Absolute negative conductivity in two-dimensional electron systems associated with acoustic scattering stimulated by microwave radiation

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We discuss the feasibility of absolute negative conductivity (ANC) in two-dimensional electron systems (2DES's) stimulated by microwave radiation in a transverse magnetic field. The mechanism of ANC under consideration is associated with electron scattering on acoustic phonons accompanied by absorption of microwave photons. It is demonstrated that the dissipative components of the 2DES dc conductivity can be negative ($\sigma_{xx}=\sigma_{yy}<0$) due to negative values of the dc photoconductivity caused by microwave radiation at certain ratios of the microwave frequency Ω and the electron cyclotron frequency Ω_c. The phase of the oscillations of the dissipative dc photoconductivity associated with photon-assisted electron scattering on acoustic phonons is quite different from that in the case of the photon-assisted impurity scattering mechanism. The concept of ANC associated with an interplay of the scattering mechanisms can be invoked to explain the nature of the occurrence of zero-resistance ''dissipationless'' states observed in recent experiments.

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I. INTRODUCTION

The effect of vanishing electrical resistance and transition to ''dissipationless'' states in a two-dimensional electron system (2DES) subjected to a magnetic field and irradiated with microwaves has recently been observed in the experiments by Mani *et al.*,[1] Zudov *et al.*,[2] and Yang *et al.*[3] The most popular scenario[4–6] of the occurrence of this effect is based on the concept of absolute negative conductivity (ANC) in 2DES proposed more than three decades ago by Ryzhii.[7] Later,[8] a detailed theory of the ANC effect in 2DES due to the photon-assisted impurity scattering was developed. Recently, a version of the theory of this effect was presented by Durst *et al.*[9] Some nonequilibrium processes in 2DES associated with electron-phonon interactions were studied theoretically as well.[10–13]

The feasibility of ANC in heterostructures with a 2DES in a magnetic field under microwave irradiation is associated with the following. The dissipative electron transport in the direction parallel to the electric field and perpendicular to the magnetic field is due to hops of the electron Larmor orbit centers caused by scattering processes. These hops result in a change in the electron potential energy $\delta\epsilon=eE\delta\xi$, where $e=|e|$ is the electron charge, $E=|\mathbf{E}|$ is the modulus of the net in-plane electric field, which includes both the applied and Hall components, and $\delta\xi$ is the displacement of the electron Larmor orbit center. If the electron Larmor orbit center shifts in the direction opposite to the electric field ($\delta\xi<0$), the electron potential energy decreases ($\delta\epsilon<0$). In equilibrium, electron Larmor orbit center hops in this direction dominate, so the dissipative electron current flows in the direction of the electric field. However, in some cases, displacements of the electron Larmor orbit centers in the direction of the electric field (with $\delta\epsilon>0$) can prevail. Indeed, under sufficiently strong microwave irradiation the main contribution to the electron scattering on impurities can be associated with processes involving the absorption of microwave photons. If an electron absorbs such a photon and transfers to a higher Landau level (LL), a portion of the absorbed energy $N\hbar\Omega_c$, where $\Omega_c=eH/mc$ is the electron cyclotron frequency, $\hbar$ is the Planck constant, m is the electron effective mass, H is the strength of the magnetic field, c is the velocity of light, and $N=1,2,3,\ldots$ is the LL index, goes to an increase of the electron kinetic energy, whereas the change in the electron potential energy is $\delta\epsilon=\hbar(\Omega-\Lambda\Omega_c)$, where $\Lambda=N'-N$. If $(\Omega-\Lambda\hbar\Omega_c)>0$, the potential energy of electrons increases with each act of their scattering. Hence, the dissipation current flows opposite to the electric field, resulting in negative (absolute, not differential) conductivity. A similar situation arises in multiple-quantum-well structures with sequential resonant tunneling in which ANC associated with photon-assisted transitions between the spatially separated states was observed experimentally.[14]

Since the probability of scattering with spatial displacements of the electron Larmor orbit center ξ exceeding the quantum Larmor radius $L=(c\hbar/eH)^{1/2}$ is exponentially small, such scattering processes are effective, and the variation of the dissipative component of the current caused by microwave radiation (i.e., the photocurrent) is substantial only in the immediate vicinities of the resonances $|\Omega-\Lambda\Omega_c|\lesssim\max\{eEL/\hbar,\,\Gamma\}$, where Γ characterizes the LL broadening.[15] One of the intriguing features of the observed experimental characteristics[1–3] is a steep temperature dependence of the maxima and minima of the 2DES resistance which, perhaps, cannot be explained by invoking solely the photon-assisted impurity scattering mechanism. Therefore, the contributions of other scattering mechanisms should be assessed. In this paper, we calculate the dc dissipative components of the 2DES conductivity (mobility) tensor, assuming that the main scattering mechanisms are electron scattering and electron photon-assisted scattering on acoustic phonons. This assumption is supported by the high quality of the samples (with the electron mobility $\mu>10^7$ cm^2/V s) used in the experiments[1–3] performed at low temperatures ($T\simeq1$ K).

