Scaling behavior of the metal-insulator transition in one-dimensional disordered systems with long-range hopping

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(Received 27 June 2003; published 11 November 2003)

We present a quantitative description on the scaling behavior of metal-insulator transition in a one-dimensional disordered model with nonrandom hopping integrals falling off as a power law of the distance $J_{nm}=1/|n-m|^{\mu}$ with m and n being site indices in the lattice, proposed by Rodríguez *et al.* [Phys. Rev. Lett. **90**, 027404 (2003)]. Using the finite size scaling analysis combined with the transfer matrix method, the conductance of a finite system with varying length M is investigated. There exists a metal-insulator transition at critical value μ_c for energies within a range near the band top. By varying the energy from the top towards the bottom of this range, μ_c monotonically increases from the lower limit (~ 1) towards the upper limit (~ 1.7), and the localization-length exponent on the insulator side decreases from ~ 2.0 towards 0, implying the variation of the effective dimensionality of the system. On the metallic side there exists an upper bound of conductance in the order of e^2/h, different from the behavior of usual multidimensional systems.

DOI: 10.1103/PhysRevB.68.174201 PACS number(s): 71.30.+h, 78.30.Ly, 36.20.Kd, 71.35.Aa

I. INTRODUCTION

Since Anderson published his famous paper[1] about disorder induced localization, extensive investigations have focused on the metal-insulator transition (MIT). From scale theory[2] it is commonly believed that in systems of dimensions not greater than 2 all states are localized and there does not exist MIT. In several low-dimensional disordered models with short-range and long-range correlations, however, specific extended states have been found.[3–8] Especially, in one-dimensional (1D) systems with long-range correlated disorder, there exists disorder-induced metal-insulator transition. Usually only the nearest-neighbor hoppings are considered in these models. The long-range hoppings can have essential effect on properties of electron states in low-dimensional systems, as illustrated from the level statistics and multifractal analysis on a small-world-like network.[9] By using a supersymmetric method combined with a renormalization group analysis, Rodríguez *et al.* have shown the existence of extended states near the top of band in 1D and 2D Anderson models with long-range hoppings which fall off as a power law of the site distance, $J_{\mathbf{m}-\mathbf{n}}=J/|\mathbf{m}-\mathbf{n}|^{\mu}$ with $\mathbf{m}$ and $\mathbf{n}$ being site coordinates. They found that the MIT occurs only within the range of $d<\mu<3d/2$, with d being the geometric dimensionality of the system. By introducing the long-range hoppings, the effective dimensionality of the system is different from d, owing to the possibility of the long-distance motion of electrons with one hopping. So it is interesting to investigate the scaling behavior of the MIT in systems with varying effective dimensionality.

In this paper we investigate the scaling behavior of the MIT in model proposed by Rodríguez *et al.* in Ref. 10. Using the standard finite size scaling analysis combined with the transfer matrix method, the conductance of a finite system with varying length M is investigated. It is found that there exists MIT at critical value μ_c for energies within a range near the band top. The position of this energy range depends on the strength of disorder. By varying the energy from the top towards the bottom of this range, μ_c monotonically increases from the lower limit (~ 1) towards the upper limit (~ 1.7), and the localization-length exponent on the insulator side decreases from ~ 2.0 towards 0, implying the variation of the effective dimensionality. On the metallic side there exists an upper bound in the order of e^2/h for the conductance.

This paper is organized as follows. In the next section the method of applying the finite-size scaling on this system is described. In Sec. III we present the numerical results of the scaling analysis. Section IV is devoted to a brief summary.

II. METHOD OF CALCULATIONS

The Hamiltonian of the 1D tight-binding model is expressed as[10]

$$H=\sum_n \epsilon_n|n\rangle\langle n|+\sum_{n\neq m} J_{nm}|n\rangle\langle m|, \tag{1}$$

where ϵ_n is energy level at site n, uniformly distributed in interval $[-W/2,W/2]$, and $J_{nm}=J/|n-m|^{\mu}$ is the long-range hopping amplitude. We will adopt J as energy units. If $W=0$, the dispersion relation is

$$E(k)=2J\sum_{n=1}^{\infty}\frac{\cos(nk)}{n^{\mu}}, \tag{2}$$

where E is the energy and k is the wave vector. It can be seen that for a given energy E within the band, there may exist more than one k's satisfying Eq. (2). Every k corresponds to a channel for the transmission of electrons. Thus, there can exist many channels depending on the value of E, in spite of the 1D lattice structure. In this sense the effective dimensionality may become larger than 1.

