Saturation of dephasing time in mesoscopic devices produced by a ferromagnetic state

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(Received 19 August 2003; revised manuscript received 17 September 2003; published 24 November 2003)

We consider an exchange model of itinerant electrons in a Heisenberg ferromagnet and we assume that the ferromagnet is in a fully polarized state. Using the Holstein-Primakoff transformation we are able to obtain a boson-fermion Hamiltonian that is well known in the interaction between light and matter. This model describes the spontaneous emission in two-level atoms that is the proper decoherence mechanism when the number of modes of the radiation field is taken increasingly large, the vacuum acting as a reservoir. In the same way one can see that the interaction between the bosonic modes of spin waves and an itinerant electron produces decoherence by spin flipping with a rate proportional to the size of the system. In this way we are able to show that the experiments on quantum dots, described by D. P. Pivin *et al.* [Phys. Rev. Lett. **82**, 4687 (1999)], and nanowires, described in D. Natelson *et al.* [Phys. Rev. Lett. **86**, 1821 (2001)], can be understood as the interaction of itinerant electrons and an electron gas in a fully polarized state.

DOI: 10.1103/PhysRevB.68.193413 PACS number(s): 73.23.−b, 75.10.Lp, 75.30.Ds, 03.65.Yz

I. INTRODUCTION

Recent experiments on saturation of dephasing time by lowering the temperature in nanowires[1,2] seem to indicate that magnetic moments are relevant to the understanding of this effect that received a great interest after an experiment by Webb *et al.*[3] In Ref. 1 it has been shown how extremely diluted magnetic impurities can explain saturation in nanowires, even if they are not able to uncover the proper signature of Kondo effect. In Ref. 2 clear evidence for a spin-glass ground state was given. Finally, an experiment by Mohanty and Webb,[4] aimed to prove that the decoherence in nanowires is due to an intrinsic mechanism, definitely has shown that indeed the effect can only be explained by a new mechanism. They reached the aim by freezing all the magnetic impurities with a very high magnetic field and still observing saturation in the dephasing time at very low temperatures. Besides, dependence on the geometry for nanowires was observed in an experiment by Natelson *et al.*[5] where it was seen that decreasing the size of the wire, the saturation of the dephasing time tends to disappear.

Similar experiments in quantum dots have given contrasting results.[6,7] Even if saturation of the dephasing time lowering the temperature is observed in both experiments, in Refs. 7 and 8 no dependence on the number of electrons in the two-dimensional electron gas (2DEG) was claimed but in Refs. 6 and 9 such a dependence was clearly proved. A possible explanation, given in Ref. 10, is that in the former experiment fully chaotic dots were employed, differently from the latter experiment.

The result of Ferry's group is striking and our aim in this paper is to give an explanation for it assuming that the 2DEG was fully polarized. A first hint of this possibility was presented in Ref. 11 but the model that was considered there is too simplified.

The Heisenberg model is essential for the understanding of ferromagnetism and rather well understood.[12,13] Besides, recently, there has been growing evidence, through numerical computations, of the existence of a ferromagnetic phase in a two-dimensional electron gas.[14,15] So, it is a sound question to ask if the effect of a fully polarized state in a ferromagnet can produce decoherence to explain recent experiments on saturation of dephasing time in quantum dots and nanowires. The extension of the model to a spin glass would be straightforward.

The main result we obtain can be stated in the form of the so-called Dicke model that describes the interaction between two-level atoms and several radiation modes.[16,17] When the number of radiation modes is taken increasingly large, the model describes spontaneous emission, a typical decaying effect, but when the radiation modes are very few, Rabi oscillations are observed instead, a coherent effect. So, the changing behavior from the latter to the former can be seen as an example of decoherence and the decaying time can be computed without difficulty.

Similarly, in quantum dots we can have a fully polarized 2DEG and the interaction between the modes of spin waves and an itinerant electron can cause a spin flip by spontaneous emission or absorption of a magnon, provoking the electron to decohere. The interesting result is that, in this case, the rate is directly proportional to the size of the dot as obtained in the experiment of Ferry *et al.*[6] Then, the implication of their findings is that they really observed a fully polarized 2DEG. This same mechanism may be certainly at work in other systems as nanowires, as observed in the recent experiment by Webb and Mohanty[4] and in agreement with the measurements by Natelson *et al.*[5]

The paper is structured in the following way. In Sec. II we present the double exchange model we use, already known in the current literature. In Sec. III we apply the Holstein-Primakoff transformation to bosonic modes and keeping only the leading term in a $1/S$ expansion, we obtain the equivalent Dicke model of the interaction between the spin of an itinerant electron and the magnons. In Sec. IV the rate of spontaneous emission (or absorption) of magnons is computed showing the linear dependence from the size of the dot in agreement with the experiment in Ref. 6 or the size of the nanowire in agreement with the experiment in Ref. 5. In Sec.

