Magnon-assisted transport and thermopower in ferromagnet–normal-metal tunnel junctions

Edward McCann and Vladimir I. Fal'ko

Department of Physics, Lancaster University, Lancaster LA1 4YB, United Kingdom

(Received 3 March 2003; revised manuscript received 28 May 2003; published 13 November 2003)

Magnon-assisted transport across a tunnel junction between a ferromagnet and a normal (nonmagnetic) metal is studied theoretically. A finite temperature difference across the junction produces a nonequilibrium magnetization that drives a charge current, mediated by electrons via electron-magnon interactions, from the ferromagnet into the normal metal. The corresponding thermopower coefficient is large, $S \sim -(k_B/e)$ $\times (k_B T/\omega_M)^{3/2} P(\Pi_+, \Pi_-, \Pi_N)$ where $P(\Pi_+, \Pi_-, \Pi_N)$, $0 \le P \le 1$, represents the degree of spin polarization of the current response to a bias voltage, and depends on the relative sizes of the majority Π_{+} and the minority Π ₋ band Fermi surface in the ferromagnet and in the normal metal, Π _{*N*}.

DOI: 10.1103/PhysRevB.68.172404 PACS number(s): 72.25. - b, 72.15.Jf, 73.40.Rw, 73.23. - b

There is intense research of spin-polarized transport, fueled by the desire to develop a form of electronics that utilizes the spin polarization of carriers.¹ A great deal of activity has focused on spin injection across junctions between ferromagnets and normal metals or semiconductors. $2-4$ Current in the ferromagnet F is carried unequally by majority and minority carriers so that a current flowing across the interface with a normal conductor is expected to have a finite degree of spin polarization P , $0 \le P \le 1$. Whereas spin injection from a ferromagnetic metal into a normal metal *N* has been measured in broad agreement with theory, $2,3$ there has been difficulty in achieving large degrees of spin injection into a semiconductor at room temperature.5 A possible limiting factor is conductivity mismatch, 6 a problem that may be overcome by introducing a tunneling barrier between the ferromagnet and semiconductor.⁷ Spin injection in allsemiconducting devices has generally been more successful,⁸ although it is currently limited to relatively low temperatures.

Spin injection may be viewed as a current of magnetization that is carried by the electric current flowing in response to a nonequilibrium electric potential. The subject of this paper is the opposite effect: the injection of charge caused by the equilibration of magnetization. The magnetization of a ferromagnet held at a finite temperature *T* is less than its maximum value due to the thermal occupation of magnons. The reduction in the absolute value of the magnetization $\delta m(T)$ obeys Bloch's law:

$$
\delta m(T) = (3.47/\xi)(k_B T/\omega_M)^{3/2},\tag{1}
$$

where ξ is the spin of the localized moments and ω_M is the magnon Debye energy. Therefore, a temperature difference ΔT held across a tunnel junction between one or more ferromagnetic electrodes is associated with a nonequilibrium magnetization. Recently we found that the equilibration of magnetization in an *F*-*F* tunnel junction may be mediated by electrons via electron-magnon interactions, resulting in a substantial charge current response to ΔT .⁹

In this paper we consider a different situation, an *F*-*N* tunnel junction in which the transfer of heat from the ferromagnet to the normal metal involves simultaneous spin and charge injection. The effect may be understood as follows.

Since a reduction in the temperature of the ferromagnet is related to a change of its magnetization, Eq. (1) , thermal equilibration of the *F*-*N* junction is accompanied by a flow of magnetization across it. The magnetization flow is mediated by conduction electrons via the electron-magnon interaction, resulting in a charge current response to ΔT . The current response to ΔT is characterized by a contribution to the thermopower *S* dependent on the difference between the size of the majority and minority band Fermi surfaces in the ferromagnet,

$$
S \approx -0.48(k_B/e)\,\delta m P(\Pi_+,\Pi_-,\Pi_N),\tag{2}
$$

and holds in a range of temperatures given by

$$
1 \geq \delta m \geq (k_B T) / \epsilon_F, \tag{3}
$$

where ϵ_F is the Fermi energy. The function $P(\Pi_+, \Pi_-, \Pi_N)$, $0 \le P \le 1$, represents the degree of spin polarization of the current response to a bias voltage, and depends on the relative sizes of the majority Π_{+} and the minority Π ₋ band Fermi surface in the ferromagnet and of the Fermi surface of the normal metal Π_N . The result, Eq. (2) , does not depend on the direction of polarization or the choice of quantization axis.