If $W\neq 0$, k is no longer a good quantum number. To study properties of electron states of the above model, we perform a scaling analysis on the conductance of system with finite size. For a system of size $M+1$, J_{nm} makes sense only for

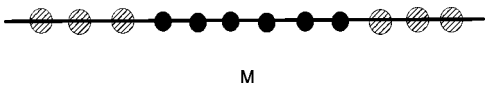

FIG. 1. Illustration of extending a finite system with length M to a very long chain. The black circles denote the original system and the shaded circles stand for the attached leads.

$|m-n| \leqslant M$. To determine the Lyapunov exponents of this system we embed it between two leads, as shown in Fig. 1, each of which is a long 1D chain with the same strength of disorder and the same long-range hoppings as those in the finite system (J_{nm} for $|m-n| \leqslant M$). The whole length of the system is extended to L. Then the Lyapunov exponents can be calculated by the use of the transfer matrix method for this long chain.[11] In the present case the transfer matrix is a $2M \times 2M$ one. A wave function ψ at energy E can be expressed as a linear combination of site orbitals

$$|\psi\rangle = \sum_n a_n |n\rangle, \tag{3}$$

where coefficients a_n obey the Schrödinger equation

$$\epsilon_n a_n + \sum_{j=1}^{M} (J_{n,n+j} a_{n+j} + J_{n,n-j} a_{n-j}) = E a_n . \tag{4}$$

Equation (4) can be rewritten in a transfer matrix form

$$\begin{pmatrix} a_{n+M} \\ a_{n+M-1} \\ \vdots \\ a_{n-M+1} \end{pmatrix} = T_n \begin{pmatrix} a_{n+M-1} \\ a_{n+M-2} \\ \vdots \\ a_{n-M} \end{pmatrix}, \tag{5}$$

where transfer matrix T_n is

$$\begin{pmatrix} -\frac{J_{n,n+M-1}}{J_{n,n+M}} & -\frac{J_{n,n+M-2}}{J_{n,n+M}} & \cdots & \frac{E-\epsilon_n}{J_{n,n+M}} & \cdots & -\frac{J_{n,n-M}}{J_{n,n+M}} \\ 1 & 0 & \cdots & \cdots & \cdots & \cdots \\ 0 & 1 & 0 & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & \cdots & 1 & 0 \end{pmatrix}. \tag{6}$$

The propagation along this wire is described by the product of the transfer matrices

$$T = \prod_{n=1}^{L} T_n . \tag{7}$$

The eigenvalues of T can be calculated by the standard algorithm of the power method with reorthogonalization procedures.[11] There are M pairs of eigenvalues, in each of which logarithms of two eigenvalues are opposite, corresponding to the forward and backward motions of electrons. Lyapunov exponents, denoted by λ_i, $i=1,\ldots,M$, are the positive logarithms of eigenvalues divided by L. Thus, the conductance of the finite system with length M can be calculated as

$$G(M) = \frac{e^2}{h} \sum_{i=1}^{M} \frac{2}{\cosh^2 \lambda_i M}. \tag{8}$$

The scaling behavior of the conductance can be investigated from its size dependence.

III. NUMERICAL RESULTS AND DISCUSSIONS

We calculate the conductance for systems with various sizes and different parameter values. In Fig. 2 we show the dependence of the conductance on μ for $E=6.0$ and $W=5.0$ in units of J and different system sizes. It clearly shows that there exists a critical value of μ, $\mu_c \sim 1.46$, at which the curves intersect. For $\mu < \mu_c$, the conductance increases with the system size, corresponding to the metallic behavior, while $\mu > \mu_c$ is the insulator regime. The decrease of μ leads to longer hoppings in the system and results in the increase of the effective dimensionality and the delocalization of wave functions. However, μ should be larger than 1, since $\mu \leqslant 1$ corresponds to unphysical situations where the band

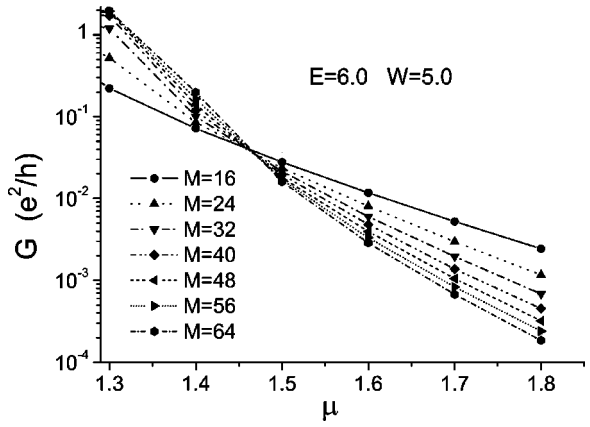

FIG. 2. Conductance as a function of μ for different system size. E and W are in units of J.

