Quantum dynamics of tunneling between ferromagnets

Yu-Li Lee*
Physics Department, National Changhua University of Education, Changhua, Taiwan, Republic of China

Yu-Wen Lee†
Physics Department, Tunghai University, Taichung, Taiwan, Republic of China
(Received 30 June 2003; published 18 November 2003)

We study the Josephson-like spin currents between two ferromagnetic metals by deriving the effective action of the junction. A dc spin Josephson current with the full O(3) symmetry is obtained. We also show that a time-independent uniform magnetic field can serve as the source of the ac spin Josephson effect. That is, the spin current in a uniform magnetic field becomes a periodic function of the time with the period proportional to the inverse of the magnitude of the external magnetic field.

DOI: 10.1103/PhysRevB.68.184413 PACS number(s): 75.47.−m, 73.40.Gk, 74.50.+r

I. INTRODUCTION

One of the striking phenomena about the superconductors is the Josephson effect[1] in the superconducting (SC) tunnel junctions. The Josephson effect arises from the fact that the phases of the SC order parameters of the two superconductors tend to become uniform when they are coupled to each other. A natural question associated with the Josephson effects is what happens when two systems with different types of long range orders are weakly coupled? We shall partly address this question by considering the tunnel junction between two ferromagnets. This is the simplest extension of the SC tunnel junctions because the underlying symmetry behind the ferromagnets is O(3), while the occurrence of the SC long range order is a realization of the spontaneous U(1) symmetry breaking.

In recent years, the possibility of using the spin degrees of freedom in the electronic devices, known as spintronics, receives considerable attention and is a rapidly developing research topic. In this field, the manipulation of the spin current is a subject of extensive investigation. An interesting extreme case of a finite spin current without charge currents has been investigated by several groups.[2–4] Also, the spin transport without dissipation in thin film ferromagnets was discussed recently.[5] In analogy with the SC junctions, a Josephson-like spin current may occur in the ferromagnetic (FM) tunnel junctions. Therefore, the study of the FM junctions is intimately connected with the control of the spin transport.

Indeed, a dc Josephson-like spin current occurring in the FM junctions was predicted recently.[6] However, in Ref. 6, only the effects of the U(1) phase, which corresponds to a subgroup of the full O(3) symmetry, were explored. In the present paper, we will treat the bulk ferromagnets within the framework of the Stoner ferromagnetism[7] and study this problem by taking the effective action approach, parallel to the one in the investigation of the SC junctions.[8] This approach has several advantages. First, the O(3) symmetry is manifestly respected, and thus the effects of other degrees of freedom in addition to the U(1) phase considered in Ref. 6 can be revealed. Next, the roles of the quasiparticles and collective modes, especially the dissipation due to the quasiparticle tunneling, are explicitly disentangled. Moreover, a renormalization group analysis about the relevancy of the tunneling action can be performed. Finally, with the help of the effective action, the calculations of the spin current and its correlation functions become straightforward. Our main results are as follows. (i) We derive an effective action of the FM tunnel junction. (ii) The dc spin Josephson current and current noise are obtained. The latter exhibits the Johnson-Nyquist form at low temperature. (iii) We show that a time-independent uniform magnetic field can serve as the source of the ac spin Josephson effect. That is, the spin current in a uniform magnetic field becomes a periodic function of the time with the period proportional to the inverse of the magnitude of the external magnetic field.

The rest of the paper is organized as follows. The derivation of the effective action of the tunnel junction is given in Sec. II. We calculate the dc and ac spin Josephson currents as well as the current noise in Sec. III. Section IV is devoted to the perturbative renormalization group (RG) analysis of the effective action, where the possible role of the quasiparticle tunneling played in the dissipation is examined. Finally, we discuss our results and experimental implications in the last section.

II. EFFECTIVE ACTION

We start with the action

$$S=\int_0^{\beta} d\tau\left[\int d^d x(\mathcal{L}_l+\mathcal{L}_r)+L_T\right], \tag{1}$$

where d is the spatial dimensions. Here $\mathcal{L}_l$ and $\mathcal{L}_r$ describe the ferromagnetic metals on the left and right of the junction. We adopt the Stoner model for the itinerant ferromagnetism.[7] The corresponding Lagrangian is given by

$$\mathcal{L}_l=\psi_l^{\dagger}\left[\partial_{\tau}-\frac{\nabla^2}{2m_l}-\mu-\Delta_l\boldsymbol{\Omega}_l\cdot\sigma_z\right]\psi_l \tag{2}$$

and a similar expression with $l\to r$. Here $\psi_{l(r)\alpha}$ is the electron operator with spin α on the left (right) side of the junction, $\boldsymbol{\Omega}_{l(r)}$ is a unit vector, and $\Delta_{l(r)}>0$ is proportional to the magnitude of the bulk magnetization on the left (right) of the junction. In Eq. (2), the term proportional to Δ_l^2 is not written down explicitly because it is only related to the determination of Δ_l and is not important for the following discussions. We will treat $\Delta_{l(r)}$ as a given number which is determined, for example, by the mean-field theory, and consider the fluctuations of $\mathbf{\Omega}_{l(r)}$ only. L_T is the tunneling Lagrangian which is of the form

$$L_T = \int_{\substack{\boldsymbol{x}\in l\\ \boldsymbol{x}'\in r}} d^dx d^dx' [T(\boldsymbol{x},\boldsymbol{x}')\psi_l^\dagger(\boldsymbol{x})\psi_r(\boldsymbol{x}') + \text{H.c.}]. \quad (3)$$

We shall follow a procedure similar to Ref. 8 to derive the effective action of the FM tunnel junction.

