Crossed Andreev reflection in structures consisting of a superconductor with ferromagnetic leads

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A theory of crossed Andreev reflection in structures consisting of a superconductor with two ferromagnetic leads is presented. The electric current due to the crossed Andreev reflection strongly depends on the relative orientation of the magnetization of two ferromagnetic leads. It is shown that the dependence of the electric current and magnetoresistance on the distance between two ferromagnetic leads is understood by considering the interference between the wave functions in ferromagnets. The current and magnetoresistance are calculated as functions of the exchange field and height of the interfacial barriers.

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I. INTRODUCTION

Much attention has been focused on the spin-dependent transport through magnetic nanostructures.[1] Tunnel magnetoresistance (TMR) was observed in ferromagnet/ferromagnet (FM/FM) tunnel junctions.[2–5] In ferromagnet/superconductor (FM/SC) tunnel junctions, the current flowing thorough the tunnel junctions is spin polarized.[6] When spin-polarized quasiparticles (QP's) are injected into the SC from the FM, the superconducting gap is suppressed due to the spin accumulation in FM/SC and FM/SC/FM junctions.[7–14] Detailed studies of the spin transport and relaxation in the SC have been done.[15–17]

In recent years, many theoretical and experimental studies in relation to Andreev reflection[18] in FM/SC metallic contacts have been done because the spin polarization of conduction electrons is estimated by measuring the conductance in this system.[19–29] In FM/SC/FM double-junction systems, the coherence length in the SC is extracted by measuring the magnetoresistance (MR).[30,31] In a system consisting of SC's with two ferromagnetic leads FM1 and FM2 (see Fig. 1), there is a novel quantum phenomenon called crossed Andreev reflection:[32–43] When an electron with energy below the superconducting gap in FM1 is injected into a SC, the electron captures an electron in FM2 to form a Cooper pair in the SC. As a result, a hole is created in FM2. Deutscher and Feinberg[33] have discussed crossed Andreev reflection and MR by using the theory of Blonder, Tinkham, and Klapwijk (BTK).[44] They argued that crossed Andreev reflection should occur when the distance between FM1 and FM2 is of the order of or less than the size of the Cooper pairs (the coherence length) and calculated the probability of crossed Andreev reflection in the case that both ferromagnetic leads are half metals and the spatial separation of FM1 and FM2 is neglected (one-dimensional model); i.e., the effect of the distance between two ferromagnetic leads on the crossed Andreev reflection is not incorporated. Subsequently, Falci *et al.*[35] have discussed crossed Andreev reflection and elastic cotunneling in the tunneling limit by using a lowest-order perturbation of the tunneling Hamiltonian. However, to elucidate the effect of crossed Andreev reflection on the spin transport more precisely, it is important to explore how the crossed Andreev reflection depends on the distance between two ferromagnetic leads as well as on the exchange field of FM1 and FM2, for arbitrary transparency of the interface from the metallic limit to the tunneling limit.

In the present paper, we present a theory of crossed Andreev reflection in structures consisting of SC's with two ferromagnetic leads. By extending the BTK theory to this system, we derive an expression of the electric current and calculate the current and MR originating from crossed Andreev reflection. The dependence of the current and MR on the distance (L) between FM1 and FM2 is examined. It is shown that the dependence of the crossed Andreev reflection on the distance L comes from the interference between the wave functions in FM1 and FM2, and the probability decreases rapidly as $(k_F L)^{-3}$ with increasing $k_F L$, but not the coherence length of the SC,[33] where k_F is the Fermi wave number. The current and MR are calculated as functions of the exchange field and height of the interfacial barriers in order to clarify crossed Andreev reflection in the spin transport of the present system.

II. MODEL AND FORMULATION

We consider a system consisting of a superconductor with two ferromagnetic leads (FM1 and FM2) as shown in Fig. 1. FM1 and FM2 with width W_F are connected to a SC with width W_S at $x=0$. The distance between FM1 and FM2 is L. The system we consider is described by the following Bogoliubov–de Gennes (BdG) equation:[45]

$$\begin{pmatrix} H_0-\sigma h_{ex} & \Delta \\ \Delta^* & -(H_0+\sigma h_{ex}) \end{pmatrix} \begin{pmatrix} f_\sigma(\mathbf{r}) \\ g_\sigma(\mathbf{r}) \end{pmatrix} = E \begin{pmatrix} f_\sigma(\mathbf{r}) \\ g_\sigma(\mathbf{r}) \end{pmatrix}, \quad (1)$$

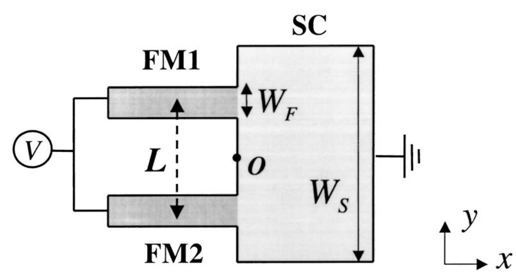

FIG. 1. Schematic diagram of a superconductor (SC) with two ferromagnetic leads (FM1 and FM2). FM1 and FM2 with width W_F are connected to the SC with width W_S at $x=0$. The distance between FM1 and FM2 is L.

where $H_0 \equiv -(\hbar^2/2m)\nabla^2 - \mu_F$ is the single-particle Hamiltonian measured from the Fermi energy μ_F, E is the QP excitation energy, and $\sigma = +\ (-)$ is for the up (down) spin band. The exchange field h_{ex} is given by

$$h_{ex}(\mathbf{r}) = \begin{cases} h_0 & (x<0,|y-L/2|<W_F/2), \\ 0 & (x>0), \\ \pm h_0 & (x<0,|y+L/2|<W_F/2), \end{cases} \tag{2}$$

where $+h_0$ and $-h_0$ represent the exchange fields in FM2 for parallel and antiparallel alignments of the magnetization, respectively. The superconducting gap is expressed as

$$\Delta(\mathbf{r}) = \begin{cases} \Delta & (x>0,|y|<W_S/2), \\ 0 & (x<0). \end{cases} \tag{3}$$

