is the cavity-mode width.[12–14] The new function $D_s(r,r';\omega)$ is given by

$$D_s(r,r';\omega)=D_s^{(0)}(r,r';\omega)D_s(0,0;\omega)/D_s^{(0)}(0,0;\omega), \tag{7a}$$

$$D_s(0,0;\omega)=D_s^{(0)}(0,0;\omega)/[1-D_s^{(0)}(0,0;\omega)\Sigma(\omega)], \tag{7b}$$

where $\Sigma(\omega)$ is the radiative self-energy for the MC, $\Sigma(\omega)=N\omega_c(2\epsilon_b L)^{-1}d_{cv}^2|U|^2(\omega-\omega_c+i\gamma_c)^{-1}$. It is useful to parametrize $\Sigma(\omega)$ in terms of the vacuum Rabi splitting Ω between the CPs associated with the lowest-lying $(1s)$ exciton $\varphi_{00}(r)$ as $\Sigma(\omega)=(\Omega/2)^2|\varphi_{00}(0)|^{-2}(\omega-\omega_c+i\gamma_c)^{-1}$. Note that $(\Omega/2)^2\propto|\varphi_{00}(0)|^2=8(\pi a_0^2)^{-1}$. In other words, Ω is the vacuum Rabi splitting if only the $1s$ exciton is accounted for. Below we show that the function $D_s(r,r';\omega)$ is the *s*-wave part of the GF for the interband polarization accounting self-consistently for the radiation of the dipole moment and its back-action on the polarization itself, i.e., renormalized by the coupling to the MC electromagnetic field.

The expression for T_c may be back-propagated using the transfer matrices to the MC center to obtain $\mathcal{E}_\omega(0)$:

$$\begin{bmatrix}\mathcal{E}_\omega^+(0)\\ \mathcal{E}_\omega^-(0)\end{bmatrix}=\begin{bmatrix}e^{ik_zL/2} & 0\\ 0 & e^{-ik_zL/2}\end{bmatrix}\begin{bmatrix}\frac{1}{T^*} & \frac{R}{T}\\ \frac{R^*}{T^*} & \frac{1}{T}\end{bmatrix}\begin{bmatrix}0\\ T_c\end{bmatrix}. \tag{8}$$

Here, $\mathcal{E}_\omega^{\pm}(0)$ is the field amplitude at the QW location $z=0$ for the component wave traveling in the same or opposite direction as the incident field, i.e., $\mathcal{E}_\omega(0)=\mathcal{E}_\omega^+(0)+\mathcal{E}_\omega^-(0)$. One finds $\mathcal{E}_\omega(0)=[R\exp(ik_zL/2)+\exp(-ik_zL/2)]T_c/T$. In the $R\to1$ limit for the λ cavity with $\omega\approx\omega_c$, $\mathcal{E}_\omega(0)=-(2/T)T_c\mathcal{E}_\omega^+(-z_0)$ is obtained.

Let us return to Eqs. (1) and (2) in order to understand better the significance of D and Σ. Denote the field in the MC at the QW location *but in the absence of the QW* (Ref. 15) as $\mathcal{E}_\omega^{(0)}(0)$, so that $\mathcal{E}_\omega(0)=\mathcal{E}_\omega^{(0)}(0)+\mathcal{E}_\omega^{\rm self}(0)$ where the self-field associated with coupling to the interband polarization is $\mathcal{E}_\omega^{\rm self}(0)=\mathcal{E}_\omega(0)-\mathcal{E}_\omega^{(0)}(0)$. Using the previous results, we obtain $\mathcal{E}_\omega^{\rm self}(0)=\mathcal{E}_\omega(0)-\mathcal{E}_\omega^{(0)}(0)=-(d_{cv}U)^{-1}\Sigma(\omega)P(0)$. We can therefore recast Eq. (1) as $[\omega-\hat{\mathcal{H}}_{eh}-\Sigma(\omega)\delta^2(\mathbf{r})]P(\mathbf{r})=-d_{cv}U\mathcal{E}_\omega^{(0)}(0)\delta^2(\mathbf{r})$, from which $D_s(r,r';\omega)$, introduced above, is shown to be the *s*-wave part of the GF of

$$[\omega+i0^+-\hat{\mathcal{H}}_{eh}-\Sigma(\omega)\delta^2(\mathbf{r})]D(\mathbf{r},\mathbf{r}';\omega)=\delta^2(\mathbf{r}-\mathbf{r}'). \tag{9}$$