To proceed, we make the gauge transformation

$$\tilde{\psi}_{l(r)}(x) = g_{l(r)}^\dagger(x)\psi_{l(r)}(x),$$

$$\tilde{\psi}_{l(r)}^\dagger(x) = \psi_{l(r)}^\dagger(x) g_{l(r)}(x), \quad (4)$$

where $x_\mu = (\tau, \boldsymbol{x})$ and $g_{l(r)}$ is an SU(2) matrix which satisfies the relation

$$g_{l(r)}\sigma_z g_{l(r)}^\dagger = \mathbf{\Omega}_{l(r)}\cdot\boldsymbol{\sigma}. \quad (5)$$

Then, $\mathcal{L}_{l(r)}$ and L_T become

$$\mathcal{L}_{l(r)} = \tilde{\psi}_{l(r)}^\dagger\left[\partial_\tau - \frac{\nabla^2}{2m_{l(r)}} - \mu - \Delta_{l(r)}\sigma_z\right]\tilde{\psi}_{l(r)} + \rho_{l(r)\alpha\beta}(g_{l(r)}^\dagger\partial_\tau g_{l(r)})_{\alpha\beta} - i\boldsymbol{J}_{l(r)\alpha\beta}\cdot(g_{l(r)}^\dagger\boldsymbol{\nabla} g_{l(r)})_{\alpha\beta} - \frac{1}{2m_{l(r)}}\rho_{l(r)\alpha\beta}[(g_{l(r)}^\dagger\boldsymbol{\nabla} g_{l(r)})^2]_{\alpha\beta}, \quad (6)$$

$$L_T = \int_{\substack{\boldsymbol{x}\in l\\ \boldsymbol{x}'\in r}} d^dx d^dx' [T(\boldsymbol{x},\boldsymbol{x}')\tilde{\psi}_l^\dagger(\boldsymbol{x}) g_l^\dagger(\boldsymbol{x}) g_r(\boldsymbol{x}')\tilde{\psi}_r(\boldsymbol{x}') + \text{H.c.}], \quad (7)$$

where $\rho_{l(r)\alpha\beta} = \tilde{\psi}_{l(r)\alpha}^\dagger\tilde{\psi}_{l(r)\beta}$ and $\boldsymbol{J}_{l(r)\alpha\beta} = [1/2m_{l(r)}][\tilde{\psi}_{l(r)\alpha}^\dagger \times(-i\boldsymbol{\nabla})\tilde{\psi}_{l(r)\beta} + i\boldsymbol{\nabla}\tilde{\psi}_{l(r)\alpha}^\dagger\tilde{\psi}_{l(r)\beta}]$. By integrating out the fermion fields, the partition function is written as

$$\mathcal{Z} = \int D[g_l^\dagger]D[g_l]D[g_r^\dagger]D[g_r]\exp\{-\mathcal{A}[g,g^\dagger]\},$$

where

$$\mathcal{A}[g,g^\dagger] = -\,\text{tr}[\ln(\underline{\mathcal{G}}^{-1})]. \quad (8)$$

Here we have introduced a four-component fermion space by adding the spinor spaces of the left and right ferromagnets. In particular,

$$\underline{\mathcal{G}}^{-1} = \begin{pmatrix} \hat{\mathcal{G}}_l^{-1} & -\hat{\mathcal{T}} \\ -\hat{\mathcal{T}}^\dagger & \hat{\mathcal{G}}_r^{-1}\end{pmatrix}, \quad (9)$$

$$\hat{\mathcal{T}} = T(\boldsymbol{x},\boldsymbol{x}') g_l^\dagger(x) g_r(x')\,\delta(\tau-\tau'), \quad (10)$$

and

$$\hat{\mathcal{G}}_{l(r)}^{-1} = -\left\{\partial_\tau + g_{l(r)}^\dagger\partial_\tau g_{l(r)} - \frac{1}{2m_{l(r)}}[\boldsymbol{\nabla} + g_{l(r)}^\dagger\boldsymbol{\nabla} g_{l(r)}]^2 - \mu - \Delta_{l(r)}\sigma_z\right]\delta(x-x'). \quad (11)$$

Note that we indicate matrices in the spinor space of one ferromagnet by carets, and matrices acting on the four-component fermion space by underlines.

Now we expand the right-hand side of Eq. (8) in powers of the tunneling matrix elements, namely, the off-diagonal parts of Eq. (9). Keeping the lowest nonvanishing terms, we obtain

$$\mathcal{A} = \mathcal{A}_l + \mathcal{A}_r + \text{tr}[\hat{\mathcal{G}}_l\hat{\mathcal{T}}\hat{\mathcal{G}}_r\hat{\mathcal{T}}^\dagger], \quad (12)$$

where $\mathcal{A}_{l(r)}$ is the bulk action of the left (right) ferromagnet. For simplicity, we shall consider the pointlike junction, i.e., $T(\boldsymbol{x},\boldsymbol{x}') = \tilde{T}\delta(\boldsymbol{x})\delta(\boldsymbol{x}')$. In terms of the parametrization $g_l(x) = h_l(x) g_{0l}$ and a similar expression with $l\to r$, where $g_{0l(r)}$ satisfies the relation $g_{0l(r)}\sigma_z g_{0l(r)}^\dagger = \boldsymbol{n}_{l(r)}\cdot\boldsymbol{\sigma}$ and $\boldsymbol{n}_{l(r)}$ is a unit vector along the direction of the magnetization in the left (right) ferromagnet, we can expand $\mathcal{A}_{l(r)}$ in powers of $h_{l(r)}^\dagger\partial_\mu h_{l(r)}$. [Here $g_{l(r)}$ or $\mathbf{\Omega}_{l(r)}$ are decomposed into two parts: $g_{0l(r)}$ or $\boldsymbol{n}_{l(r)}$, which gives the direction of the magnetization in the bulk, is fixed and the matrix field $h_{l(r)}$ describes the quantum (spin-wave) fluctuations.] Keeping the lowest nonvanishing term and integrating out the degrees of freedom away from the position of the junction, the resulting action is given by

$$\mathcal{A}_l = \mathcal{A}_l^0 + M_l\int_0^\beta d\tau\,\boldsymbol{n}_l\,\text{tr}[\boldsymbol{\sigma}h_l^\dagger\partial_\tau h_l], \quad (13)$$

and the similar expression with $l\to r$. Here $M_{l(r)}$ is the magnetization per volume of the left (right) ferromagnet. The effective action of the ferromagnetic junction is given by the last term in Eq. (12), the last term in Eq. (13), and a similar term with $l\to r$. Before examining the third term in Eq. (12), two points should be mentioned. First, to arrive at Eq. (13), the interactions between the spin waves are neglected. Second, the bulk action of the FM tunnel junction starts with the first-order time derivative due to the Berry phase of quantum spins, whereas for the SC tunnel junction, the bulk action starts with the second-order time derivative.

