

Quantum phase transitions in dissipative tunnel junctions

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The Ueda-Guinea model of a dissipative tunnel junction is investigated. This model accounts for final-state effects associated with single-electron tunneling. A quantum phase transition emerges, marking a boundary between insulating (Coulomb blockade) and conducting phases. The system is analyzed by large- N techniques, self-consistent harmonic approximation, and Monte Carlo methods.

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I. INTRODUCTION

Over 30 years ago, Mahan¹ and Nozieres and deDominicis² predicted the existence of power-law behavior in the absorption edges for x-ray transitions in metals. This phenomenon is due to the influence of a suddenly switched-on potential, due to the (screened) core hole, on the electrons. There are two aspects to this physics. One is the orthogonality catastrophe³ due to the change of electronic wave functions in the presence of the core hole. The other is formation of an excitonic resonance between the liberated electron and the core hole.^{1,2} Recently, several authors have noted the relevance of such nonequilibrium effects to mesoscopic systems such as tunnel junctions^{5,4} and quantum dots.^{7,6,8}

The model we will study has been described in the recent work of Bascones *et al.*,⁹ based on the original work of Ueda and Guinea.⁵ Briefly, we consider a junction consisting of two banks, left (L) and right (R), each described by a non-interacting Hamiltonian of the form $\mathcal{H}_\alpha = \sum_{\mathbf{k}} \epsilon_\alpha(\mathbf{k}) c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$, where $\alpha = L, R$. The tunneling between these banks is described by $\mathcal{H}_{\text{tunnel}} = -t e^{i\phi} \sum_{\mathbf{k}, \mathbf{k}'} c_{\mathbf{k}\mathbf{R}}^\dagger c_{\mathbf{k}'\mathbf{L}} + \text{H.c.}$ The phase variable ϕ is a collective coordinate conjugate to the charge transfer Q between left and right banks: $[Q, e^{i\phi}] = e^{i\phi}$. Associated with this charge transfer is a Coulomb energy $\mathcal{H}_Q = (Q - Q_{\text{offset}})^2 / 2C$, where C is the capacitance and $Q_{\text{offset}} \equiv \nu e$ accounts for the charge environment of the junction. Finally, the interaction between electrons and the global charge is written as^{5,9,10}

$$\mathcal{H}_{\text{int}} = (Q - Q_{\text{offset}}) \sum_{\mathbf{k}, \mathbf{k}', \alpha} U_{\mathbf{k}\mathbf{k}'}^\alpha c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}'\alpha}. \quad (1)$$

The fermionic degrees of freedom are quadratic in the Hamiltonian $\mathcal{H} = \mathcal{H}_L + \mathcal{H}_R + \mathcal{H}_{\text{tunnel}} + \mathcal{H}_Q + \mathcal{H}_{\text{int}}$, and can be integrated out,¹¹ leaving an effective model whose only dynamical degree of freedom is the phase field ϕ . When $U_{\mathbf{k}\mathbf{k}'}^\alpha = 0$, the resultant effective action is that obtained by Ben-Jacob, Mottola, and Schön (BMS).¹¹ This approach implicitly assumes that the energy-level spacing in each of the banks is small on the scale of the charging energy $E_c \equiv e^2/2C$ and temperature $k_B T$. When the effects of \mathcal{H}_{int} are accounted for, one obtains a modified Euclidean BMS action of the form^{5,9}

$$S[\phi(s)] = \frac{1}{4} \int_0^L ds \left(\frac{\partial \phi}{\partial s} \right)^2 + \alpha \int_0^L ds \int_0^L ds' K(s-s') \times \{1 - \cos(\phi(s) - \phi(s'))\}. \quad (2)$$

The kernel $K(s-s')$ is given by

$$K(s) = \left[\frac{\pi}{L} \text{csc} \left(\frac{\pi |s|}{L} \right) \right]^{2-\epsilon}. \quad (3)$$

Here, $L = E_c / k_B T$ is the dimensionless inverse temperature and $\alpha = g_\infty / 4\pi^2$, where g_∞ is the high-temperature conductance of the junction¹² in units of e^2/h . The BMS model is recovered for $\epsilon = 0$. The parameter ϵ is a sum over contributions from the two banks $j = L, R$, with $\epsilon_j = -(\delta_j / \pi)^2$ due to the orthogonality catastrophe and $\epsilon_j = (2\delta / \pi) - (\delta_j / \pi)^2$ if excitonic effects are relevant.

When the tunnel junction is placed in series with a capacitor, forming a single-electron box,¹³ the external charge environment is accounted for by a topological term in the action (2), $\Delta S_{\text{top}} = -2\pi i \nu W[\phi]$, where $W[\phi] = [\phi(L) - \phi(0)] / 2\pi$ is the winding number of the phase field ϕ .

The model is a version of the ubiquitous dissipative quantum rotor.¹⁴ The $\epsilon = 0$ case has been studied by several authors.¹⁵⁻¹⁹ Physical quantities are smoothly dependent on α and there is no phase transition at any finite α .²⁰ For $\epsilon > 0$, however, a quantum ($T = 0$) phase transition is present at a critical value $\alpha_c(\epsilon)$, as first noted by Kosterlitz.²¹ Here, we investigate this phase transition using a large- N expansion, the self-consistent harmonic approximation, and finally Monte Carlo simulations.

II. LARGE- N THEORY

The large- N generalization of this problem was first discussed in an unpublished work by Renn.²² Consider the action

$$S[\mathbf{n}(s), \lambda(s)] = \int_0^L ds \left\{ \frac{1}{4} \left(\frac{\partial \mathbf{n}}{\partial s} \right)^2 + \lambda(s) (\mathbf{n}^2(s) - qN) \right\} + \frac{1}{2} \alpha \int_0^L ds \int_0^L ds' K(s-s') |\mathbf{n}(s) - \mathbf{n}(s')|^2, \quad (4)$$

where $\mathbf{n}(s)$ is a real N -component vector. The field $\lambda(s)$ serves as a Lagrange multiplier, which enforces the constraint $\mathbf{n}^2(s) = qN$ for all s ; typically we take $q = 1/N$, so the \mathbf{n} field is of unit length. When $N = 2$, one can eliminate the constraint with the parametrization $\mathbf{n} = (\cos \phi, \sin \phi)$, whence one recovers the action of Eq. (2).