II. GENERAL FORMULAS

The density of the in-plane dc current in a 2DES in a transverse magnetic field is given by the following equations:

$$j_x = \sigma(E)E_x + \sigma_H E_y, \quad j_y = \sigma(E)E_y - \sigma_H E_x, \tag{1}$$

where $\sigma(E)$ and σ_H are the in-plane dissipative and Hall components of the dc conductivity tensor, respectively, with $\sigma(E)=\sigma_{xx}=\sigma_{yy}$ and, considering the classical Hall effect, $\sigma_H=\sigma_{xy}=-\sigma_{yx}=ec\Sigma/H$. Here, Σ is the electron sheet concentration, and the directions x and y are in the 2DES plane. Positive values of $\sigma(E)$ correspond to electron drift in the direction opposite to the direction of the electric field, i.e., to the usual conductivity, whereas the case $\sigma(E)<0$ is referred to as ANC.

We shall assume that $\hbar\Omega$, $\hbar\Omega_c > T$, where T is the temperature in energy units. In this case, one can disregard the processes accompanied by the emission of microwave photons.

As follows from Eq. (1), the density of the dissipative current— i.e., the current parallel to the net electric field **E**—can be presented as $j_D(E)=\sigma(E)E$. The dissipative current is determined by the transitions of electrons between their quantum states with a change in the coordinate of the electron Larmor orbit center ξ. The energies of the electron states in a 2DES in crossed magnetic and electric fields (neglecting Zeeman splitting) are given by

$$\epsilon_{N,\xi} = \left(N+\frac{1}{2}\right)\hbar\Omega_c + eE\xi. \tag{2}$$

Adjusting the standard "orthodox" approach[16] for calculation of the dissipative current in a 2DES (see, for example, Refs. 7,10,12,17, and 18), $j_D(E)$ can be presented in the form

$$j_D(E) = \frac{e}{\hbar}\sum_{N,N'} f_N(1-f_{N'}) \int d^3\mathbf{q}\; q_y |V_\mathbf{q}|^2 |Q_{N,N'}(L^2 q_\perp^2/2)|^2 \{\mathcal{N}_q \delta[(N-N')\hbar\Omega_c + \hbar\omega_q + eEL^2 q_y] + (\mathcal{N}_q+1)\delta[(N-N')\hbar\Omega_c - \hbar\omega_q + eEL^2 q_y] + I_\Omega(q_x,q_y)\mathcal{N}_q \delta[\hbar\Omega + (N-N')\hbar\Omega_c + \hbar\omega_q + eEL^2 q_y] + I_\Omega(q_x,q_y)(\mathcal{N}_q+1)\delta[\hbar\Omega + (N-N')\hbar\Omega_c - \hbar\omega_q + eEL^2 q_y]\}. \tag{3}$$

Here, f_N is the filling factor of the Nth Landau level given by the Fermi distribution function, $\mathbf{q}=(q_x,q_y,q_z)$, $\omega_q = sq$, and $\mathcal{N}_q = [\exp(\hbar\omega_q/T)-1]^{-1}$ are the phonon wave vector, frequency, and distribution function, respectively, s is the velocity of sound, $q=\sqrt{q_x^2+q_y^2+q_z^2}$, $q_\perp = \sqrt{q_x^2+q_y^2}$, $\delta(\omega)$ is the LL form factor which at small definiteness Γ can be assumed to be the Dirac delta function, $|V_\mathbf{q}|^2 = w(q)\exp(-l^2 q_z^2/2)$ characterizes the interaction of electrons with acoustic phonons with $w(q)\propto q^{-1}$ and $w(q)\propto q$ for the piezoelectric and deformation interactions, respectively,[19] l is the width of the electron localization in the z direction perpendicular to the 2DES plane ($l \ll L$), $|Q_{N,N'}(L^2 q_\perp^2/2)|^2 = |P_N^{N'-N}(L^2 q_\perp^2/2)|^2 \exp[-L^2 q_\perp^2/2]$ is the matrix element determined by the overlap of the electron wave functions before and after the hop caused by the scattering, and $|P_N^{N'-N}(L^2 q_\perp^2/2)|^2$ is proportional to a Laguerre polynomial. We shall assume in the following that the dominant mechanism of the acoustic scattering in the temperature range under consideration is the piezoelectric interaction (see Ref. 20).

For the processes with the absorption of a single real photon in each act taken into account in Eq. (3), the quantity $I_\Omega(q_x,q_y) = J_1^2(\sqrt{\mathcal{J}_\Omega} L q_\perp)$, where $J_1(t)$ is the Bessel function and $\mathcal{J}_\Omega = (\mathcal{E}_\Omega/\tilde{\mathcal{E}}_\Omega)^2$ is proportional to the incident microwave power, $\mathcal{E}_\Omega$ is the microwave electric field amplitude, and $\tilde{\mathcal{E}}_\Omega$ is some characteristic microwave field $\tilde{\mathcal{E}}_\Omega$.[21,22] Here for simplicity we have neglected the effect of microwave radiation polarization The ratio $\mathcal{E}_\Omega/\tilde{\mathcal{E}}_\Omega$ characterizes the effect of microwave field on the in-plane electron motion. When this ratio is small, the Larmor orbit center oscillation amplitude in the microwave field is smaller than L, and radiative processes with the participation of more than one microwave photon are insignificant. In such a case, $I_\Omega(q_x,q_y) \simeq \mathcal{J}_\Omega L^2 q_\perp^2/4$. The inclusion of polarization effects leads to the appearance of some anisotropy of the dissipative and Hall conductivities at frequencies far from the cyclotron resonance.[21,22]