Working out the third term in Eq. (12), we find that it is given by

$$\mathcal{A}_T = |\tilde{T}|^2\int_0^\beta d\tau_1 d\tau_2\int\frac{d^dp_1}{(2\pi)^d}\frac{d^dp_2}{(2\pi)^d} \times\text{tr}[h_r^\dagger(\tau_1)h_l(\tau_1)\hat{D}_l(\tau_1-\tau_2,\boldsymbol{p}_1) \times h_l^\dagger(\tau_2)h_r(\tau_2)\hat{D}_r(\tau_2-\tau_1,\boldsymbol{p}_2)], \quad (14)$$

where

$$\hat{D}_l(\tau,\boldsymbol{p})=\int d^dx e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} g_{0l}\hat{\mathcal{G}}_l(\tau,\boldsymbol{x})g_{0l}^\dagger$$
$$=G_l(\tau,\boldsymbol{p})\sigma_0-F_l(\tau,\boldsymbol{p})\boldsymbol{n}_l\cdot\boldsymbol{\sigma}, \quad (15)$$

with

$$G_l(\tau,\boldsymbol{p})=\frac{1}{\beta}\sum_n e^{-i\omega_n\tau}\frac{i\omega_n-\epsilon_{l\boldsymbol{p}}+\mu}{(i\omega_n-\epsilon_{l\boldsymbol{p}}+\mu)^2-\Delta_l^2},$$

$$F_l(\tau,\boldsymbol{p})=\frac{1}{\beta}\sum_n e^{-i\omega_n\tau}\frac{\Delta_l}{(i\omega_n-\epsilon_{l\boldsymbol{p}}+\mu)^2-\Delta_l^2}, \quad (16)$$

and a similar expression with $l\to r$. In the above, σ_0 is the 2×2 identity matrix and $\omega_n=(2n+1)\pi T$. With the help of Eq. (15), one may find that $\mathcal{A}_T$ is composed of four terms

$$\mathcal{A}_T=\int_0^\beta d\tau_1 d\tau_2\{\mathcal{I}_1(\tau_1-\tau_2)\mathrm{tr}[\mathcal{M}^\dagger(\tau_1)\mathcal{M}(\tau_2)]$$
$$+\mathcal{I}_2(\tau_1-\tau_2)\mathrm{tr}[(\boldsymbol{n}_r\cdot\boldsymbol{\sigma})\mathcal{M}^\dagger(\tau_1)(\boldsymbol{n}_l\cdot\boldsymbol{\sigma})\mathcal{M}(\tau_2)]$$
$$+\mathcal{I}_3(\tau_1-\tau_2)\mathrm{tr}[\mathcal{M}^\dagger(\tau_1)(\boldsymbol{n}_l\cdot\boldsymbol{\sigma})\mathcal{M}(\tau_2)]$$
$$+\mathcal{I}_4(\tau_1-\tau_2)\mathrm{tr}[(\boldsymbol{n}_r\cdot\boldsymbol{\sigma})\mathcal{M}^\dagger(\tau_1)\mathcal{M}(\tau_2)]\}, \quad (17)$$

where $\mathcal{M}(\tau)=h_l^\dagger(\tau)h_r(\tau)$ and

$$\mathcal{I}_1(\tau)=|\tilde{T}|^2\int\frac{d^dp_1}{(2\pi)^d}\frac{d^dp_2}{(2\pi)^d}\,G_l(\tau,\boldsymbol{p}_1)G_r(-\tau,\boldsymbol{p}_2),$$

$$\mathcal{I}_2(\tau)=|\tilde{T}|^2\int\frac{d^dp_1}{(2\pi)^d}\frac{d^dp_2}{(2\pi)^d}\,F_l(\tau,\boldsymbol{p}_1)F_r(-\tau,\boldsymbol{p}_2),$$

$$\mathcal{I}_3(\tau)=-|\tilde{T}|^2\int\frac{d^dp_1}{(2\pi)^d}\frac{d^dp_2}{(2\pi)^d}\,F_l(\tau,\boldsymbol{p}_1)G_r(-\tau,\boldsymbol{p}_2),$$

$$\mathcal{I}_4(\tau)=-|\tilde{T}|^2\int\frac{d^dp_1}{(2\pi)^d}\frac{d^dp_2}{(2\pi)^d}\,G_l(\tau,\boldsymbol{p}_1)F_r(-\tau,\boldsymbol{p}_2).$$

For $\epsilon_F^{-1}\ll|\tau|\ll T^{-1}$, we have

$$\mathcal{I}_i(\tau)\approx-\alpha_i\frac{\pi^2T^2}{\sin^2(\pi T\tau)},\quad i=1,\ldots,4, \quad (18)$$

where

$$\alpha_1=\frac{1}{4}\,N_l(0)N_r(0)|\tilde{T}|^2,$$

$$\alpha_2=n_l(0)n_r(0)|\tilde{T}|^2,$$

$$\alpha_3=\frac{1}{2}\,n_l(0)N_r(0)|\tilde{T}|^2,$$

$$\alpha_4=\frac{1}{2}\,N_l(0)n_r(0)|\tilde{T}|^2.$$

In the above, $N_{l(r)}(0)=N_{l(r)\uparrow}(0)+N_{l(r)\downarrow}(0)$ and $n_{l(r)}(0)=[N_{l(r)\uparrow}(0)-N_{l(r)\downarrow}(0)]/2$, where $N_{l(r)\sigma}(\epsilon)$ is the density of states for spin σ electrons in the left (right) ferromagnet and $\epsilon=0$ denotes the Fermi surface.

