

Persistent current in ballistic mesoscopic rings with Rashba spin-orbit coupling

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The presence of spin-orbit coupling affects the spontaneously flowing persistent currents in mesoscopic conducting rings. Here we analyze their dependence on magnetic flux with emphasis on identifying possibilities to prove the presence and extract the strength of Rashba spin splitting in low-dimensional systems. Effects of disorder and mixing between quasi-one-dimensional ring subbands are considered. The spin-orbit coupling strength can be inferred from the values of flux where sign changes occur in the persistent charge current. As an important consequence of the presence of spin splitting, we identify a nontrivial persistent spin current that is not simply proportional to the charge current. The different flux dependences of persistent charge and spin currents are a *unique* signature of spin-orbit coupling affecting the electronic structure of the ring.

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I. INTRODUCTION

The interplay between spin-orbit (SO) coupling and quantum confinement in semiconductor heterostructures has recently attracted great interest. It provides a useful tool to manipulate the spin degree of freedom of electrons by coupling to their orbital motion and vice versa. As a result, spin-orbit coupling has become one of the key ingredients for phase-coherent spintronics applications.^{1,2} Various sources of broken inversion symmetry give rise to intrinsic (zero-field) spin splitting in semiconductor heterostructures.³ We focus here on the one induced by structural inversion asymmetry: i.e., the Rashba effect.⁴ It is typically important in small-gap zinc-blende-type semiconductors and can be tuned by external gate voltages.⁵⁻⁷

Many proposals have been put forward recently for devices based on spin-dependent transport effects due to the Rashba SO coupling in low-dimensional systems.⁸ To explore the possibilities for their realization, it is desirable to have a reliable way to determine experimentally the strength α of the Rashba SO coupling. Transport experiments have been performed in two-dimensional (2D) electron systems, and α was extracted from beating patterns in Shubnikov-de Haas oscillations⁵⁻⁷ as well as the SO relaxation time obtained from weak-antilocalization behavior in the resistance.⁹ The only previous experimental studies of SO coupling in *quasi-1D* systems have measured transport through mesoscopic rings.^{10,11} Beating patterns in the Aharonov-Bohm (AB) oscillations of the ring's conductance are expected to arise from quantum phases¹²⁻¹⁵ induced by the presence of SO coupling.

In practice, it turns out,¹⁶ however, that the signature of the Rashba effect in AB oscillations can be masked by features arising due to the ring's nonideal coupling to external leads. As an alternative, we explore here the possibility to

obtain a direct measure of the Rashba SO coupling strength from the persistent current^{17,18} induced by a magnetic flux perpendicular to the ring. This approach would have the advantage of circumventing entirely any problems arising from contacting the ring.

There is a vast literature of theoretical¹⁷⁻²⁵ and experimental²⁶⁻²⁸ studies on persistent currents. From the theoretical point of view, the effect of SO coupling on the Fourier transform of observables has been addressed in Refs. 22-24. Measurements of the persistent charge current have been performed both in an ensemble of metallic rings²⁶ and on single isolated rings realized in nanostructured 2D electron systems.^{27,28} So far, persistent currents have not yet been studied in rings where the Rashba effect is likely to be important. From our study, we find features in the flux dependence of the persistent charge current that allow for a direct quantitative determination of the Rashba SO coupling strength. We discuss how averaging over rings with different numbers of particles and mixing between different 1D subbands affects these features. An unambiguous signature of SO coupling is obtained from a comparison of the persistent *spin* current with the persistent *charge* current. In the absence of SO coupling, the persistent spin current is finite only for an odd number of particles in the ring and is proportional to the persistent charge current. With SO coupling, the persistent spin current is finite also for an even electron number. For an odd number of electrons in the ring, the persistent spin current is sizable only for small values of the SO coupling strength. The flux dependence of the persistent spin current is generally strikingly different from that of the charge current. Observability of the persistent spin current by its induced electric field²⁹⁻³² should enable an unambiguous identification of SO effects in low-dimensional mesoscopic rings.

The effect of electron-electron interactions on the persis-

tent charge current in mesoscopic rings has been the subject of a long-standing debate, especially in the diffusive limit.³³ However, in a ballistic quasi-1D ring, it appears to be quite clear that, at low temperatures, interactions only affect the current amplitude, leaving its flux dependence essentially unaltered.³⁴ Signatures of SO coupling discussed in our work presented here should therefore remain largely unchanged when interactions are switched on. A proper quantitative treatment of interaction effects could employ a Luttinger-liquid model for a clean interacting ring³⁴ that has to be generalized to the case with SO coupling present.³⁵ We postpone a discussion along this line to a later publication.

The paper is organized as follows. In Sec. II, we write down and discuss the model Hamiltonian used to describe the ring. Electronic properties and persistent currents of a purely 1D ring are computed in the following Sec. III. Sec. IV is devoted to the effect of higher radial subbands. Conclusions are presented in Sec. V.

