

Conductance fluctuations in the presence of spin scattering

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Electron transport through disordered systems that include spin scatterers is studied numerically. We consider three kinds of magnetic impurities: the Ising, the XY, and the Heisenberg. By extending the transfer matrix method to include the spin degree of freedom, the two terminal conductance is calculated. The variance of conductance is halved as the number of spin components of the magnetic impurities increases. Application of the Zeeman field in the lead causes a further halving of the variance under certain conditions.

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I. INTRODUCTION

Quantum transport phenomena that involve the carrier's spin degree of freedom have attracted a lot of attention during the past decade.^{1,2} A number of studies have analyzed the spin polarized transport in ballistic regime and reported intriguing phenomena,^{3,4} and have opened up the possibility of new spintronic devices. It is also interesting to study the spin-dependent transport in diffusive and chaotic regimes because the interference of coherent electron waves shows a number of characteristic effects when the spin degree of freedom is taken into consideration. One of the special characteristics of diffusive and chaotic systems is the fluctuating nature of transport coefficients such as conductance.⁵ It is well understood that such fluctuations do not depend on the details of the sample parameters, but depend only on the symmetry of the system.^{5,6} The two relevant symmetries here are time-reversal symmetry (TRS) and spin-rotation symmetry (SRS). TRS is broken by applied magnetic fields or by magnetic scattering due to magnetic impurities or magnetic domain walls. If TRS is broken, systems are classified as unitary, regardless of whether or not SRS is broken. The spin-orbit interaction breaks SRS but preserves TRS, and in this case the systems are classified as symplectic.

When the spin degree of freedom is taken into account, the description of the conductance fluctuations becomes more complex. Conductance fluctuations of two-dimensional systems which are coupled to the Ising spin glass is reported.⁷ It has been reported that the reduction of variance of conductance takes place due to the Zeeman splitting in the sample region.⁸⁻¹⁰ Altshuler and Shklovskii had shown that the variance of the conductance is described by⁸

$$\langle \delta G^2 \rangle = s^2 \frac{3K}{\beta} \left(\frac{e^2}{\hbar \pi^3} \right)^2 b_d, \quad (1)$$

where b_d is a dimension-dependent factor that is of the order of unity. β is equal to 1, 2, and 4 for orthogonal, unitary and symplectic systems, respectively. The quantity K is equal to the number of noninteracting series of energy levels with s -fold spin degeneracy.

Recent works have pointed out that the universal conductance fluctuations (UCF's) in a chaotic quantum dot in the presence of spin-orbit scattering shows new features.¹¹⁻¹³ They have shown that the UCF in the presence of spin de-

pendent scattering is interesting not only from the theoretical point of view but also from the experimental view point.

The transport properties of mesoscopic systems depend not only on the sample but also on the states in the leads through which currents flow into and out of the sample and through which voltages are measured.¹⁴ How the transport properties and the universality class are changed by modulating lead states is a very interesting question, especially when the spin degree of freedom plays a role.

In this paper, we investigate the influence of the spin scattering on transport properties in disordered systems. We consider magnetic impurities in sample region and the Zeeman field in the lead. Three types of magnetic impurities are considered. We call these Ising, XY, and Heisenberg, depending on the number of spin components. In order to calculate two terminal conductance, we employ the transfer matrix method¹⁵ that is extended to include the spin degree of freedom. Magnetic impurities remove the spin degeneracy and break TRS in certain cases.

We find that the variance of the conductance is halved as the number of spin components of the magnetic impurities increases. When the Zeeman field is applied in a lead, a further reduction of the variance is observed. In order to observe the crossover of the universality class with the increase of the Zeeman field in the lead, we study the level spacing distribution of transmission eigenvalues. Part of this work has been presented in the international conference "Localisation 2002."^{16,17}

II. MAGNETIC SCATTERING

We consider a two dimensional (2D) system connected to two electrodes. The 2D system is constructed in the x - and y -directions and the current flows in the x direction. There is an exchange interaction between the electron spin and the static local spins in the system. The one-electron Hamiltonian is

$$H = H_0 + H', \quad (2)$$

$$H_0 = \sum_{i,\sigma} W_i c_{i,\sigma}^\dagger c_{i,\sigma} - \sum_{\langle i,j \rangle, \sigma, \sigma'} V_{i\sigma, j\sigma'} c_{i,\sigma}^\dagger c_{j,\sigma'}, \quad (3)$$

$$H' = -J \sum_{i,\sigma,\sigma'} c_{i,\sigma}^\dagger \boldsymbol{\sigma}_{\sigma,\sigma'} c_{i,\sigma'} \cdot \mathbf{S}_i, \quad (4)$$

where $c_{i,\sigma}^\dagger$ ($c_{i,\sigma}$) denotes the creation (annihilation) operator of electron at the site i with spin σ ($= \pm 1$) on the 2D square lattice. On-site energy W_i denotes the nonmagnetic random potential distributed independently and uniformly in the range $[-W/2, W/2]$. The hopping is restricted to nearest neighbors.