Deriving Eq. (3), we have taken into account that the displacement of the electron Larmor orbit center is $\delta\xi = -L^2 q_y$. First two terms on the right-hand side of Eq. (3) correspond to the electron-phonon interactions whereas the third and fourth terms are associated with such interactions accompanied by the absorption of a photon. It is worth pointing out that, as can be seen from Eq. (3), at the resonances $\Omega = (N'-N)\Omega_c$, the contribution of the photon-assisted processes to the dissipative conductivity becomes zero (see Refs. 7 and 8). The characteristic amplitude can be presented as[21,22]

$$\tilde{\mathcal{E}}_\Omega = \frac{\sqrt{2} m\Omega |\Omega_c^2 - \Omega^2| L}{e\sqrt{\Omega_c^2+\Omega^2}}. \tag{4}$$

At the cyclotron resonance $\Omega=\Omega_c$, the quantity $\tilde{\mathcal{E}}_\Omega$ tends to zero. However, the quantity $I_\Omega(q_x,q_y)$ in the immediate vicinity of the cyclotron resonance is limited either due to a specific behavior of the Bessel function or by the LL broadening.

Assuming that $eE|\xi|\ll\hbar|\Omega-\Lambda\Omega_c|<\hbar\Omega,\hbar\Omega_c$, one can expand the expression for the dissipating current given by Eq. (3)in powers of $(eEL^2/\hbar s)$ and present the dissipative dc conductivity in the following form:

$$\sigma(E)\simeq\sigma_{dark}+\sigma_{ph}, \tag{5}$$

in which terms of the order of $(eEL^2/\hbar s)^3$ and higher have been neglected. As a result, from Eq. (3), we obtain

$$\sigma_{dark}=\left(\frac{e^2L^2}{\hbar^3s^2}\right)\sum_{N,\Lambda}\int d^3\mathbf{q}\;\; q_y^2|V_\mathbf{q}|^2|Q_{N,N+\Lambda}(L^2q_\perp^2/2)|^2\delta'(q-q^\Lambda)[f_N(1-f_{N+\Lambda})\mathcal{N}_q-f_{N+\Lambda}(1-f_N)(\mathcal{N}_q+1)], \tag{6}$$

$$\sigma_{ph}=\mathcal{J}_\Omega\left(\frac{e^2L^4}{\hbar^3s^2}\right)\sum_{N,\Lambda>0}\int d^3\mathbf{q}\;\; q_y^2q_\perp^2|V_\mathbf{q}|^2|Q_{N,N+\Lambda}(L^2q_\perp^2/2)|^2[\mathcal{N}_q\delta'(q+q_\Omega^\Lambda)-(\mathcal{N}_q+1)\,\delta'(q-q_\Omega^\Lambda)]f_N(1-f_{N+\Lambda}), \tag{7}$$

where $\delta'(q)=d\delta(q)/dq$, $q^{(\Lambda)}=\Lambda\Omega_c/s$, $q_\Omega^{(\Lambda)}=(\Omega-\Lambda\Omega_c)/s$, and $\Lambda>0$. Substituting the integration over $d^3\mathbf{q}$ for the integration over $dqdq_\perp d\theta$, where $\sin\theta=q_y/q_\perp$, Eqs. (6)and (7) can be rewritten as

$$\sigma_{dark}\propto\sum_{N\Lambda}\int_0^\infty\frac{dqdq_\perp q_\perp^3}{\sqrt{q^2-q_\perp^2}}\exp\left[-\frac{l^2(q^2-q_\perp^2)}{2}\right]|Q_{N,N+\Lambda}(L^2q_\perp^2/2)|^2\delta'(q-q^{(\Lambda)})$$
$$\times[f_N(1-f_{N+\Lambda})\mathcal{N}_q-f_{N+\Lambda}(1-f_N)(\mathcal{N}_q+1)], \tag{8}$$

$$\sigma_{ph}\propto\mathcal{J}_\Omega L^2\sum_{N,\Lambda}\int_0^\infty\frac{dqdq_\perp q_\perp^5}{\sqrt{q^2-q_\perp^2}}\exp\left[-\frac{l^2(q^2-q_\perp^2)}{2}\right]|Q_{N,N+\Lambda}(L^2q_\perp^2/2)|^2[\mathcal{N}_q\delta'(q+q_\Omega^{(\Lambda)})$$
$$-(\mathcal{N}_q+1)\,\delta'(q-q_\Omega^{(\Lambda)})]f_N(1-f_{N+\Lambda}) \tag{9}$$

and reduced to

$$\sigma_{dark}\propto\sum_{N,\Lambda}f_N(1-f_{N+\Lambda})\int_0^\infty dq\exp\left(-\frac{l^2q^2}{2}\right)G_N^\Lambda(q)[\mathcal{N}_q-\exp(-\Lambda\hbar\Omega_c/T)(\mathcal{N}_q+1)]\delta'(q-q^{(\Lambda)}), \tag{10}$$

$$\sigma_{ph}\propto\mathcal{J}_\Omega\sum_{N,\Lambda}f_N(1-f_{N+\Lambda})\int_0^\infty dq\exp\left(-\frac{l^2q^2}{2}\right)H_N^{(\Lambda)}(q)[\mathcal{N}_q\delta'(q+q_\Omega^{(\Lambda)})-(\mathcal{N}_q+1)\,\delta'(q-q_\Omega^{(\Lambda)})], \tag{11}$$

where

$$G_N^{(\Lambda)}(q)=\int_0^{Lq}dtt^3\frac{\exp\left[\frac{(l^2-L^2)}{2L^2}t^2\right]}{\sqrt{L^2q^2-t^2}}|P_N^\Lambda(t^2/2)|^2, \tag{12}$$

$$H_N^{(\Lambda)}(q)=\int_0^{Lq}dtt^5\frac{\exp\left[\frac{(l^2-L^2)}{2L^2}t^2\right]}{\sqrt{L^2q^2-t^2}}|P_N^\Lambda(t^2/2)|^2. \tag{13}$$