II. MODEL OF A MESOSCOPIC RING WITH RASHBA SPIN-ORBIT COUPLING

For completeness and to introduce the notation used later in our work, we outline here briefly the derivation of the Hamiltonian describing the motion of an electron in a realistic quasi-1D ring.³⁶ We consider 2D electrons in the xy plane that are further confined to move in a ring by a radial potential $V_c(r)$. The electrons are subject to the Rashba SO coupling, which reads

$$H_{so} = \frac{\alpha}{\hbar} [\sigma_x(\vec{p} - e\vec{A})_y - \sigma_y(\vec{p} - e\vec{A})_x]. \quad (1)$$

Here \vec{A} is the vector potential of an external magnetic field applied in the z direction. The coupling strength α defines the spin-precession length $l_{so} = \pi\hbar^2/(m\alpha)$. The full single-electron Hamiltonian reads

$$H = \frac{(\vec{p} - e\vec{A})_x^2 + (\vec{p} - e\vec{A})_y^2}{2m} + V_c(r) + H_{so} + \hbar\omega_z\sigma_z, \quad (2)$$

where the Zeeman splitting from the external magnetic field is included as the last term. Due to the circular symmetry of the problem, it is natural to rewrite the Hamiltonian in polar coordinates:³⁶

$$H = -\frac{\hbar^2}{2m} \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{r^2} \left(i \frac{\partial}{\partial \varphi} + \frac{\Phi}{\Phi_0} \right)^2 \right] + V_c(r) - \frac{\alpha}{r} \sigma_r \left(i \frac{\partial}{\partial \varphi} + \frac{\Phi}{\Phi_0} \right) + i\alpha\sigma_\varphi \frac{\partial}{\partial r} + \hbar\omega_z\sigma_z, \quad (3)$$

where Φ is the magnetic flux threading the ring, Φ_0 the flux quantum, $\sigma_r = \cos\varphi\sigma_x + \sin\varphi\sigma_y$, and $\sigma_\varphi = -\sin\varphi\sigma_x + \cos\varphi\sigma_y$. In the case of a thin ring—i.e., when the radius a of the ring is much larger than the radial width of the wave function—it is convenient to project the Hamiltonian on the eigenstates of

$$H_0 = -\frac{\hbar^2}{2m} \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right] + V_c(r).$$

To be specific, we use a parabolic radial confining potential

$$V_c(r) = \frac{1}{2} m\omega^2(r-a)^2, \quad (4)$$

for which the radial width of the wave function is given by $l_\omega = \sqrt{\hbar/m\omega}$. In the following, we assume $l_\omega/a \ll 1$ and neglect contributions of order l_ω/a . In this limit, H_0 reduces to

$$H_0 = -\frac{\hbar^2}{2m} \left[\frac{\partial^2}{\partial r^2} \right] + \frac{1}{2} m\omega^2(r-a)^2. \quad (5)$$

We now calculate matrix elements of the Hamiltonian, Eq. (3), in the basis of eigenfunctions of Eq. (5) that correspond to quasi-1D radial subbands, labeled here by the quantum number n . The diagonal matrix elements are given by

$$H_{n,n} = \frac{\hbar^2}{2ma^2} \left(i \frac{\partial}{\partial \varphi} + \frac{\Phi}{\Phi_0} \right)^2 - \frac{\alpha}{a} \sigma_r \left(i \frac{\partial}{\partial \varphi} + \frac{\Phi}{\Phi_0} \right) - i \frac{\alpha}{2a} \sigma_\varphi + \hbar\omega_z\sigma_z + \hbar\omega \left(n + \frac{1}{2} \right). \quad (6)$$

The only nonvanishing off-diagonal matrix elements are those coupling adjacent radial subbands:

$$H_{n,n+1} = H_{n+1,n}^\dagger = i\sigma_\varphi \sqrt{\frac{n+1}{2}} \frac{\alpha}{l_\omega}. \quad (7)$$

III. PROPERTIES OF IDEAL 1D RINGS

The ideal 1D limit for a mesoscopic ring is realized when only the lowest radial subband is occupied by electrons and all relevant energy scales as, e.g., temperature, voltage, and disorder broadening are small enough such that interband excitations can be neglected. In the following section, we focus on this situation that can be realized in recently fabricated ring structures.^{37–39}

A. Energy spectrum of a 1D ring with impurity

Straightforward algebra yields the eigenenergies of $H_{0,0}$ which are usually labeled by an integer number q :

$$E_{q,\pm} = \hbar\omega_a \left(q - \frac{\Phi}{\Phi_0} + \frac{1}{2} \mp \frac{1}{2 \cos\theta_q} \right)^2 + \frac{\hbar\omega_a}{4} \left(1 - \frac{1}{\cos^2\theta_q} \right) \pm \frac{\hbar\omega_z}{\cos\theta_q}. \quad (8)$$

Here we have introduced the frequency $\omega_a = \hbar/(2ma^2)$ and omitted the constant energy shift of the radial subband bottom. The eigenvectors corresponding to the eigenenergies given in Eq. (8) are

$$\Psi_{q,\pm} = e^{i(q+1/2)\varphi} \chi_{q,\pm}, \quad (9)$$

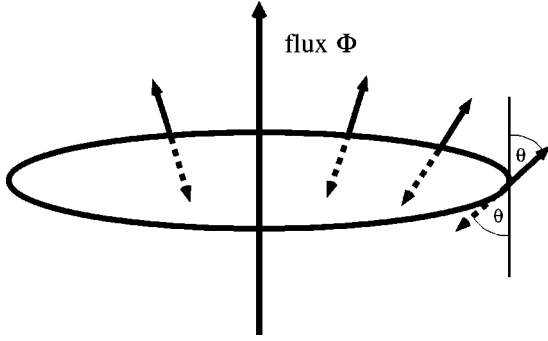


FIG. 1. Schematic illustration of the spin texture exhibited by the eigenstates of the ideal one-dimensional ring.