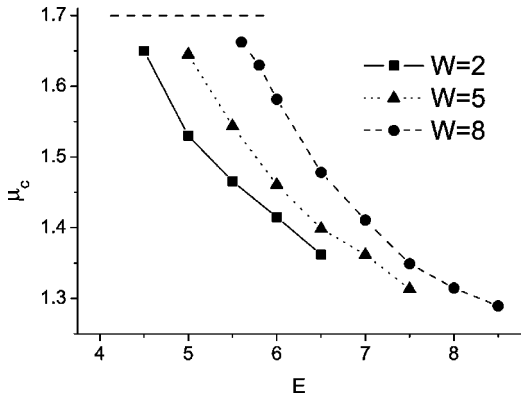

FIG. 3. μ_c as a function of E for different values of W. Estimated errors are less than the size of symbols. The dashed line on the top indicates the upper limit for μ_c.

top is at the infinity. In Fig. 3 we plot μ_c as a function of energy E for different values of W. It can be seen that μ_c increases when the energy is varied from the band top towards to the band center. This implies that the states near the band center are more delocalized than the states near the band edge, coinciding with the general behavior in 3D disordered systems. In the present model, however, if the energy goes too far from the band top, the critical point can not be found from this scaling analysis and the MIT disappears. This is shown by the dashed line at $\mu_c \sim 1.7$, where all the curves stop. In fact, if the energy goes farther towards the band center, μ_c is too large and, as a consequence, the system has too low an effective dimensionality that cannot support the MIT. For a given energy, μ_c is increased upon increasing the strength of disorder W. It implies a delocalization effect of W. Such an unusual behavior is due to the fact that by increasing W the band top is raised and the energy E becomes more distant from the top. These results provide a quantitative description of conclusions in Ref. 10.

Then we carry out a scaling analysis in the vicinity of μ_c. By the use of the scaling transformation $M' = M/\xi$, with ξ being an appropriate factor for each $\ln G$-$\ln M$ curve, all the $\ln G$-$\ln M'$ curves for parameters E and W that give the same μ_c can merge together. Here, ξ is the correlation length on the metallic side and is referred as the localization length on the insulator side. This procedure is illustrated in Fig. 4 where all $\ln G$-$\ln(M/\xi)$ curves for values E and W giving $\mu_c = 1.4$ in the vicinity of μ_c are shown. The value of G at the limit $M/\xi \to 0$ corresponds the critical conductance G_c at μ_c. The curves with values of E and W giving different μ_c can not merge together because G_c varies with μ_c. In fact, the effective dimensionality of the system at the critical point depends on the value of μ_c, so the critical behavior can be tuned by changing E and W to adjust μ_c. Another noteworthy point of Fig. 4 is the saturation of G on the metallic side at the large M/ξ limit, different from the situation of real 3D systems where the conductance on the metallic side is unlimited obeying the Ohm's law. The saturation value is in the order of quantum conductance e^2/h, suggesting that the number of available channels is still limited by the one geometric dimension, although the effective dimensionality may be larger. This saturation has less effect on the scaling analysis because G_c is usually smaller than e^2/h, but it reminds us that the ''metallic phase'' is an unusual one with a limited conductance.

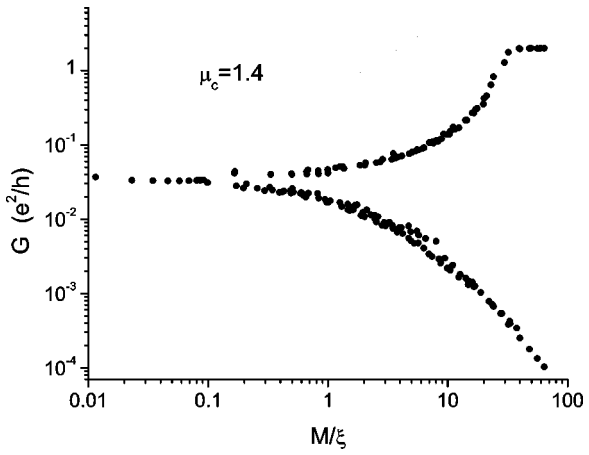

FIG. 4. Conductance as a function of rescaled size M/ξ. The curves include data for various values of E and W that give the same μ_c. μ runs over a range of values in vicinity of μ_c.