We assume that the temperature dependence of the superconducting gap is given by $\Delta = \Delta_0 \tanh(1.74\sqrt{T_c/T-1})$,[46] where Δ_0 is the superconducting gap at $T=0$ and T_c is the superconducting critical temperature. In order to capture the effect of the interfacial scattering, we employ the following potential at the interfaces, $x=0$:

$$H_B(\mathbf{r}) = H\delta(x)\{\theta_1(y)+\theta_2(y)\}, \tag{4}$$

where $\delta(x)$ is the delta function and $\theta_{1(2)}(y) = \theta(W_F/2 - |y-(+)L/2|)$, $\theta(x)$ being the step function. Throughout this paper, we neglect the impurity scattering in SC's and the proximity effect near the interfaces.[33,47–51]

The solution of the BdG equation in the SC region is given by

$$\Psi_{\pm k_l^+}(\mathbf{r}) = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} e^{\pm i k_l^+ x} \Phi_{\mathrm{SC},l}(y),$$

$$\Psi_{\pm k_l^-}(\mathbf{r}) = \begin{pmatrix} v_0 \\ u_0 \end{pmatrix} e^{\pm i k_l^- x} \Phi_{\mathrm{SC},l}(y), \tag{5}$$

where $\mathbf{r} = (x,y)$, and u_0 and v_0 are the coherence factors,

$$u_0^2 = 1 - v_0^2 = \frac{1}{2}\left[1 + \frac{\sqrt{E^2-\Delta^2}}{E}\right]. \tag{6}$$

For $E<\Delta$, u_0 and v_0 are complex conjugates. $\Phi_{\mathrm{SC},l}(y)$ is the wave function in the y direction,

$$\Phi_{\mathrm{SC},l}(y) = \sqrt{\frac{2}{W_s}} \sin\frac{l\pi}{W_s}\left[y + \frac{W_s}{2}\right], \tag{7}$$

where l is the quantum number which defines the channel. The eigenvalue of the y mode for channel l is

$$E_l = \frac{\hbar^2}{2m}\left(\frac{l\pi}{W_S}\right)^2. \tag{8}$$

The x component of the wave number of an electron (hole) like QP, $k_l^{+(-)}$, is expressed as

$$k_l^{\pm} = \frac{\sqrt{2m}}{\hbar}\sqrt{\mu_F \pm \sqrt{E^2-\Delta^2} - E_l}. \tag{9}$$

In the FM1 (FM2) region, the solutions are given by

$$\Psi_{\pm p_{\sigma,l}^+}(\mathbf{r}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{\pm i p_{\sigma,l}^+ x} \Phi_{\mathrm{FM1(FM2)},l}(y),$$

$$\Psi_{\pm p_{\sigma,l}^-}(\mathbf{r}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{\pm i p_{\sigma,l}^- x} \Phi_{\mathrm{FM1(FM2)},l}(y), \tag{10}$$

where $\Phi_{\mathrm{FM1(FM2)},l}(y)$ is the wave function in the y direction,

$$\Phi_{\mathrm{FM1(FM2)},l}(y) = \sqrt{\frac{2}{W_F}} \sin\frac{l\pi}{W_F}\left[y - (+)\frac{L}{2} + \frac{W_F}{2}\right], \tag{11}$$

and $p_{\sigma,l}^{+(-)}$ is the x component of the wave number of an electron (hole) with σ spin:

$$p_{\sigma,l}^{\pm} = \frac{\sqrt{2m}}{\hbar}\sqrt{\mu_F \pm E \pm \sigma h_{ex} - E_l}. \tag{12}$$

We consider the scattering of an electron with σ spin in channel n injected into the SC from FM1. There are the following six processes: ordinary Andreev reflection and normal reflection at the interface of FM1/SC, crossed Andreev reflection, crossed normal reflection, transmission to the SC as an electronlike QP, and the one as a holelike QP. Therefore, the wave function in each region is expressed as follows: in the FM1 region,

$$\Psi_{\mathrm{FM1}}(\mathbf{r}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{i p_{\sigma,n}^+ x} \Phi_{\mathrm{FM1},n}(y) + \sum_{l=1}^{\infty} \left[a_{\sigma,ln} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i p_{\sigma,l}^- x} + b_{\sigma,ln} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i p_{\sigma,l}^+ x} \right] \Phi_{\mathrm{FM1},l}(y), \tag{13}$$

in the FM2 region,

$$\Psi_{\mathrm{FM2}}(\mathbf{r}) = \sum_{l=1}^{\infty} \left[c_{\sigma,ln} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i q_{\sigma,l}^- x} + d_{\sigma,ln} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i q_{\sigma,l}^+ x} \right] \Phi_{\mathrm{FM2},l}(y), \tag{14}$$

and in the SC region,

$$\Psi_{\mathrm{SC}}(\mathbf{r}) = \sum_{l=1}^{\infty} \left[\alpha_{\sigma,ln} \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} e^{i k_l^+ x} + \beta_{\sigma,ln} \begin{pmatrix} v_0 \\ u_0 \end{pmatrix} e^{-i k_l^- x} \right] \Phi_{\mathrm{SC},l}(y). \tag{15}$$

Here, $p_{\sigma,l}^{\pm}$, $q_{\sigma,l}^{\pm}$, and $k_l^{\pm}$ are the wave numbers in FM1, FM2, and SC, respectively.