with the spinors

$$\chi_{q,+} = \begin{pmatrix} \cos\left(\frac{\theta_q}{2}\right) e^{-i\varphi/2} \\ \sin\left(\frac{\theta_q}{2}\right) e^{i\varphi/2} \end{pmatrix}, \quad (10a)$$

$$\chi_{q,-} = \begin{pmatrix} -\sin\left(\frac{\theta_q}{2}\right) e^{-i\varphi/2} \\ \cos\left(\frac{\theta_q}{2}\right) e^{i\varphi/2} \end{pmatrix}. \quad (10b)$$

The angle θ_q is given by¹³

$$\tan(\theta_q) = -\frac{\frac{\alpha}{a}\left(q - \frac{\Phi}{\Phi_0} + \frac{1}{2}\right)}{\hbar\omega_a\left(q - \frac{\Phi}{\Phi_0} + \frac{1}{2}\right) - \hbar\omega_z}. \quad (11)$$

The spinors $\chi_{q,\pm}$ are the eigenstates of the operator

$$\sigma_{\theta_q} = \sigma_z \cos \theta_q + \sigma_r \sin \theta_q \quad (12)$$

and constitute a basis in spin space with space-dependent quantization direction, as shown in Fig. 1. We will refer to this φ -dependent spin basis as the *local spin frame*. θ_q is the angle between the local quantization axis and the direction perpendicular to the ring (z axis). The tilt angle described by Eq. (11) becomes independent of the quantum number q when the Zeeman energy is negligible, i.e., when $|\hbar\omega_a(q - \Phi/\Phi_0 + \frac{1}{2})| \gg \hbar\omega_z$. For typical realizations of mesoscopic rings with many electrons present, states contributing importantly to the persistent current fulfill this requirement. Therefore, in the following, we focus exclusively on the limit where Zeeman splitting vanishes and $\theta_q \rightarrow \theta = \lim_{\omega_z \rightarrow 0} \theta_q$. Then all eigenstates have the same local spin frame, to which we can transform using the SU(2) matrix

$$\mathcal{U} = \begin{pmatrix} e^{-i\varphi/2} \cos \frac{\theta}{2} & -e^{-i\varphi/2} \sin \frac{\theta}{2} \\ e^{i\varphi/2} \sin \frac{\theta}{2} & e^{i\varphi/2} \cos \frac{\theta}{2} \end{pmatrix}. \quad (13)$$

This yields $H_{1D} \equiv \mathcal{U}^\dagger (H_{0,0} - \hbar\omega/2)_{\omega_z=0} \mathcal{U}$ where

$$H_{1D} = \hbar\omega_a \left(-i \frac{\partial}{\partial \varphi} - \frac{\Phi}{\Phi_0} - \frac{1}{2 \cos \theta} \sigma_z \right)^2 + \frac{\hbar\omega_a}{4} \left(1 - \frac{1}{\cos^2 \theta} \right). \quad (14)$$

Here $\cos \theta$ parametrizes the strength of the SO coupling. The eigenstates in the local spin frame are simply $e^{i(q+1/2)\varphi} |\pm\rangle$, where $|\pm\rangle$ denote the eigenspinors of σ_z and the eigenenergies are given by Eq. (8) with $\theta_q \rightarrow \theta$ and $\omega_z = 0$. Note that the orbital part of the eigenstates obeys anti-periodic boundary conditions to compensate for the anti-periodicity of the spinors of Eq. (10).

To discuss the effect of a nonmagnetic impurity, we exploit the formal analogy between a ring with an impurity and a 1D periodic potential.¹⁷ The latter is described by a Kronig-Penney model,⁴⁰ with the magnetic flux playing the role of the quasimomentum of the 1D crystal. The impurity is modeled by its energy-dependent transmission amplitude $t = |t| \exp(i\delta)$. The energy spectrum for the electrons with spin $|\pm\rangle$ can now be obtained by solving the transcendental secular equation

$$|t| \cos \left[2\pi \left(\frac{\Phi}{\Phi_0} \pm \frac{1}{2 \cos(\theta)} \right) \right] = -\cos(2\pi\kappa_{\pm} + \delta), \quad (15)$$

complemented by the relation

$$E_{\pm} = \hbar\omega_a \left[\kappa_{\pm}^2 + \frac{1}{4} \left(1 - \frac{1}{\cos^2 \theta} \right) \right]. \quad (16)$$

In general, the secular equation (15) cannot be solved analytically for arbitrary transmission function t . To simplify the problem, we will now assume that the impurity is a delta-function barrier $V_0 \delta(\varphi)$. The transmission coefficient for a state $\exp(i\kappa\varphi) |\pm\rangle$ is $t = 2\kappa / [2\kappa + iV_0 / (\hbar\omega_a)]$. For states close to the Fermi level, Eq. (15) can be written as

$$\cos \left(2\pi \frac{\Phi_{\pm}}{\Phi_0} \right) = \cos(2\pi\kappa_{\pm}) + \text{sgn}(\kappa_{\pm}) A \sin(2\pi\kappa_{\pm}), \quad (17)$$

with a constant $A = V_0 / (\hbar\omega_a \mathcal{N})$, where \mathcal{N} is the total number of electrons. We also defined the effective fluxes

$$\Phi_{\pm} = \Phi + \Phi_0 \left(\frac{1}{2} \pm \frac{1}{2 \cos(\theta)} \right). \quad (18)$$

Equation (17) with constant A would be exact for a barrier with energy-independent transmission amplitude $t = [1 - iA \text{sgn}(\kappa)] / (A^2 + 1)$. The approximated secular equation (17) has the solution

$$\kappa_{q,\pm} = q + \frac{1}{2\pi} \arccos \left[\frac{\cos(2\pi\Phi_{\pm}/\Phi_0) - \text{sgn}(q) \sqrt{A^2[\sin^2(2\pi\Phi_{\pm}/\Phi_0) + A^2]}}{1 + A^2} \right]. \quad (19)$$

Equation (19) together with Eq. (16) yields the single-particle energy spectrum for a ring with an idealized impurity. Note that, in the representation of the local spin frame, the impurity problem maps onto that of electrons without SO coupling but with an effective spin-dependent flux^{22,23} given by Eq. (18). This is illustrated in an example spectrum shown in Fig. 2.

