Perturbative results on localization for a driven two-level system

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(Received 30 March 2003; revised manuscript received 4 August 2003; published 15 October 2003)

Using perturbation theory in the strong coupling regime, that is, the dual Dyson series, and renormalization-group techniques to resum secular terms, we obtain the perturbation series of the two-level system driven by a sinusoidal field till second order. The third order correction to the energy levels is obtained by proving how this correction does not modify at all the localization condition for a strong field as arising from the zeros of the zeroth Bessel function of integer order. A comparison with weak coupling perturbation theory is done showing how the latter is contained in the strong coupling expansion in the proper limits. The strong coupling expansion we obtain proves to be accurate in the regime of high-frequency driving field. This computation gives an explicit analytical form to Floquet eigenstates and quasienergies for this problem, for high-frequency driving fields, supporting recent theoretical and experimental findings for quantum devices expected to give a representation for qubits in quantum computation.

DOI: 10.1103/PhysRevB.68.165315 PACS number(s): 72.20.Ht, 73.40.Gk, 03.67.Lx

I. INTRODUCTION

Quantum computation,[1–4] intended as information theory performed using qubits, that is, two-level systems evolving by the unitary evolution as obtained by the laws of quantum mechanics, demands an increasing control on such systems.

Recent experimental findings[5–9] proved that qubits can be realized by solid-state devices, making the realization of a quantum computer even more possible. Such solid-state devices performs in a way like two-level systems driven externally by some time-varying field. This means that it is very important to have a clear theoretical understanding of such systems to make their control easier.

Advances in this field have been realized by devising new approaches to the solution of the Schrödinger equation in different regimes.[10–15] These methods are perturbative in nature but permit us to study a quantum system in different physical regimes. So, there seems to be a proper framework to make two-level systems theoretically manageable in any situation. These methods give to the Floquet method, for periodical perturbations, a strong analytical support.

The proper control of a qubit, realized by some solid-state device, relies on the possibility to obtain localization of a particle in each one of the two available states. This effect is also known as *coherent destruction of tunneling* (CDT)[16,17] as tunneling between one state and the other is destroyed by making unity the probability of staying of the particle in the initial state. So in recent years, this question has been the main motivation for several works on two-level systems.[18–21] Some of these papers have put forward an interesting nonperturbative result showing a set of curves for the crossing of the quasienergies of the Floquet solution for two-level systems with different periodical external fields. As it stands, such a result, being nonperturbative in its very nature, appears rather difficult to recover with perturbation theories.

The aim of this paper is to show how perturbation theory can give definite results for localization in different regimes, putting the study of two-level systems on a sound ground from an analytical point of view. Besides, an expression is given for localization, after that an effective time-independent Hamiltonian is obtained using perturbation theory. Our results confirm till third order the well-known result that the zeros of the zeroth order Bessel function are the points where CDT occurs. Besides, our result recovers the small perturbation theory computation in the proper limits. This is in agreement with the recent numerical computations.[20,21] But, it is essential to emphasize that the main result of the paper, the strong coupling expansion, is very accurate for high-frequency driving fields.

The paper is so structured. In Sec. II we discuss localization on a quite general ground, using Floquet theory. In Sec. III we give a presentation of the perturbation methods that we used to obtain our results. In Sec. IV we present the computation for the two-level system driven by a sinusoidal field giving the effective time-independent Hamiltonian obtained by a computation till second order. In Sec. V we give the third-order correction to the effective Hamiltonian showing how the localization condition at high fields is kept in agreement with all numerical results as also happens to the small field condition. Then we discuss the conditions under which localization happens comparing the perturbation series obtained with the weak perturbation series showing the way one result contains the other. Finally, in Sec. VI conclusions are given.