In the $N \rightarrow \infty$ limit, the action becomes dominated by the saddle point $\lambda(s) = \lambda$, a constant function. Then one has

$$S = \sum_{\omega_n} \left[\frac{1}{4} \omega_n^2 + \lambda + \alpha [\hat{K}(0) - \hat{K}(\omega_n)] \right] |\hat{\mathbf{n}}(\omega_n)|^2 - qNL\lambda, \quad (5)$$

where $\omega_n = 2\pi n/L$ is a bosonic Matsubara frequency and

$$\mathbf{n}(s) \equiv \frac{1}{\sqrt{L}} \sum_{\omega_n} \hat{\mathbf{n}}(\omega_n) e^{-i\omega_n s}, \quad (6)$$

$$\hat{K}(\omega_n) = \int_0^L ds K(s) e^{i\omega_n s}. \quad (7)$$

The saddle point theory is thus a Gaussian theory, with correlation functions

$$\langle n_i(s) n_j(s') \rangle = G(s) \delta_{ij}$$

$$G(s) = \frac{1}{2L} \sum_{\omega_n} \frac{e^{-i\omega_n s}}{\frac{1}{4} \omega_n^2 + \lambda + \alpha [\hat{K}(0) - \hat{K}(\omega_n)]}. \quad (8)$$

Extremizing the free energy $F = -L^{-1} \text{Tr}_{\hat{\mathbf{n}}} \exp(-S[\hat{\mathbf{n}}])$ with respect to λ yields the equation $G(0) = q$, which is to be solved for $\lambda(\alpha, L)$.

In the zero-temperature ($L \rightarrow \infty$) limit, $\omega_n \rightarrow \omega$ becomes a continuous quantity and $\hat{K}(0) - \hat{K}(\omega) = C_\epsilon |\omega|^{1-\epsilon}$, with

$$C_\epsilon = \frac{\pi}{\Gamma(2-\epsilon) \cos\left(\frac{1}{2}\pi\epsilon\right)}. \quad (9)$$

Straightforward analysis of the integral shows that λ vanishes for $\alpha > \alpha_c(\epsilon)$, where

$$\alpha_c(\epsilon) = \frac{1}{4C_\epsilon} \left(\frac{2A_\epsilon}{\pi q \epsilon} \right)^{1+\epsilon} \quad (10)$$

$$A_\epsilon \equiv \int_0^\infty \frac{dt}{1+t^{1+\epsilon^{-1}}}. \quad (11)$$

From $\lim_{\epsilon \rightarrow 0} A_\epsilon = 1$ and $C_{\epsilon=0} = \pi$, we find, restoring $q = 1/N$, that for small ϵ the critical point occurs at $\alpha_c(\epsilon) = N/2\pi^2\epsilon$. This is to be contrasted with the renormalization group result of Kosterlitz,²¹ $\alpha_c^{\text{RG}}(\epsilon) = (N-1)/2\pi^2\epsilon$. Note that these results agree to leading order in $1/N$.

Further analysis reveals the critical behavior of $\lambda(\alpha)$ in the vicinity of the critical point:

$$\lambda(\alpha) \simeq D_\epsilon [\alpha_c(\epsilon) - \alpha]^\nu, \quad (12)$$

with

$$D_\epsilon = [2^{1-\epsilon} C_\epsilon \alpha_c(\epsilon)]^{2/1+\epsilon} \left(\frac{A_\epsilon/B_\epsilon}{(1+\epsilon)\alpha_c(\epsilon)} \right)^\nu,$$

$$B_\epsilon = \int_0^\infty \frac{dt}{1+t^{\epsilon^{-1}-1}} \quad \text{if } 0 \leq \epsilon < \frac{1}{2}$$

$$= \frac{\epsilon}{2\epsilon-1} \int_0^\infty \frac{dt}{(1+t^{1+\epsilon/2\epsilon-1})^2} \quad \text{if } \frac{1}{2} \leq \epsilon, \quad (13)$$

where $\nu = \max(1, \epsilon^{-1} - 1)$.

A. Kubo Formula

When $N = 2$ the symmetry group $O(2)$ has a single generator, hence there is one vector potential. For the $O(N)$ -symmetric case, a local gauge is effected by $\mathbf{n}(s) \rightarrow \mathcal{R}(s) \mathbf{n}(s)$, where $\mathcal{R}(s) \in O(N)$ is given by

$$\mathcal{R}(s) = \exp[iA_a(s)T^a], \quad (14)$$

where the T^a generate the Lie algebra $\mathfrak{o}(N)$. These $\frac{1}{2}N(N-1)$ generators are $N \times N$ Hermitian antisymmetric tensors; they may be chosen to satisfy the normalization $\text{Tr}(T^a T^b) = N \delta^{ab}$. (For $N = 2$, the sole generator is $T = \sigma^y$.) There are therefore $\frac{1}{2}N(N-1)$ vector potentials $A_a(s)$, variation with respect to which defines the $O(N)$ currents

$$I_a(s) = \frac{\delta S_{\text{int}}[\mathcal{Y}(s)\mathbf{n}(s)]}{\delta A_a(s)} \Big|_{\mathbf{A}=0}$$

$$= 2i\alpha n_k(s) T_{kl}^a \int_0^L ds K(u-s) n_l(s) \quad (15)$$

and the noise current-current correlation function

$$\langle \mathcal{T} I_a^n(s) I_b^n(0) \rangle = \left\langle \frac{\delta^2 S_{\text{int}}}{\delta A_a(s) \delta A_b(0)} \right\rangle_{\mathbf{A}=0}$$

$$= 2N\alpha \delta^{ab} \left[\delta(s) \int_0^L du K(u) G(u) - K(s) G(s) \right]. \quad (16)$$