B. Persistent charge currents

Having calculated the single-particle electronic properties of the ring, we proceed to evaluate the persistent charge current. At zero temperature, it is given by¹⁷

$$I = - \frac{\partial E_{\text{gs}}}{\partial \Phi} = - \sum_{i \in \text{occupied}} \frac{\partial E_i}{\partial \Phi}, \quad (20)$$

where E_{gs} is the ground-state energy and E_i are the single-particle eigenenergies. Here i stands for a set of quantum numbers used to label corresponding eigenstates, including here the spin projection in the local spin frame. The second equality in Eq. (20) is valid only in the absence of electron-electron interactions, which we neglect here. The zero-temperature formula applies when the thermal energy $k_B T$ is smaller than the energy difference between the last occupied state and the first unoccupied one. Generally, temperature induces a rounding of the sharp features (jumps) in the flux dependence of the persistent current that are due to level crossings. In the following, we will always consider the number \mathcal{N} of electrons in the ring to be fixed—i.e., work in the canonical ensemble. This is the relevant situation for an isolated ring.

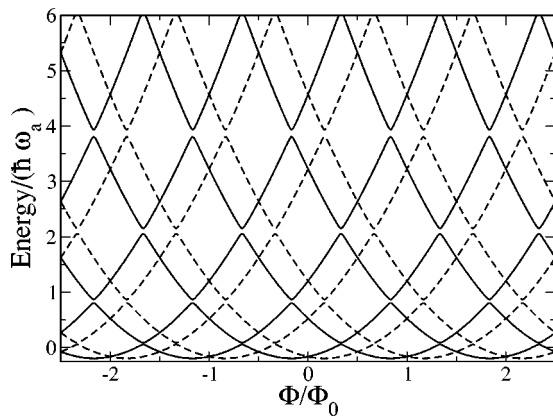


FIG. 2. Single-particle energy spectrum of an ideal 1D ring with a model δ -barrier impurity. Parameters are $\cos \theta = 2/5$ and $A = 0.1$. Energy levels for states corresponding to spin-up (solid line) and spin-down (dashed line) in the local-spin-frame basis are shifted, in flux direction, by $1/\cos \theta$.

For spinful electrons, the flux dependence of the persistent charge current is distinctly different for the following cases:¹⁹ (i) $\mathcal{N} = 4N$, (ii) $\mathcal{N} = 4N + 2$, and (iii) $\mathcal{N} = 2N + 1$, where N denotes a positive integer. When \mathcal{N} is large enough, the persistent charge current in units of $I_0 = \hbar \omega_x \mathcal{N} / \Phi_0$ has a universal behavior independent of \mathcal{N} . We start discussing the weak barrier limit (small A in our model), shown in Fig. 3. In case (i) where $\mathcal{N} = 4N$, the numbers of spin-up and spin-down electrons (spin projection in the local spin frame) are both even, resulting in jumps of the persistent current at $\Phi/\Phi_0 = M + 1/2 \pm 1/(2 \cos \theta)$, with M being integer. This is simply the superposition of the even-number spinless-electron persistent current characteristics for each spin direction, shifted in flux by $\pm 1/(2 \cos \theta)$. Case (ii) corresponds to an odd number of spin-up and spin-down electrons and exhibits jumps of the persistent charge current at $\Phi/\Phi_0 = M \pm 1/(2 \cos \theta)$, which is the analogous superposition of the appropriately flux-shifted spinless odd-electron currents for each spin direction. Note that the case $\mathcal{N} = 4N + 2$ is obtained from the $\mathcal{N} = 4N$ case simply by shifting flux by $1/2\Phi_0$. It is apparent that, for both cases (i) and (ii), the minimum distance between jumps of the persistent charge current within the periodic flux interval is a measure of $1/\cos \theta$ and, hence, of the SO coupling strength. In contrast, for case (iii)—i.e., an odd number of electrons in the ring—jumps appear at the same values of flux ($\Phi/\Phi_0 = 0$ and $\pm 1/2$) as in the absence of spin-orbit coupling. The only effect of SO coupling turns out to be a suppression of impurity rounding for these jumps. This can be explained quite easily. Inspection shows that, for finite SO coupling, jumps in the persistent charge current in the case of an odd number of electrons are due to a crossing of levels with opposite spin, while those in the case of an even electron number arise from crossings of levels having the same spin. As a spin-independent impurity cannot couple levels with opposite spin, only the jumps in the case of an even electron number get rounded because of impurity-induced anticrossings. For an odd number of electrons, jumps in the persistent charge current get broadened only by temperature. The effect of increasing the impurity (barrier) strength can be seen by comparing Figs. 3 and 4, where the persistent charge current is shown for different SO coupling strengths, occupancy of the ring, and disorder.