There are two points which should be emphasized. First, the quasiparticle tunneling results in nonlocal terms in $\mathcal{A}_T$. Secondly, compared with the SC tunnel junctions, there are no terms similar to $\cos\phi$, which corresponds to $\mathrm{tr}[\mathcal{M}^2]+\mathrm{H.c.}$ in the present case. In the expansion in powers of the tunneling matrix elements, such a term arises from the nonvanishing anomalous Green functions of electrons. However, in the FM case, the anomalous Green functions of electrons vanish. Consequently, terms such as $\mathrm{tr}[\mathcal{M}^n]+\mathrm{H.c.}$ with some positive integer n cannot appear at any order in the expansion of the tunneling matrix elements. This is a crucial distinction between the SC and FM tunnel junctions. In conclusion, we have derived the effective action of a FM tunnel junction, which is given by

$$\mathcal{A}_{\mathrm{eff}}=\int_0^\beta d\tau\{M_l\boldsymbol{n}_l\cdot\mathrm{tr}[\boldsymbol{\sigma}h_l^\dagger\partial_\tau h_l]+(l\to r)\}+\mathcal{A}_T. \quad (19)$$

III. SPIN JOSEPHSON EFFECT

Now we are able to study the spin currents and noises with the help of the effective action $\mathcal{A}_{\mathrm{eff}}$ [Eq. (19)]. The spin current operator is defined by

$$\boldsymbol{I}=\frac{d\boldsymbol{S}_l}{dt}=-i[\boldsymbol{S}_l,H], \quad (20)$$

where $\boldsymbol{S}_l$ is the spin operator in the left ferromagnet which is defined by $\boldsymbol{S}_l=\int d^dx\frac{1}{2}\psi_l^\dagger(\boldsymbol{x})\boldsymbol{\sigma}\psi_l(\boldsymbol{x})$. (Note that $\boldsymbol{I}=d\boldsymbol{S}_l/dt=-d\boldsymbol{S}_r/dt$ due to the conservation of the total spin.) After straightforward algebra, one may find that

$$\boldsymbol{I}=\int_{\substack{\boldsymbol{x}_1\in l\\ \boldsymbol{x}_2\in r}}d^dx_1d^dx_2\left[T(\boldsymbol{x}_1,\boldsymbol{x}_2)\frac{1}{2i}\psi_l^\dagger(\boldsymbol{x}_1)\boldsymbol{\sigma}\psi_r(\boldsymbol{x}_2)+\mathrm{H.c.}\right]. \quad (21)$$

Therefore, the spin current and its correlation functions can be calculated in terms of the generating functional

$$\mathcal{Z}[\boldsymbol{\eta}]=\int D[u]\exp\left[-S+\int_0^\beta d\tau\;\boldsymbol{\eta}(\tau)\cdot\boldsymbol{I}(\tau)\right], \quad (22)$$

where the integration measure is defined by $D[u]=D[\psi_l^\dagger]D[\psi_l]D[\psi_r^\dagger]D[\psi_r]$, the action S is given by Eq. (1), and $\boldsymbol{\eta}$ is a real source field. Using Eq. (22), the spin current and its two-point correlation function are given by

$$\langle I_a(\tau)\rangle=\frac{1}{\mathcal{Z}[0]}\left.\frac{\delta\mathcal{Z}[\boldsymbol{\eta}]}{\delta\eta_a(\tau)}\right|_{\boldsymbol{\eta}=0}, \quad (23)$$

$$\langle I_a(\tau_1)I_b(\tau_2)\rangle=\frac{1}{\mathcal{Z}[0]}\left.\frac{\delta^2\mathcal{Z}[\boldsymbol{\eta}]}{\delta\eta_b(\tau_2)\delta\eta_a(\tau_1)}\right|_{\boldsymbol{\eta}=0}. \quad (24)$$

The calculation of the generating functional $\mathcal{Z}[\boldsymbol{\eta}]$ is parallel to the derivation of the effective action of the tunnel junction

$\mathcal{A}_{\rm eff}$. The effect of the addition of the source term $\boldsymbol{\eta}(\tau)\cdot\boldsymbol{I}(\tau)$ is to replace $T(\boldsymbol{x}_1,\boldsymbol{x}_2)$ by $T(\boldsymbol{x}_1,\boldsymbol{x}_2)[1+(i/2)\boldsymbol{\sigma}\cdot\boldsymbol{\eta}(\tau)]$. This amounts to replacing $\mathcal{M}(\tau)$ in Eq. (17) by $\mathcal{M}(\tau)+(i/2)\boldsymbol{\eta}(\tau)\cdot h_l^\dagger(\tau)\boldsymbol{\sigma}h_r(\tau)$.

dc Josephson effect. We first compute the spin current. According to Eq. (23), it is given by

$$\langle \boldsymbol{I}(\tau)\rangle=\frac{1}{\mathcal{Z}[0]}\int D[u]e^{-\mathcal{A}_{\rm eff}}\boldsymbol{I}[h_l,h_r;\tau], \tag{25}$$

where $D[u]=D[h_l^\dagger]D[h_l]D[h_r^\dagger]D[h_r]$ and

$$\begin{aligned}\boldsymbol{I}[h_l,h_r;\tau]&\\ =-\frac{i}{2}\int_0^\beta d\tau'\{&\mathcal{I}_1(\tau-\tau')\mathrm{tr}[\mathcal{M}^\dagger(\tau')h_l^\dagger(\tau)\boldsymbol{\sigma}h_r(\tau)]\\ &+\mathcal{I}_2(\tau-\tau')\mathrm{tr}[(\boldsymbol{n}_r\cdot\boldsymbol{\sigma})\mathcal{M}^\dagger(\tau')(\boldsymbol{n}_l\cdot\boldsymbol{\sigma})h_l^\dagger(\tau)\boldsymbol{\sigma}h_r(\tau)]\\ &+\mathcal{I}_3(\tau-\tau')\mathrm{tr}[\mathcal{M}^\dagger(\tau')(\boldsymbol{n}_l\cdot\boldsymbol{\sigma})h_l^\dagger(\tau)\boldsymbol{\sigma}h_r(\tau)]\\ &+\mathcal{I}_4(\tau-\tau')\mathrm{tr}[(\boldsymbol{n}_r\cdot\boldsymbol{\sigma})\mathcal{M}^\dagger(\tau')h_l^\dagger(\tau)\boldsymbol{\sigma}h_r(\tau)]-\mathrm{H.c.}\}.\end{aligned} \tag{26}$$

Within the semiclassical approximation where $h_l(\tau)=\sigma_0=h_r(\tau)$, the spin current becomes

$$\langle \boldsymbol{I}(\tau)\rangle=2(\boldsymbol{n}_r\times\boldsymbol{n}_l)\int_0^\beta d\tau'\mathcal{I}_2(\tau-\tau'). \tag{27}$$

