Anderson localization of light in inverted opals

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We describe a photonic crystal disordered by stacking faults which can be realized in the laboratory and which exhibits an Anderson localization of light over certain regions of frequency. The localization length is obtained from a calculation of the transmittance of a slab of the material as a function of the thickness of the slab. The localization length depends on the frequency and the angle of incidence of the incident light. It appears that there are regions of frequency where all states of the electromagnetic field are either localized or delocalized, but we also find regions where localized states coexist with delocalized ones.

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It has been realized for some time that Anderson localization is not restricted to the Schrödinger field of an electron in a disordered solid, that it can happen in a classical wavefield as well; electromagnetic (EM) or elastic waves may become localized in a medium whose dielectric function or elastic coefficients, respectively, vary randomly in space (see, e.g., Ref. 1, and references therein). It has been argued that such classical wavefields can provide a surer testing ground of Anderson localization, because photons and phonons, unlike electrons, are indeed independent excitations, practically free of the correlation effects which are never entirely absent in the case of electrons in solids. We know that in semiconductors localization sets in at energies about the edges of a band. With increasing disorder localization spreads from the edges of the band toward its center, and if the disorder is sufficiently strong all states of the band become localized. By analogy, if we have a photonic crystal which exhibits an absolute frequency gap, it is likely that with the introduction of disorder the states about the band edges will become localized.

According to the scaling theory of localization a small amount of disorder is sufficient to localize all the eigenstates of a one-dimensional (1D) system.[2–4] This may be also true for 2D systems, as shown by numerical simulations in disordered photonic crystals.[5,6] In 3D systems the situation is more complicated and the experimental evidence of photon localization due to disorder is still uncertain.[7–11] In this paper we deal with a system, realizable in the laboratory, which exhibits a specific type of disorder (it remains periodic in two dimensions) and is, therefore, amenable to large-scale calculations of the localization length.

Theory tells us that defect-free inverted opals, which consist of close-packed air spheres in a high-refractive-index material, have absolute frequency gaps,[12,13] and recent improvements in fabrication techniques promise that such can be prepared in the laboratory.[14] Moreover, stacking faults can be introduced in a controlled manner during fabrication,[15] so that an inverted opal with stacking faults appears to be a disordered system for which a direct comparison between theory and experiment can be made. It also exhibits a number of particular features which are quite interesting and worth noting. In previous work[16,17] it has been shown that an absolute frequency gap of an inverted opal, as seen in optical transmission experiments, will in general appear wider in the presence of stacking faults. Here we confirm the existence of an Anderson localization of light in this system and point out aspects of the phenomenon which have not been dealt with previously, to our knowledge.

A fcc (inverted) opal grown in the [111] direction (the z axis) consists, in the absence of disorder, of parallel planes of spheres (layers) in a sequence: ...ABCABC... The layers denoted by A, B, and C are identical and have the same 2D periodicity (a hexagonal lattice of lattice constant a_0) in the xy plane, but each layer is displaced relative to the one preceding it by $\mathbf{d}_+=(\mathbf{d}_\parallel,d)=a_0(1/2,\sqrt{3}/6,\sqrt{6}/3)$. When this sequence is destroyed, as in ...ABC$\underline{\text{B}}$CAB..., we have a stacking fault: the layer $\underline{\text{B}}$ is obtained from C by a displacement $\mathbf{d}_-=(-\mathbf{d}_\parallel,d)$ instead of $\mathbf{d}_+$. In a disordered structure there is a certain (percent) probability, n, that a layer is displaced relative to the one preceding it by $\mathbf{d}_-$ instead of $\mathbf{d}_+$. We note that n lies between 0 and 50%.

The calculation of the transmittance of a slab containing any number of stacking faults can be done efficiently using the method of Ref. 18. The results presented here were obtained using a cutoff in the angular-momentum expansions at $l_{max}=7$, and 37 2D reciprocal-lattice vectors $\mathbf{g}$ in the plane-wave expansions, which ensured a fractional relative accuracy of 10^{-5} for the transmittance of an individual slab.