The conductance (in units of e^2/h) is computed according to the Kubo formula,²³

$$g_{ab}(i\omega_n) = \frac{2\pi}{\omega_n} \int_0^L ds e^{i\omega_n s} \langle \mathcal{T} I_a^n(s) I_b^n(0) \rangle. \quad (17)$$

We define $g(i\omega_n)$ by $g_{ab}(i\omega_n) \equiv Ng(i\omega_n) \delta_{ab}$. We have no expression for the analytic continuation of the conductance to real frequencies. As a diagnostic of any phase transition, we will examine the quantity $g_A \equiv g(i\omega_{n=1})$. We find

$$g_A = \frac{4\pi^2\alpha}{L} \left(\frac{L}{\pi} \right)^\epsilon \int_0^L ds \sin^\epsilon(\pi s/L) \langle \cos(\phi(s) - \phi(0)) \rangle. \quad (18)$$

A similar expression was proposed by Bascones *et al.*⁹

$$g_B = 4\pi^2\alpha \left(\frac{L}{\pi}\right)^\epsilon \left\langle \cos\left(\phi\left(\frac{1}{2}L\right) - \phi(0)\right) \right\rangle. \quad (19)$$

At the quantum critical point, we can evaluate dc ($\omega \rightarrow 0$) limit of $g(i\omega_n, \alpha_c, T=0)$. We obtain

$$g_c = \pi(1-\epsilon) \text{ctn}\left(\frac{1}{2}\pi\epsilon\right). \quad (20)$$

Note that while the critical coupling α_c is q dependent, the critical conductance $g_c(\epsilon)$ is universal and independent of q . The approximations g_A and g_B also take on universal values at criticality:

$$g_A^c = \frac{1}{\pi} \int_0^\pi d\theta \left(\frac{\sin\theta}{\theta}\right)^\epsilon \times g_c(\epsilon), \quad (21)$$

$$g_B^c = \left(\frac{2}{\pi}\right)^{1+\epsilon} \times g_c(\epsilon). \quad (22)$$

III. SELF-CONSISTENT HARMONIC APPROXIMATION

The fully nonlinear theory with action

$$S[\phi(s)] = \int_0^L ds \frac{1}{4} \left(\frac{\partial\phi}{\partial s}\right)^2 + \alpha \int_0^L ds \int_0^L ds' K(s-s') \times [1 - \cos(\phi(s) - \phi(s'))] \quad (23)$$

is replaced with a trial Gaussian theory described by the quadratic action

$$S_0[\phi(s)] = \frac{1}{2} \int_0^L ds \int_0^L ds' V(s-s') [\phi(s) - \phi(s')]^2 = \frac{1}{2} \sum_{\omega_n} \hat{G}^{-1}(\omega_n) |\hat{\phi}(\omega_n)|^2, \quad (24)$$

where

$$\hat{G}(\omega_n) \equiv \frac{1}{2[\hat{V}(0) - \hat{V}(\omega_n)]}. \quad (25)$$

$V(s)$, or equivalently $\hat{G}(\omega_n)$, is treated variationally, so we extremize by setting

$$\frac{\partial}{\partial \hat{G}(\omega_n)} \left\{ F_0 + \frac{1}{L} \langle S - S_0 \rangle_0 \right\} \quad (26)$$

where $F_0 = -L^{-1} \text{Tr}_\phi \exp(-S_0[\phi])$. This leads to the following self-consistent equation:

$$\frac{1}{\hat{G}(\omega_n)} = \frac{1}{2} \omega_n^2 + 2\alpha \int_0^L ds [1 - \cos(\omega_n s)] K(s) \Gamma(s), \quad (27)$$

where

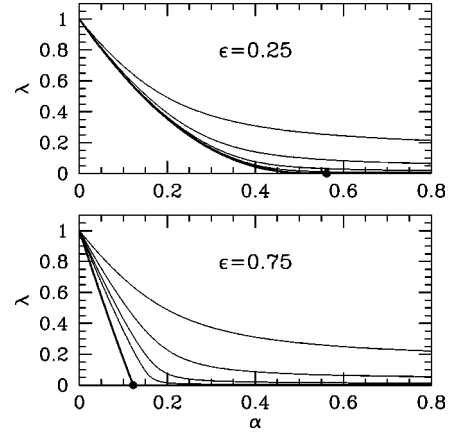


FIG. 1. Solution to the large- N model at finite L with $\epsilon=0.25$ (top) and $\epsilon=0.75$ (bottom). λ versus α for $L=8, 32, 128, 512$ (thin lines) and $L=\infty$ (thick line). $\alpha_c(\epsilon=0.25)=0.5619$, and $\alpha_c(\epsilon=0.75)=0.1226$.

$$\Gamma(s) \equiv \exp\left(-\frac{1}{2} \langle [\phi(s) - \phi(0)]^2 \rangle_0\right) = \exp\left(-\frac{2}{L} \sum_{n=1}^{\infty} [1 - \cos(\omega_n s)] \hat{G}(\omega_n)\right). \quad (28)$$

We iterate these equations to self-consistency.

The dimensionless conductance $g_A(\alpha, L)$ is plotted for $\epsilon=0.1$ and 0.2 in Fig. 1. As the dimensionless inverse temperature L is increased from $L=16$ to $L=256$, the curves apparently cross at a critical point $\alpha_c(\epsilon)$. For $\alpha > \alpha_c$, the conductance increases as the temperature is lowered, indicating a conducting phase. For $\alpha < \alpha_c$, the conductance vanishes as the temperature is lowered, i.e., the Coulomb gap survives. For larger values of ϵ , however, a spurious first-order transition preempts this critical behavior, as shown in Fig. 2. The solution to the SCHA is hysteretic, and discon-

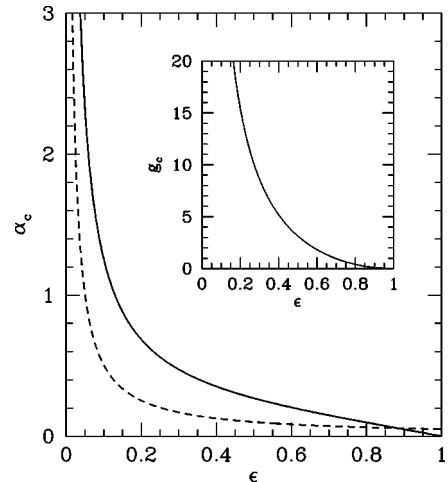


FIG. 2. Large- N results for $\alpha_c(\epsilon)$ (solid) compared with the Kosterlitz RG value $\alpha_c^{\text{RG}} = 1/2\pi^2\epsilon$ (dashed). Inset: critical conductance $g_c(\epsilon)$ within the large- N approximation, with $N=2$.

tinuous, provided L is large enough. For $\epsilon \lesssim 0.6$, however, this discontinuity becomes apparent only at relatively low temperatures $L^{-1} \lesssim \frac{1}{64}$.