The boundary conditions at the interfaces $(x=0)$ are as follows:

$$\Psi_{\mathrm{FM1}}\theta_1(y) + \Psi_{\mathrm{FM2}}\theta_2(y) = \Psi_{\mathrm{SC}}\theta_S(y), \tag{16}$$

$$\frac{d\Psi_{\rm SC}}{dx}\theta_S(y)-\frac{d}{dx}[\Psi_{\rm FM1}\theta_1(y)+\Psi_{\rm FM2}\theta_2(y)] = \frac{2mH}{\hbar^2}[\Psi_{\rm FM1}\theta_1(y)+\Psi_{\rm FM2}\theta_2(y)], \tag{17}$$

where $\theta_S(y)=\theta(W_S/2-|y|)$. From the boundary conditions, the coefficients $a_{\sigma,ln}$, $b_{\sigma,ln}$, $c_{\sigma,ln}$, $d_{\sigma,ln}$, $\alpha_{\sigma,ln}$, and $\beta_{\sigma,ln}$ are determined (see the Appendix)[20,52–54]. The probabilities of Andreev reflection $R^{1,he}_{\sigma,mn}$, normal reflection $R^{1,ee}_{\sigma,mn}$, crossed Andreev reflection $\tilde{R}^{1,he}_{\sigma,mn}$, crossed normal reflection $\tilde{R}^{1,ee}_{\sigma,mn}$, transmission to the SC as an electronlike QP, $T^{1,e'e}_{\sigma,mn}$, and the one as a holelike QP, $T^{1,h'e}_{\sigma,mn}$, are written as

$$R^{1,he}_{\sigma,mn}=\frac{p^-_{\sigma,m}}{p^+_{\sigma,n}}|a_{\sigma,mn}|^2,\quad R^{1,ee}_{\sigma,mn}=\frac{p^+_{\sigma,m}}{p^+_{\sigma,n}}|b_{\sigma,mn}|^2,$$

$$\tilde{R}^{1,he}_{\sigma,mn}=\frac{q^-_{\sigma,m}}{p^+_{\sigma,n}}|c_{\sigma,mn}|^2,\quad \tilde{R}^{1,ee}_{\sigma,mn}=\frac{q^+_{\sigma,m}}{p^+_{\sigma,n}}|d_{\sigma,mn}|^2,$$

$$T^{1,e'e}_{\sigma,mn}=\begin{cases}\dfrac{k^+_{\sigma,m}}{p^+_{\sigma,n}}(u_0^2-v_0^2)|\alpha_{\sigma,mn}|^2, & E>\Delta,\\ 0, & E<\Delta,\end{cases}$$

$$T^{1,h'e}_{\sigma,mn}=\begin{cases}\dfrac{k^-_{\sigma,m}}{p^+_{\sigma,n}}(u_0^2-v_0^2)|\beta_{\sigma,mn}|^2, & E>\Delta,\\ 0, & E<\Delta,\end{cases} \tag{18}$$

where the superscripts e' (h') and 1 in Eq. (18) indicate the electron (hole) like QP in the SC and injection from FM1, respectively.

Let us evaluate the current in FM1. When the bias voltage V is applied to the system (see Fig. 1), the current carried by electrons with σ spin in channel m is given by

$$I^{1,e}_{\sigma,m}=\frac{e}{h}\int_0^\infty[f^{1,e}_{\sigma,m,\rightarrow}(E)-f^{1,e}_{\sigma,m,\leftarrow}(E)]dE, \tag{19}$$

where h is Planck's constant, and $f^{1,e}_{\sigma,m,\rightarrow}(E)$ is the distribution function of an electron with positive group velocity in the x direction and is expressed as

$$f^{1,e}_{\sigma,m,\rightarrow}(E)=f_0(E-eV), \tag{20}$$

where $f_0(E)$ is the Fermi distribution function. The distribution function of electrons with negative group velocity in the x direction, $f^{1,e}_{\sigma,m,\leftarrow}(E)$, is given by

$$\begin{aligned}f^{1,e}_{\sigma,m,\leftarrow}(E)=&\sum_{l=1}^{\infty}[R^{1,ee}_{\sigma,ml}f_0(E-eV)+R^{1,eh}_{\sigma,ml}f_0(E+eV)]\\&+\sum_{l=1}^{\infty}[\tilde{R}^{2,ee}_{\sigma,ml}f_0(E-eV)+\tilde{R}^{2,eh}_{\sigma,ml}f_0(E+eV)]\\&+\sum_{l=1}^{\infty}\frac{v_{S,l}N_{S,l}}{v^\sigma_{F,m}N^\sigma_{F,m}}[T^{1,ee'}_{\sigma,ml}+T^{1,eh'}_{\sigma,ml}]f_0(E),\end{aligned} \tag{21}$$

where $v_{S,l}$ and $v^\sigma_{F,l}$ are the group velocity of an electron in channel l in the SC and the one with σ spin in channel l in FM1, respectively, $N_{S,l}$ and $N^\sigma_{F,l}$ are the density of states in channel l in the SC and the one of the σ spin band in channel l in FM1, respectively. Using the relations

$$R^{1,ee(eh)}_{\sigma,ml}=R^{1,ee(he)}_{\sigma,lm},\quad \tilde{R}^{2,ee(eh)}_{\sigma,ml}=\tilde{R}^{1,ee(he)}_{\sigma,lm},$$
$$v_{S,l}N_{S,l}T^{1,ee'(eh')}_{\sigma,ml}=v^\sigma_{F,m}N^\sigma_{F,m}T^{1,e'e(h'e)}_{\sigma,lm} \tag{22}$$

and the conservation law of the probability,

$$\sum_{l=1}^{\infty}[(R^{1,ee}_{\sigma,lm}+R^{1,he}_{\sigma,lm}+\tilde{R}^{1,ee}_{\sigma,lm}+\tilde{R}^{1,he}_{\sigma,lm})+(T^{1,e'e}_{\sigma,lm}+T^{1,h'e}_{\sigma,lm})]=1, \tag{23}$$

we obtain

$$\begin{aligned}I^{1,e}_{\sigma,m}=\frac{e}{h}\sum_{l=1}^{\infty}\int_0^\infty&[(R^{1,he}_{\sigma,lm}+\tilde{R}^{1,he}_{\sigma,lm})[f_0(E)-f_0(E+eV)]\\&+(1-R^{1,ee}_{\sigma,lm}-\tilde{R}^{1,ee}_{\sigma,lm})[f_0(E-eV)-f_0(E)]]dE.\end{aligned} \tag{24}$$