In the absence of stacking faults the structure we consider is a fcc inverted opal consisting of touching air spheres in silicon (dielectric constant $\epsilon=11.9$). This crystal has an absolute frequency gap extending from $\omega a_0/c=3.452$ to $\omega a_0/c=3.623$, where ω denotes the angular frequency and c is the speed of light in vacuum.[13,16] We have selected two $\mathbf{k}_\parallel=(k_x,k_y)$ points: the center $\bar{\Gamma}$ [$\mathbf{k}_\parallel=(0,0)$] and the vertex $\bar{\text{K}}$ [$\mathbf{k}_\parallel=(4\pi/3a_0,0)$], of the surface Brillouin zone of the (111) plane of the crystal (a regular hexagon). For each of these points we present the frequency bands of the ordered structure in the dimensionless form: $\omega a_0/c$ versus $k_z d/\pi$, where k_z is the component of the Bloch wave vector normal to the (111) surface in the reduced zone: $-\pi/d<k_z\leqslant\pi/d$. We consider the region from $\omega a_0/c=3.0$ to $\omega a_0/c=4.0$ which includes the above-mentioned absolute gap. We then calculate the transmittance of a (111) slab of the material consisting of N_L layers, over the same frequency region,

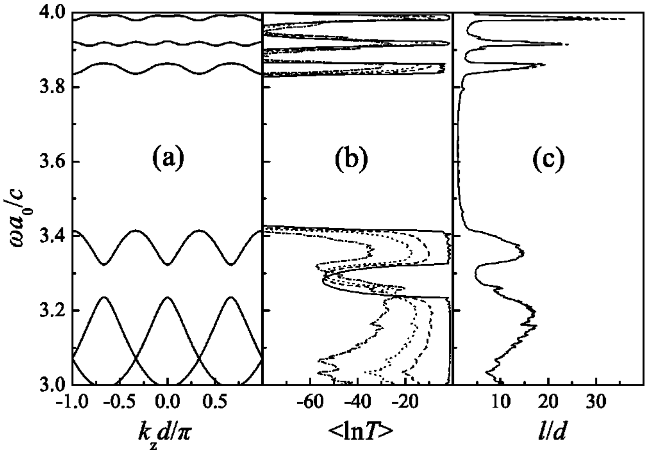

FIG. 1. (a) The frequency band structure of the inverted opal described in the text, for $\mathbf{k}_{\parallel}$ at the $\overline{\mathrm{K}}$ point. (b) $\langle \ln T \rangle$ of s-polarized light with the same $\mathbf{k}_{\parallel}$ incident on a slab of the material, 128 layers thick, with $n=0\%$ (solid line), $n=5\%$ (dashed line), $n=10\%$ (dotted line), and $n=20\%$ (dash-dotted line). (c) l/d for slabs with $n=10\%$ and $N_{\mathrm{L}}=128$ (solid line), and $N_{\mathrm{L}}=256$ (broken line).

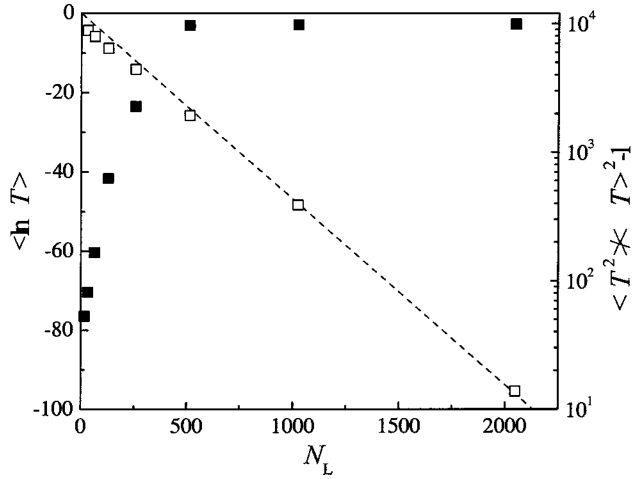

FIG. 2. $\langle \ln T \rangle$ (empty squares) and $\langle T^2 \rangle / \langle T \rangle^2 - 1$ (filled squares) for $\mathbf{k}_{\parallel}$ at the $\overline{\mathrm{K}}$ point for slabs with $n=10\%$ as a function of N_{L}, at $\omega a_0/c=3.9825$.

keeping $\mathbf{k}_{\parallel}$ constant. We do this for a slab of the (ordered) crystal and for slabs with a different degree of disorder, and different thickness. For the sake of simplicity we assume that the silicon host extends to infinity on either side of the slab.