At $N\gg1$, using the asymptotic expressions for the Laguerre polynomials,[23] one can obtain $|P_N^{(\Lambda)}(t^2/2)|^2\simeq J_\Lambda^2(\sqrt{2N_m}t)$, where $J_\Lambda(t)$ is the Λth Bessel function, so that $|P_N^{(\Lambda)}(t^2/2)|^2\simeq\cos^2[\sqrt{2N}t-(2\Lambda+1)\pi/4]/\pi Nt$ if $t\gg1/\sqrt{2N}$ and $|P_N^{(\Lambda)}(t^2/2)|^2\simeq t^{2\Lambda}$ when $t<1/\sqrt{2N}\ll1$. Therefore, $G_N^{(\Lambda)}(q)$ and $H_N^{(\Lambda)}(q)$ become

$$G_N^{(\Lambda)}(q)\simeq\frac{1}{\pi NLq}\int_0^\infty dtt^2\exp\left[\frac{(l^2-L^2)}{2L^2}t^2\right]\cos^2\left[\sqrt{2N}t-\frac{(2\Lambda+1)\pi}{4}\right],$$

$$H_N^{(\Lambda)}(q) \simeq \frac{1}{\pi N L q} \int_0^\infty dt t^4 \exp\left[\frac{(l^2-L^2)}{2L^2} t^2\right] \cos^2\left[\sqrt{2N}t - \frac{(2\Lambda+1)\pi}{4}\right],$$

at $Lq \gg 1$, and

$$G_N^{(\Lambda)}(q) \simeq \frac{1}{\pi N} \int_0^{Lq} dt \frac{t^2}{\sqrt{L^2q^2-t^2}} \cos^2\left[\sqrt{2N}t - \frac{(2\Lambda+1)\pi}{4}\right],$$

$$H_N^{(\Lambda)}(q) \simeq \frac{1}{\pi N} \int_0^{Lq} dt \frac{t^4}{\sqrt{L^2q^2-t^2}} \cos^2\left[\sqrt{2N}t - \frac{(2\Lambda+1)\pi}{4}\right],$$

at $1/\sqrt{2N} < Lq < 1$. After integration involving averaging over fast oscillations, one can obtain $G_N^{(\Lambda)}(q) \simeq [2\sqrt{2\pi}(1-l^2/L^2)^{3/2}NLq]^{-1} \simeq (2\sqrt{2\pi}NLq)^{-1}$ at $Lq \gg 1$ and $G_N^{(\Lambda)}(q) \simeq (Lq)^2/8N$ in the range $1/\sqrt{2N} < Lq < 1$. Analogously, for $H_N^{(\Lambda)}(q)$ in these ranges one obtains, respectively, $H_N^{(\Lambda)}(q) \simeq 3[2\sqrt{2\pi}(1-l^2/L^2)^{5/2}NLq]^{-1} \simeq 3(2\sqrt{2\pi}NLq)^{-1}$ and $H_N^{(\Lambda)}(q) \simeq 3(Lq)^4/32N$. Derivative of the function $H_N^{(\Lambda)}(q)$ is proportional to the average displacement of the electron Larmor orbit center in the direction of the electric field associated with the electron transition between the Nth and $(N+1)$th LL's involving a microwave photon and an acoustic phonon with energies $\hbar\Omega$ and $\hbar s q$, respectively. As an example, the function $H_{50}^{(1)}(q)$ calculated using Eq. (13), in which the asymptotic $|P_N^{(\Lambda)}(t^2/2)|^2 \simeq J_\Lambda^2(\sqrt{2N_m}t)$ was used, is shown in Fig. 1.

III. DARK CONDUCTIVITY

Considering the explicit formula for the phonon distribution (Planck's) function, Eq. (10) can be presented in the following form:

$$\sigma_{dark} \propto \sum_{N,\Lambda>0} f_N(1-f_{N+\Lambda}) \int_0^\infty dq \exp\left(-\frac{l^2q^2}{2}\right) G_N^\Lambda(q) \times \frac{1-\exp[\hbar(sq-\Lambda\Omega_c)/T]}{\exp(\hbar sq/T)-1} \delta'\left(q - \frac{\Lambda\Omega_c}{s}\right). \quad (14)$$

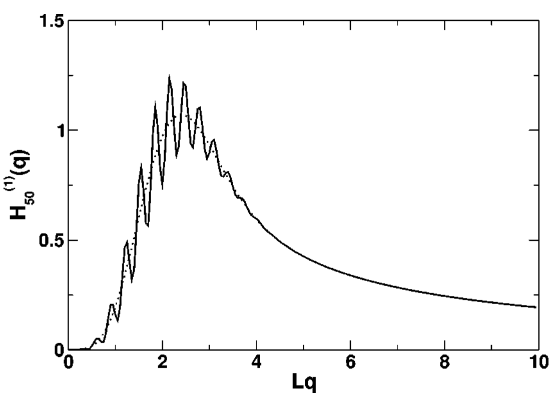

FIG. 1. Function $H_{50}^{(1)}(q)$ given by Eq. (13). The dashed line corresponds to averaging over fast oscillations.