$\Sigma(\omega)\delta^2(\mathbf{r})$ is seen to be the self-energy that accounts for the back action (accounting for the possible presence of a MC) of the radiated field on the polarization. The connection between D_s, $D_s^{(0)}$, and Σ was derived quantum mechanically first by Agranovich and Dubovskii[10] and further developed by Citrin.[11] Equation (9) is to be compared with Eq. (2) which defines $D^{(0)}(\mathbf{r},\mathbf{r}';\omega)$.

Let us contrast the case in which the MC is absent with that in which it is present. In the absence of the MC, $\Sigma(\omega)\to\Sigma_{\rm QW}(\omega)$. The relevant quantity associated with the radiative renormalization of excitonic resonances $\varphi_{n0}^*(0)\Sigma_{\rm QW}(\omega)\varphi_{n'0}(0)$ is typically small (<10 μeV), and varies slowly with ω. In this case, the separate treatment of the *e-h* Coulomb interaction and radiative effects is justified. For the MC, however, $\Sigma(\omega)$ exhibits a resonance at ω_c; both the magnitude of $\varphi_{n0}^*(0)\Sigma(\omega)\varphi_{n'0}(0)$ and its frequency dependence can be substantial—in the range of the binding energy of the ground-state exciton.

Here a brief discussion of the key results is presented beginning with Eq. (7a). This states that the spatial correlations in the MC are the same as those in the QW in the absence of the MC, but with the new energy-dependent weights $D_s(0,0;\omega)/D_s^{(0)}(0,0;\omega)$. Thus, the MC acts as a filter to modify the bare-QW *e-h* spatial correlations that are seen at the MC resonances. Were only the $1s$ state retained, we would have $D_s(r,0;\omega)\propto\exp(-2r/a_0)$; however, for frequencies above E_g, $D_s(r,0;\omega)$ falls off with r as the continuum portion of the GF $D_s^{(0)}(r,0;\omega)$ which is spatially quite flat. Thus, for the MC in which $\Omega/2\gg4E_0$, the lower CP's spatial characteristics will be dominated by the $1s$ exciton, while those of the upper CP will be dominated by the continuum. For the lower CP, as Ω increases, $D_s(r,0;\omega)$ falls off more rapidly due to the absence of bare exciton eigenstates below ω_0.

We consider a MC using parameters achievable for III-V and II-VI semiconductors, with $\mu=0.05m_0$ (m_0 is the free-electron mass), $\gamma=1$ meV, $E_0=2.5$ meV, $a_0=136$ meV, and $\gamma_c=1$ meV. In Fig. 1(a) we show $G(r,\omega)=\ln|D_s^{(0)}(r,0;\omega)/D_s^{(0)}(0,0;\omega)|$ as a contour plot, which gives the frequency dependence of the *e-h* spatial correlation normalized at $\mathbf{r}=0$. Below ω_{00}, $\partial G(r,\omega)/\partial r$ decreases with increasing ω; i.e., the spatial correlation decreases as expected. Near ω_{00} and the other bound-state energies, this quantity increases (the contours in the vertical direction are closely spaced), while well above E_g, the spatial correlation is weak. One notes that there is a significant enhancement of the spatial correlation within the range E_g to $\sim E_g+E_0$ associated with the Sommerfeld factor. Also note the strong departure from exponential decay as a function of r in the vicinity of E_g.

This said, one must use caution in describing a frequency-dependent length scale associated with the spatial correlation. An analysis of the data in Fig. 1(a) shows that the spatial decay is strongly nonexponential except near the $1s$-exciton resonance. Clearly, the contribution of the continuum gives a long tail far after the bound-state contribution has decayed. We find that a fit of the exact formula over several Bohr radii agrees qualitatively with the calculations [Fig. 2(b) of Ref. 2]. Thus, the variational approach of Ref. 2 indeed indicates the important trends insofar as the optical properties are concerned. The single-exponential variational wave function used there may, however, miss important aspects specifically associated with the nonexponential nature of the decay. In particular, probes that are sensitive to the long-range part of the excitation, such as far-infrared absorption, may require the fuller treatment given here.