Measurements are often performed on ensembles of many rings.²⁶ The measured persistent charge current is then an average over different occupation numbers, with even and odd occupations occurring with the same probability. Among cases with even electron numbers $\mathcal{N} = 4N$ and $4N + 2$ would also be equiprobable. An example of average persistent charge current is shown in Fig. 5. It exhibits the well-known period halving^{20,21} which must occur irrespective of the presence of SO coupling. Most importantly, however, all features present for the single ring and discussed above for different

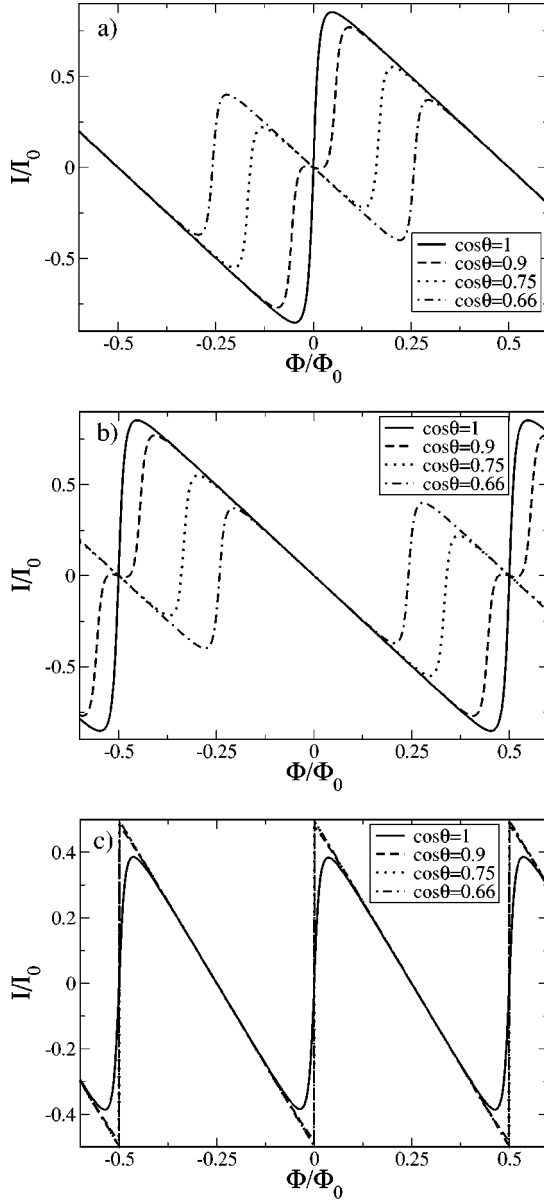


FIG. 3. Persistent charge current vs magnetic flux for a set of values for the spin-orbit coupling strength. The total number of electrons is set to $4N$ in panel (a), to $4N+2$ in panel (b), and to $2N+1$ in panel (c) in the regime of large-enough N such that the persistent current is universal. A dimensionless barrier strength of $A=0.1$ was assumed. The persistent current is measured in units of $I_0 = \hbar \omega_a N / \Phi_0$.

occupancy are still visible. It should therefore be possible to obtain the Rashba SO coupling strength from a measurement of the ensemble-averaged persistent charge current.

C. Persistent spin currents

As electrons carry spin as well as charge, their motion gives rise also to a spin current besides the charge current. Very often, the difference of charge currents carried by spin-up and spin-down electrons is identified with the spin current. While this is appropriate in many contexts, it has to be kept in mind^{12,32} that the spin current is actually a tensor.

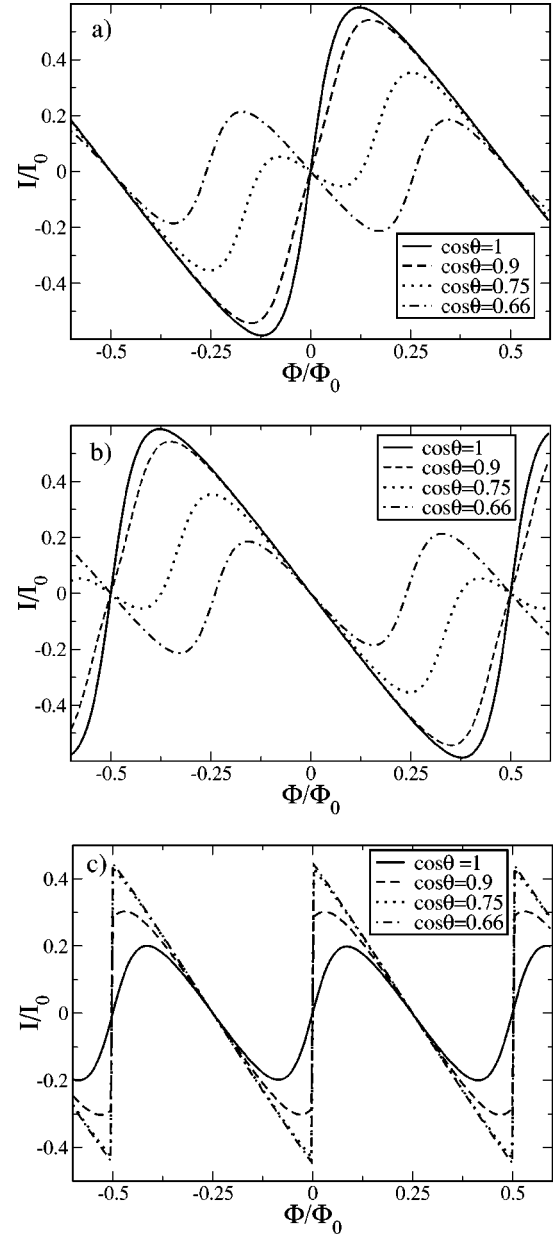


FIG. 4. Same as Fig. 3 but with different impurity parameter $A=0.5$. Note the remaining sharpness of jumps in the case of an odd electron number and finite spin-orbit coupling even at this rather large value of A .