II. LOCALIZATION IN DRIVEN TWO-LEVEL SYSTEMS

The model we consider has the Hamiltonian

$$H=\frac{\Delta}{2}\sigma_3+g\sigma_1 f(t), \tag{1}$$

Δ being the separation between the two levels, g denotes the coupling, σ_1 and σ_3 are Pauli matrices, and $f(t)=f(t+T)$ with T the period of the perturbation. The theory of CDT[16,17] can be straightforwardly applied. Given the unitary evolution operator $U(t)$, one can compute the probability of the system of being in the initial state as

$$P(t)=|\langle\psi(0)|U(t)|\psi(0)\rangle|^2, \tag{2}$$

with $|\psi(0)\rangle$ being one of the eigenstates of σ_3. One has CDT when this probability is equal to unity. As the Hamiltonian is periodic in time, one can apply the Floquet theory and take[22]

$$\overline{P}=\frac{1}{T}\int_0^T P(t)dt \tag{3}$$

to ascertain that we have true localization by having unity also in this case. The consequence is that one can show, again from Floquet theory, that the conditions for CDT arise from crossing of quasienergies. This condition is necessary but not sufficient.

Our aim in the following section is to derive such conditions that apply to the quasienergies for the sinusoidal case, that is, assuming $f(t)=\cos(\omega t)$ with $\omega=2\pi/T$. We give an explicit analytical expression for the Floquet eigenstates and quasi-energies by using perturbation theory in the strong coupling regime with the method described in the following section.

III. DUAL DYSON SERIES AND RESUMMATION TECHNIQUES

It is generally believed that perturbation theory, being plagued by secularities, that is, unbounded terms in a perturbation series that increase as powers of time, $t, t^2, \ldots$, should be avoided to treat a model like the one we consider in this paper. Indeed, the methods devised so far for removing such singular terms proved to be generally not very easy to apply. Recently, a new approach, that can be called dynamical renormalization-group method,[23–28] has given an algorithmic way to remove secularities, making computation in perturbation theory though tedious but very easy to accomplish. This method, coupled with the dual Dyson series,[29–33] gives a straightforward method to compute higher-order corrections to well-known results.[17] Besides, we will get an explicit analytical expression for the Floquet modes and the quasienergies in the given approximation and we will prove that the small coupling result is recovered. This is a property of the dual Dyson series.

The dynamical renormalization-group method was first formulated by the Urbana Group[23,24] based on the observation that renormalization group methods can be seen as a means of asymptotic analysis.[23] They showed how their approach can be proved equivalent to other methods that resum secularities, such as the multiple time scale method,[34] for many cases. This method, for its very nature, can be described by an example. So, let us consider the well-known equation of the forced harmonic oscillator

$$\ddot{x}(t)=-x(t)-\epsilon x(t)^3, \tag{4}$$

ϵ being the strength of the anharmonic term assumed to be small. A naive perturbation expansion in ϵ till first order gives the well-known result

$$x(t)=A_0\sin(t+\phi_0)+\epsilon\frac{3}{8}A_0(t-t_0)\cos(t+\phi_0) -\epsilon\frac{A_0}{16}\sin(3t+3\phi_0)+O(\epsilon^2), \tag{5}$$

A_0 and ϕ_0 being constants depended on the initial conditions. We recognize a secular term $t-t_0$ that makes this series useless as it breaks down for $\epsilon(t-t_0)>1$. The situation can be improved if we interpret the time t_0 as the logarithm of the ultraviolet cutoff in quantum field theory and introduce A and ϕ as the renormalized counterpart of A_0 and ϕ_0 due to the fact that the nonlinearity may change these constants. Then, we introduce another renormalization point by splitting $t-t_0$ in $t-\tau+\tau-t_0$ and adsorb the terms containing $\tau-t_0$ into A and ϕ. We introduce a multiplicative renormalization constant $Z_1=1+\sum_{n=1}^{\infty}a_n\epsilon^n$ and an additive one $Z_2=\sum_{n=1}^{\infty}b_n\epsilon^n$ so that $A_0(t_0)=Z_1(t_0,\tau)A(\tau)$ and $\phi_0(t_0)=\phi(\tau)+Z_2(t_0,\tau)$ with the coefficients a_n and b_n to be computed order by order to remove the terms with $\tau-t_0$ as happens in standard renormalization group.[35,36] It is easily seen that, in our case, a possible choice to first order is $a_1=0$ and $b_1=-\frac{3}{8}(\tau-t_0)$ removing the secular term and we are left with

$$x(t)=A\sin(t+\phi)+\epsilon\frac{3}{8}A(t-\tau)\cos(t+\phi) -\epsilon\frac{A}{16}\sin(3t+3\phi)+O(\epsilon^2), \tag{6}$$