The current carried by holes with σ spin in channel m in FM1, $I^{1,h}_{\sigma,m}$, and the currents carried by electrons and holes in FM2, $I^{2,e}_{\sigma,m}$ and $I^{2,h}_{\sigma,m}$, respectively, are calculated in a similar way as

$$\begin{aligned}I^{1,h}_{\sigma,m}=\frac{e}{h}\sum_{l=1}^{\infty}\int_0^\infty&[(R^{1,eh}_{\sigma,lm}+\tilde{R}^{1,eh}_{\sigma,lm})[f_0(E-eV)-f_0(E)]\\&+(1-R^{1,hh}_{\sigma,lm}-\tilde{R}^{1,hh}_{\sigma,lm})[f_0(E)-f_0(E+eV)]]dE,\end{aligned} \tag{25}$$

$$\begin{aligned}I^{2,e}_{\sigma,m}=\frac{e}{h}\sum_{l=1}^{\infty}\int_0^\infty&[(R^{2,he}_{\sigma,lm}+\tilde{R}^{2,he}_{\sigma,lm})[f_0(E)-f_0(E+eV)]\\&+(1-R^{2,ee}_{\sigma,lm}-\tilde{R}^{2,ee}_{\sigma,lm})[f_0(E-eV)-f_0(E)]]dE,\end{aligned} \tag{26}$$

$$\begin{aligned}I^{2,h}_{\sigma,m}=\frac{e}{h}\sum_{l=1}^{\infty}\int_0^\infty&[(R^{2,eh}_{\sigma,lm}+\tilde{R}^{2,eh}_{\sigma,lm})[f_0(E-eV)-f_0(E)]\\&+(1-R^{2,hh}_{\sigma,lm}-\tilde{R}^{2,hh}_{\sigma,lm})[f_0(E)-f_0(E+eV)]]dE.\end{aligned} \tag{27}$$

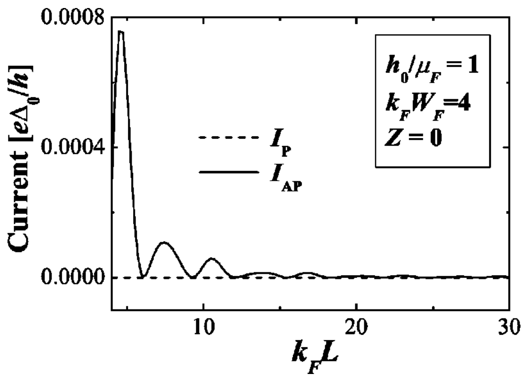

FIG. 2. The current as a function of L. FM1 and FM2 are half metals ($h_0/\mu_F=1$). The solid and dashed lines are for the currents in the antiparallel and parallel alignment of the magnetizations, respectively.

By using Eqs. (24)–(27), we obtain the total current in the system:

$$I=\sum_{\sigma,m}[I^{1,e}_{\sigma,m}+I^{1,h}_{\sigma,m}+I^{2,e}_{\sigma,m}+I^{2,h}_{\sigma,m}]. \tag{28}$$

We define the magnetoresistance as

$$\mathrm{MR}\equiv\frac{R_{\mathrm{AP}}-R_{\mathrm{P}}}{R_{\mathrm{P}}}=\frac{I_{\mathrm{P}}-I_{\mathrm{AP}}}{I_{\mathrm{AP}}}, \tag{29}$$

where $R_{\mathrm{P(AP)}}=V/I_{\mathrm{P(AP)}}$ is the resistance in parallel (antiparallel) alignment.

III. RESULTS

In the following calculation, we take the temperature, the applied bias voltage, the width of the SC, and the superconducting order parameter to be $T/T_c=0.01$, $eV/\Delta_0=0.01$, $W_S=1000/k_F$, and $\mu_F/\Delta_0=200$, respectively, where k_F is the Fermi wave number. First, we consider the case that FM1 and FM2 are half metals ($h_0/\mu_F=1$) and the strength of the interfacial barrier $Z=mH/\hbar^2k_F=0$. The width of FM1 and FM2 is taken to be $W_F=4/k_F$. In this case, there is only one propagating mode [$l=1$ in Eq. (10)]. We obtain the maximum possible value of MR, i.e., MR$=-1$, independently of L. In order to understand this behavior, we consider the L dependence of the currents in parallel and antiparallel alignment as shown in Fig. 2. When an electron with up spin in FM1 is injected into the SC, ordinary Andreev reflection does not occur because electrons with down spin are absent in FM1. In parallel alignment, crossed Andreev reflection does not occur either because there are no electrons with down spin in FM2. Therefore, no current flows in the system as shown in Fig. 2. On the other hand, in antiparallel alignment, while ordinary Andreev reflection is absent, crossed Andreev reflection occurs because there are electrons with down spin in FM2, which is a member of a Cooper pair, for an incident electron with up spin from FM1, and therefore finite current flows in the system as shown in Fig. 2. As a result, we find MR$=-1$ irrespective of L in the case of half metallic FM1 and FM2. The current in the antiparallel alignment decreases, oscillating with increasing L. The behavior of the current is understood as follows. From Eqs. (24)–(28), the current in antiparallel alignment at low temperatures and low applied bias voltage is expressed as

$$I_{\mathrm{AP}}\sim\frac{e^2V}{h}[\tilde{R}^{1,he}_{\uparrow,11}+\tilde{R}^{1,eh}_{\downarrow,11}+\tilde{R}^{2,he}_{\downarrow,11}+\tilde{R}^{2,eh}_{\uparrow,11}]. \tag{30}$$