IV. COULOMB GAS REPRESENTATION AND MONTE CARLO

Including the offset charge, the Euclidean action for our system is $S = S_0 + S_{\text{int}} + S_{\text{top}}$, where

$$S_0[\phi] = \frac{1}{4} \int_0^L ds \left(\frac{\partial \phi}{\partial s} \right)^2, \quad (29)$$

$$S_{\text{int}}[\phi] = -\alpha \int_0^L ds \int_0^L ds' K(s-s') \cos[\phi(s) - \phi(s')], \quad (30)$$

$$S_{\text{top}} = -i\nu[\phi(L) - \phi(0)] \equiv -2\pi i\nu W[\phi], \quad (31)$$

where $W[\phi] = [\phi(L) - \phi(0)]/2\pi$ is the winding number of the phase field. In the phase representation, the topological term represents a purely imaginary contribution to the action. However, in the Coulomb gas representation, the action remains purely real.

We have discarded a formally divergent constant from our action

$$\Delta S = L \int_0^L ds K(s), \quad (32)$$

which may be rendered finite through an ultraviolet regularization of $K(s)$, viz.

$$K_b(s) = \left\{ \frac{\pi}{L} \csc\left(\frac{\pi|s|}{L}\right) \left[1 - e^{-L \sin(\pi|s|/L)/\pi b} \right] \right\}^{2-\epsilon}, \quad (33)$$

so that $K_b(0) = b^{-(1-\epsilon)}$.

The partition function for our problem is given by

$$\Xi(\nu) = \Xi_0 \left\langle e^{i\nu[\phi(L) - \phi(0)]} \sum_{N=0}^{\infty} \frac{1}{N!} (-S_{\text{int}})^N \right\rangle_0 \quad (34)$$

$$\begin{aligned} &= \Xi_0 \sum_{N=0}^{\infty} \frac{\alpha^N}{N!} \int_0^L ds_1^+ \cdots \int_0^L ds_N^- \prod_{j=1}^N K(s_j^+ - s_j^-) \\ &\quad \times \left\langle e^{i\nu\phi(L)} e^{-i\nu\phi(0)} e^{i \sum_{k=1}^N [\phi(s_k^+) - \phi(s_k^-)]} \right\rangle_0, \end{aligned} \quad (35)$$

where the average is with respect to the bare action S_0 . This allows us to transform the complex action (due to the topological term) in the phase representation to a purely real action in a dipole gas representation.²⁵ The coordinates $\{s_k^\pm\}$ are interpreted as locations of positive and negative charges. Each factor of $(-S_{\text{int}})$ introduces another pair of such charges, i.e., a dipole. We now write the field $\phi(s)$ as a sum of winding plus a periodic pieces:

$$\phi(s) = \frac{2\pi Ws}{L} + \eta + \varphi(s), \quad (36)$$

$$\varphi(s) = \frac{1}{\sqrt{L}} \sum'_{\omega_n} \hat{\varphi}(\omega_n) e^{-i\omega_n s}, \quad (37)$$

where η is a constant and W is the winding number. The prime on the sum denotes exclusion of the $n=0$ term, which is accounted for by η . The bare action is then

$$S_0 = \frac{\pi^2 W^2}{L} + \frac{1}{4} \sum'_{\omega_n} \omega_n^2 |\hat{\varphi}(\omega_n)|^2. \quad (38)$$

Thus, $\langle |\hat{\varphi}(\omega_n)|^2 \rangle_0 = 2/\omega_n^2$, and

$$\begin{aligned} C(s) &\equiv \frac{1}{2} \langle [\varphi(s) - \varphi(0)]^2 \rangle_0 \\ &= \frac{L}{\pi^2} \sum_{n=1}^{\infty} \frac{1 - \cos(2\pi n s/L)}{n^2} \\ &= \left[s \left(1 - \frac{s}{L} \right) \right]_{\text{per}}, \end{aligned} \quad (39)$$

where the subscript indicates that the expression is to be periodically extended from its value on the interval $s \in [0, L]$. Summing over the winding number W and averaging over $\phi(s)$, we obtain the N -dipole pair Boltzmann weight,

$$\begin{aligned} \varrho_N(s_1^+, \dots, s_N^-) &= \frac{\alpha^N}{N!} \prod_{j=1}^N K(s_j^+ - s_j^-) \\ &\quad \times \exp \left[\frac{1}{2} \sum_{\substack{j, j' \\ \sigma, \sigma'}} \sigma \sigma' C(s_j^\sigma - s_{j'}^{\sigma'}) \right] \\ &\quad \times \frac{\vartheta_3(\pi P/L + \pi\nu | i\pi/L)}{\vartheta_3(0 | i\pi/L)}, \end{aligned} \quad (40)$$

where P is the total dipole moment,

$$P = \sum_{j=1}^N (s_j^+ - s_j^-), \quad (41)$$

and $\vartheta_3(z|\tau)$ is the Jacobi θ function.²⁶

We now use the ϑ -function identity

$$\vartheta_3(z|\tau) = \left(\frac{i}{\tau} \right)^{1/2} e^{-iz^2/\pi\tau} \vartheta_3\left(-\frac{z}{\tau} \middle| -\frac{1}{\tau} \right) \quad (42)$$