now A and ϕ being function of τ. But τ does not appear in the original problem and so $x(t)$ must be independent of it. This gives us the condition

$$\left.\frac{\partial x}{\partial\tau}\right|_{\tau=t}=0, \tag{7}$$

that is, the renormalization-group equation in our case gives the equations

$$\frac{\partial A}{\partial t}=O(\epsilon^2), \tag{8}$$

$$\frac{\partial\phi}{\partial t}=\frac{3}{8}\epsilon A+O(\epsilon^2)$$

where the well-known shift in the frequency of the oscillator is recovered. Besides, the secularity is completely removed by taking in Eq. (6) the condition $\tau=t$ with $A(t)$ and $\phi(t)$ given by the solutions of the renormalization-group equations. Finally we have

$$x(t)=A(0)\sin\left[\left(1+\frac{3}{8}\epsilon A(0)\right)t+\phi(0)\right] -\epsilon\frac{A(0)}{16}\sin\left[3\left(1+\frac{3}{8}\epsilon A(0)\right)t+3\phi(0)\right]+O(\epsilon^2). \tag{9}$$

So, a straightforward application of renormalization-group methods was permitted to remove a secular term in the perturbation series. But there is a way to make the computation simpler by noting that we have done nothing else than to compute the envelope of Eq. (5) using known renormalization techniques.

Then, the method of renormalization group to resum secularities in a perturbation series that we present here is obtained by the mathematical theory of envelopes and is due to Kunihiro.[25–27] In order to describe it, let us consider the following equation

$$\dot{x}(t)=f(x(t),t), \tag{10}$$

with $x(t)$ that can also be a vector. The initial condition is given by $x(t_0)=X(t_0)$. At this stage we assume $X(t_0)$, not yet specified. We write the solution of this equation as $x(t;t_0,X(t_0))$ which is exact. If we change t_0 to t_0' we are able to determine $X(t_0)$ by assuming that the solution should not change

$$x(t;t_0,X(t_0))=x(t;t_0',X(t_0')) \tag{11}$$

and in the limit $t_0 \to t_0'$ becomes

$$\frac{dx}{dt_0}=\frac{\partial x}{\partial t_0}+\frac{\partial x}{\partial X}\frac{\partial X}{\partial t_0}=0 \tag{12}$$

giving the evolution equation or flow equation of the initial value $X(t_0)$. We see again the renormalization-group equation proper to this approach.

Till now, all our equations are exact and no perturbation theory entered in any part of our argument. But, except for a few cases, the solution $x(t;t_0,X(t_0))$ is only known perturbatively and such a solution is generally valid only locally, i.e., for $t\sim t_0$ and $t\sim t_0'$ and a more restrictive request should be demanded for our renormalization-group equation

$$\left.\frac{dx}{dt_0}\right|_{t_0=t}=\left.\frac{\partial x}{\partial t_0}\right|_{t_0=t}+\left.\frac{\partial x}{\partial X}\frac{\partial X}{\partial t_0}\right|_{t_0=t}=0. \tag{13}$$

But this equation can be interpreted by the mathematical theory of envelopes.[25] Indeed, varying t_0 we have that $x(t;t_0,X(t_0))$ is a family of curves with t_0 as a characterizing parameter. Then, Eq. (13) becomes an equation to compute the envelope of such a family of curves. Such an envelope is given by the initial condition $x(t;t_0=t)=X(t)$. It can be proved that $X(t)$ satisfies Eq. (10) in a global domain up to the order $x(t;t_0)$ for $t\sim t_0$.

Now, we can describe our approach in some steps to show how such an algorithmic perturbation method indeed works

1. Consider the following unitary transformation on Hamiltonian (1) (here and in the following, we set $\hbar=1$) to remove the perturbation[29,30]

$$U_F(t)=\exp\left[-ig\sigma_1\int_0^t f(t')dt'\right], \tag{14}$$

giving the transformed Hamiltonian

$$H_F(t)=\frac{\Delta}{2}\sigma_3\exp\left[-i2g\sigma_1\int_0^t f(t')dt'\right]; \tag{15}$$

the dual Dyson series is computed by[29–32]

$$S_D(t,t_0)=\mathcal{T}\exp\left[-i\epsilon\int_{t_0}^t H_F(t')dt'\right], \tag{16}$$

as usual $\mathcal{T}$ being the time ordering operator and an ordering parameter ϵ has been introduced that will be taken unity at the end of computation. It is fundamental for our argument that the computation of this series is performed at a different starting point t_0.