In Fig. 1(a) we show the frequency bands for $\mathbf{k}_{\parallel}$ at the $\overline{\mathrm{K}}$ point. None of these bands is degenerate. In Fig. 1(b), by a solid line we show the logarithm of the transmission coefficient of light of the same $\mathbf{k}_{\parallel}$, for a slab of the crystal consisting of 128 layers. We see that this is practically unity for frequencies within the bands and drops to vanishingly small values for frequencies in the gaps (we note that for the ordered slab there is no need for any averaging and $\langle \ln T \rangle$ means $\ln T$ in this case). The dashed, dotted, and dash-dotted lines in Fig. 1(b) show the ensemble-averaged logarithm of the transmittance, $\langle \ln T \rangle$, for slabs of the same thickness, disordered by a random distribution of stacking faults with $n=5\%$, 10%, and 20%, respectively. Actually, $\langle \ln T \rangle$ of Fig. 1(b), and the same applies to the rest of the figures, has been obtained by averaging over an ensemble of 100 different realizations of random (according to our definition of n) sequences of the given in each case number of layers. The standard error of the mean does not exceed 6% in our calculations.

We see that the transmission of light through the disordered slab of the 128 layers is reduced by many orders of magnitude over the band regions, though not uniformly, and that this reduction increases with disorder. Over the gap regions the transmittance may be larger than that of the corresponding ordered system, due to the introduction of defect states in the gaps, and this naturally increases with disorder. However these states are highly localized and the transmittance remains very small. In order to decide whether localization is at work, we need to calculate a decay length l,

$$l/d=-2N_{\mathrm{L}}/\langle \ln T \rangle, \tag{1}$$

for disordered slabs of different thickness. In a regime of Anderson localization, l will be independent of thickness for N_{L} sufficiently large ($l \ll N_{\mathrm{L}} d$) and we can then refer to it as the localization length (a function of ω independent of N_{L}).[2,3,5] We should remember that in the present case localization occurs only in the z direction (normal to the surface of the slab), and that the wave functions are Bloch waves parallel to the surface of the slab. In Fig. 1(c), we show l versus ω for two slabs with the same amount of disorder ($n=10\%$), but of different thickness: $N_{\mathrm{L}}=128$ for the solid line, and $N_{\mathrm{L}}=256$ for the broken one. We see that the two lines practically coincide at most frequencies which implies that Anderson localization is at work in the present case. However, at some frequencies, e.g., at $\omega a_0/c=3.9825$, the two lines in Fig. 1(c) do not coincide, which suggests a larger localization length. In Fig. 2 we show $\langle \ln T \rangle$ at this frequency as a function of the slab thickness. We observe that the apparent decay length for thin slabs is smaller than the true localization length, but as N_{L} increases, and the criterion $l \ll N_{\mathrm{L}} d$ is satisfied, the exponential decay of the transmittance characteristic of the Anderson localization is established. In this particular case we deduce a localization length of 43 layers. It is worth noting that the variance of the relative transmittance $T/\langle T \rangle$ increases exponentially with the thickness for thin slabs[10] and tends to saturate for slabs much thicker than the localization length (see Fig. 2). We should say, however, that our numerical results show that the variance $\sigma^2=[\langle (\ln T)^2 \rangle - \langle \ln T \rangle^2]/(2N_{\mathrm{L}}d)^2$ does not relate to $1/l$ in the manner $\sigma^2=1/(lN_{\mathrm{L}}d)$, expected from the single-parameter-scaling theory. That this may be so has, of course, been pointed out by others.[19]

The results in Figs. 1(b) and 1(c) were obtained with s-polarized light. We have obtained similar results for p-polarized light. On a logarithmic scale the transmittance for p polarization is practically the same with that of Fig. 1(b). Correspondingly the localization length for p polarization is practically the same as that of Fig. 1(c).