to obtain

$$\begin{aligned} \varrho_N(s_1^+, \dots, s_N^-) &= \frac{\alpha^N}{N!} \prod_{j=1}^N K(s_j^+ - s_j^-) \\ &\quad \times \frac{H_L(P + \nu L)}{H_L(0)} e^{-U} e^{-2\nu P} e^{-L\nu^2}, \end{aligned} \quad (43)$$

where U is given by

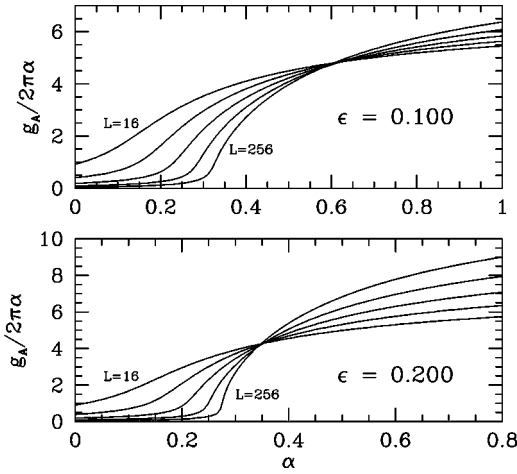


FIG. 3. Results of the self-consistent harmonic approximation for $\epsilon=0.10$ and $\epsilon=0.20$, with dimensionless inverse temperature $L=16, 32, 64, 128$, and 256 . The crossing of the curves at a single-point indicates a second order phase transition.

$$U = -\frac{1}{2} \sum_{\substack{j,j' \\ \sigma\sigma'}} \sigma\sigma' |s_j^\sigma - s_{j'}^{\sigma'}| \quad (44)$$

and

$$H_L(y) = \vartheta_3(iy|iL/\pi) = \sum_{m=-\infty}^{\infty} e^{-m^2L} e^{-2my}, \quad (45)$$

and where the location of each charge is restricted to the interval $s_j^\sigma \in [0, L]$, so that $P \in [-NL, NL]$.

Interpreting Eq. (43) for $\nu=0$, we have that the system consists of a gas of N dipole pairs. Each of the $2N$ charges interacts with every other charge according to a one-dimensional Coulomb potential $-\sigma\sigma'|s-s'|$. In addition, each charge is logarithmically attracted to its unique mate—the other member of its dipole pair—via a potential $V_d(s) = -\ln K(s) \approx (2-\epsilon)\ln|s|$. For $\nu \neq 0$, there is an additional electric field of strength 2ν present. α is the dipole fugacity; $\alpha \rightarrow 0$ will suppress the appearance of dipoles. Hence, in the absence of any short-time cutoff b , short-distance dipole pairs will proliferate without limit. A simple estimation of the dipole density, ignoring the Coulomb interactions, is $n_{\text{dip}} \sim b^{-(1-\epsilon)}/(1-\epsilon)$. In our Monte Carlo simulations, we have used $b=1$; the essential physics is rather weakly dependent on b , though. Grand canonical averages are to be computed in the usual way, i.e., $\langle A \rangle = \text{Tr}(\varrho A) / \text{Tr} \varrho$, where the trace entails a sum over all possible numbers N of dipole pairs and integration over their $2N$ charge coordinates.

In Fig. 3, we plot the Coulomb energy for a single-dipole pair,

$$V_C(s) = |s| + \ln \frac{H_L(s + \nu L)}{H_L(0)} + 2\nu s \quad (46)$$

as well as the total dipole energy $V_C(s) + V_d(s)$ for $\nu=0$, $\nu=\frac{1}{4}$, and $\nu=\frac{1}{2}$ for $L=5$ and $L=25$. Note how the interaction becomes asymmetric (yet still properly periodic) when ν

is neither integer nor half-odd integer. Furthermore, the Coulomb interaction is effectively cancelled when $\nu=\frac{1}{2}$.

The XY phase correlation function

$$G(s) = \langle e^{i\phi(s)} e^{-i\phi(0)} \rangle \quad (47)$$

is simply related to the dipole separation correlation function

$$h(s) = \left\langle \sum_{i=1}^N \delta(s_i^+ - s_i^- - s) \right\rangle. \quad (48)$$

To see this, define the quantity

$$\begin{aligned} Q_N(\tau^+, \tau^-) &= (N+1) \int_0^L ds_1^+ \int_0^L ds_1^- \cdots \int_0^L ds_N^+ \int_0^L ds_N^- \\ &\times \varrho_{N+1}(s_1^+, s_1^-, \dots, s_N^+, s_N^-, \tau^+, \tau^-). \end{aligned} \quad (49)$$

Then one has

$$\alpha K(s) G(s) = \frac{1}{L\Xi} \sum_{N=0}^{\infty} \int_0^L d\tau Q_N(\tau+s, \tau) = h(s). \quad (50)$$

What is computed in the Coulomb gas Monte Carlo calculation is the dipole separation correlation function $h(s)$, from which the XY phase correlator is obtained via $G(s) = h(s) / \alpha K(s)$. The conductance is then obtained using Eq. (18).

The renormalized charging energy E_c^* is given by

$$\begin{aligned} \frac{E_c^*}{E_c} &= \frac{1}{2} \frac{\partial^2 F}{\partial \nu^2} \Big|_{\nu=0} \\ &= 1 - \frac{L}{2} \left\langle \frac{H_L''(P)}{H_L(P)} \right\rangle + 2 \left\langle P \frac{H_L'(P)}{H_L(P)} \right\rangle - \frac{2}{L} \langle P^2 \rangle. \end{aligned} \quad (51)$$

Note that when the dipole fugacity α vanishes, there are no pairs at all and one obtains

$$\frac{E_c^*}{E_c} \Big|_{\alpha=0} = 1 - \frac{L}{2} \frac{H_L''(0)}{H_L(0)}, \quad (52)$$

which also follows from an analysis of the noninteracting Hamiltonian $\mathcal{H}_{\alpha=0} = E_c(\hat{n} + \nu)^2$. In this case, the renormalized charging energy E_c^* interpolates between its low-temperature value of $E_c^* = E_c$ and its high-temperature limit of $E_c^* = 0$.