We investigate how the variance of the conductance distribution of H_0 changes due to the presence of H' . The variance of conductance distribution for H_0 is determined by the symmetry, which is controlled by the hopping term, $V_{i\sigma,j\sigma'}$. If it is set to 2×2 unit matrix, H_0 belongs to the orthogonal class. H_0 belongs to the unitary class when

$$V_{i,i+\hat{y}} = \begin{pmatrix} \exp(i\phi) & 0 \\ 0 & \exp(i\phi) \end{pmatrix}, \quad (5)$$

where ϕ is a Peierls phase which is distributed uniformly in the range $[0, 2\pi)$. To realize systems belonging to the symplectic class,¹⁸ we set the hopping

$$V_{i,i+\hat{x}} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad (6)$$

and

$$V_{i,i+\hat{y}} = \begin{pmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{pmatrix}, \quad (7)$$

where the parameter θ denotes the strength of the spin-orbit interaction.

The additional term H' is the spin scattering term. σ is the Pauli spin matrix and \mathbf{S}_i is the static local spin. We consider three types of magnetic impurities: Ising, XY, and Heisenberg where $\mathbf{S}_i = (0, 0, S_z)$, $(S_x, S_y, 0)$, and (S_x, S_y, S_z) , respectively. The absolute value of \mathbf{S}_i is set to unity, and their components are distributed randomly according to the uniform distribution on n -dimensional sphere n ($= 1, 2$ or 3) being the number of spin components.

In order to calculate the conductance, we employ the transfer matrix method¹⁵ and extend it to include the spin degree of freedom. The conductance G is given by the Landauer formula^{19,20} as

$$G = \frac{e^2}{h} \text{tr}(tt^\dagger) = \frac{e^2}{h} \sum_i \tau_i, \quad (8)$$

where t is the transmission matrix including the spin degree of freedom and τ_i is the transmission eigenvalue. In the present simulation, the system size is 30×30 in units of the lattice spacing.

In Fig. 1 we show the variance of conductance in the presence of magnetic impurities where H_0 belongs to the orthogonal class. W is set to be 4.0 and $J = 0.4 \cdot 10^4$ ensemble averages are taken for each data. When $J = 0$, there is no magnetic scattering and the variance is close to that expected for the universal conductance fluctuation of orthogonal systems.⁵ For convenience, let us denote the variance of $\sum_i \tau_i$ for the conventional orthogonal class as V_{orth} and $G/(e^2/h) = \tilde{G}$, so that in the case of $J = 0$, $\text{Var } \tilde{G} = V_{\text{orth}}$. V_{orth} is $1/2$

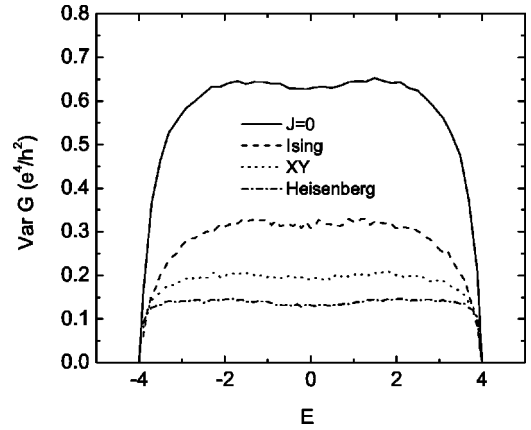


FIG. 1. Variance of conductance in the presence of magnetic impurities. The system size is 30×30 . W is set to be 4.0 and $J = 0.4$.

for the chaotic cavity, $8/15$ for the quasi-1D wire, 0.74 for 2D, and 1.2 for 3D systems, respectively.^{5,6}

In the presence of the Ising type impurities, no spin flip processes occur as in the conventional nonmagnetic system. However, the up spin and down spin electrons are described by different wave functions, because the exchange field of magnetic impurities lifts the spin degeneracy. With sufficiently large J , these wave functions are no longer correlated, and the variance is represented by

$$\text{Var } \tilde{G} = \text{Var } \tilde{G}_\uparrow + \text{Var } \tilde{G}_\downarrow, \quad (9)$$

where $\tilde{G}_{\uparrow(\downarrow)}$ is the conductance through the up (down) spin channel. Since both of the variances \tilde{G}_\uparrow and \tilde{G}_\downarrow are $V_{\text{orth}}/4$, the sum \tilde{G} becomes $V_{\text{orth}}/2$.