Neglecting terms containing higher powers of exp $(-\hbar\Omega_c/k_BT)$ and denoting

$$G(q) = f_{N_m}(1-f_{N_m+1}) G_{N_m}^{(1)}(q)$$

and

$$F(q) = 1 - \exp\left[\frac{\hbar(\Lambda\Omega_c - sq)}{T}\right],$$

where N_m is the index of the LL immediately below the Fermi level, we present Eq. (14) in the following form:

$$\sigma_{dark} \propto \exp\left(-\frac{\hbar\Omega_c}{T}\right) \frac{d}{Ldq}\left[\exp\left(-\frac{l^2q^2}{2}\right) G(q) F(q)\right]\Big|_{q=q^{(1)}}$$

$$= \overline{G}\left(\frac{\hbar s}{TL}\right)\left(\frac{s}{L\Omega_c}\right) \exp\left(-\frac{\hbar\Omega_c}{T}\right) \exp\left(-\frac{l^2\Omega_c^2}{2s^2}\right). \quad (15)$$

We have taken into account that $Lq^{(1)} = L\Omega_c/s \gg 1$. For example, if $H = 2$ kG, one obtains $Lq^{(1)} \simeq 10$. Hence, for $q \simeq q^{(1)}$ one can set $G(q) \simeq \overline{G}/Lq$, where $\overline{G} \simeq 1/2\sqrt{2\pi}N$. A similar formula for σ_{dark} can be derived also for the deformation mechanism of the electron-phonon interaction. One can see from Eq. (15) that the expression for σ_{dark} contains a small exponential factor $\exp(-\hbar\Omega_c/T)$. This factor arises because the dark conductivity is associated with the absorption of acoustic phonons with the energy $\hbar sq$ close to the LL spacing by electrons transferring from almost a fully filled LL (just below the Fermi level, $N = N_m$) to the next one which is nearly empty. However, the number of such photons at $T \ll \hbar\Omega_c$ is small. The rate of processes with emission of acoustic phonons accompanying electron transitions from an upper LL is also exponentially small due to a low occupancy of this level and the Pauli exclusion principle.

Equation (15) differs from that calculated previously by Erukhimov[18] for the dark conductivity associated with acoustic phonon scattering by the last exponential factor which, according to Eq. (15), depends on the width of the electron localization l. This dependence is attributed to a significant contribution of the electron scattering processes with $q_z \neq 0$ which were neglected in Ref. 18. As can be seen from a comparison of Eq. (15) and that obtained in Ref. 18, the inclusion of such scattering processes results in the replacement of factor $\exp(-L^2\Omega_c^2/2s^2) = \exp(-\hbar\Omega_c^2/2ms^2)$

(as in Ref. 18) by a much larger factor $\exp(-l^2\Omega_c^2/2s^2)$ [as in Eq. (15)]. This yields a substantially higher (exponentially) value of σ_{dark} than if the transitions with $q_z \neq 0$ were neglected. However, the temperature dependences of the dark conductivity obtained in Ref. 18 and in the present paper coincide (compare also with Ref. 24).

IV. PHOTOCONDUCTIVITY

Using Eq. (9), we arrive at the following formula for the dissipative dc photoconductivity:

$$\sigma_{ph} = \sum_{\Lambda} \sigma_{ph}^{(\Lambda)}, \tag{16}$$

where, at relatively low microwave powers ($\mathcal{J}_\Omega < 1$),

$$\sigma_{ph}^{(\Lambda)} \propto \mathcal{J}_\Omega \sum_N f_N(1-f_{N+\Lambda}) \int_0^\infty dq \exp\left(-\frac{l^2q^2}{2}\right) H_N^{(\Lambda)}(q) \times [\mathcal{N}_q \delta'(q+q_\Omega^{(\Lambda)}) - (\mathcal{N}_q+1)\delta'(q-q_\Omega^{(\Lambda)})]$$
$$\simeq \mathcal{J}_\Omega \int_0^\infty dq \exp\left(-\frac{l^2q^2}{2}\right) H^{(\Lambda)}(q) \times [\mathcal{N}_q \delta'(q+q_\Omega^{(\Lambda)}) - (\mathcal{N}_q+1)\delta'(q-q_\Omega^{(\Lambda)})]. \tag{17}$$

Integrating on the right-hand side of Eq. (17), we obtain

$$\sigma_{ph}^{(\Lambda)} \propto -\mathcal{J}_\Omega \frac{d}{Ldq}\left[\exp\left(-\frac{l^2q^2}{2}\right) H^{(\Lambda)}(q)\mathcal{N}_q\right]\Bigg|_{q=-q_\Omega^{(\Lambda)}} \tag{18}$$

at $\Omega - \Lambda\Omega_c < 0$ and

$$\sigma_{ph}^{(\Lambda)} \propto \mathcal{J}_\Omega \frac{d}{Ldq}\left[\exp\left(-\frac{l^2q^2}{2}\right) H^{(\Lambda)}(q)(\mathcal{N}_q+1)\right] f\Bigg|_{q=q_\Omega^{(\Lambda)}} \tag{19}$$

at $\Omega - \Lambda\Omega_c > 0$. Here,

$$H^{(\Lambda)}(q) \simeq \sum_{N=N_m-\Lambda+1}^{N_m} f_N(1-f_{N+\Lambda}) H_N^{(\Lambda)}(q).$$