A particular case where this fact matters is the one to be considered here. As the electron velocity in the presence of SO coupling turns out to be an operator in spin space⁴¹ and eigenstates for electrons of the ring correspond to eigenspinors of a spatially varying spin matrix [σ_θ as defined in Eq. (12)], the proper expression for the spin current has to be derived carefully. After presenting details of this derivation, we proceed to show results for the persistent spin currents of electrons in a ring with Rashba SO coupling.

The operator of the ν component of spin density in a real-space representation is given by $s_\nu(\vec{r}) = \sigma_\nu(\vec{r}^\top) \delta(\vec{r} - \vec{r}^\top)$, with σ_ν being the SU(2) spin matrix whose eigenstates form the basis for projection of spin in the ν direction.

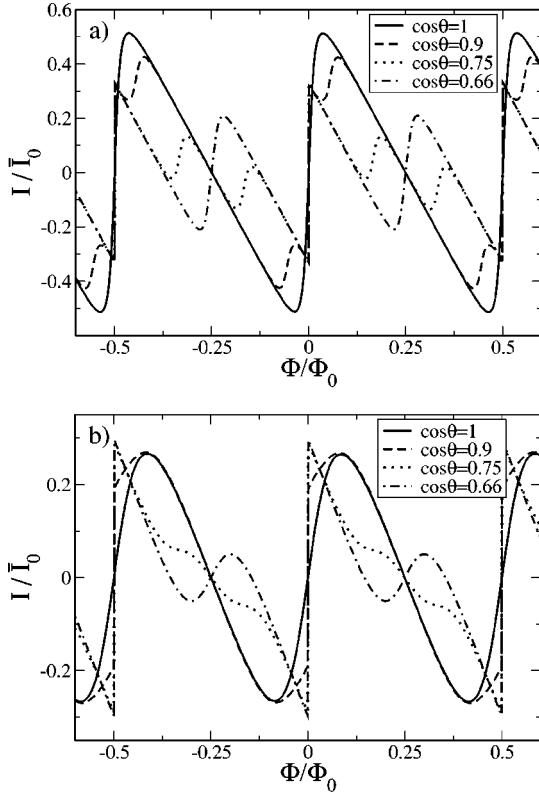


FIG. 5. Average persistent charge current for an ensemble of identical rings with different electron numbers, shown as function of magnetic flux for different values of the spin-orbit coupling strength. The impurity parameter is $A=0.1$ in panel (a) and $A=0.5$ in panel (b). The current unit is $I_0 = \hbar \omega_a \bar{N} / \Phi_0$, where \bar{N} denotes the average number of electrons.

In general, this projection direction can vary in space. The equation of motion for the spin-density operator is given by the familiar Heisenberg form

$$\frac{d}{dt} s_\nu(\vec{r}) = \frac{i}{\hbar} [H, s_\nu(\vec{r})] \quad (21a)$$

$$= \left(\frac{d}{dt} \sigma_\nu(\vec{r}) \right) \delta(\vec{r} - \vec{r}') - \vec{\nabla}_r \cdot [\sigma_\nu(\vec{r}') \vec{v}(\vec{r})]. \quad (21b)$$

Here $\vec{\nabla}_r$ denotes the gradient operator acting on the coordinate \vec{r} , and $\vec{v}(\vec{r})$ is the electron velocity operator. The latter differs from its expression \vec{v}_0 in the absence of SO coupling by a spin-dependent term⁴¹ $\vec{v} = \vec{v}_0 + \alpha(\hat{z} \times \vec{\sigma}) / \hbar$.

Straightforward calculation for the case of *spatially constant* σ_ν and vanishing Zeeman splitting yields the continuity equation

$$\frac{d}{dt} s_\nu(\vec{r}) + \vec{\nabla} \cdot \vec{J}_\nu(\vec{r}) = \frac{2\alpha}{\hbar^2} [\hat{v} \times (\hat{z} \times \vec{\sigma})] \cdot (\vec{p} - e\vec{A}), \quad (22a)$$

with the ν component of the spin-current tensor given by

$$\vec{J}_\nu(\vec{r}) = \vec{v}(\vec{r}) \sigma_\nu. \quad (22b)$$

We have used the symbols \hat{z} and \hat{v} to denote unit vectors in the z and ν directions, respectively. Note that the expression (22b) and the source term on the right-hand side (RHS) of Eq. (22a) have been written in the usual shorthand notation where it is understood that the real part has to be taken in the expectation value. As an example, we fix $\nu=z$ and consider the case of electrons moving in the lowest quasi-1D radial ring subband. We find, after transformation into the representation of the local spin frame, for the continuity equation (22a) the simple expression

$$\frac{d}{dt} s_z(\varphi) + \frac{1}{a} \frac{\partial}{\partial \varphi} j_z^\varphi(\varphi) = 2\omega_a \sigma_y \left(i \frac{\partial}{\partial \varphi} + \frac{\phi}{\phi_0} \right) \tan \theta. \quad (23a)$$