While Ising type impurities do not rotate the spin direction, the spin flip process occurs in XY type impurities. Then the variance is simply given by $\text{Var } \tilde{G} = V_{\text{orth}}/4$, since the factor of 4 coming from the spin degeneracy in the conventional nonmagnetic orthogonal class is missing. Though the Hamiltonian is complex due to σ_y , the statistics of the transmission eigenvalues as well as the energy level statistics are that of the orthogonal class. This can be seen if we define the time-reversal operator T as $T = i\sigma_x K$, where K denotes the complex conjugation. This operator is antiunitary, satisfies $T^2 = 1$ and commutes with the Hamiltonian of the system including XY type impurities.

For Heisenberg and XY type impurities, spin flips occur. However, when H' includes the Heisenberg type scatterers, the Hamiltonian no longer commutes with the time-reversal operator, and the system is classified into the unitary class. Therefore, the variance is further reduced by a factor of 2.

From these results, the variance of conductance in the presence of impurities is given by

$$\text{Var } \tilde{G} = \frac{V_{\text{orth}}}{2^n}. \quad (10)$$

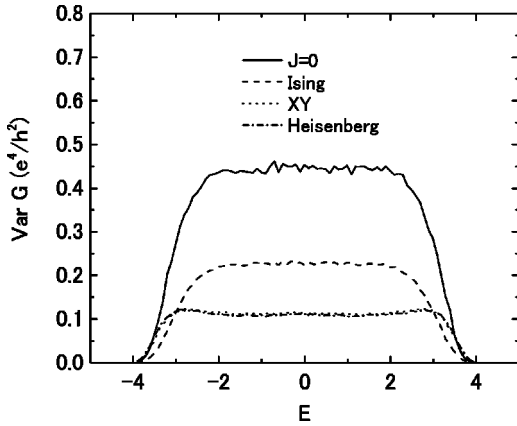


FIG. 2. Variance of conductance when the sample contains both the magnetic field and magnetic impurities. Parameters are the same as in Fig. 1.

We have numerically investigated a square 2D system, but the argument above is general, and this relation should be valid in other dimensions.¹⁶

Figure 2 shows the change of the variance when H_0 is in the unitary class. Parameters are the same as in Fig. 1. We consider the random magnetic field and the phase in the transfer integral is given by Eq. (2). Without magnetic impurities, the system belongs to the unitary class with spin degeneracy and the variance is $V_{\text{orth}}/2$. Ising type magnetic impurities removes the spin degeneracy and the variance becomes $V_{\text{orth}}/4$. Spin flips occur due to the XY type magnetic impurities and the variance becomes half of the Ising case. As shown in Fig. 2, the variance of the conductance when H' includes the Heisenberg type impurities is the same as in the case of the XY type, because the Hamiltonian of neither system commutes with the time reversal operator.

The reduction of the variance is also obtained when H_0 includes the spin-orbit interaction and the Hamiltonian belongs to the symplectic class (Fig. 3). Once the magnetic impurities are included, irrespectively of the type of the magnetic scatterers, the variance becomes $V_{\text{orth}}/8$ from $V_{\text{orth}}/4$.

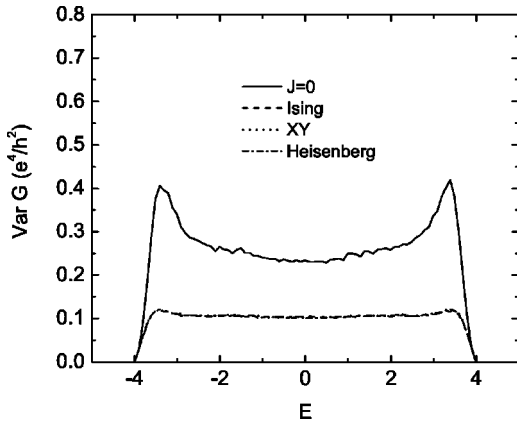


FIG. 3. Variance of conductance when the sample contains both the spin-orbit interaction and magnetic impurities. Parameters are the same as in Fig. 1.