V we present a comparison of the theory with the present status of experiments on dephasing in mesoscopic devices. The conclusions are given in Sec. VI.

II. EXCHANGE MODEL

Our aim is to give a realistic model for electrons interacting with a ferromagnetic 2DEG in a quantum dot. The model that we consider is a double exchange model well known in literature[18] and can be described by (here and the following $\hbar=1$)

$$H=H_0+H_h+H_e \tag{1}$$

being

$$H_0=\sum_{\mathbf{p}\sigma} E_{\mathbf{p}} c^{\dagger}_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} \tag{2}$$

the Hamiltonian describing the itinerant electrons. This part of the Hamiltonian will be considered as a small perturbation with respect to the exchange term, assuming the coupling between spins to be larger. This is in order to favor the tendency of the conduction electron to align.[18] So,

$$H_h=-J_h\sum_{\langle ij\rangle} \mathbf{S}_i\cdot\mathbf{S}_j \tag{3}$$

is the Heisenberg term of ferromagnetic type, $J_h>0$, representing the interaction between the spins of the gas. Finally,

$$H_e=J\sum_i \mathbf{S}_i\cdot\mathbf{s}_i \tag{4}$$

is the exchange term (a Kondo term as from the first Hund's rule), being

$$\mathbf{s}_i=\sum_{\alpha\beta} c^{\dagger}_{i\alpha}\mathbf{s}_{\alpha\beta}c_{i\beta} \tag{5}$$

with $\mathbf{s}_{\alpha\beta}$ spin matrices whose components for spin $\frac{1}{2}$ are given by $\boldsymbol{\sigma}_{\alpha\beta}/2$ with $\boldsymbol{\sigma}_{\alpha\beta}$ the Pauli matrices. The sign of the coupling constant J in the exchange term will be determined in the following.

This model can be proved to be equivalent to a Heisenberg model at the leading order in $1/S$ with S being much larger than zero[18] and under the condition that the exchange term is much larger than the Hamiltonian of itinerant electrons. Our aim here is simpler, we want to show how, by emission or absorption of magnons, an electron interacting with a ferromagnet can undergo decoherence on the spin degree of freedom proving that the corresponding rate is proportional to the size of the ferromagnet.

III. FERMION-BOSON MODEL IN A FERROMAGNET

The standard approach with the model we consider, assuming that the electron gas is in a ferromagnetic state (e.g., after a quantum phase transition[19]), is to make a Holstein-Primakoff transformation to bosonize the spin degrees of freedom of the Heisenberg Hamiltonian. So, we put

$$\begin{aligned} S_i^+ &= a_i^{\dagger}(2S-a_i^{\dagger}a_i)^{1/2},\\ S_i^- &= (2S-a_i^{\dagger}a_i)^{1/2}a_i,\\ S_i^+ &= S-a_i^{\dagger}a_i \end{aligned} \tag{6}$$

and we do an expansion with $1/S$ keeping just the leading term. After introducing the Fourier series as

$$f_{\mathbf{k}}=\frac{1}{\sqrt{N}}\sum_i f_i e^{i\mathbf{k}\cdot\mathbf{r}_i} \tag{7}$$

being N the number of sites, we arrive at the following expression, omitting H_0 as assumed initially,

$$\begin{aligned} H' = &-2zNJ_hS^2+\sum_{\mathbf{k}} \epsilon_{\mathbf{k}} a^{\dagger}_{\mathbf{k}}a_{\mathbf{k}}+JS\sum_i s_i^z\\ &+J\sqrt{\frac{S}{2}}\sum_{\mathbf{k}} (a^{\dagger}_{\mathbf{k}}s^-_{\mathbf{k}}+a_{\mathbf{k}}s^+_{\mathbf{k}}) \end{aligned} \tag{8}$$

being $\epsilon_{\mathbf{k}}=2zJ_hS(1-\gamma_{\mathbf{k}})$ and $\gamma_{\mathbf{k}}=1/z\Sigma_{\mathbf{a}}e^{i\mathbf{k}\cdot\mathbf{a}}$ with $\mathbf{a}$ the vector linking two nearest-neighbor spins and z the number of nearest-neighbor spins. It is straightforward to prove that the operators $\Sigma_i s_i^z$, $\sqrt{N}s^+_{\mathbf{k}}$, and $\sqrt{N}s^-_{\mathbf{k}}$ form the algebra of angular momentum.