To evaluate the thermopower we write down a balance equation for the current $I(V, \Delta T)$ response to a bias voltage *V* and a temperature drop ΔT across the junction, by taking into account competing elastic and inelastic electron transfer processes. Then we determine the thermopower coefficient $S = -V/\Delta T$ by satisfying the relation $I(V, \Delta T) = 0$. The magnon-assisted processes we consider for an *F*-*N* tunnel junction, shown schematically in Fig. 1, are similar to those discussed in relation to transport in the *F*-*F* tunnel iunctions $9,10$ and we refer the reader to Ref. 9 for additional details. In the figure we assume that the majority electrons are ''spin up'' and the minority electrons are ''spin down'' in the ferromagnetic reservoir on the left-hand side (this arbitrary choice does not influence the result), whereas in the normal (nonmagnetic) metallic reservoir on the right, the density of states of the spin-up and spin-down conduction electrons are equal. A typical process in Fig. 1, process (i) , begins with a spin-down electron on the right, which then tunnels through the barrier (without additional spin flip) into

FIG. 1. Schematic of the four processes of magnon-assisted tunneling between a ferromagnet on the left and a normal metal on the right. Process (i) begins with a spin-down electron on the right, which then tunnels through the barrier (without spin flip) into an intermediate, virtual state with spin-down minority polarization on the left. Finally, the electron emits a magnon (wavy line) and incorporates itself into a previously unoccupied state in the spin-up majority band on the left. Process (ii) is the reverse of (i). Processes (iii) and (iv) are the same as (i) and (ii) , respectively, except that the electronic spin states are opposite and therefore magnon emission is replaced by absorption (or vice versa).

an intermediate, virtual state with spin-down minority polarization on the left. Finally, the electron in the virtual state emits a magnon (indicated by a wavy line) and incorporates itself into a previously unoccupied state in the spin-up majority band on the left.

The Hamiltonian of the ferromagnet H_F can be written in terms of Fermi $\{c^{\dagger}, c\}$ and magnon $\{b^{\dagger}, b\}$ creation and annihilation operators as^{9,11} $H_F = \sum_{\mathbf{k}\alpha} \epsilon_{\mathbf{k}\alpha}^L c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha} + \sum_{\mathbf{q}} \omega_{\mathbf{q}} b_{\mathbf{q}}^\dagger b_{\mathbf{q}}$ H_{em} where the index $\alpha = \{+, -\}$ takes into account splitting of the conduction-band electrons into majority $\epsilon_{\mathbf{k}+}^{L}$ and minority $\epsilon_{\mathbf{k}-}^L$ subbands $\epsilon_{\mathbf{k}\alpha}^L = \epsilon_{\mathbf{k}}^L - \alpha \Delta/2$, where $\epsilon_{\mathbf{k}}^L$ is the bare energy and Δ is the spin splitting energy. The electronmagnon interaction term is

$$
H_{em} = -\frac{\Delta}{\sqrt{2\xi\mathcal{N}}} \sum_{\mathbf{k}\mathbf{q}} [c_{\mathbf{k}-\mathbf{q}+}^{\dagger} c_{\mathbf{k}-} b_{\mathbf{q}}^{\dagger} + c_{\mathbf{k}-}^{\dagger} c_{\mathbf{k}-\mathbf{q}+} b_{\mathbf{q}}], \quad (4)
$$