A. Results

Typical raw data for $g_A(\alpha, L)$ are shown in Fig. 4. Two phases are identified for every positive value of ϵ : a small α insulating phase in which the Coulomb gap persists and g_A decreases as $T \rightarrow 0$ ($L \rightarrow \infty$), and a large α conducting phase in which g_A diverges as $T \rightarrow 0$, indicating a nonlinear I - V relation at $T=0$. The boundary between these phases is marked by a critical value $\alpha_c(\epsilon)$, at which there is a quantum phase transition. At $\alpha = \alpha_c$, the conductance becomes temperature independent (provided T is low enough that the sys-

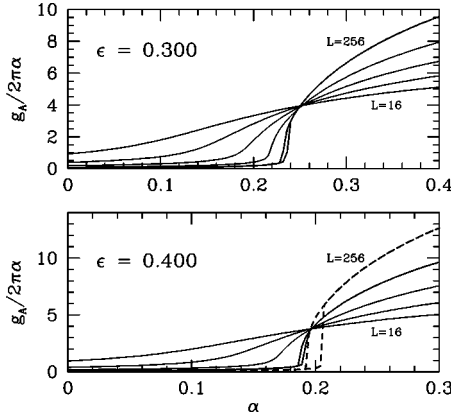


FIG. 4. Results of the self-consistent harmonic approximation for $\epsilon=0.30$ and $\epsilon=0.40$, with dimensionless inverse temperature $L=16,32,64,128,256$. For high temperatures, the curves seem to cross at a single point. However, a spurious first-order transition intervenes at a low T (large L).

tem is in the scaling regime). Monte Carlo results for $\alpha_c(\epsilon)$ and $g_A^c(\epsilon)$ are presented in Fig. 5, along with comparisons to SCHA and large- N results.

Our results for $\alpha_c(\epsilon)$ differ significantly from those of Bascones *et al.*,⁹ who obtained α_c through analysis of the renormalized charging energy, assuming $E_c^*(\alpha) \sim (\alpha_c - \alpha)^{1/\epsilon}$. [Our values for $\alpha_c(\epsilon)$ are approximately four times smaller throughout the range $0 < \epsilon \leq \frac{1}{2}$.] In fact, we find that our raw data for E_c^*/E_c versus α are in good agreement with those of Bascones *et al.*²⁷ This agreement is noteworthy since their Monte Carlo was carried out in the phase representation, whereas our is in the Coulomb gas representation. However, it is very difficult to reliably extract α_c from the charging energy data, as our results shown in Fig. 6 show. At a temperature corresponding to $L=50$ (Ref. 9) or $L=40$ (this work), there is no detectable signature of the phase transition at $\alpha = \alpha_c$. A similar behavior is found in the large- N results of Fig. 7. In the large- N theory, $\lambda(\alpha)$ plays the role of an energy gap, similar to E_c^* . As is evident from Fig. 1, even at

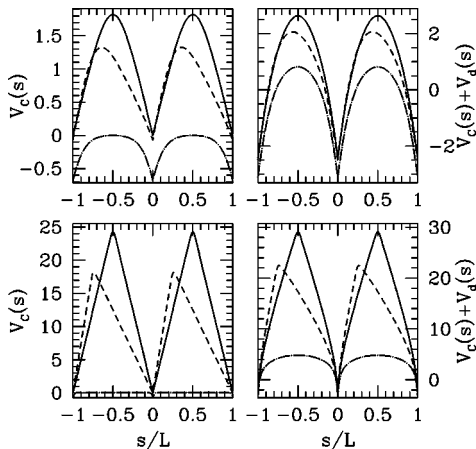


FIG. 5. Single pair potentials $V_C(s)$ and $V_C(s)+V_d(s)$ for $\nu=0$ (solid), $\nu=0.25$ (dashed), and $\nu=0.5$ (dot-dashed). Upper panels have $L=5$; lower panels have $L=25$. In all cases $b=0.2$.

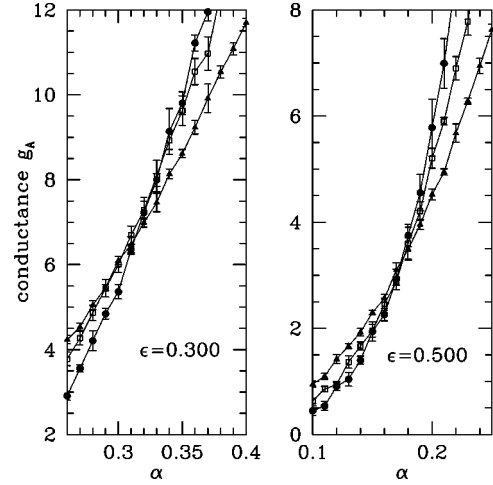


FIG. 6. Coulomb gas Monte Carlo results for conductance $g_A(\alpha)$ at $L=10$ (filled triangles), $L=20$ (open squares), and $L=40$ (filled circles) for $\epsilon=0.30$ (left panel) and $\epsilon=0.50$ (right panel). A critical point marks the boundary between a high α conducting phase and a low α insulating (Coulomb blockade) phase.

relatively low temperatures of $1/L \approx 1/32$, extrapolation of α_c based on the zero-temperature critical (i.e., power-law) behavior is problematic (Fig. 8). Indeed, for a fixed $1/L$, varying α takes the system through renormalized classical, quantum critical, and quantum disordered regimes,²⁸ and the single parameter $L=\infty$ behavior of $\lambda(\alpha)$ is insufficient to extract α_c .²⁹

Finally, we plot conductance versus charge offset ν for $\epsilon=0.2$ and $\epsilon=0.8$ in Figs. 9 and 10, respectively. Two values of α on either side of α_c are chosen, corresponding to oppo-