Note that the contributions to the spin current arising from all terms except the second one in $\boldsymbol{I}[h_l,h_r;\tau]$ [Eq. (26)] vanish. Using Eq. (18), the integral in Eq. (27) can be evaluated and the result is

$$\langle \boldsymbol{I}\rangle=\pi T\tau_0\cot(\pi T\tau_0)I_0(\boldsymbol{n}_l\times\boldsymbol{n}_r)\rightarrow I_0(\boldsymbol{n}_l\times\boldsymbol{n}_r), \tag{28}$$

where $I_0=4n_l(0)n_r(0)|\tilde{T}|^2/\tau_0$ is the critical current and $\tau_0\sim\epsilon_F^{-1}$ is an IR cutoff. Equation (28), which is valid only at low temperature, is the analogy of the dc Josephson effect in the SC tunnel junctions. Note that $\langle \boldsymbol{I}\rangle=0$ when the directions of the magnetizations in the left and right ferromagnets are parallel or antiparallel to each other. Moreover, the critical current is proportional to the difference between the densities of states of electrons with spin up and down at the Fermi surface. Therefore, the existence of the spin current we obtained requires that the metals on both sides of the junction must exhibit long range FM orders simultaneously.

ac Josephson effect. For the SC tunnel junctions, an applied dc bias will induce an ac Josephson current. The effect of the dc bias in that case is to make a time-dependent phase rotation or a U(1) gauge transformation on the SC order parameter. In the FM case, an analogous ac spin Josephson effect may be induced by making a spin SU(2) gauge transformation on the magnetization. One of the way to achieve this goal is to add uniform magnetic fields. Based on this observation, we consider the effects of uniform magnetic fields, which is described by the Zeeman term

$$\begin{aligned}H_Z=&-\int_{\boldsymbol{x}\in l}d^dx\boldsymbol{B}_l\cdot\frac{1}{2}\psi_l^\dagger(\boldsymbol{x})\boldsymbol{\sigma}\psi_l(\boldsymbol{x})\\ &-\int_{\boldsymbol{x}\in r}d^dx\boldsymbol{B}_r\cdot\frac{1}{2}\psi_r^\dagger(\boldsymbol{x})\boldsymbol{\sigma}\psi_r(\boldsymbol{x}),\end{aligned} \tag{29}$$

where $\boldsymbol{B}_{l(r)}$ is the external magnetic field exerted on the left (right) ferromagnet. (Here, for simplicity, the constant $g\mu_B$ is absorbed into $\boldsymbol{B}_{l(r)}$, where g is the gyromagnetic ratio and μ_B is the Bohr magneton. Furthermore, the orbital effects are neglected.) Equation (29) can be eliminated by performing the gauge transformation in the real-time formalism

$$\begin{aligned}\psi_{l(r)}&\rightarrow\exp\left\{\frac{i}{2}t\boldsymbol{B}_{l(r)}\cdot\boldsymbol{\sigma}\right\}\psi_{l(r)},\\ \psi_{l(r)}^\dagger&\rightarrow\psi_{l(r)}^\dagger\exp\left\{-\frac{i}{2}t\boldsymbol{B}_{l(r)}\cdot\boldsymbol{\sigma}\right\}.\end{aligned} \tag{30}$$

In the imaginary-time formulation, t and $\boldsymbol{B}_{l(r)}$ in Eq. (30) are replaced by $-i\tau$ and $i\boldsymbol{B}_{l(r)}$, respectively. Under the gauge transformation (30), the tunneling matrix $\hat{\mathcal{T}}$ [Eq. (10)] becomes

$$\hat{\mathcal{T}}\rightarrow T(\boldsymbol{x},\boldsymbol{x}')g_l^\dagger(x)U_l^\dagger(\tau)U_r(\tau)g_r(x')\delta(\tau-\tau'), \tag{31}$$

where $U_{l(r)}(\tau)=\exp\{(i/2)\tau\boldsymbol{B}_{l(r)}\cdot\boldsymbol{\sigma}\}$. After integrating out the fermion fields, we perform the following gauge transformation:

$$\begin{aligned}h_{l(r)}(\tau)&\rightarrow U_{l(r)}^\dagger(\tau)h_{l(r)},\\ h_{l(r)}^\dagger(\tau)&\rightarrow h_{l(r)}^\dagger(\tau)U_{l(r)}(\tau).\end{aligned} \tag{32}$$

Then, the only effect of the external magnetic fields on the effective action of the tunnel junction is that the bulk action [Eq. (13)] turns into

$$\mathcal{A}_l=\mathcal{A}_l^0+M_l\int_0^\beta d\tau\boldsymbol{n}_l\,\mathrm{tr}\left[\boldsymbol{\sigma}h_l^\dagger\left(\partial_\tau-\frac{i}{2}\boldsymbol{B}_l\cdot\boldsymbol{\sigma}\right)h_l\right], \tag{33}$$

and a similar expression with $l\rightarrow r$, and $\mathcal{A}_T$ [Eq. (17)] remains intact. Eq. (33) pins the value of h_l and gives $h_l(\tau)=U_l(\tau)$, and a similar expression with $l\rightarrow r$. Inserting this into Eq. (25), we find that the last two terms in $\boldsymbol{I}[h_l,h_r;\tau]$ [Eq. (26)] vanish after taking the trace, whereas the first term gives a time-independent component to the spin current. By choosing $\boldsymbol{B}_l=\boldsymbol{B}_r\equiv\boldsymbol{B}$, the latter also vanishes. Hereafter, for simplicity, we shall focus on this situation. (We restrict ourselves to the case where the gyromagnetic ratio and the effective mass of electrons are identical for the left and right FM metals.) Under this condition, the spin current arises solely from the second term in $\boldsymbol{I}[h_l,h_r;\tau]$ and the result is

$$\begin{aligned}\langle \boldsymbol{I}(\tau)\rangle/I_0=&\cos(B\tau)\{(\boldsymbol{n}_l\times\boldsymbol{n}_r)-\boldsymbol{e}[\boldsymbol{e}\cdot(\boldsymbol{n}_l\times\boldsymbol{n}_r)]\}\\ &-\sin(B\tau)[\boldsymbol{e}\times(\boldsymbol{n}_l\times\boldsymbol{n}_r)]+\boldsymbol{e}[\boldsymbol{e}\cdot(\boldsymbol{n}_l\times\boldsymbol{n}_r)],\end{aligned} \tag{34}$$

where $\boldsymbol{B}=B\boldsymbol{e}$ and $B=|\boldsymbol{B}|$.