We now consider transmission at normal incidence. In this case the incident light can only be s polarized (in our calcu-

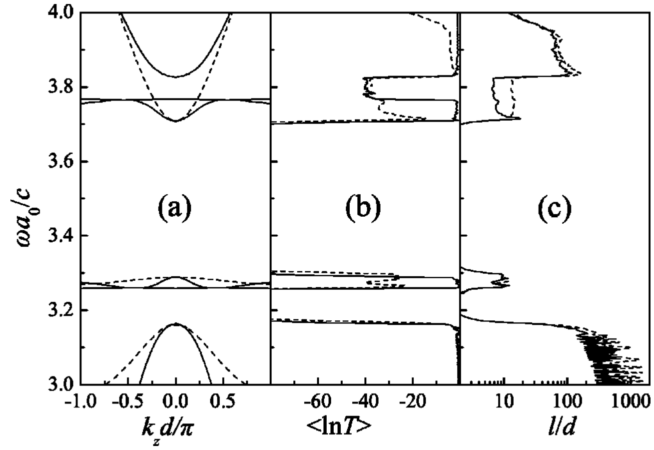

FIG. 3. (a) The frequency band structure of the inverted opal described in the text, for $\mathbf{k}_\parallel = \mathbf{0}$. The solid lines refer to the degenerate bands and the dashed lines to the nondegenerate bands (b) $\langle \ln T \rangle$ of light incident normally on a slab of the material, 128 layers thick, with $n = 0\%$ (solid line) and $n = 20\%$ (dashed line). (c) l/d for slabs with $n = 20\%$ and $N_{\mathrm{L}} = 128$ (solid line), $N_{\mathrm{L}} = 256$ (broken line).

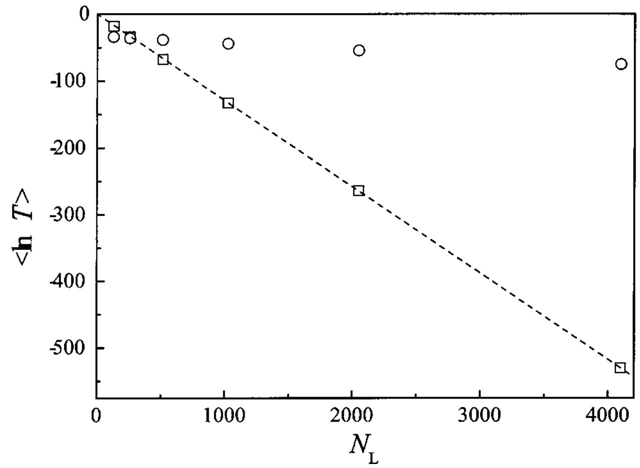

FIG. 4. $\langle \ln T \rangle$ for $\mathbf{k}_\parallel = \mathbf{0}$ for slabs with $n = 20\%$ as a function of N_{L}, at $\omega a_0/c = 3.74$ (circles). The squares show the corresponding results when each stacking-fault displacement $(-\mathbf{d}_\parallel, d)$ is replaced by $(-0.8\mathbf{d}_\parallel, d)$.