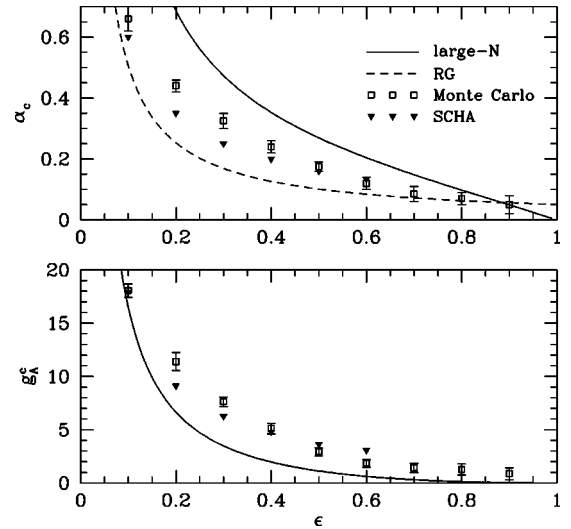


FIG. 7. Critical coupling $\alpha_c(\epsilon)$ (top) and critical conductance $g_A^c(\epsilon)$ (bottom), comparing data from Monte Carlo (open squares, with error bars), self-consistent Harmonic approximation (filled triangles), and large- N (smooth curve) calculations. As discussed in the text, the SCHA yields a discontinuous transition at the lowest temperatures; the SCHA results shown are obtained from crossings of conductance for $L \leq 64$. The dashed line in the top figure is the Kosterlitz RG result $\alpha_c^{\text{RG}} = 1/2\pi^2\epsilon$.

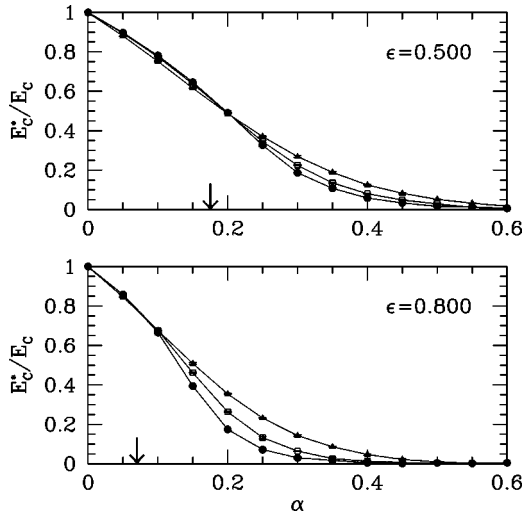


FIG. 8. Dimensionless renormalized charging energy E_c^*/E_c versus α at $L=10$ (filled triangles), $L=20$ (open squares), and $L=40$ (filled circles) for $\epsilon=0.50$ (top panel) and $\epsilon=0.80$ (bottom panel). Arrows indicate the location of the phase transition as obtained from crossing of conductance curves.

site temperature dependences at $\nu=0$. We find a curious double peak structure in the vicinity of $\nu=\frac{1}{2}$ at higher temperatures, but it disappears as $T\rightarrow 0$. For $\nu=\frac{1}{2}$ and $\epsilon=0.8$, the conducting state prevails even at small values of α . This feature is emphasized in Fig. 11, where $g_A(\alpha, \nu=\frac{1}{2})$ is contrasted for $\epsilon=0.8$ and $\epsilon=0.1$. For $\epsilon=0.8$, a conducting state is observed down to values of α as small as 0.01, well below the $\nu=0$ critical value of $\alpha_c\approx 0.07$. Hence, it is possible for the junction to exhibit opposite temperature dependences in the troughs ($\nu\approx 0$) and peaks ($\nu\approx \frac{1}{2}$) of the conductance as the gate voltage is varied. This result is to be contrasted with the behavior at $\epsilon=0.1$, where the conductance at $\nu=0.5$ is very weakly temperature dependent. Indeed, at $\epsilon=0$ and ν

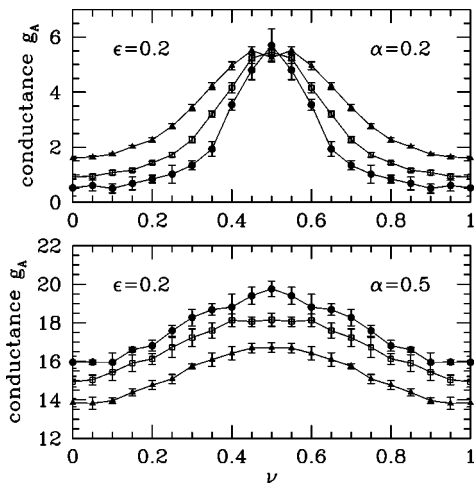


FIG. 9. Conductance g_A versus charge offset ν at $\epsilon=0.2$ for $\alpha=0.2$ (top panel) and $\alpha=0.5$ (bottom panel) at dimensionless inverse temperatures $L=10$ (filled triangles), $L=20$ (open squares), and $L=40$ (filled circles). The critical coupling for $\nu=0$ is $\alpha_c\approx 0.44$.

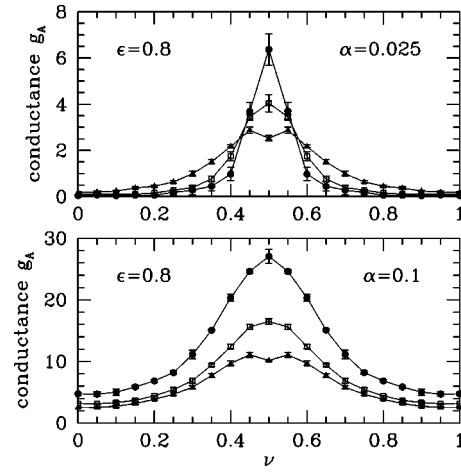


FIG. 10. Conductance g_A versus charge offset ν at $\epsilon=0.8$ for $\alpha=0.025$ (top panel) and $\alpha=0.10$ (bottom panel) at dimensionless inverse temperatures $L=10$ (filled triangles), $L=20$ (open squares), and $L=40$ (filled circles). The critical coupling for $\nu=0$ is $\alpha_c\approx 0.07$.

$=\frac{1}{2}$, the inverse charging energy is known to diverge very weakly.³⁰ While the experiments of Joyez *et al.*³¹ seem to be perfectly consistent with the more familiar $\epsilon=0$ behavior, anomalous temperature dependence in the troughs has been observed in the conductance of quantum dots,³² although it is hardly clear that the nonequilibrium effects which we consider are uniquely responsible for this phenomenon.