To understand the meaning of the various terms in Eq. (34), we first note that the external magnetic fields will induce a spin current on account of the precession of the magnetization around the axis of the magnetic field, which takes the form $\boldsymbol{I}(\tau)\propto\sin(|\boldsymbol{B}|\tau)[\boldsymbol{n}-(\boldsymbol{n}\cdot\boldsymbol{e})\boldsymbol{e}]+\cos(|\boldsymbol{B}|\tau)(\boldsymbol{e}\times\boldsymbol{n})$, where $\boldsymbol{n}$ and $\boldsymbol{e}$ are the unit vectors along the directions of the magnetization and the magnetic field, respectively. Compared with Eq. (34), one may recognize that the first two terms in Eq. (34) arise from the precession of $\boldsymbol{n}_l\times\boldsymbol{n}_r$ around the axis of the external magnetic field, while the last term is the component of the dc spin Josephson current parallel to the direction of magnetic fields, which does not perform the precession. As a consequence, we will identify the spin current given by Eq. (34) as the ac spin Josephson current. To sum up, we have shown that a time-independent uniform magnetic field can induce the ac spin Josephson effect.

Current noise. Finally, we would like to compute the noise spectrum of the spin current, which can be extracted from the two-point correlation function of the spin current. According to Eq. (24), the latter is given by

$$\langle I_a(\tau_1)I_b(\tau_2)\rangle=\frac{1}{\mathcal{Z}[0]}\int D[u]e^{-\mathcal{A}_{\rm eff}}\times\left\{I_a[h_l,h_r;\tau_1]I_b[h_l,h_r;\tau_2]-\frac{1}{4}K_{ab}[h_l,h_r;\tau_1,\tau_2]\right\},\tag{35}$$

where $D[u]=D[h_l^\dagger]D[h_l]D[h_r^\dagger]D[h_r]$ and

$$\begin{aligned}K_{ab}[h_l,h_r;\tau_1,\tau_2]=&\mathcal{I}_1(\tau_1-\tau_2)\{\mathrm{tr}[h_r^\dagger(\tau_1)\sigma_a h_l(\tau_1)h_l^\dagger(\tau_2)\sigma_b h_r(\tau_2)]+(a\leftrightarrow b,\tau_1\leftrightarrow\tau_2)\}+\mathcal{I}_2(\tau_1-\tau_2)\\&\times\{\mathrm{tr}[(\boldsymbol{n}_r\cdot\boldsymbol{\sigma})h_r^\dagger(\tau_1)\sigma_a h_l(\tau_1)(\boldsymbol{n}_l\cdot\boldsymbol{\sigma})h_l^\dagger(\tau_2)\sigma_b h_r(\tau_2)]+(a\leftrightarrow b,\tau_1\leftrightarrow\tau_2)\}+\mathcal{I}_3(\tau_1-\tau_2)\\&\times\{\mathrm{tr}[h_r^\dagger(\tau_1)\sigma_a h_l(\tau_1)(\boldsymbol{n}_l\cdot\boldsymbol{\sigma})h_l^\dagger(\tau_2)\sigma_b h_r(\tau_2)]+(a\leftrightarrow b,\tau_1\leftrightarrow\tau_2)\}+\mathcal{I}_4(\tau_1-\tau_2)\\&\times\{\mathrm{tr}[(\boldsymbol{n}_r\cdot\boldsymbol{\sigma})h_r^\dagger(\tau_1)\sigma_a h_l(\tau_1)h_l^\dagger(\tau_2)\sigma_b h_r(\tau_2)]+(a\leftrightarrow b,\tau_1\leftrightarrow\tau_2)\}.\end{aligned}$$

In terms of the semiclassical approximation, the connected two-point correlation function of the spin current is given by

$$\begin{aligned}D_{ab}(\tau_1-\tau_2)&\equiv-[\langle I_a(\tau_1)I_b(\tau_2)\rangle-\langle I_a\rangle\langle I_b\rangle]\\&=\delta_{ab}\mathcal{I}_1(\tau_1-\tau_2)+R_{ab}\mathcal{I}_2(\tau_1-\tau_2),\end{aligned}\tag{36}$$

where $R_{ab}=n_{la}n_{rb}+n_{lb}n_{ra}-\delta_{ab}(\boldsymbol{n}_l\cdot\boldsymbol{n}_r)$. Note that the third and fourth terms in $\mathcal{A}_T$ only contribute to the higher order correlation functions of the spin current within the semiclassical approximation.

The noise spectrum is determined by the Fourier transform of the autocorrelation function of the spin current

$$\begin{aligned}S_{ab}(\omega)&\equiv\int_{-\infty}^{\infty}dte^{i\omega t}\frac{1}{2}\langle[I_a(t),I_b(0)]_+\rangle\\&=\frac{1}{2}\ \rho_{ab}(\omega)\coth\left(\frac{\omega}{2T}\right),\end{aligned}\tag{37}$$

where $[,]_+$ denotes the anticommutator. In the above, $\rho_{ab}(\omega)$ is the spectral function of the current-current correlation function, which is related to the Fourier transform of $D_{ab}(\tau)$ via

$$\rho_{ab}(\omega)=-2\ \mathrm{Im}\{\mathcal{D}_{ab}(\omega+i0^+)\},\tag{38}$$

where $\mathcal{D}_{ab}(i\omega_n)=\int_0^\beta d\tau e^{i\omega_n\tau}D_{ab}(\tau)$ and $\mathcal{D}_{ab}(\omega+i0^+)$ is obtained from $\mathcal{D}_{ab}(i\omega_n)$ through the analytical continuation $i\omega_n\to\omega+i0^+$. From Eq. (36), we see that to get $\rho_{ab}(\omega)$, we need the Fourier transforms of $\mathcal{I}_1(\tau)$ and $\mathcal{I}_2(\tau)$ which are, respectively, given by