The photoconductivity given by Eqs. (16)–(19) as a function of microwave frequency exhibits pronounced oscillations in which the photoconductivity sign alternates. At the resonances $\Omega = \Lambda\Omega_c$, the photoconductivity $\sigma_{ph} = 0$. Near the resonances $(s/\sqrt{2N_m}L)$, $(T/\sqrt{2N_m}) < \hbar|\Omega - \Lambda\Omega_c| < (s/L)$, $(T/\hbar)$, the value of the photoconductivity varies as a power of $(\Omega - \Lambda\Omega_c)$:

$$\sigma_{ph}^{(\Lambda)} \propto -\mathcal{J}_\Omega \left(\frac{TL}{\hbar s}\right) \frac{L^2(\Omega - \Lambda\Omega_c)^2}{s^2} < 0 \tag{20}$$

at $\Omega - \Lambda\Omega_c < 0$ and

$$\sigma_{ph}^{(\Lambda)} \propto \mathcal{J}_\Omega \left(\frac{TL}{\hbar s}\right) \frac{L^2(\Omega - \Lambda\Omega_c)^2}{s^2} > 0 \tag{21}$$

when $\Omega - \Lambda\Omega_c > 0$.

Outside the resonances, i.e., in the ranges (s/L), $(T/\hbar) < |\Omega - \Lambda\Omega_c| < \Omega_c$, Eqs. (18) and (19) lead to

$$\sigma_{ph}^{(\Lambda)} \propto \mathcal{J}_\Omega \exp\left[\frac{\hbar(\Omega - \Lambda\Omega_c)}{T}\right] \exp\left[-\frac{l^2(\Omega - \Lambda\Omega_c)^2}{2s^2}\right] > 0 \tag{22}$$

at $\Omega - \Lambda\Omega_c < 0$ and

$$\sigma_{ph}^{(\Lambda)} \propto -\exp\left[-\frac{l^2(\Omega - \Lambda\Omega_c)^2}{2s^2}\right] < 0 \tag{23}$$

at $\Omega - \Lambda\Omega_c > 0$. In this frequency range, the photoconductivity is determined by two components: $\sigma_{ph}^{(\Lambda)}$ (which is negative) and $\sigma_{ph}^{(\Lambda+1)}$ (whose contribution is positive). As a result, when $\Lambda\Omega_c < \Omega < (\Lambda+1)\Omega_c$, we obtain

$$\sigma_{ph} \simeq \sigma_{ph}^{(\Lambda)} + \sigma_{ph}^{(\Lambda+1)}, \tag{24}$$

so that

$$\sigma_{ph} \propto \mathcal{J}_\Omega \Bigg\{ -\exp\left[-\frac{l^2(\Omega - \Lambda\Omega_c)^2}{2s^2}\right] + \exp\left[\frac{\hbar(\Omega - \Lambda\Omega_c - \Omega_c)}{T}\right] \times \exp\left[-\frac{l^2(\Omega - \Lambda\Omega_c - \Omega_c)^2}{2s^2}\right]\Bigg\}$$
$$= \mathcal{J}_\Omega W\left(\frac{\Omega - \Lambda\Omega_c}{\Omega_c}, T\right). \tag{25}$$

V. DISCUSSION

The negativity of σ_{ph} associated with photon-assisted acoustic scattering near the resonances at $\Omega \lesssim \Lambda\Omega_c$ and in the ranges $\Lambda\Omega_c < \Omega(\Lambda+1)\Omega_c$ is attributed to the following. When $\Omega \lesssim \Lambda\Omega_c$, the electron transitions between LL's contributing to $\sigma_{ph}^{(\Lambda)}$ are accompanied by the absorption of acoustic phonons with energies $\hbar\omega_q \simeq \Lambda\Omega_c - \Omega$ which are rather small. In this situation, the probability of the photon-assisted phonon absorption decreases with decreasing phonon energy $\hbar\omega_q \propto q$ because of a decrease in the matrix elements of the scattering in question with decreasing q (see Fig. 1). Since the energies of phonons absorbed near the resonances are small, the distinction between $\mathcal{N}_q$ with sufficiently small and very close values of q is insignificant. Due to this, the rate of the absorption of acoustic phonons with $\hbar\omega_q$ slightly higher than $\hbar(\Lambda\Omega_c - \Omega)$ exceeds that of acoustic phonons with $\hbar\omega_q \lesssim \hbar(\Lambda\Omega_c - \Omega)$. In the first case, the electron Larmor orbit center displacement $\delta\xi = \hbar(\Omega - \Lambda\Omega_c + \omega_q)/eE > 0$ and therefore the change in the electron potential energy $\delta\epsilon > 0$, so that such an act of the electron scattering provides a negative contribution to the dissipative current, i.e., to ANC. In contrast, near the resonances but at Ω slightly larger than $\Lambda\Omega_c$, the scattering acts with $\delta\xi < 0$ dominate.

Sufficiently far from the resonances the energies of the emitted (absorbed) phonons are relatively large. The matrix elements of the scattering involving such phonons and the phonon number steeply decrease with increasing phonon energy (momentum) as shown in Fig. 1. In this situation, the electron Larmor orbit center displacements $\delta\xi = \hbar(\Omega - \Lambda\Omega_c - \omega_q)/eE > 0$ corresponding to the emission of less energetic phonons prevail, which results in $\sigma_{ph} < 0$ if $\Omega > \Lambda\Omega_c$ but Ω is still not too close to $(\Lambda+1)\Omega_c$. When Ω increases approaching to and passing the next resonance, the situation repeats, leading to an oscillating dependence of σ_{ph} on Ω if Ω_c is kept constant or on Ω_c at fixed Ω.