2. Assume, at the start, that the time evolution operator has the form

$$U(t,t_0)=U_F(t)S_D(t,t_0)U_R(t_0), \tag{17}$$

where $U_R(t_0)$ is a "renormalizable" part of the unitary evolution.

3. At the given order, one gets $S_D(t,t_0)$ as

$$S_D(t,t_0)=I-i\epsilon f_1(t,t_0)-\epsilon^2 f_2(t,t_0)+\cdots \tag{18}$$

and, at this stage, if some oscillating functions in t_0 appear like $e^{-i\omega t_0}$ then introduce the phase $\phi(t_0)=-t_0$ as a "renormalizable" parameter rewriting it as $e^{i\omega\phi(t_0)}$.[28] We make this choice assuming that any initial phase of the system can be changed by the dynamics. The minus sign is fixed arbitrarily. The secularities must be left untouched.

4. Eliminate the dependence on t_0 by requiring[25,26]

$$\left.\frac{dU(t,t_0)}{dt_0}\right|_{t_0=t}=0, \tag{19}$$

and one obtains the renormalization group equation

$$\begin{aligned}\frac{dU_R(t)}{dt}&=\epsilon g_1 U_R(t)+\epsilon^2 g_2 U_R(t)+O(\epsilon^3),\\ \frac{d\phi(t)}{dt}&=\epsilon\phi_1\phi(t)+\epsilon^2\phi_2\phi(t)+O(\epsilon^3),\end{aligned} \tag{20}$$

where, at some stage to obtain such equations at the second order, we have to use their expressions at the first order, and to compute their form at nth order, one has to use these equations at the order $(n-1)$th, into condition (19). This is a step toward the computation of the envelope of the perturbation series.[25,26]

5. Finally, the renormalization equations should be solved and substituted into equation

$$U(t,t_0)|_{t_0=t} \tag{21}$$

giving the solution, i.e., the envelope we were looking for without secularities at the order we made the computation.

We will give an explicit application of this procedure in the following section choosing $f(t)=\cos(\omega t)$, that is, a sinusoidal driving.

IV. STRUCTURE OF THE PERTURBATION SERIES TO SECOND ORDER

The unitary transformation $U_F(t)$ for a sinusoidal driving field takes the form

$$U_F(t)=\exp\left[-i\frac{g}{\omega}\sin(\omega t)\sigma_1\right] \tag{22}$$

and the transformed Hamiltonian takes the form

$$H_F(t)=\frac{\Delta}{2}\sigma_3\exp[-iz\sin(\omega t)\sigma_1] \tag{23}$$

having put $z=2g/\omega$. This Hamiltonian gives the term $S_D(t,t_0)$ till first order as

$$S_D(t,t_0)=I-i\frac{\Delta}{2}\sigma_3 J_0(z)(t-t_0)+\frac{\Delta}{2}\sigma_3\sigma_1\sum_{n\neq 0}J_n(z)\frac{e^{-in\omega t\sigma_1}-e^{-in\omega t_0\sigma_1}}{n\omega}+\cdots, \tag{24}$$

where $J_n(z)$ are the Bessel functions of nth order n being an integer. We see immediately that a secular term, proportional to $t-t_0$, plagues our computation. To remove it, we rewrite the above expression as

$$S_D(t,t_0)=I-i\frac{\Delta}{2}\sigma_3 J_0(z)(t-t_0)+\frac{\Delta}{2}\sigma_3\sigma_1\sum_{n\neq 0}J_n(z)\frac{e^{-in\omega t\sigma_1}-e^{in\omega\phi(t_0)\sigma_1}}{n\omega}+\cdots \tag{25}$$

introducing in the last oscillating term the renormalizable parameter $\phi(t_0)$. So, one has