where N is the number of localized moments in the ferromagnet and ξ is the spin per unit magnetic cell. We assume a quadratic magnon dispersion, $\omega_q = Dq^2 + \omega_0$, $D \sim \Delta$, and $\omega_0 \ll k_B T \ll \omega_M$, where $\omega_M = D(\frac{\pi^2}{v})^{2/3}$ is the Debye magnon energy, *v* is the volume of a unit cell, and ω_0 is the magnon anisotropy gap. For the nonmagnetic metal on the right-hand side $H_N = \sum_{\mathbf{k}\alpha} \epsilon_{\mathbf{k}\alpha}^R c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$ where the conduction band is spin degenerate $\epsilon_{\mathbf{k}+}^{R} = \epsilon_{\mathbf{k}-}^{R} = \epsilon_{\mathbf{k}}^{R}$.

Following the tunneling Hamiltonian approach, $12-14$ the amplitude for an electron with wave number \mathbf{k}_R on the right and spin orientation parallel to the minority spins of the ferromagnet to finish in a majority state $(+, \mathbf{k}')$ on the left after emitting a spin wave with wave number **q** can be estimated using second-order perturbation theory with respect to the electron-magnon interaction and the tunneling matrix element $t_{\mathbf{k}_{\mathbf{R}},\mathbf{k'}+\mathbf{q}}$:

$$
A_{\mathbf{k}_{\mathbf{R}},\mathbf{k'}+\mathbf{q}} = \frac{t_{\mathbf{k}_{\mathbf{R}},\mathbf{k'}+\mathbf{q}}\Delta}{\sqrt{2\xi\mathcal{N}}(\epsilon_{\mathbf{k'}+\mathbf{q}+}^{L} - \epsilon_{\mathbf{k}_{\mathbf{R}}}^{R} + \Delta)} \approx \frac{t_{\mathbf{k}_{\mathbf{R}},\mathbf{k'}+\mathbf{q}}}{\sqrt{2\xi\mathcal{N}}}.
$$
 (5)

For $k_B T$, $eV \ll \Delta$, when both initial and final electron states should be taken close to the Fermi level, only longwavelength magnons can be emitted, so that the energy deficit in the virtual states can be approximated as $\epsilon_{\mathbf{k}' + \mathbf{q}+}^L$ $-\epsilon_{\mathbf{k}_{\mathbf{R}}}^{R} + \Delta \approx \Delta$. As noticed in Refs. 11,15, this cancels out the large exchange parameter since the same electron-core spinexchange constant appears both in the splitting between minority and majority bands and in the electron-magnon coupling.

We take into account all four magnon-assisted tunneling processes depicted in Fig. 1. Process (i) begins with a \parallel electron on the right with wave vector \mathbf{k}_R [with occupation number $n_R(\epsilon_{\mathbf{k_R}}^R)$], which tunnels through the barrier into an intermediate virtual \downarrow state on the left (\mathbf{k}_L). Then, this electron flips spin by emitting a magnon with wave vector **q** [this process is stimulated by the occupancy factor of thermal magnon excitations $1 + N_L(q)$, and, thus, incorporates itself into the majority spin band on the left, provided the final ↑ state $(\mathbf{k}' = \mathbf{k}_L - \mathbf{q})$ is not occupied [which has probability 1 $-n_L(\epsilon^L_{\mathbf{k}'}+)$. Process (ii) involves transitions between the same states as (i), but in the reverse order. It begins with a \uparrow electron on the left with occupation number $n_L(\epsilon_{\mathbf{k}'}^L)$, which absorbs a magnon [probability $N_L(q)$] to flip its spin and move into an intermediate virtual ↓ state on the left, before tunneling into an empty final \downarrow state on the right [which is unoccupied with probability $1 - n_R(\epsilon_{\mathbf{k}_R}^R)$. The balance between these two processes contributes to the total current as