The only nonvanishing (φ) component of the spin current turns out to be

$$j_z^\varphi(\varphi) = \frac{\hbar}{ma} \left\{ \left(-i \frac{\partial}{\partial \varphi} - \frac{\phi}{\phi_0} - \frac{1}{2 \cos \theta} \sigma_z \right) \sigma_z \cos \theta - \left(-i \frac{\partial}{\partial \varphi} - \frac{\phi}{\phi_0} \right) \sigma_x \sin \theta \right\}. \quad (23b)$$

Eigenstates on the ring which are labeled by quantum numbers q and σ carry a current for the z projection of spin given by

$$I_z^{(q\sigma)} = \frac{1}{2\pi a} \langle j_z^\varphi(\varphi) \rangle_{q\sigma} = -\frac{1}{e} \frac{\partial E_{q,\sigma}}{\partial \Phi} \sigma \cos \theta, \quad (24)$$

which is just the charge current multiplied by the magnetization in z direction of the corresponding state.⁴²

As an important example for the current of a spatially varying projection of the magnetization, we consider the case of the local spin frame: i.e., $\sigma_\nu(\vec{r}) = \sigma_\theta(\varphi)$. [See Eq. (12).] Additional terms arising from derivatives of σ_θ with respect to polar angle φ appear in the continuity equation for $s_\theta(\vec{r})$. After transformation into the local spin frame, it has the extremely simple form

$$\frac{d}{dt} s_\theta(\varphi) + \frac{1}{a} \frac{\partial}{\partial \varphi} j_\theta^\varphi(\varphi) = 0, \quad (25a)$$

with the current

$$j_\theta^\varphi(\varphi) = \frac{\hbar}{ma} \left(-i \frac{\partial}{\partial \varphi} - \frac{\phi}{\phi_0} - \frac{1}{2 \cos \theta} \sigma_z \right) \sigma_x. \quad (25b)$$

The current of magnetization parallel to the quantization axis in the local spin frame carried by eigenstates is therefore given by

$$I_\theta^{(q\sigma)} = -\frac{1}{e} \frac{\partial E_{q,\sigma}}{\partial \Phi} \sigma. \quad (26)$$

Comparison with results from above yields the relation $I_z^{(q\sigma)} = I_\theta^{(q\sigma)} \cos \theta$, and we have derived also the related one $I_r^{(q\sigma)} = I_\theta^{(q\sigma)} \sin \theta$.

We now present results for the total persistent spin current $I_\theta = \sum_{q\sigma} I_\theta^{(q\sigma)}$ for the projection onto the quantization axis of

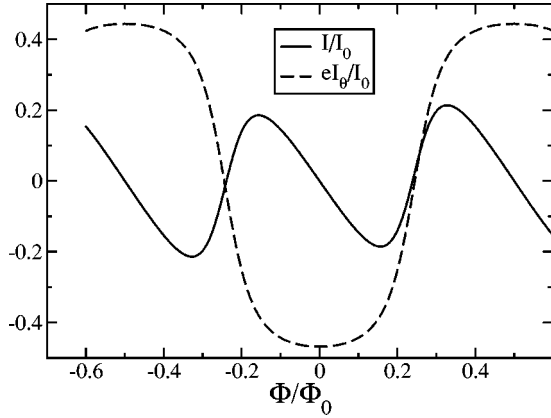


FIG. 6. Persistent spin current for spin projection onto the local spin frame (dashed curve) and persistent charge current (solid curve) vs magnetic flux for the case with electron number $4N+2$. The barrier strength is $A=0.5$ and $\cos\theta=0.66$. The current is measured in units of $I_0=\hbar\omega_a N/\Phi_0$.

the local spin frame. As shown above, spin currents for certain other projections can be easily obtained from I_θ . The fact that flux dependences for the persistent current contributions from opposite-spin eigenstates are shifted according to Eq. (18) results in large spin currents at certain flux values. In particular, this is realized when the currents carried by electrons with opposite spin flow in opposite directions. In Fig. 6, we show the persistent spin current for an even number of electrons. For comparison, the persistent charge current is plotted as well. Both exhibit strikingly different flux dependences. Note also that, in the absence of SO coupling, the persistent spin current vanishes for even electron number in the ring. Only the relative shift of energy bands in flux direction caused by SO coupling enables a finite persistent spin current in this case. For an odd number of electrons, the persistent spin current is finite both with and without SO coupling present. We find it to be sizable, however, only for small values of SO coupling strength. We show a comparison of even and odd electron number cases in Fig. 7.

The persistent spin current would be a mere theoretical curiosity if no detectable effect of it could be found. Fortunately, this is not so. Recently, it has been pointed out by several authors^{29–32} that a spin current, being a magnetization current, gives rise to an electric field. This is easily proved by making a Lorenz transform to the rest frame of spin. For example, the electrostatic potential for a point at a distance $z\ll a$ from the plane of the ring on the vertical from the center of the ring is

$$\phi(z)\approx\frac{\mu_0}{4\pi}g\mu_B I_\theta\sin\theta\frac{a}{z^2}, \quad (27)$$

where μ_0 is the vacuum permeability, g the gyromagnetic ratio, μ_B the Bohr magneton, a the radius of the ring, and θ the tilt angle due to SO coupling. This result is identical with the one derived in Ref. 30 for the electric field resulting from persistent spin currents in Heisenberg rings.