It is shown from Eq. (A1)–(A16) in the Appendix that the L dependence of the probability of crossed Andreev reflection $\tilde{R}^{1,he}_{\uparrow,11}$ originates from the interference term between the wave functions of FM1 and FM2, and

$$\tilde{R}^{1,he}_{\uparrow,11}\propto[1-\sin(2k_FL+\phi)](k_FL)^{-3}\exp(-L/\xi), \tag{31}$$

where $\xi=\xi_{GL}(\pi\Delta/2\sqrt{\Delta^2-E^2})$, $\xi_{GL}=\hbar v_F/\pi\Delta$ being the Ginzburg-Landau (GL) coherence length, v_F is the Fermi velocity, and ϕ is a phase defined as Eq. (A16). The probabilities $\tilde{R}^{1,eh}_{\downarrow,11}$, $\tilde{R}^{2,he}_{\downarrow,11}$, and $\tilde{R}^{2,eh}_{\uparrow,11}$ show the same L dependence as $\tilde{R}^{1,he}_{\uparrow,11}$ in Eq. (31), and therefore the current in antiparallel alignment, Eq. (30), decreases rapidly with a rate of $(k_FL)^{-3}$, oscillating with period of π with increasing k_FL. Note that the k_FL dependence of the probabilities is dominated by the term $(k_FL)^{-3}$, not the exponential term exp $(-L/\xi)$,[33] since $k_F\xi\gg1$.[55,56]

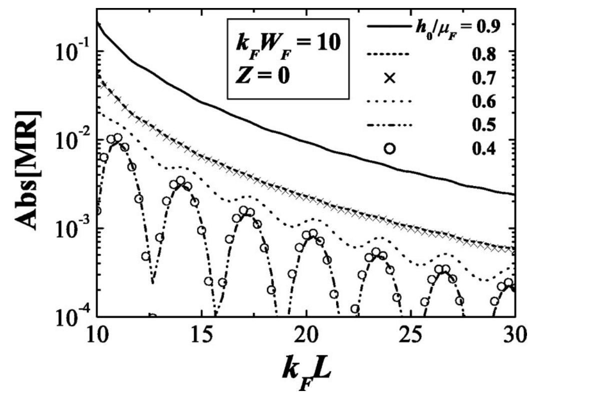

FIG. 3. The absolute value of MR as a function of L in the case that the exchange field h_0/μ_F are 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9.

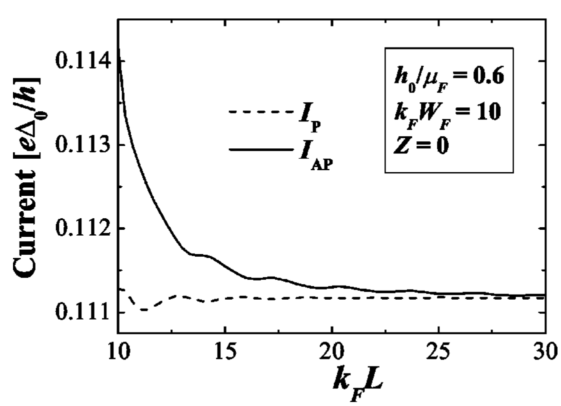

FIG. 4. The current as a function of L in the case of $h_0/\mu_F=0.6$. The solid and dashed lines are for the currents in the antiparallel and parallel alignment, respectively.

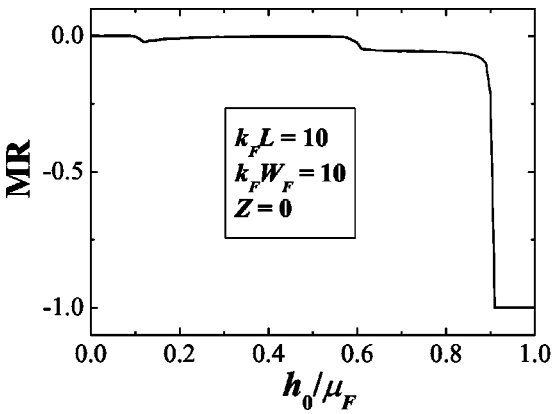

FIG. 5. MR as a function of h_0 for $L=10/k_F$.

We next consider the L dependence of MR for several values of the exchange field in the case that $W_F=10/k_F$ and $Z=0$ (Fig. 3). In this case, there are several propagating modes in FM1 and FM2. The magnitude of MR decreases with increasing L for each value of the exchange field. This behavior of MR is understood by considering the L dependence of the current in parallel and antiparallel alignment. As shown in Fig. 4, in the case that $h_0=0.6\mu_F$, the finite current in the parallel alignment flows because ordinary Andreev reflection occurs and is almost independent of L. On the other hand, the current in antiparallel alignment decreases with increasing L since the contribution of the crossed Andreev reflection process to the current decreases with increasing L, and therefore the magnitude of MR decreases with increasing L. In this case, the oscillation of the current in antiparallel alignment is suppressed because electrons and holes in the several propagating modes l in Eq. (10) contribute to the current and wash out the oscillation. The reason why MR for $h_0=0.8\mu_F(0.5\mu_F)$ are almost equal to MR for $h_0=0.7\mu_F(0.4\mu_F)$ is as follows. In Fig. 5, the h_0 dependence of MR is plotted. We find three drops in MR at $h_0\sim 0.12\mu_F$, $0.62\mu_F$, and $0.92\mu_F$. MR for $h_0=0.8\mu_F(0.5\mu_F)$ and $h_0=0.7\mu_F(0.4\mu_F)$ are in the same plateau. This plateau structure is understood by considering the denominator I_{AP} and the numerator I_P-I_{AP} in Eq. (29) separately. As shown in Fig. 6, I_{AP} is mainly given by ordinary Andreev reflection and decreases with increasing h_0 because the number of channels for the minority spin decreases by 1 when passing across $h_0\sim 0.12\mu_F$, $0.62\mu_F$, and $0.92\mu_F$. Especially, in the range of $h_0/\mu_F=0.92$–1, there is no open channel for minority spin and ordinary Andreev reflection is completely prohibited. Therefore, we find MR$=-1$ (see Fig. 5). Figure 7 shows the h_0 dependence of I_P-I_{AP}, which is mainly due to the crossed Andreev reflection. The magnitude of I_P-I_{AP} is much smaller than that of I_{AP}, and therefore MR shows the plateau structure as shown in Fig. 5, and MR for $h_0=0.8\mu_F(0.5\mu_F)$ are almost equal to MR for $h_0=0.7\mu_F(0.4\mu_F)$.