$$
I^{(i,ii)} = -4\pi^2 \frac{e}{h} \int_{-\infty}^{+\infty} d\epsilon \sum_{\mathbf{k}'\mathbf{k}_{\mathbf{R}}\mathbf{q}} |A_{\mathbf{k}_{\mathbf{R}},\mathbf{k}'+\mathbf{q}}|^2 \delta(\epsilon - eV - \epsilon_{\mathbf{k}_{\mathbf{R}}}^R) \delta(\epsilon
$$

$$
- \epsilon_{\mathbf{k}'+}^L - \omega_{\mathbf{q}} \left\{ -n_R(\epsilon_{\mathbf{k}_{\mathbf{R}}}^R) \left[1 - n_L(\epsilon_{\mathbf{k}'+}^L) \right] \left[1 + N_L(\mathbf{q}) \right] + \left[1 - n_R(\epsilon_{\mathbf{k}_{\mathbf{R}}}^R) \right] n_L(\epsilon_{\mathbf{k}'+}^L) N_L(\mathbf{q}) \right\}, \tag{6}
$$

which is written in terms of the occupation numbers of electrons on the left- (right-) hand side of the junction $n_{L(R)}(\epsilon_{\mathbf{k}\alpha}^{L(R)}) = (\exp[(\epsilon_{\mathbf{k}\alpha}^{L(R)} - \epsilon_F^{L(R)})/(k_B T_{L(R)})] + 1)^{-1}$ and of magnons $N_L(\mathbf{q}) = (\exp[\omega_{\mathbf{q}} / (k_B T_L)] - 1)^{-1}$ in the ferromagnet. Here $T_{L(R)}$ is the temperature on the left- (right-) hand side, $\epsilon_F^L - \epsilon_F^R = -eV$, ω_q is the energy of a magnon of wave vector **q**, and, in the following, we set $T_L = T + \Delta T$ and $T_R = T$.

Processes (iii) and (iv) in Fig. 1 are the same as (i) and (ii), respectively, except that the electronic spin states are opposite and therefore magnon emission is replaced by absorption (or vice versa): (iii) and (iv) involve transitions into or from a minority state on the left via an intermediate, virtual state in the majority band. Their contribution to the total current is

$$
I^{(\text{iii},\text{iv})} = -4\pi^2 \frac{e}{h} \int_{-\infty}^{+\infty} d\epsilon \sum_{\mathbf{k}' \mathbf{k}_{\mathbf{R}} \mathbf{q}} |A_{\mathbf{k}_{\mathbf{R}},\mathbf{k}'+\mathbf{q}}|^2 \delta(\epsilon - eV - \epsilon_{\mathbf{k}_{\mathbf{R}}}^R) \delta(\epsilon - \epsilon V - \epsilon_{\mathbf{k}_{\mathbf{R}}}^R) \delta(\epsilon - \epsilon V - \epsilon_{\mathbf{k}'-}^L) \delta(\epsilon - \epsilon V - \epsilon_{\mathbf{k}'-}^R) \delta(\epsilon - \epsilon V - \epsilon_{\mathbf{k}'-}
$$

After combining them together into an expression for the total inelastic contribution to the current $I_{in} = I^{(i,ii)} + I^{(iii,iv)}$, performing summation over wave numbers and integration over initial electron energies, and keeping only terms linear in *V* and ΔT , we arrived at the following expression:

$$
I_{in} = \frac{e}{h} \frac{3}{4\xi} \left(\frac{k_B T}{\omega_M}\right)^{3/2} \left\{ eV \Gamma\left(\frac{5}{2}\right) \zeta\left(\frac{3}{2}\right) \left[\mathcal{T}_{+N} + \mathcal{T}_{-N}\right] - \frac{1}{2} k_B \Delta T \Gamma\left(\frac{7}{2}\right) \zeta\left(\frac{5}{2}\right) \left[\mathcal{T}_{+N} - \mathcal{T}_{-N}\right] \right\},
$$
\n(8)