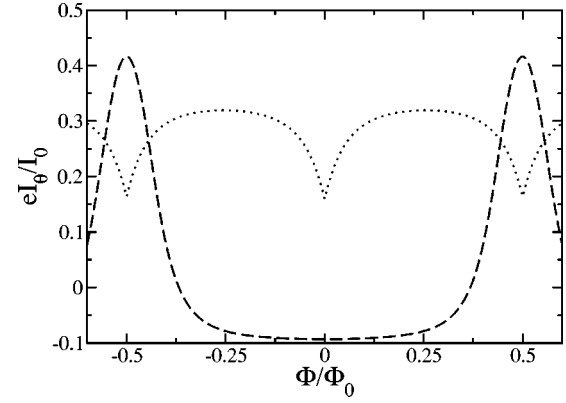


FIG. 7. Comparison of persistent spin currents for electron number equal to $4N+2$ (dashed curve) and $2N+1$ (dotted curve). The barrier strength is $A=0.5$ and $\cos\theta=0.9$ corresponding to a small spin-orbit coupling strength. The magnitude of persistent spin current decreases rapidly for an odd electron number as $\cos\theta$ approaches 0.66.

IV. EFFECT OF MANY RADIAL SUBBANDS

In the previous section, we have analyzed the persistent current in a strictly 1D ring—i.e., a ring with only the lowest radial subband occupied by electrons and a sufficiently large subband energy splitting. We now generalize this discussion to the case where higher subbands are important. SO coupling introduces coupling between neighboring radial subbands as described in Eq. (7). More specifically, the Hamiltonian, Eq. (7), couples radial subbands with opposite spin in the local spin frame, leading to nonparabolicity of energy dispersions and to hybridization of opposite-spin bands. The physics in the limit of strong subband coupling is analogous to what happens in a quantum wire with Rashba SO coupling; this has been discussed in Refs. 43 and 44. Here it is sufficient to notice that $H_{n,n+1}$ is negligible if $l_\omega/l_{so}\ll 1$ —i.e., if the radial width of the wave function is much smaller than the spin-precession length. This condition is fulfilled in realistic samples. Therefore, we neglect in the following the coupling term, Eq. (7). For the sake of simplicity we now consider only the two lowest subbands. Furthermore, we introduce a barrier in the same way as in Sec. III A. Assuming that the barrier does not couple different subbands and that the transmission coefficient is the same for both radial subbands and is given by $t=[1-i\text{sgn}(\kappa)A]/(A^2+1)$, we find for the energy spectrum

$$E_{q,\pm,n}=\hbar\omega_a\left[\kappa_{q,\pm}^2+\frac{1}{4}\left(1-\frac{1}{\cos^2\theta}\right)\right]+\hbar\omega\left(n+\frac{1}{2}\right), \quad (28)$$

where $n=0,1$ is the subband index and $\kappa_{q,\pm}$ is still given by Eq. (19). In Fig. 8, we show the average persistent current with and without SO coupling. In comparison to the single-subband case, additional fine structure appears due to crossing of levels with different radial quantum numbers. The jumps arising from these extra crossings are very sharp due to the way we model the barrier and occur at flux values that are strongly dependent on the ring occupancy. All other fea-

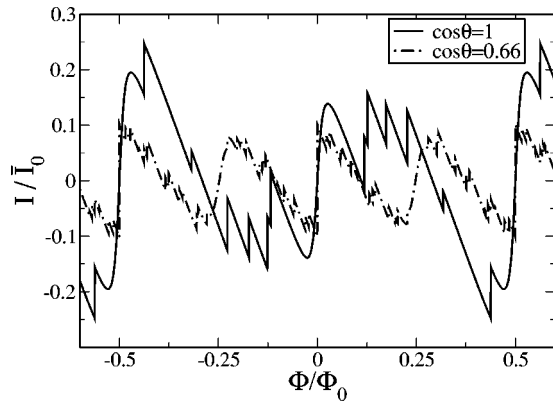


FIG. 8. Average persistent current vs magnetic flux for a ring with two occupied radial subbands. The barrier strength is $A = 0.1$. The average is performed on an ensemble containing rings with occupancy ranging from 60 to 80 electrons.

tures discussed for the strictly 1D case occur at the same flux values for all radial subbands. Hence, upon averaging, the latter are magnified and the former demagnified, as it is evident comparing Fig. 8 with Fig. 5(a). The dependence of the average persistent current on the SO coupling and barrier strength is the same as for the 1D case; hence, we do not show it again for the many-subband case. The presence of many radial subbands, although it introduces some additional fine structure, essentially yields, after averaging over differ-

ent electron numbers, the same SO-related features discussed in the purely 1D case.

V. CONCLUSIONS

We have investigated the effect of Rashba spin-orbit coupling on the persistent spin and charge currents circling in ballistic quasi-one-dimensional rings. The flux dependence of persistent charge currents exhibits features that allow for a direct measurement of the spin-orbit coupling strength. These features survive averaging over different electron number configurations as well as the inclusion of higher subbands. The most striking effect of spin-orbit coupling discussed here is the occurrence of finite persistent *spin* currents for even electron numbers. We have carefully derived the correct general form of spin currents in the presence of spin-orbit coupling. The possibility to measure persistent *spin* currents via the electric field generated by their transported magnetization should make it possible to unambiguously verify the presence and magnitude of spin-orbit coupling—namely, by the different flux dependences of persistent spin and charge currents.

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