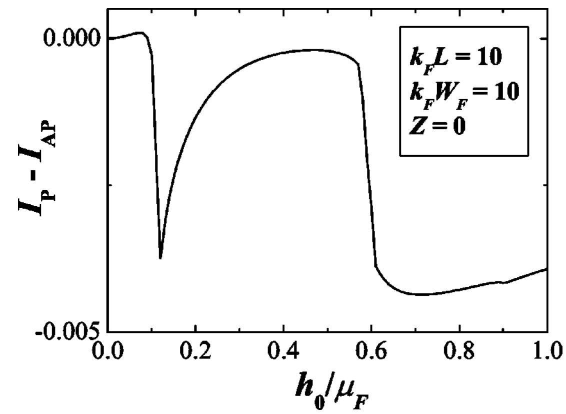

FIG. 7. The difference between the currents in the parallel and antiparallel alignment as a function of h_0.

Finally, we investigate the effect of the interfacial barriers on the transport in this system. Figure 8 shows the L dependence of MR for $h_0=0.6\mu_F$ and several values of interfacial barrier parameter Z. As seen in Fig. 8, MR approaches zero with increasing L and shows a strong dependence on the height of the interfacial barrier Z. The decrease of MR with increasing L is explained by the same way as in the case of no interfacial barriers (Fig. 3). To investigate the Z dependence of MR in detail, we calculate the Z dependence of MR for $k_FL=10$, 15, and 20 as shown in Fig. 9. The magnitude of MR decreases with increasing Z in the range of $Z\lesssim 0.5$

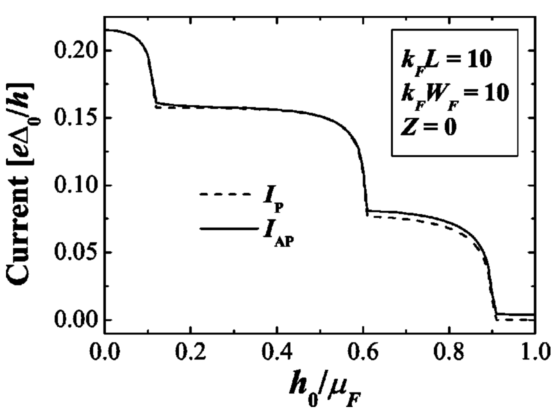

FIG. 6. The current as a function of h_0 for $L=10/k_F$. The solid and dashed lines are for the current in the antiparallel and parallel alignment, respectively.

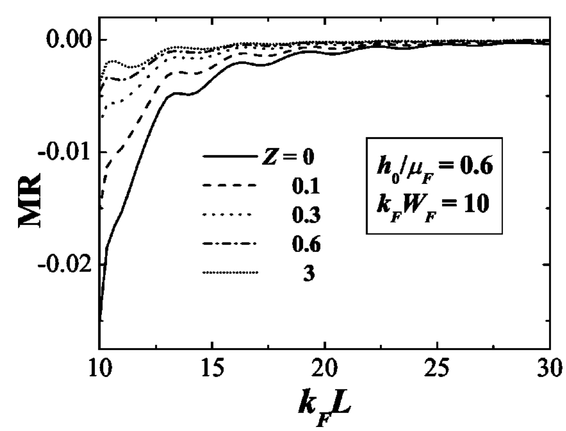

FIG. 8. MR as a function of L for various values of the interfacial barrier parameter Z and $h_0/\mu_F=0.6$.

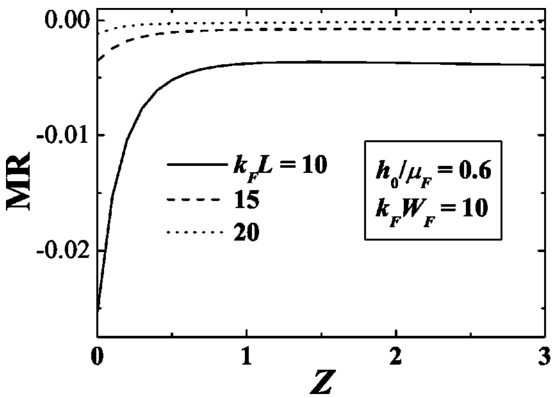

FIG. 9. MR as a function of the height of the interfacial barriers Z for $h_0/\mu_F = 0.6$. The solid, dashed, and dotted lines represent the case of $k_FL = 10$, 15, and 20.

and is almost constant for L in the range of $Z \gtrsim 0.5$. This dependence is understood as follows. MR consists of the denominator I_{AP} and the numerator $I_{\mathrm{P}} - I_{\mathrm{AP}}$, which mainly come from the process of ordinary Andreev reflection and crossed Andreev reflection, respectively. Crossed Andreev reflection is more sensitive to scattering at the interfacial barriers than ordinary Andreev reflection, and therefore the value of $I_{\mathrm{P}} - I_{\mathrm{AP}}$ decreases more rapidly than that of I_{AP} in the range of $Z \lesssim 0.5$, and therefore the magnitude of MR decreases with increasing Z for $k_FL = 10$, 15, and 20 as shown in Fig. 9.