where $\Gamma(x)$ is the gamma function and $\zeta(x)$ is Riemann's zeta function. For convenience we have grouped all the information about the quality of the interface into a parameter $\mathcal{T}_{\alpha N}(\epsilon_F),$

$$
T_{\alpha N}(\epsilon) \approx 4 \pi^2 \sum_{\mathbf{k}_{\mathbf{L}} \mathbf{k}_{\mathbf{R}}} |t_{\mathbf{k}_{\mathbf{L}},\mathbf{k}_{\mathbf{R}}}|^2 \delta(\epsilon - \epsilon_{\mathbf{k}_{\mathbf{L}} \alpha}^L) \delta(\epsilon - \epsilon_{\mathbf{k}_{\mathbf{R}}}^R), \quad (9)
$$

which is equivalent to the sum over all scattering channels, from states with spin α on the left to states on the right (where both spin channels are equivalent), of the transmission eigenvalues usually introduced in the Landauer formula, $16-18$ although we restrict ourselves to the tunneling regime in this paper.

In order to find the total current, we also take into account the contribution of elastic processes that involve transitions without any spin flip from either the majority or the minority band in the ferromagnet to the normal metal. To lowest order in *V* and ΔT , the elastic contribution is

$$
I_{\rm el} \approx \frac{e^2}{h} V [T_{+N} + T_{-N}] + O\left(\frac{e}{h} \frac{k_B T}{\epsilon_F} k_B \Delta T\right). \tag{10}
$$

The term linear in *V* corresponds to the contribution to the electrical conductance $G = (e^2/h)[T_{+N} + T_{-N}]$ whereas the term linear in ΔT is responsible for the Mott formula¹⁹ contribution to the thermopower, typically with a small parameter $k_B T / \epsilon_F$ in metallic systems.

In the regime of temperatures given in Eq. (3) the total current $I = I_{el} + I_{in}$ may be approximated by

$$
I \approx \frac{e^2}{h} V[T_{+N} + T_{-N}] - \frac{e}{h} \frac{3}{8\xi} \Gamma\left(\frac{7}{2}\right) \zeta\left(\frac{5}{2}\right) \left(\frac{k_B T}{\omega_M}\right)^{3/2}
$$

$$
\times k_B \Delta T[T_{+N} - T_{-N}], \qquad (11)
$$

where the leading term proportional to *V* arises from the elastic processes, Eq. (10) , and the leading term proportional to ΔT comes from the magnon-assisted processes, Eq. (8) . The corresponding thermopower $S = -V/\Delta T$, found by setting $I=0$, is

$$
S \approx -0.48(k_B/e)\,\delta m(T)P(\Pi_+, \Pi_-, \Pi_N). \tag{12}
$$

The function $\delta m(T)$ is the change in the magnetization due to thermal magnons at temperature *T* (Bloch's $T^{3/2}$ law) as given in Eq. (1) and *P* is the degree of spin-polarized current that flows in response to a bias voltage:

$$
P = (T_{+N} - T_{-N})/(T_{+N} + T_{-N}).
$$
\n(13)

It is a function of the relative sizes of the majority Π_+ and the minority Π ₋ band Fermi surface in the ferromagnet and of the Fermi surface of the normal metal Π_N . Its exact form depends on the nature of the interface between the electrodes.

For a uniformly transparent interface of area *A* we account for conservation of the parallel component of momentum $\mathbf{k}_{\text{L}(\text{R})}^{||}$ by assuming that the tunneling matrix element has the form

$$
|t_{\mathbf{k}_\mathbf{L},\mathbf{k}_\mathbf{R}}|^2 = |t|^2 L^{-2} |h^2 v_L^z(\mathbf{k}_\mathbf{L}) v_R^z(\mathbf{k}_\mathbf{R})| \delta_{\mathbf{k}_\mathbf{L},\mathbf{k}_\mathbf{R}^{\parallel}} \tag{14}
$$