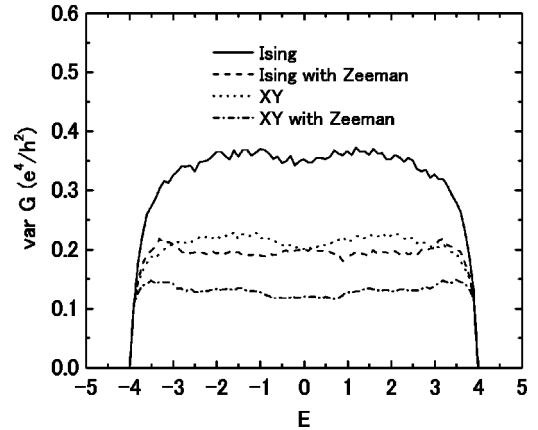


FIG. 4. The effect of the Zeeman coupling in a lead on the variance. H_0 belongs to the orthogonal class, $W=3.0$ and $Z=3.0$.

III. EFFECT OF LEADS

We then address the question of whether or not transport properties and the universality class can change as a result of an asymmetric spin population in the leads, i.e., when the number of up and down spin channels in the leads becomes asymmetric as a result of Zeeman splitting. We show that even if the sample is unchanged, the universality class can change under certain conditions.

We consider the Zeeman field in one of the leads, setting the other lead Zeeman field free. The transverse energy of the channel (i, σ) in the lead ε_i^σ is given by

$$\varepsilon_i^\sigma = -2 \cos\left(\frac{i\pi}{L+1}\right) - Z\sigma \quad (i=1, 2, \dots, L), \quad (11)$$

where L denotes the number of sites in the transverse direction (y direction), and Z denotes the strength of Zeeman splitting and $\sigma(\pm 1)$ is the spin index. The channel (i, σ) is a propagating mode if $|E_F - \varepsilon_i^\sigma| < 2.0$, E_F being the Fermi energy. For example, when we set $E_F = -1.1$ and $Z=1.0$, the number of up (down) spin channels becomes 27 (15) for the sample of width 30 sites.

Figure 4 shows the variance in the presence of Ising and XY type impurities in a system including Zeeman splitting in a lead. In this simulation, we set $W=3.0$, $\theta = \pi/4$, and $E_F = -1.1$. The system size is again set to be 30×30 in units of the lattice spacing. The population of up and down spins in one lead is always set to be symmetric ($Z=0$), and that in the other lead is varied ($Z=0, 3.0$).

From this figure, we observe that the variance becomes almost half due to the Zeeman splitting in the leads, even if the sample region is not changed at all. The effect of Zeeman splitting is similar to that of increasing the number of spin component of the magnetic impurities. It should be noted that if the direction of the Zeeman field is the same as one of the spin components of the scatters, the reduction is not observed. For example, in the case of Ising type scatterers, applying the Zeeman field in the z direction does not change the variance while the halving of the variance is observed if we apply the Zeeman field in the x or y directions. The same

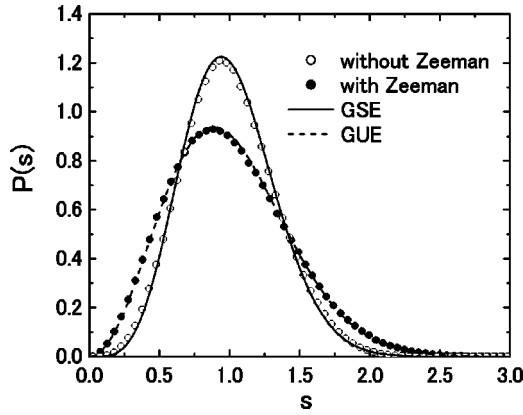


FIG. 5. Spacing distribution of τ for the sample with the spin-orbit interaction. We set $W=2.0$ and $Z=3.0$. The distribution without Zeeman splitting in a lead (\circ) fits to the form of GSE while that with Zeeman (\bullet) fits to the GUE.

is true for the case of the XY type scatterers. We need to apply the Zeeman field in the z direction to observe the halving of the variance.