Although the impurity scattering in the SC and the proximity effect are neglected in our theory, these assumptions are justified as follows. First, as shown in the present calculations, the crossed Andreev reflection process occurs on the scale which is less than several nanometers for $k_F \sim 1$ Å^{-1} (Ref. 57). This scale is much smaller than the mean free path of the SC,[56] and therefore the effect of impurity scattering in the SC on the crossed Andreev reflection is neglected. Second, in the present paper, we consider the case that the area of the contacts of FM1 and FM2 with the SC are several nanometers and thus the proximity effect can be neglected.[33,47–49]

IV. CONCLUSION

We present a theory of crossed Andreev reflection in structures consisting of a superconductor with two ferromagnetic leads. By extending the BTK theory to this system, we calculate the current and magnetoresistance due to crossed Andreev reflection. It is shown that the dependence of the crossed Andreev reflection on the distance between two ferromagnetic leads, L, is given by the interference between the wave functions in ferromagnetic leads. The probability of crossed Andreev reflection follows $(k_FL)^{-3}$, where k_F is the Fermi wave number, and therefore the magnetoresistance due to crossed Andreev reflection strongly decreases with increasing k_FL except for the case of half metallic ferromagnets. It is also presented that the dependences of the magnetoresistance on the exchange field show a plateau structure and the magnitude of the magnetoresistance rapidly decreases with increasing the height of the interfacial barriers. These dependences are explained by considering the relation between the probabilities of ordinary Andreev reflection and crossed Andreev reflection.

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APPENDIX: BOUNDARY CONDITIONS

The coefficients $a_{\sigma,ln}$, $b_{\sigma,ln}$, $c_{\sigma,ln}$, $d_{\sigma,ln}$, $\alpha_{\sigma,ln}$, and $\beta_{\sigma,ln}$ in Eqs. (13), (14), and (15) are determined from the boundary conditions (16) and (17) as follows.[20,52–54] Substituting the wave functions (13), (14), and (15) for the boundary conditions (16) and (17), we obtain

$$\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Phi_{\mathrm{FM1},n}(y) + \sum_{l=1}^{\infty} \left[a_{\sigma,ln} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + b_{\sigma,ln} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right] \Phi_{\mathrm{FM1},l}(y) \right\} \times \theta_1(y) + \sum_{l=1}^{\infty} \left[c_{\sigma,ln} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + d_{\sigma,ln} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right] \Phi_{\mathrm{FM2},l}(y)\, \theta_2(y) = \sum_{l=1}^{\infty} \left[\alpha_{\sigma,ln} \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} + \beta_{\sigma,ln} \begin{pmatrix} v_0 \\ u_0 \end{pmatrix} \right] \Phi_{\mathrm{SC},l}(y)\, \theta_S(y) \tag{A1}$$

and

$$\left\{ \left(p^{+}_{\sigma,n} - i\frac{2mH}{\hbar^2} \right) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Phi_{\mathrm{FM1},n}(y) + \sum_{l=1}^{\infty} \left[a_{\sigma,ln} \left(p^{-}_{\sigma,l} - i\frac{2mH}{\hbar^2} \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} - b_{\sigma,ln} \left(p^{+}_{\sigma,l} + i\frac{2mH}{\hbar^2} \right) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right] \Phi_{\mathrm{FM1},l}(y) \right\} \theta_1(y) + \sum_{l=1}^{\infty} \left[c_{\sigma,ln} \left(q^{-}_{\sigma,l} - i\frac{2mH}{\hbar^2} \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} - d_{\sigma,ln} \left(q^{+}_{\sigma,l} + i\frac{2mH}{\hbar^2} \right) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right] \Phi_{\mathrm{FM2},l}(y)\, \theta_2(y) = \sum_{l=1}^{\infty} \left[\alpha_{\sigma,ln} k^{+}_{l} \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} - \beta_{\sigma,ln} k^{-}_{l} \begin{pmatrix} v_0 \\ u_0 \end{pmatrix} \right] \Phi_{\mathrm{SC},l}(y)\, \theta_S(y). \tag{A2}$$

First, by multiplying both sides of Eq. (A1) by $\Phi_{\mathrm{SC},m}(y)$ and integrating them with respect to y, we obtain

$$\begin{pmatrix}1\\0\end{pmatrix}\Lambda_{1,nm}+\sum_{l=1}^{\infty}\left[a_{\sigma,ln}\begin{pmatrix}0\\1\end{pmatrix}+b_{\sigma,ln}\begin{pmatrix}1\\0\end{pmatrix}\right]\Lambda_{1,lm}+\sum_{l=1}^{\infty}\left[c_{\sigma,ln}\begin{pmatrix}0\\1\end{pmatrix}+d_{\sigma,ln}\begin{pmatrix}1\\0\end{pmatrix}\right]\Lambda_{2,lm}=\alpha_{\sigma,mn}\begin{pmatrix}u_0\\v_0\end{pmatrix}+\beta_{\sigma,mn}\begin{pmatrix}v_0\\u_0\end{pmatrix}, \tag{A3}$$

where $\Lambda_{1(2),lm}(L)$ is the overlap integral between the wave functions in FM1 (FM2) and SC, and is given by

$$\Lambda_{1,lm}(L)=\int_{(L-W_F)/2}^{(L+W_F)/2}\Phi_{\mathrm{FM1},l}(y)\Phi_{\mathrm{SC},m}(y)dy, \tag{A4}$$

$$\Lambda_{2,lm}(L)=\int_{(-L-W_F)/2}^{(-L+W_F)/2}\Phi_{\mathrm{FM2},l}(y)\Phi_{\mathrm{SC},m}(y)dy. \tag{A5}$$