We recognize at this stage the fermion-boson Hamiltonian typical of radiation-matter interaction generally used in quantum optics (Dicke model).[16,17] The only nontrivial difference is the dependence on $\mathbf{k}$ of the spin operators. Besides, if we take just one mode we can transform the above Hamiltonian into the Jaynes-Cummings form that describes Rabi oscillations proper to a coherent evolution. The presence of more modes makes coherence looser and we can observe decay by emission of a spin-wave mode, that is, a magnon. This is a form of decoherence induced by increasing the number of bosonic modes, with the vacuum acting as a reservoir, interacting with a fermion field.

The spin operators we have identified in this way have the following property on the wave function of the itinerant electron. They can be explicitly written as

$$s^+_{\mathbf{k}}=\frac{1}{\sqrt{N}}\sum_i s_i^+ e^{i\mathbf{k}\cdot\mathbf{r}_i} \tag{9}$$

and similarly for $s^-_{\mathbf{k}}$. So, when they act on the wave function of the itinerant electron they change it to the wave function in the $\mathbf{k}$ space flipping the spin part of it. Then, we can stipulate to work in the $\mathbf{k}$ space looking just at the flipping spin. Thus, instead of itinerant electrons, we have quasiparticles being spin excitations, described by the Hamiltonian

$$H_S=JS\sum_i s_i^z=\frac{JS}{2}\sum_{\mathbf{k}} (c^{\dagger}_{\mathbf{k}\uparrow}c_{\mathbf{k}\uparrow}-c^{\dagger}_{\mathbf{k}\downarrow}c_{\mathbf{k}\downarrow}), \tag{10}$$

interacting with magnons. This is one of the main results of the paper.

Finally, we can pass to the interaction picture and we obtain the following Hamiltonian

$$H_I = J\sqrt{\frac{S}{2}}\sum_{\mathbf{k}} \left(a_{\mathbf{k}}^{\dagger} s_{\mathbf{k}}^{-} e^{i(\epsilon_{\mathbf{k}}-JS)t} + a_{\mathbf{k}} s_{\mathbf{k}}^{+} e^{-i(\epsilon_{\mathbf{k}}-JS)t}\right) \quad (11)$$

and we can immediately identify to the leading order the processes that can induce decoherence, that is, we can have an itinerant electron to flip its spin by emitting a magnon or, being a magnon present, by absorption. We can conclude that the only possible choice for the coupling is $J>0$.

It is important to emphasize that Hamiltonian (11) holds just when the approximations for the Holstein-Primakoff approximation hold and assuming that the Hamiltonian of the itinerant electrons could be neglected at the leading order assuring ferromagnetic or antiferromagnetic ordering.

IV. COMPUTATION OF THE DECOHERENCE TIME

The computation of the decoherence time is straightforward by the Fermi golden rule. We have an itinerant electron interacting with the vacuum of the bosonic modes and this is enough to get the spin flipped by spontaneous emission of a magnon. The emission rate is

$$\Gamma = 2\pi \frac{J^2 S}{2} \sum_{\mathbf{k}} \delta(\epsilon_{\mathbf{k}} - JS) \quad (12)$$

where we have summed on the final states. Changing the sum with an integral we obtain

$$\Gamma = 2\pi \frac{J^2 S}{2} V \int \frac{d^d k}{(2\pi)^d} \delta(\epsilon_{\mathbf{k}} - JS) \quad (13)$$

with V the volume. We realize that it is the phase space that introduces the requested dependence on the size and so, it is crucial to have the possibility to change the sum into an integral. For the experiments with dots and nanowires this approximation is rather good.

Being the Hamiltonian invariant for time reversal, the rate of absorption of a magnon is the same as the rate of spontaneous emission.