V. CONCLUSIONS

Nonequilibrium shakeup processes have the potential to drastically affect the physics of tunnel junction behavior, by allowing for a conducting phase in which the Coulomb blockade is completely suppressed. The phase transition is made manifest within a large- N approach, although it was first predicted over 25 years ago by Kosterlitz²¹ in his renor-

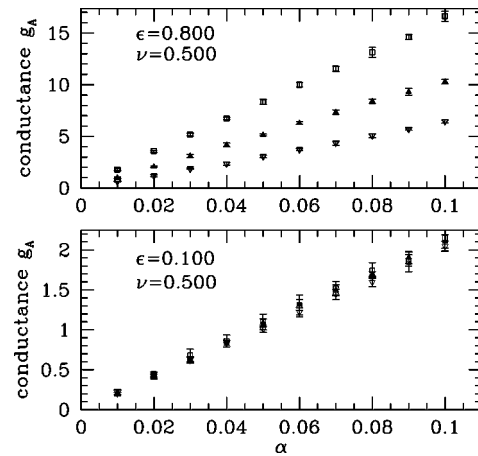


FIG. 11. Conductance versus coupling α at charge offset $\nu=\frac{1}{2}$ for $\epsilon=0.80$ (top) and $\epsilon=0.1$ (bottom), for $L=5$ (open down-pointing triangles), $L=10$ (filled triangles), and $L=20$ (open squares). For reference, at $\nu=0$, $\alpha_c(\epsilon=0.8)\approx 0.07$ and $\alpha_c(\epsilon=0.1)\approx 0.66$.

malization group studies of $O(N)$ classical spin chains with long-ranged (power-law) interactions. We have applied three methods in studying the phase transition: large- N approach, self-consistent harmonic approximation, and Coulomb gas Monte Carlo. The results from all three methods are in good qualitative agreement with each other (see Fig. 7), and the agreement between SCHA and Monte Carlo is unexpectedly good, given the crudeness of the SCHA and its neglect of the proper topology of the phase field.

The phase transition has two principal signals. First, the renormalized charging energy E_c^* vanishes for $\alpha \geq \alpha_c$. Second, while the conductance vanishes for $\alpha < \alpha_c$ and diverges for $\alpha > \alpha_c$, precisely at the transition $g_c = g(\alpha_c)$ is finite and universal (although ϵ dependent). Extracting the critical value α_c from numerical data at finite temperature can be tricky, we have found. In particular, the renormalized charging energy shows little signal at α_c even at dimensionless inverse temperatures of the order of $L = E_c/k_B T \sim 40$; this behavior is borne out explicitly in our large- N studies. It is more reliable to obtain α_c from the crossing of the conductance curves $g(\alpha, L)$.

An extension of the model investigated here to the case of granular systems has been recently considered.³³

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APPENDIX: ABSENCE OF LONG-RANGED ORDER FOR $\epsilon=0$

Applying Mermin's classic Bogoliubov inequality arguments,³⁴ Šimánek³⁵ has proven the absence of long-ranged order in the $1/n^2$ XY chain. Here, we (trivially) extend Šimánek's work to the continuum, and show how the $\epsilon=0$ case is marginal, i.e., absence of order cannot be so proven for $\epsilon > 0$.

We begin with an action

$$\begin{aligned} S[P(s), \phi(s)] = & \int_0^L ds \left[\frac{1}{2} P^2(s) + \frac{1}{4} (\partial_s \phi)^2 - h \cos \phi(s) \right] \\ & + \alpha \int_0^L ds \int_0^L ds' K(s-s') \\ & \times \{1 - \cos[\phi(s) - \phi(s')]\}. \end{aligned} \quad (\text{A1})$$

The Poisson bracket of A and B is defined by

$$\{A, B\} \equiv \int_0^L ds \left(\frac{\delta A}{\delta P(s)} \frac{\delta B}{\delta \phi(s)} - \frac{\delta A}{\delta \phi(s)} \frac{\delta B}{\delta P(s)} \right), \quad (\text{A2})$$

and the average of a functional $A[P(s), \phi(s)]$ is

$$\langle A \rangle = \frac{\int \mathcal{D}P \int \mathcal{D}\phi A[P, \phi] e^{-S[P, \phi]}}{\int \mathcal{D}P \int \mathcal{D}\phi e^{-S[P, \phi]}}. \quad (\text{A3})$$

Note that when A is a functional of $\phi(s)$ alone, the functional integral over the momentum field $P(s)$ cancels between numerator and denominator, i.e., we recover the modified BMS model.

The Bogoliubov inequality guarantees³⁴

$$\langle A^* A \rangle \geq \frac{|\langle \{C, A^*\} \rangle|^2}{\langle \{C, \{C^*, S\}\} \rangle}, \quad (\text{A4})$$

for an arbitrary functional $C[P, \phi]$. Following Šimánek, we take

$$\begin{aligned} A[P, \phi] &= \int_0^L ds \sin \phi(s) e^{-i\omega_n s}, \\ C[P, \phi] &= \int_0^L ds P(s) e^{-i\omega_n s}, \end{aligned} \quad (\text{A5})$$

where $\omega_n = 2\pi n/L$ is a Matsubara frequency. In $L \rightarrow \infty$ limit, the Bogoliubov inequality then establishes the following identity:

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{m^2}{hm + \frac{1}{2}\omega^2 + 2\alpha C_\epsilon |\omega|^{1-\epsilon}} \leq 1, \quad (\text{A6})$$

where $m = \langle \cos \phi(0) \rangle$ is the average magnetization density. When $\epsilon=0$, the integral diverges as $-m^2 \ln(hm)$, hence the inequality demands $m=0$ when $h=0$, i.e., no long-ranged order. However, the case $\epsilon=0$ is marginal, and for any $\epsilon > 0$ the integral is convergent when $h=0$. While this does not rigorously prove the existence of an ordered phase for $\epsilon > 0$, it is at least consistent with our findings.

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