$$\begin{aligned}\mathcal{J}_1(i\omega_n)&=\frac{|\tilde{T}|^2}{8}\int d\epsilon_1 d\epsilon_2\frac{N_l(\epsilon_1)N_r(\epsilon_2)}{i\omega_n-\epsilon_1+\epsilon_2}\times\left[\tanh\left(\frac{\epsilon_1}{2T}\right)-\tanh\left(\frac{\epsilon_2}{2T}\right)\right],\\ \mathcal{J}_2(i\omega_n)&=\frac{|\tilde{T}|^2}{2}\int d\epsilon_1 d\epsilon_2\frac{n_l(\epsilon_1)n_r(\epsilon_2)}{i\omega_n-\epsilon_1+\epsilon_2}\times\left[\tanh\left(\frac{\epsilon_1}{2T}\right)-\tanh\left(\frac{\epsilon_2}{2T}\right)\right],\end{aligned}\tag{39}$$

where $\mathcal{J}_l(i\omega_n)=\int_0^\beta d\tau e^{i\omega_n\tau}\mathcal{I}_l(\tau)$ with $l=1,2$. At low temperature, i.e., $T\ll\epsilon_F$, the densities of states can be approximated as the ones at the Fermi surface, and we obtain $\mathrm{Im}\{\mathcal{J}_l(\omega+i0^+)\}=-\pi\omega\alpha_l$ with $l=1,2$. Inserting this into Eq. (37) gives

$$S_{ab}(\omega)=(\delta_{ab}\alpha_1+R_{ab}\alpha_2)\pi\omega\coth\left(\frac{\omega}{2T}\right).\tag{40}$$

We see that the noise spectrum of the spin Josephson current takes the Johnson-Nyquist form at low temperature. This result is not surprising because the Johnson-Nyquist form is a direct consequence of a linear circuit, and in our calculations the semiclassical configuration dominates the path (functional) integral. If there is some tunable structure in the junction or the quantum fluctuations are strong, then a deviation from the Johnson-Nyquist form should be expected.

IV. RENORMALIZATION GROUP ANALYSIS

The underlying assumption behind the semiclassical approximation is that $\mathcal{A}_T$ in Eq. (19) is not relevant under the RG flow, and thus we can treat it as a perturbation in the weak tunneling limit. This has to be verified by a renormalization group (RG) analysis.

At zero temperature, $\mathcal{I}_i(\tau)$ with $i=1,\ldots,4$ are of the form

$$\mathcal{I}_i(\tau)\approx-\frac{\alpha_i}{\tau^2},\quad i=1,\ldots,4. \tag{41}$$

Therefore, $\mathcal{A}_T$ is a nonlocal action in the time space. To proceed, we define the following operators:

$$\mathcal{Q}_1(\mathcal{T})=\int d\tau\frac{1}{\tau^2}\,\mathrm{tr}[\mathcal{M}^\dagger(\mathcal{T}+\tau/2)\mathcal{M}(\mathcal{T}-\tau/2)], \tag{42}$$

$$\mathcal{Q}_2(\mathcal{T})=\int d\tau\frac{1}{\tau^2}\,\mathrm{tr}[(\boldsymbol{n}_r\cdot\boldsymbol{\sigma})\mathcal{M}^\dagger(\mathcal{T}+\tau/2)(\boldsymbol{n}_l\cdot\boldsymbol{\sigma})\times\mathcal{M}(\mathcal{T}-\tau/2)], \tag{43}$$

$$\mathcal{Q}_3(\mathcal{T})=\int d\tau\frac{1}{\tau^2}\,\mathrm{tr}[\mathcal{M}^\dagger(\mathcal{T}+\tau/2)(\boldsymbol{n}_l\cdot\boldsymbol{\sigma})\mathcal{M}(\mathcal{T}-\tau/2)], \tag{44}$$

$$\mathcal{Q}_4(\mathcal{T})=\int d\tau\frac{1}{\tau^2}\,\mathrm{tr}[(\boldsymbol{n}_r\cdot\boldsymbol{\sigma})\mathcal{M}^\dagger(\mathcal{T}+\tau/2)\mathcal{M}(\mathcal{T}-\tau/2)], \tag{45}$$

and then, $\mathcal{A}_T$ can be written as

$$\mathcal{A}_T=-\int d\mathcal{T}[\alpha_1\mathcal{Q}_1(\mathcal{T})+\alpha_2\mathcal{Q}_2(\mathcal{T})+\alpha_3\mathcal{Q}_3(\mathcal{T})+\alpha_4\mathcal{Q}_4(\mathcal{T})]. \tag{46}$$

The relevancy of $\mathcal{A}_T$ is determined by the scaling dimensions of these operators at the fixed point described by the action

$$\mathcal{A}_0=\int_0^\beta d\tau\{M_l\boldsymbol{n}_l\,\mathrm{tr}[\boldsymbol{\sigma}h_l^\dagger\partial_\tau h_l]+(l\to r)\}. \tag{47}$$

The scaling dimensions of the operators $\mathcal{Q}_i$'s can be extracted from the long-time behaviors of their two-point correlation functions $\langle\mathcal{Q}_i(\mathcal{T}_1)\mathcal{Q}_i(\mathcal{T}_2)\rangle$. At the fixed point, they are given by

$$\langle\mathcal{Q}_i(\mathcal{T}_1)\mathcal{Q}_i(\mathcal{T}_2)\rangle\sim\frac{1}{(\mathcal{T}_1-\mathcal{T}_2)^{2+4d_M}},\quad i=1,\ldots,4, \tag{48}$$

where d_M is the scaling dimension of $\mathcal{M}$. As a result, the scaling dimensions of the operators $\mathcal{Q}_i$'s are identical and are given by $d_Q=1+2d_M$. By the definition of $\mathcal{M}$, we have $d_M=d_l+d_r$ where $d_{l(r)}$ is the scaling dimension of $h_{l(r)}$. From Eq. (47), we get $d_l=0=d_r$. Therefore, $d_Q=1$, and we conclude that all terms in $\mathcal{A}_T$ are marginal perturbations with respect to $\mathcal{A}_0$.