At this stage we already have proved the main assertion of the paper. But we can have a more explicit expression by assuming just long-wavelength spin waves with a dispersion relation

$$\epsilon_{\mathbf{k}} = \frac{\mathbf{k}^2}{2m^*} \quad (14)$$

being m^* the effective mass of the magnon given by the Heisenberg Hamiltonian in the Holstein-Primakoff approximation. Then, the integral can be computed, assuming the dimensionality to be two, to give

$$\Gamma_{d=2} = \tfrac{1}{2} V m^* J^2 S \quad (15)$$

or, taking into account that experiment by Ferry *et al.* was done with the density of the 2DEG being constant and varying the geometry, we get

$$\Gamma_{dot} = N m^* \frac{J^2 S}{2 n_{2DEG}} \quad (16)$$

being N the number of electrons in the 2DEG and n_{2DEG} its density. It is easily seen that the results of Fermi liquid theory are recovered by reducing the size of the sample, as found in both the experiments by Ferry *et al.* and Natelson *et al.*, increasing in this way the decoherence time.

The introduction of a magnetic field into the system adds a gap Δ into the dispersion relation of the magnons. In the long-wavelength approximation and two dimensions, the gap plays no role into the computation of the decoherence time.

V. ANALYSIS OF EXPERIMENTS ON SATURATION IN DEPHASING TIME

The experiments on quantum dots[6,7] have the greatest advantage that a direct measurement of the dephasing time is obtained. In other experiments as the one by Natelson *et al.*,[5] using weak localization theory and measuring magnetoresistance, the coherence phase length L_ϕ is measured and then, the dephasing time τ_ϕ is obtained by the relation $L_\phi = \sqrt{D\tau_\phi}$ being D the diffusion constant. So, as a rule, a precise measurement of D should be warranted. But we will assume that this is generally done (for a review about experimental studies see Ref. 20).

The main point here is that the dependence on geometry can be observed if, for more samples, the diffusion constant is always the same. This is exactly what happens in the experiment of Natelson *et al.*[5] These means that, from the point of view of our theory, the comparison is possible and satisfying as already observed in Sec. IV.

Recent measurements by Bird *et al.* on Pt nanowires[21] seem to support both our theoretical findings and the work by Natelson *et al.*[5] But the problem on the diffusion constant can also be found here.[22] So, it seems that if the problem of the diffusion constant is not properly set, a comparison becomes truly difficult.

The paper that started a large number of studies on this matter was that by Mohanty *et al.*[3] From Table I in their paper is it easy to see a large variation of the diffusion constant on all their samples with the possibility that a dependence on geometry as the one we obtained could be masked. But the authors of this paper proved that the saturation of dephasing time is to be considered an intrinsic effect and this is obtained considering also preceding experiments. On this ground we have reconsidered some of these experiments for our aims.

The papers by Lin and Giordano[23,24] reports on AuPd films and wires. The results, the conclusion holds just for films, seem to agree with the more recent paper on three-dimensional (3D) polycrystalline metals[25] where a dependence on geometry is found, but not the same as ours, proving that a different mechanism may be at work in this case. A recent review by Lin *et al.*[26] presents an extended discussion about.

In a paper by Hiramoto *et al.*[27] AlGaAs/GaAs nanowires are considered. The same problem about the diffusion constant can be found but a dependence of τ_ϕ on the electron density is suggested.

We would like to point out that, for a 2D device, we do not expect a dependence on the applied magnetic field as shown in Sec. IV. So, we can conclude that, at the present stage of the experimental situation, there exist hints for a possible ferromagnetic state of the electron gas in a mesoscopic device but a clear experimental research in this direction should be accomplished.

VI. DISCUSSION AND CONCLUSIONS

By an exchange model for itinerant electrons in a ferromagnet we have shown how an effective Hamiltonian can be derived having spin excitations interacting with magnons. This is a typical fermion-boson Hamiltonian as seen in radiation-matter interaction in quantum optics.

The effect of the interaction of spin excitations and magnons, due to spontaneous emission, having the bosonic vacuum as a reservoir, or absorption of magnons can flip the spin causing decoherence.

This model is relevant for the understanding of geometry dependent results seen in the experiments by Ferry *et al.*[6] and Natelson *et al.*[5] We would like to point out that these experimental results give hints for our findings as, e.g., in the Ferry's group experiment,[6] the dependence on the number of electrons in the 2DEG is not seen in all the samples.[9]

It is worth emphasizing that different mechanisms may be at work in other systems such as polycrystalline disordered metals.[25] But the results observed in quantum dots and nanowires seem to point out toward a similar effect originating from polarization of an electron gas.

This means that measurements dependent on geometry should be done extensively to verify our hypothesis. The experimental verification of the existence of a fully polarized electron gas is a striking result itself and then, proving its existence inside samples as quantum dots or nanowires should be considered a breakthrough.

ACKNOWLEDGMENTS

I would like to thank Federico Casagrande for precious help and Jon Bird for very useful information about his experiments on mesoscopic devices.

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