where $v_{L,R}^z(\mathbf{k}) = \partial \epsilon_{L,R}(\mathbf{k})/\partial(\hbar k_z)$ are components of electron velocity perpendicular to the interface and *t* represents the interface transparency and is independent of momentum. This leads to

$$
T_{\alpha N}^{\text{flat}} \approx 4 \pi^2 |t|^2 \frac{A}{h^2} \text{min} \{ \Pi_\alpha, \Pi_N \},\tag{15}
$$

$$
Pflat \approx \frac{(\min\{\Pi_+, \Pi_N\} - \min\{\Pi_-, \Pi_N\})}{(\min\{\Pi_+, \Pi_N\} + \min\{\Pi_-, \Pi_N\})},
$$
(16)

where Π_{α} is the area of the maximal cross section of the Fermi surface of spins α in the ferromagnet, $\Pi_+ \ge \Pi_- \ge 0$, and Π_N is the area of the maximal cross section of the Fermi surface in the normal metal. For an isotropic Fermi surface in three dimensions $\Pi_{\alpha} = \pi \hbar^2 k_{F\alpha}^2$, where $k_{F\alpha}$ is the Fermi wave vector, although the form of Π_α may be different for more complicated Fermi surfaces. Within the model of a uniformly transparent interface, the result does not depend on the form of the electronic dispersion. As an opposite extreme, it is possible to consider a strongly nonuniform interface which is transparent in a finite number of points only.²⁰

We assume that, in a small-area contact, the bottleneck of both charge and heat transport lies in the tunnel contact between the electrodes held at different temperatures and/or electric potentials. The magnon-assisted response to ΔT |Eq. (8)] results in a nonzero spin current across the interface that corresponds to a flow of magnetization into the normal metal:³ every electron has a finite magnetic moment as well as charge $-e$. The resulting nonequilibrium spin polarization spreads into the normal metal and decays due to spinorbit scattering:

$$
(D\partial_x^2 - \tau_S^{-1})M(x) = 0,
$$

$$
D\partial_x M(0) = -(\alpha/h)\delta m(T)k_B \Delta T[T_{+N} + T_{-N}]
$$

where $M(x)$ is the nonequilibrium spin polarization per unit length of the normal-metal wire, τ_s is the spin-relaxation time in it, $L_s = \sqrt{D \tau_s}$, and $\alpha \sim 1$. In the normal metal, 0 $\leq x \leq \infty$, the magnitude of the nonequilibrium spin polarization is given by

$$
M(x) \approx \frac{\alpha}{h} \sqrt{\frac{\tau_S}{D}} \delta m(T) k_B \Delta T [T_{+N} + T_{-N}] e^{-x/L_S}.
$$

- ¹G.A. Prinz, Phys. Today 48(4), 58 (1995); Science 282, 1660 (1998), and references therein.
 2^2 M. Johnson and R.H. Silsbee, Phys. Rev. Lett. **55**, 1790 (1985).
-
- ³A.G. Aronov, Pis'ma Zh. Éksp. Teor. Fiz. **24**, 37 (1976) [JETP Lett. 24, 32 (1976)]; M. Johnson and R.H. Silsbee, Phys. Rev. B **35**, 4959 (1987); Phys. Rev. Lett. **60**, 377 (1988); P.C. van Son, H. van Kempen, and P. Wyder, *ibid.* **58**, 2271 (1987).
- 4 S. Datta and B. Das, Appl. Phys. Lett. 56 , 665 (1990).
- 5W.Y. Lee, S. Gardelis, B.-C. Choi, Y.B. Xu, C.G. Smith, C.H.W. Barnes, D.A. Ritchie, E.H. Linfield, and J.A.C. Bland, J. Appl. Phys. 85, 6682 (1999); P.R. Hammar, B.R. Bennett, M.J. Wang, and M. Johnson, Phys. Rev. Lett. 83, 203 (1999); M. Ramsteiner, H.J. Zhu, H.-P. Schönherr, and K.H. Ploog, Physica E (Amsterdam) 13, 529 (2002).
- 6G. Schmidt, D. Ferrand, L.W. Molenkamp, A.T. Filip, and B.J. van Wees, Phys. Rev. B 62, R4790 (2000).
- ⁷E.I. Rashba, Phys. Rev. B 62 , R16 267 (2000); A. Fert and H. Jaffrès, *ibid.* **64**, 184420 (2001).
- 8R. Fiederling, M. Keim, G. Reuscher, W. Ossau, G. Schmidt, A. Waag, and L.W. Molenkamp, Nature (London) 402, 787 (1999); Y. Ohno, D.K. Young, B. Beschoten, F. Matsukura, H. Ohno, and D.D. Awschalom, *ibid.* **402**, 790 (1999); B.T. Jonker, Y.D. Park, B.R. Bennett, H.D. Cheong, G. Kioseoglou, and A. Petrou, Phys. Rev. B 62, 8180 (2000).
- 9 E. McCann and V.I. Fal'ko, Appl. Phys. Lett. **81**, 3609 (2002);