lations the electric field is parallel to the x axis). The frequency band structure for $\mathbf{k}_\parallel = \mathbf{0}$ ($\bar{\Gamma}$ point) is shown in Fig. 3(a). The solid lines refer to doubly degenerate bands, the broken lines to the nondegenerate ones. The latter are practically inactive bands: they couple very weakly with the incident light,[20] but they are interesting in relation to localization. In order to explain better what follows it is worth remembering at this point the form of the wavefield corresponding to an eigenmode of the infinite crystal. In the host region between two consecutive layers, this can be expanded into plane waves with wave vectors $\mathbf{K}_{\mathbf{g}}^{\pm} = (\mathbf{k}_\parallel + \mathbf{g}, \pm[\epsilon\omega^2/c^2 - (\mathbf{k}_\parallel + \mathbf{g})^2]^{1/2})$.[18,20] At the $\bar{\Gamma}$ point which interests us here, $\mathbf{k}_\parallel = \mathbf{0}$. For the nondegenerate bands the coefficients $\mathbf{E}_{\mathbf{g}=\mathbf{0}}^{\pm} = \mathbf{0}$ in the plane-wave expansion of the electric field vanish identically for symmetry reasons. On the other hand, a normally incident wave has only one component, corresponding to $\mathbf{g} = \mathbf{0}$. Therefore, most of it will be reflected at the surface of the slab, if there are no bands with $\mathbf{E}_{\mathbf{g}=\mathbf{0}}^{\pm} \neq \mathbf{0}$ at the frequency of the incident wave. Diffraction at the surface will excite only to a very small degree a mode of a nondegenerate band with $\mathbf{E}_{\mathbf{g}=\mathbf{0}}^{\pm} = \mathbf{0}$. This low excitation (the amplitude $\mathbf{E}_{\mathbf{g}\neq\mathbf{0}}^{\pm}$ of the wavefield will be very small compared to that of the incident wave) will be carried to the other side of the slab and, if $\sqrt{\epsilon}\omega/c > |\mathbf{g}|$ for some of the $\mathbf{g} \neq \mathbf{0}$ components, will be partly transmitted beyond it. The transmittance obtained in this way is about 10^{-18} ($\ln T \approx -40$) in a typical case, as shown in Fig. 3(b) in the region about $\omega a_0/c = 3.8$ where only a nondegenerate band which satisfies the above criterion exists. Now it turns out that these bands are almost insensitive to disorder. For example, we have obtained practically the same inactive band above $\omega a_0/c = 3.7$, as the one described by the broken line of Fig. 3(a), for crystals where the period ABC along the [111] direction, which characterizes the fcc structure, is replaced by different periods such as AB, ABAC, etc. This can be understood as follows. For a single layer we obtain eigenstates of the EM field which are highly localized on the layer and decay rapidly to a vanishingly small amplitude on either side of it. When many layers are brought together to build an infinite crystal such states of neighboring layers overlap slightly with each other and generate a narrow band. In a close-packed arrangement, such as the one we consider here, the first-neighbor overlap will be the same, because in any stacking configuration the layers have equivalent positions relative to each other. A difference arises from the interaction between second nearest neighbors which is very weak. Therefore, the inactive bands are insensitive to the stacking sequence because they are essentially determined by the interaction of nearest-neighbor layers, which is the same in all possible stacking configurations. By the same token we should expect the random distribution of layers (implied by stacking faults) not to have a significant effect on the transmission of light by modes of these bands. It is small, about 10^{-18} for the ordered structure, but it will not be reduced much further by disorder. This is clearly demonstrated in Fig. 3(b) for frequencies around $\omega a_0/c = 3.8$. Accordingly, the corresponding l, defined by Eq. (1), increases (practically linearly) with N_{L}, as shown in Fig. 3(c). The above argument suggests that an arbitrary sliding disorder which introduces a variation in the interaction between nearest-neighbor layers will produce a considerable variation in the inactive band, and in a randomly disordered slab the corresponding states will become localized. This is indeed what happens. As we see in Fig. 4, for a disordered slab with an arbitrary sliding disorder the transmittance goes exponentially to zero with increasing thickness in contrast to a disordered slab with stacking faults where the transmittance decreases very slowly with thickness.

We have found that if over a given frequency region degenerate and nondegenerate bands coexist, a random distribution of stacking faults reduces the transmittance through the degenerate bands (this goes exponentially to zero as the thickness increases) but some transmittance remains because of the nondegenerate bands. Therefore, over regions where a nondegenerate band exists, which permits a low but practically insensitive to disorder transmittance to occur, we do not have l as defined by Eq. (1) independent of thickness, as shown in Fig. 3(c). However, localization is at work in the present case reducing the transmittance to the low level permitted by the nondegenerate band. This is a rather interesting situation: it appears that we have at the same frequency localized and delocalized modes of the EM field, and that the two do not interact. This is certainly very different from the commonly accepted rule, that localized and delocalized states in a disordered system cannot exist at the same frequency.[2] We should note that, it is in principle possible that localization occurs for the inactive bands also, but with a localization length much larger than the thicknesses we have considered. A final note in relation to Fig. 3(b): over a narrow region of frequency between $\omega a_0/c=3.250$ and 3.275 we have a degenerate band by itself. There we expect and find a true localization length independent of thickness.

Finally, we would like to comment on the significance of light absorption, which could in principle complicate the experimental proof of Anderson localization.[8–10] Assuming silicon-based inverted opals with a (fcc) lattice constant $a=1.070\ \mu\text{m}$,[14] we find that, in the frequency region we have been considering, the imaginary part of the dielectric constant of crystalline silicon does not exceed 10^{-5} and thus the absorbance is negligible.[21] Therefore, we may say that inverted opals disordered by stacking faults provide an excellent testing ground for studying the localization of light due to disorder.

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