We then show that the Zeeman field in the lead changes the universality class of the system. To detect the crossover of the universality classes, we investigate the spacing distribution $P(s)$ where s is the interval between neighboring transmission eigenvalues τ 's. An ensemble of about 10^6 samples is simulated to get good statistics.

Figure 5 shows the spacing distribution $P(s)$ of the transmission eigenvalue τ for the sample with spin-orbit interaction. In the absence of Zeeman splitting in the leads (\circ), $P(s)$ is close to the Wigner surmise for the Gaussian symplectic ensemble (GSE). On the other hand, with Zeeman

TABLE I. Variance of the system in the presence of magnetic impurities and the Zeeman field.

Universality class of H_0	Additional spin scattering	Zeeman field in a lead	Variance/ V_{orth}
Orthogonal	0	no	1
	0	yes	1/2
	Ising	no	1/2
	Ising	yes	1/4
	XY	no	1/4
	XY	yes	1/8
	Heisenberg	irrelevant	1/8
Unitary	0	no	1/2
	0	yes	1/4
	Ising	no	1/4
	Ising	yes	1/8
	XY	irrelevant	1/8
	XY	irrelevant	1/8
	Heisenberg	irrelevant	1/8
Symplectic	0	no	1/4
	0	yes	1/8
	Ising	irrelevant	1/8
	XY	irrelevant	1/8
	XY	irrelevant	1/8
	Heisenberg	irrelevant	1/8

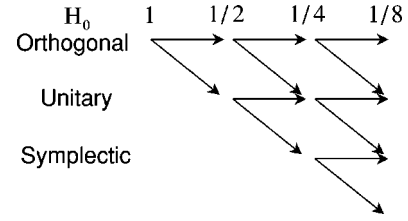


FIG. 6. Schematic diagram of the variance of the conductance. The horizontal arrow indicates the increase of the spin component of the magnetic scatterers, while the slanted arrow indicates the addition of the Zeeman field. The Zeeman field is assumed to contain a component different from that of the magnetic scatterers.

splitting in the leads (\bullet), $P(s)$ is close to that for the Gaussian unitary ensemble (GUE). The reason for this crossover is that the asymmetry of up and down spins in the lead destroys the self-dual property of the scattering matrix.⁶ To be specific, we consider the (i, j) component of the S -matrix $S_{i,j}$ which describes the transmission from $(i\uparrow, i\downarrow)$ to $(j\uparrow, j\downarrow)$. The self-duality requires

$$S_{i,j} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, S_{j,i} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \quad (12)$$

which means that the transmission probability from $i\uparrow$ to $j\uparrow$ is the same as that from $j\downarrow$ to $i\downarrow$. This symmetry is broken when we apply the Zeeman field in the lead, which lifts the degeneracy of $i\uparrow$ and $i\downarrow$.

IV. SUMMARY

We have studied the effect of spin scattering in disordered systems on the fluctuating nature of the conductance. We have considered magnetic impurities in a sample and calculated transport properties. Our results show that the variance of conductance is halved as the number of spin components of the magnetic impurities increases. Halving of the variance of the conductance is also obtained when the sample includes magnetic field or spin-orbit interaction.

We have also investigated the effect of the Zeeman splitting in a lead. Halving of the variance of conductance is obtained when the direction of the Zeeman field contains the component different from the component(s) of the magnetic scatterers. This behavior is reminiscent of the change of the variance in the superconducting-normal junction.^{6,21,22} Analyzing the transmission eigenvalues, the universality class has been shown to be changed by the Zeeman field in the lead. The results are summarized in Table I and schematically shown in Fig. 6.

Before concluding, we relate our results with that of Aleiner and Fal'ko.¹³ They have obtained for the chaotic system

$$\text{Var } \tilde{G} = \frac{s}{4\beta\Sigma}, \quad (13)$$

where $s=1,2$ indicates the Kramers degeneracy and $\Sigma=2$ if the spin flip process is present and $\Sigma=1$ otherwise. $\beta=1, 2,$ or 4 is determined by the universality class. Setting

$V_{\text{orth}}=1/2$ we recover their interesting result. For example, when H_0 is classified into the orthogonal class, and H' includes the XY type magnetic impurities $\beta=1, s=1$, and $\Sigma=2$, which gives $\text{Var } \tilde{G}=\frac{1}{8}=V_{\text{orth}}/4$. Therefore, the present results are the extension of Aleiner and Fal'ko¹³ to higher dimensions and to the inclusion of the effect of the Zeeman splitting in the lead.

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