On basis of the scaling analysis we can calculate the localization-length exponent ν on the insulator side

$$\xi = A(\mu - \mu_c)^{-\nu}, \tag{9}$$

where A is a constant. It is found that ν is also μ_c dependent. In Fig. 5 we plot the μ_c dependence of exponent ν. One can see that ν is decreased from ~ 2 towards 0 by increasing μ_c from the lower limit 1 to the upper limit 1.7. Since the critical exponent crucially depends on the dimensionality, the variation of ν provides further evidence that the effective dimensionality of the system varies upon changing μ_c. When μ_c goes towards the upper limit, the localization-length exponent approaches zero, leading to the disappearance of the MIT. To illustrate the change of the effective dimensionality further, in the inset we plot the scaling function β, defined as the logarithmic derivatives of the conductance with respect to the size,[2] as a function of the logarith-

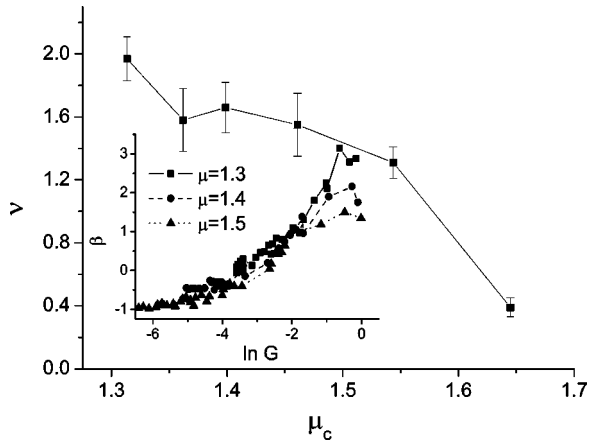

FIG. 5. Dependence of localization-length exponent on μ_c. Inset: Scaling function β as a function of the logarithmic conductance (in units of e^2/h) for different values of μ.

mic conductance. In spite of fluctuations in the curves due to numerical errors in calculation of the derivatives for different E and W, under the limit $G<e^2/h$ they still exhibit the basic features predicted by the scaling theory for multidimensional systems.[2] The slope of the curves at the critical point ($\beta=0$) is about 0.58, approximately coinciding with the inverse of ν shown in the main panel. From the saturation value of β on the metallic side one can estimate the effective dimensionality, $d_{\mathrm{eff}}\sim\beta_{\mathrm{sat}}+2$ (see Ref. 2). From this we can see that the effective dimensionality approximately increases from 3.4 to 5 as μ decreases from 1.5 to 1.3. However, it should be emphasized that this effective dimensionality can make sense only below the upper bound of the conductance.

The error bars for ν shown in Fig. 5 are estimated from the statistics of the data in the fitting procedure. Generally speaking, the numerical errors are controllable for quantities at the critical points due to the good scaling behavior of the system in the investigated parameter ranges. Owing to the upper bound of conductance shown in Fig. 4, for a system on the metallic side with large side the calculated value of G may be saturated by this bound and deviate from the features expected with the standard scaling theory, but the results still obey the scaling behavior shown in Fig. 4, independent of the size. This guarantees the applicability of the present analysis.

IV. SUMMARY

In conclusion, a finite-size scaling analysis together with the use of the transfer-matrix method shows that there exists a metal-insulator transition in varying exponent μ of a 1D disordered model with long-range hoppings in a power-law form $J_{n,n+m}=J/|n-m|^{\mu}$. Since lower and upper limits exist for μ, $1\lesssim\mu\lesssim1.7$, the MIT occurs only in an energy range near the band top. The critical point, μ_c, monotonically increases from the lower limit towards the upper limit, and the localization-length exponent decreases from 2 towards 0, when the energy is shifted from the band top towards the band center within the allowed range. This behavior is originated from the change of the effective dimensionality of the system in changing μ. The results provide a quantitative description of critical properties in systems which are geometrically one dimensional but effectively multidimensional. Since the conductance on the metallic side has an upper bound in the order of e^2/h, the model is still essentially different from usual multidimensional systems in the scaling behavior.

ACKNOWLEDGMENTS

This work was supported by the National Foundation of Natural Science in China Grant Nos. 10074029 and 60276005, and by the China State Key Projects of Basic Research (Grant No. G1999064509).

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