According to the formulas obtained above,

$$\sigma_{dark} \sim \bar{\sigma} \exp\left(-\frac{\hbar\Omega_c}{T}\right), \tag{26}$$

$$\sigma_{ph} \sim \bar{\sigma} \mathcal{J}_\Omega \sum_\Lambda W\left(\frac{\Omega - \Lambda\Omega_c}{\Omega_c}, T\right), \tag{27}$$

where $\bar{\sigma}$ is the characteristic value of the conductance due to the acoustic scattering and the function $W(\omega,T)$ is determined by Eq. (25). The net dissipative conductivity and its sign are determined by both σ_{dark} and σ_{ph}. As seen from Eqs. (15) and (26), the expression for σ_{dark} comprises an exponential factor $\exp(-\hbar\Omega_c/T)$ which is small at $T \ll \hbar\Omega_c$. Due to this factor, the photoconductivity associated with the electron-phonon scattering accompanied by the absorption of microwave photons can dominate at rather small values of $\mathcal{J}_\Omega$. The W versus ω dependences at different temperatures for $L\Omega_s/s = 10$, $l/L = 0.1$ are shown in Fig. 2. One can see that the amplitude of the oscillations increases with increasing temperature. At sufficiently high temperatures ($T > \hbar s/L \simeq 0.4$ K), the amplitude of the oscillations is nearly a linear function of the temperature. This is due to an almost linear increase in the number of acoustic phonons with $q \sim L^{-1}$. However, when T becomes comparable with $\hbar\Omega_c$, the rising temperature dependence of σ_{ph} is affected by the processes of the emission of microwave radiation with the electron transitions to lower LL's (these processes which become essential at $T \gtrsim \hbar\Omega$ were not taken into account in our model).

Due to a significant decrease of $\mathcal{J}_\Omega$ in the range of microwave frequencies off the cyclotron resonance, at relatively low microwave power the maxima and minima corresponding to higher resonances are very weak. Indeed, comparing $\mathcal{J}_\Omega$ at $\Omega/\Omega_c = 2.25$ and at $\Omega/\Omega_c = 1.25$, for the ratio of the maxima for $\Lambda = 2$ and $\Lambda = 1$ we obtain a value of about 0.1. At elevated microwave powers, the magnitudes of the maximum and minimum near the cyclotron resonance as functions of the power tend to saturation. In this case, for a rough estimate $\mathcal{J}_\Omega/4$ in Eqs. (25) and (27) can be replaced by $J_1^2(\sqrt{\mathcal{J}_\Omega})$. Due to this, at high microwave powers, the ratio of the magnitudes of the maxima and minima corresponding to higher resonances ($\Lambda > 1$) and those corresponding to the cyclotron resonance ($\Lambda = 1$) can be markedly larger. The dependence of the photoconductivity on Ω/Ω_c calculated for different temperatures and normalized microwave powers are

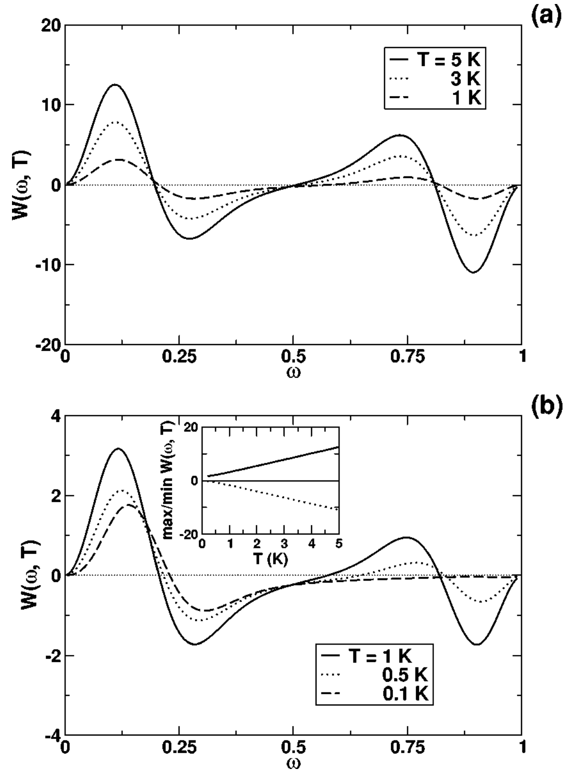

FIG. 2. Frequency dependence of $W(\omega,T)$ at different temperatures for ranges (a) $T > \hbar s/L$ and (b) $T \lesssim \hbar s/L$. The inset shows the temperature dependence of the near-resonance maximum (solid line) and minimum (dashed line) magnitudes.

shown in Fig. 3. Here the microwave power is normalized by $\bar{P} = (m\Omega^3/2\pi)(\hbar c/e^2)$. Figure 3(b) exhibits a relatively slow increase (logarithmic like) in the magnitudes of the maximum and minimum near the cyclotron resonance at high powers.