Second, by multiplying both sides of Eq. (A2) by $\Phi_{\mathrm{FM1},m}(y)$ and integrating them with respect to y, we obtain

$$\left(p^{+}_{\sigma,m}-i\frac{2mH}{\hbar^2}\right)\begin{pmatrix}1\\0\end{pmatrix}\delta_{mn}+a_{\sigma,mn}\left(p^{-}_{\sigma,m}-i\frac{2mH}{\hbar^2}\right)\begin{pmatrix}0\\1\end{pmatrix}-b_{\sigma,mn}\left(p^{+}_{\sigma,m}+i\frac{2mH}{\hbar^2}\right)\begin{pmatrix}1\\0\end{pmatrix}=\sum_{l=1}^{\infty}\left[\alpha_{\sigma,ln}k_l^{+}\begin{pmatrix}u_0\\v_0\end{pmatrix}-\beta_{\sigma,ln}k_l^{-}\begin{pmatrix}v_0\\u_0\end{pmatrix}\right]\Lambda_{1,ml}, \tag{A6}$$

where δ_{mn} is a Kronecker delta defined as

$$\delta_{mn}=\begin{cases}1 & (m=n),\\ 0 & (m\neq n).\end{cases} \tag{A7}$$

Third, by multiplying both sides of Eq. (A2) by $\Phi_{\mathrm{FM2},m}(y)$ and integrating them with respect to y, we obtain

$$c_{\sigma,mn}\left(q^{-}_{\sigma,m}-i\frac{2mH}{\hbar^2}\right)\begin{pmatrix}0\\1\end{pmatrix}-d_{\sigma,mn}\left(q^{+}_{\sigma,m}+i\frac{2mH}{\hbar^2}\right)\begin{pmatrix}1\\0\end{pmatrix}=\sum_{l=1}^{\infty}\left[\alpha_{\sigma,ln}k_l^{+}\begin{pmatrix}u_0\\v_0\end{pmatrix}-\beta_{\sigma,ln}k_l^{-}\begin{pmatrix}v_0\\u_0\end{pmatrix}\right]\Lambda_{2,ml}. \tag{A8}$$

By solving the Eqs. (A3), (A6), and (A8), the coefficients $a_{\sigma,ln}$, $b_{\sigma,ln}$, $c_{\sigma,ln}$, $d_{\sigma,ln}$, $\alpha_{\sigma,ln}$, and $\beta_{\sigma,ln}$ are determined. In the numerical calculation, we truncate the number of channels in the ferromagnetic leads (FM1 and FM2) and SC by the cutoff constants M_F and M_S, respectively. The M_F and M_S are taken to be large enough to make the calculation results converge.

Especially, in the half metallic ($h_0/\mu_F=1$) FM1 and FM2 with width $W_F=4/k_F$, there is only one propagating mode $l=1$ in Eq. (10). In this case, we can neglect the evanescent mode $l\geqslant 2$ and take the cutoff constant in FM1 and FM2 to be $M_F=1$. In the case of no interfacial barriers ($Z=0$), at the low-energy region ($E\sim 0$), the coefficient of the crossed Andreev reflection part in the wave function of FM2, $c_{\uparrow,11}$, is written as

$$c_{\uparrow,11}\sim\frac{C_1\Gamma^{+}+C_2\Gamma^{-}}{C_3}, \tag{A9}$$

where

$$C_1=ip^{+}(p^{-}+\Omega_1^{-})(q^{+}-\Omega_2^{-}), \tag{A10}$$

$$C_2=ip^{+}(p^{-}-\Omega_1^{+})(q^{+}+\Omega_2^{+}), \tag{A11}$$

$$C_3=[p^{+}p^{-}+\Omega_1^{+}\Omega_1^{-}-(p^{+}-p^{-})(\Omega_1^{+}-\Omega_1^{-})/2]\times[q^{+}q^{-}+\Omega_2^{+}\Omega_2^{-}-(q^{+}-q^{-})(\Omega_2^{+}-\Omega_2^{-})/2], \tag{A12}$$

where $p^{\pm}=p^{\pm}_{\uparrow,1}$, $q^{\pm}=q^{\pm}_{\uparrow,1}$, and $\Gamma^{\pm}$ is the interference term between the wave functions of FM1 and FM2 through SC, which strongly depends on L, given by

$$\Gamma^{\pm}(L)=\sum_{m}^{\infty}k_m^{\pm}\Lambda_{1,1m}(L)\Lambda_{2,1m}(L)\sim\frac{k_F\sqrt{2k_FW_F}}{4}(k_FL)^{-3/2}\exp(-L/2\xi)\times\exp\left[\pm i\left(k_FL-\frac{3\pi}{4}\right)\right] \tag{A13}$$

and

$$\Omega^{\pm}_{1(2)}=\sum_{m}^{\infty}k_m^{\pm}\Lambda^2_{1(2),1m}. \tag{A14}$$

Substituting Eq. (A9) for Eq. (18), we obtain

$$\tilde{R}^{1,he}_{\uparrow,11}=k_F^3W_F\frac{|C_1|^2+|C_2|^2+C_1C_2^{*}e^{i(2k_FL-3\pi/2)}+C_1^{*}C_2e^{-i(2k_FL-3\pi/2)}}{8|C_3|^2}(k_FL)^{-3}\exp(-L/\xi)\sim\frac{k_F^3W_F|C_1|^2}{4|C_3|^2}[1-\sin(2k_FL+\phi)](k_FL)^{-3}\exp(-L/\xi), \tag{A15}$$

where ϕ is a phase given by

$$C_1C_2^{*}=|C_1C_2^{*}|e^{i\phi}, \tag{A16}$$

and we use the relation $|C_1|^2\simeq|C_2|^2\simeq|C_1C_2^{*}|$.

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