Figures 1(b)–1(d) show contour plots of the spectral density $A(\omega)=-2\,{\rm Im}\,D_s(0,0;\omega)$ as a function of ω (horizontal axis) and detuning $\Delta=\omega_c-\omega_0$ (vertical axis) for $\Omega=10$

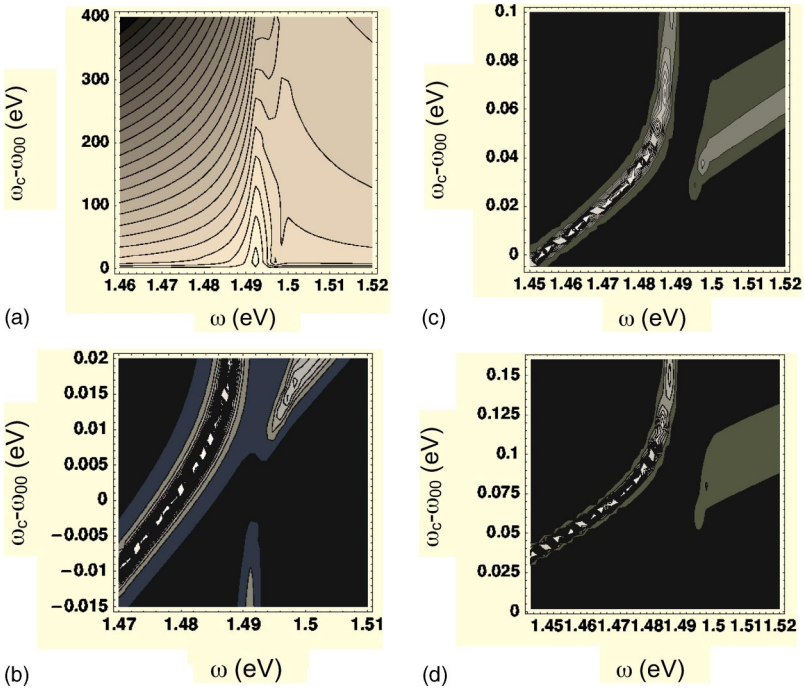

FIG. 1. (Color online) On a logarithmic contour plot $G(r,\omega)$, the r and ω dependence of the spatial e-h correlation normalized to the value at $r=0$. (b)-(d) The spectral density $A(\omega)$ as a function of detuning Δ for various Ω [(b) $\Omega=10$ meV, (c) 20 meV, (d) 30 meV]. The lighter shaded contours indicate higher values in all frames.

meV (b), 20 meV (c), and 30 meV (d). Note that the peak height of the upper CP is attenuated with respect to that of the lower CP. This effect has apparantly been observed in Ref. 9 in the very strong coupling regime, as pointed out in Ref. 2. For increasing Ω, the upper-CP maximum is strongly suppressed.[9] For $\Omega/2>E_0$, the upper CP strongly mirrors the underlying continuum optical density. At negative detunings, the upper CP virtually dissappears [Fig. 1(d)]—a result born out by experiment (see Fig. 2 of Ref. 9). This strong asymmetry of the MC spectrum arises in the absence of any additional scattering mechanism, and is intrinsic to Wannier excitons in QWs. Also worth pointing out, the smallest CP splitting occurs for $\Delta>0$ due to the participation of excitonic states lying above the 1s level.