As can be drawn from the above formulas at low temperatures ($T \lesssim \hbar s/L$), when Ω increases in the range $\Lambda\Omega_c \leqslant \Omega \leqslant \Lambda\Omega_c$, σ_{ph} sequentially turns to zero at $\Omega = \Lambda\Omega_c$, $\Omega = \Lambda\Omega_c + \delta_0$, and $\Omega = \Lambda\Omega_c + \Delta_0$. The photoconductivity becomes rather small in the range $\Lambda\Omega_c + \Delta_0 < \Omega \leqslant (\Lambda+1)\Omega_c$. The quantity δ_0 can be estimated as

$$\delta_0 \simeq \frac{2s}{L}. \tag{28}$$

As follows from Eq. (25),

$$\Delta_0 = \Omega_c\left[1 - \frac{1}{2(1+\gamma)}\right], \tag{29}$$

where $\gamma = (\hbar s/Tl)(s/l\Omega_c)$. One can see from Eq. (29) that $\Delta_0/\Omega_c > 1/2$. The value Δ_0 increases with decreasing temperature. At sufficiently low temperature ($T \ll \hbar s/L$), one obtains $\Delta_0/\Omega_c \lesssim 1$. For $s = 3\times10^5$ cm/s, $l = (5\text{–}10)\times10^{-7}$ cm, $H = 2$ kG, and $T = 1$ K, Eqs. (28) and (29) yield $\delta_0/\Omega_c \simeq 1/5$ and $\Delta_0/\Omega_c \simeq 3/5 - 3/4$.

Both the dark conductivity and photoconductivity depend to some extent on $|V_{\mathbf{q}}|^2$. In our calculation we assumed that for the piezoelectric acoustic scattering $|V_{\mathbf{q}}|^2$

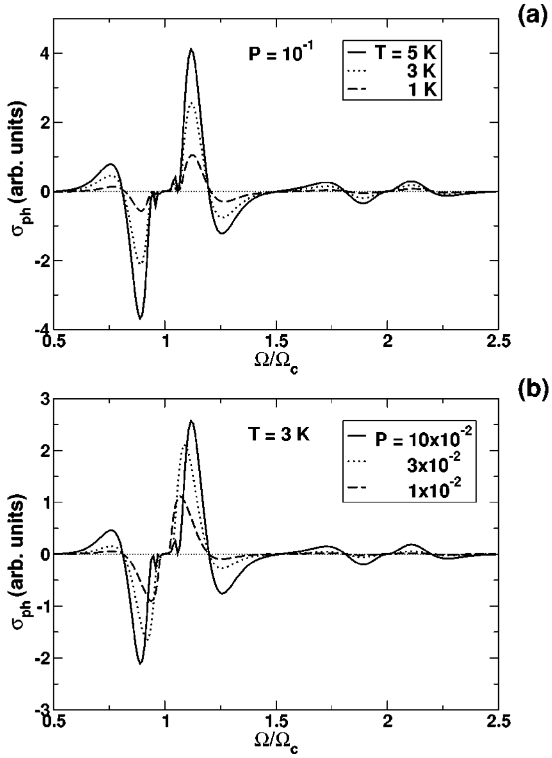

FIG. 3. Photoconductivity as a function of Ω/Ω_c at (a) different temperatures and (b) different microwave powers.

$\propto q^{-1}\exp(-l^2q_z^2/2)$. One can also use another approximation $|V_{\mathbf{q}}|^2 \propto q^{-1}(1+l^2q_z^2)^{-2}$ which corresponds to the wave functions of 2D electrons proportional to $\exp(-|z|/l)$. However, such a change in $|V_{\mathbf{q}}|^2$ does not significantly affect the obtained results.

An important feature of the obtained oscillating σ_{ph} versus Ω/Ω_c dependence is that these oscillations have the phase significantly shifted compared to that in the case of the photon-assisted impurity scattering mechanism (and observed experimentally). Moreover, the maxima (minima) of σ_{ph} determined by the photoassisted acoustic scattering at Ω/Ω_c close to the resonances approximately corresponds the minima (maxima) of σ_{ph} associated with the photon-assisted impurity scattering. Hence, one can assume that at low temperatures ($T \lesssim \hbar s/L$) the dissipative dc photoconductivity is determined primarily by the photon-assisted impurity scattering mechanism, so that maxima at $\Omega - \Lambda\Omega_c$ somewhat smaller than Ω_c and minima (with ANC) at $\Omega - \Lambda\Omega_c$ somewhat larger than Ω_c occur. However, the contribution of the photon-assisted acoustic scattering to σ_{ph} becomes essential when the temperature increases from $T < \hbar s/L$ to $T > \hbar s/L$, suppressing the maxima and minima caused by the photon-assisted impurity scattering. Thus, the combined contribution of the photon-assisted impurity and acoustic phonon scattering mechanisms complicated by sensitivity of the domains structures formed due to ANC to the characteristics of the dissipative dc photoconductivity[6] can responsible for the rather nontrivial pattern of the observed effect.

VI. CONCLUSION

We have calculated the dissipative component of the dc conductivity tensor of a 2DES in a transverse magnetic field and irradiated with microwaves. We have demonstrated that the electron transitions between the Landau levels stimulated by the absorption of microwave photons accompanied by the emission of acoustic phonons can result in absolute negative conductivity in rather wide ranges of the resonance detuning $\Omega - \Lambda\Omega_c$. Thus, the ''acoustic'' mechanism of the absolute negative conductivity can contribute to the formation of zero-resistance states.

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