For tunnel junctions, with an interface resistance greater than the resistance of a piece of metal of length L_S , the nonequilibrium spin polarization accumulated near the interface is small. In this limit, it is possible to neglect the back flow of magnetization into the ferromagnet and the current response to a temperature gradient is characterized by a large contribution to the thermopower.

The authors thank J. F. Annett, B. L. Gyorffy, T. Jungwirth, A. H. MacDonald, and G. Tkachov for discussions, and EPSRC and EC Framework 6 Project No. STREP 505587 for financial support.

Phys. Rev. B 66, 134424 (2002).

- ¹⁰ A.M. Bratkovsky, Appl. Phys. Lett. **72**, 2334 (1998); A.H. Mac-Donald, T. Jungwirth, and M. Kasner, Phys. Rev. Lett. **81**, 705 (1998); F. Guinea, Phys. Rev. B 58, 9212 (1998).
- 11 R.B. Woolsey and R.M. White, Phys. Rev. B 1, 4474 (1970); B.S. Shastry and D.C. Mattis, *ibid.* **24**, 5340 (1981); S.R. Allan and D.M. Edwards, J. Phys. C 15, 2151 (1982); M.I. Auslender and V.Y. Irkhin, *ibid.* **18**, 3533 (1985).
- 12M.H. Cohen, L.M. Falicov, and J.C. Phillips, Phys. Rev. Lett. **8**, 316 (1962).
- 13C. Caroli, R. Combescot, P. Nozieres, and D. Saint-James, J. Phys. C 4, 916 (1971).
- ¹⁴G.D. Mahon, *Many-Particle Physics* (Plenum, New York, 1981).
- 15 E. McCann and V.I. Fal'ko, Europhys. Lett. **56**, 583 (2001) ; G. Tkachov, E. McCann, and V.I. Fal'ko, Phys. Rev. B **65**, 024519 $(2002).$
- ¹⁶R. Landauer, Philos. Mag. **21**, 863 (1970).
- 17 D.S. Fisher and P.A. Lee, Phys. Rev. B 23 , 6851 (1981).
- ¹⁸ C.W.J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997).
- ¹⁹M. Cutler and N.F. Mott, Phys. Rev. **181**, 1336 (1969).
- 20 For a strongly nonuniform interface which is transparent in a finite number of points only, $(Refs. 9,15)$ each of a typical area $a \sim \lambda_F^2$ and randomly distributed over the interface area *A*, we find that $T_{\alpha N}^{\text{dis}} \approx 4 \pi^2 |t|^2 (a \Pi_{\alpha}/h^2)(a \Pi_N/h^2)$ and $P^{\text{dis}} \approx (\Pi_+$ $-\Pi_{-})/(\Pi_{+}+\Pi_{-}).$