The other way to study the effects of $\mathcal{A}_T$ is to compute its correction to the free energy.[9] It is given by

$$\Omega=\Omega_0+\Omega_1+\Omega_2+\cdots, \tag{49}$$

where Ω_0 is the unperturbed free energy and

$$\Omega_1=\frac{1}{\beta}\langle\mathcal{A}_T\rangle, \tag{50}$$

$$\Omega_2=-\frac{1}{2\beta}\langle\mathcal{A}_T\mathcal{A}_T\rangle. \tag{51}$$

In Eq. (49), $\cdots$ contains the higher order correlation functions of $\mathcal{A}_T$. At low temperature, we have

$$\Omega_1=-\sum_{i=1}^4\alpha_i\langle\mathcal{Q}_i\rangle, \tag{52}$$

$$\Omega_2=-\frac{1}{2}\sum_{i,j=1}^4\alpha_i\alpha_j\int d\tau\langle\mathcal{Q}_i(\tau)\mathcal{Q}_j(0)\rangle. \tag{53}$$

In the above, the expectation values are evaluated at the fixed point. Because $d_M=0$, one may find that

$$\langle\mathcal{Q}_i\rangle\sim\int d\tau\frac{1}{\tau^2}\sim\xi_\tau^{-1},$$

as the correlation time $\xi_\tau\to\infty$. Since the singular part of the unperturbed free energy behaves as ξ_τ^{-1}, we get $\Omega_1/\Omega_0=O(1)$. This implies that $\mathcal{A}_T$ is a marginal perturbation at the tree level. The next-order corrections to the RG flow arise from Ω_2. Because

$$\langle\mathcal{Q}_i(\tau)\mathcal{Q}_j(0)\rangle=\frac{C_{ij}}{\tau^2}, \tag{54}$$

where C_{ij} is a numerical constant, we also obtain $\Omega_2\sim\xi_\tau^{-1}$, and thus $\Omega_2/\Omega_0=O(1)$. In other words, we verify that, to the second order in α_i, $\mathcal{A}_T$ is a marginal perturbation. Thus, in the weak tunneling limit, the use of the semiclassical approximation to compute the spin current and its correlation functions is justified. Moreover, in contrast to the SC junctions, the quasiparticle tunneling does not destroy the quantum coherence between the two ferromagnets, at least in the weak tunneling limit.

V. CONCLUSIONS AND DISCUSSIONS

In this paper, we have studied the Josephson-like tunneling currents between two FM leads within the framework of Stoner ferromagnetism. In comparison with the previous work, our analysis maintains the full spin SU(2) symmetry in all intermediate steps. More importantly, we clarify the origin of the ac spin Josephson effect by utilizing an SU(2) gauge transformation to probe the nonabelian phase in calculating the ac tunneling spin current. This approach also reveals most clearly the Josephson current as a consequence of spontaneous symmetry breaking.

To obtain the ac spin Josephson current [Eq. (34)], a major simplification we made is that the orbital effects of electrons in the external magnetic fields are neglected. One way to bypass the possible complications due to the coupling between the orbital motion and magnetic fields is to use the FM thin films with an in-plane magnetic field. In that case, the orbital motion in the direction perpendicular to the films will be quenched and its omission is justified.

The explicit form of the ac spin current we obtained [Eq. (34)] should also have important experimental implications: It has been suggested that the spin current without charge currents will induce an electric dipole field.[10–12] Therefore, the measurement of the induced electric field can be used as a detection of the spin Josephson current. In the same way, the ac spin Josephson current we predicted will induce a time-dependent electric field with a period $2\pi/(g\mu_B B)$ where B is the magnitude of the external magnetic field. As a consequence, the detection of a time-dependent electric field with the above period in an applied time-independent uniform magnetic field may provide a convincing evidence of the ac spin Josephson effect.

The importance and advantage of the effective action approach also reveal themselves in clarifying the role of the quasiparticle tunneling played in the dissipation. In the case of the SC junctions, in addition to the bulk sector, the effective action consists of another two terms—a nonlocal term due to the quasiparticle tunneling and a local one arising from the Cooper-pair tunneling.[8] The latter is the origin of the Josephson effect. In that case, the RG analysis indicates that a strong coupling fixed point exists, where the Cooper-pair tunneling term becomes irrelevant.[13] This result has been interpreted as the suppression of the Josephson current or the destruction of the quantum coherence between two superconductors by the quasiparticle tunneling. As we emphasized in Sec. II, the most important distinction between the SC and FM junctions lies in the lack of a pair condensate for the ferromagnets. Consequently, the effective action for the FM junction is scale invariant up to the second-order perturbative RG analysis, where no possible IR instability was found in the perturbation theory. This implies that the spin Josephson effects we obtained are robust against the quasiparticle tunneling, at least for the weak tunneling junctions.

ACKNOWLEDGMENT

Y.L. Lee would like to thank J.C. Wu and Z.H. Wei for discussions. Y.-W. Lee is grateful to M.F. Yang for discussions. The work of Y.L. Lee is supported by the National Science Council of Taiwan under Grant No. NSC 92-2112-M-018-009. The work of Y.-W. Lee is supported by the National Science Council of Taiwan under Grant No. NSC 92-2112-M-029-008.

*Electronic address: yllee@cc.ncue.edu.tw
†Electronic address: ywlee@mail.thu.edu.tw

[1] B.D. Josephson, Phys. Lett. **1**, 251 (1962).
[2] J.E. Hirsch, Phys. Rev. Lett. **83**, 1834 (1999).
[3] A. Brataas, Y. Tserkovnyak, G.E.W. Bauer, and B.I. Halperin, Phys. Rev. B **66**, 060404 (2002).
[4] B. Wang, J. Wang, and H. Guo, Phys. Rev. B **67**, 092408 (2003).
[5] J. König, M. Chr. Bønsager, and A.H. MacDonald, Phys. Rev. Lett. **87**, 187202 (2001).
[6] F.S. Nogueira and K.H. Bennemann, cond-mat/0302528 (unpublished).
[7] E.C. Stoner, Proc. R. Soc. London, Ser. A **154**, 656 (1936).
[8] V. Ambegaokar, U. Eckern, and G. Schön, Phys. Rev. Lett. **48**, 1745 (1982).
[9] For the discussions about the long range interactions, see, for example, J. Cardy, *Scaling and Renormalization in Statistical Physics* (Cambridge University Press, Cambridge, England, 1996).
[10] F. Meier and D. Loss, Phys. Rev. Lett. **90**, 167204 (2003).
[11] F. Scgütz, M. Kollar, and P. Kopietz, Phys. Rev. Lett. **91**, 017205 (2003).
[12] Q.F. Sun, H. Guo, and J. Wang, cond-mat/0301402 (unpublished).
[13] M.P.A. Fisher and W. Zwerger, Phys. Rev. B **32**, 6190 (1985).