To what extent are our results comparable to the linear-dispersion theory of Ref. 16? In that work [cf. Eq. (1) of Ref. 16], the transmission through a cavity containing Lorentzian absorbers (atoms) was treated, although there the absorbers were distributed uniformly throughout the cavity. In our case, the QWs are spatially localized in the cavity, and thus give rise to multiple reflections—an effect not accounted for in Ref. 16. Moreover, we assume a highly non-Lorentzian absorber. It should thus be noted that some of the main results, e.g., the mode and normal-mode splittings determined by overall oscillator strength, are not strictly applicable to the MC system treated here. In this case, because the bare QW optical density is spread out over a spectral width exceeding the CP splitting, the rôle of the continuum is enhanced. It is well known that a frequency independent absorption spectrum broadens a Fabry-Perot mode without producing a splitting. In the present case, the optical susceptibility associated with the continuum electron-hole pair states has a logarithmic singularity at the QW band edge, giving rise to a step-shaped absorption spectrum. (The optical-density enhancement associated with the Sommerfeld factor only modifies this argument slightly.) Thus, if the MC splitting with the Lorentzian associated with the 1s exciton pushes a mode into the e-h continuum, a strong broadening rather than splitting is expected to result. The observation that the spectral width of the normal mode depends on the dispersion in the vicinity of the mode [cf. Eq. (7b)], however, applies.

Finally, we turn to Eq. (9) which is satisfied by the GF $D(\mathbf{r},\mathbf{r}';\omega)$. Equation (9) suggests a mechanical interpretation of these effects. The self-energy in space is proportional to $\delta^2(\mathbf{r})$. Thus, the radiative self-energy plays the rôle of a localized impurity, but whose strength is energy dependent. For the lower CP, for which $\omega<\omega_c$, the strength of the effective impurity potential is negative (attractive), while for the upper CP, the strength is positive (repulsive). The following, therefore, are worth noting: Once the e-h Hamiltonian $\hat{\mathcal{H}}_{eh}$ is diagonalized, its solutions $\varphi_{nm}(\mathbf{r})$ still satisfy Eq. (9) at $\mathbf{r}\neq 0$. The additional term proportional to $\Sigma(\omega)$ thus produces energy shifts to the exciton states. Because the e-h Coulomb potential is already singular at $\mathbf{r}=0$, the effective impurity potential does not introduce any states of new *spatial* character (in particular, new bound states) for two or three dimensions (closely connected with the vanishing of the scattering cross section for a point defect); however, due to its energy dependence, it acts as a filter so that the upper-CP resonance may occur at an energy where the states ordinarily play a relatively small spectroscopic rôle (i.e., the exciton continuum).

What systems might exhibit new states arising from $\Sigma(\omega)\delta^2(\mathbf{r})$? One might imagine quenching the singularity of the 2D e-h Coulomb interaction. The quasi-2D Coulomb matrix elements between conduction and valence subbands contain a logarithmic singularity (once the single-particle envelope functions are integrated out), which is not good enough. Nor does the singularity go away in electrically biased or spatially indirect QWs either as long as there is any e-h overlap, as $U=0$ is required for an optically allowed

transition. This is not to say systems in which the Coulomb singularity is suppressed do not exist. In such a system, the modified Coulomb potential will have a minimum; it is then possible for radiatively localized states to exist below the bottom of the potential—a sort of self-trapping. We should also note that for quasi-one-dimensional systems, the potential $\Sigma(\omega)\delta(\mathbf{r})$ will in fact modify the excitonic states themselves, since, in one dimension, the cross section of a point defect does not vanish.

To conclude, we have presented a rigorous model of the *e-h* spatial correlation in MC-embedded QWs under weak optical excitation. The main results are as follows. First, the spatial correlation is that of the bare QW, but filtered by a frequency-dependent function possessing peaks at the CPs. This means that for a resonant MC, the lower-CP spatial correlation is dominated by the $1s$ exciton independent of Ω, while the upper CP is dominated by the exciton continuum if $\Omega/2$ exceeds the $1s$-exciton binding energy $4E_0$. A closely related effect is the prediction of severe broadening of the upper-CP spectral line shape when $\Omega/2>4E_0$. Another manifestation of the change in the upper-CP spatial correlation will be in the far-infrared spectra at normal incidence of MCs excited at their upper CP, since the far-infrared properties at normal incidence at low density are dominated by exciton internal transitions. Second, the radiative self-energy in real space $\Sigma(\omega)\delta^2(\mathbf{r})$ that describes the self-consistent coupling of the interband polarization to the electromagnetic field has the form of a localized-defect potential, but with a frequency-dependent strength. As such, it does not lead to the formation of internal exciton states with a new spatial character, but nevertheless couples them radiatively.

This material is based upon work supported by the the Office of Naval Research and by the National Science Foundation through ECS-0222342 and DMR-0223770.

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