$$\begin{aligned}\frac{dU(t,t_0)}{dt_0}=&U_F(t)\left[i\frac{\Delta}{2}\sigma_3 J_0(z)-\frac{\Delta}{2}\sigma_3\sum_{n\neq 0}J_n(z)e^{in\omega\phi(t_0)\sigma_1}\frac{d\phi(t_0)}{dt_0}\right]U_R(t_0)\\&+U_F(t)\left[I-i\frac{\Delta}{2}\sigma_3 J_0(z)(t-t_0)+\frac{\Delta}{2}\sigma_3\sigma_1\sum_{n\neq 0}J_n(z)\frac{e^{-in\omega t\sigma_1}-e^{in\omega\phi(t_0)\sigma_1}}{n\omega}+\cdots\right]\frac{dU_R(t_0)}{dt_0},\end{aligned} \tag{26}$$

and imposing the condition (19), we get the renormalization-group equations

$$\frac{dU_R(t)}{dt}=-i\frac{\Delta}{2}\sigma_3 J_0(z)U_R(t)+O\left[\left(\frac{\Delta}{\omega}\right)^2\right],$$

$$\frac{d\phi(t)}{dt}=O\left[\left(\frac{\Delta}{\omega}\right)^2\right], \tag{27}$$

where the fact that these equations are at least first order has been used, transforming some terms in Eq. (26) into second order, making them negligible. We see that we have recovered a well-known result of the series for high frequency[17] with its corrections to first order. This gives the final result, using Eq. (21),

$$U(t,0)=U_F(t)\left[I+\frac{\Delta}{2}\sigma_3\sigma_1\sum_{n\neq 0}J_n(z)\frac{e^{-in\omega t\sigma_1}-1}{n\omega}+\cdots\right]e^{-i(\Delta/2)\sigma_3 J_0(z)t}. \tag{28}$$

This result is well known in literature.[11,12,31,33] It has also been proved that this is the expression of the Floquet unitary evolution at this order with the given approximations.[11,12] We see that we have a periodic part and a part that originates the quasi-energies.

So, what is really interesting is to get higher-order corrections. Particularly, we see that the quasienergies are given by $\pm(\Delta/2)J_0(z)$, a well-known fact, so that, CDT occurs, in this approximation, at the zeros of the Bessel function $J_0(z)$.[17] We now prove that higher-order corrections preserve such a result.

Applying the above procedure till second order it is straightforward to obtain

$$\begin{aligned}U(t,0)=U_F(t)\Bigg[&I+\frac{\Delta}{2}\sigma_3\sigma_1\sum_{n\neq 0}J_n(z)\frac{e^{-in\omega t\sigma_1}-1}{n\omega}\\&-i\frac{\Delta^2}{2}\sigma_1 J_0(z)\sum_{n\neq 0}J_n(z)\frac{\sin(n\omega t)}{n^2\omega^2}\\&-\frac{\Delta^2}{4}\sum_{n\neq 0}J_n^2(z)\frac{e^{in\omega t}-1}{n^2\omega^2}\\&-\frac{\Delta^2}{4}\sum_{n_1\neq 0,n_2\neq 0,n_1\neq n_2}J_{n_1}(z)J_{n_2}(z)\\&\times\left(\frac{e^{i(n_1-n_2)\omega t\sigma_1}-1}{n_2(n_1-n_2)\omega^2}-\frac{e^{in_1\omega t\sigma_1}-1}{n_1 n_2\omega^2}\right)\\&+\cdots\Bigg]e^{-i(\Delta/2)\sigma_3 J_0(z)t+i(\Delta^2/2\omega)F(z)\sigma_1 J_0(z)t},\end{aligned} \tag{29}$$

where we have introduced the function $F(z)=\sum_{n\neq 0}(J_n(z)/n)$ into the last exponential. We recognize a product of a periodic unitary operator and a term originating quasienergies. So, it is interesting to see that now we have an effective Hamiltonian giving the quasienergies with a correction term proportional to σ_1. But, the most important point is that this correction is again proportional to $J_0(z)$ and so, the zeros of this Bessel function gives CDT also at second order. The effective Hamiltonian can be written as

$$H_{eff}=\frac{\Delta J_0(z)}{2}\sigma_3-\frac{\Delta^2 J_0(z)}{2\omega}F(z)\sigma_1 \tag{30}$$

that can be easily diagonalized but we do not pursue this matter here. Indeed, our aim in the following section will be to give the third-order correction to this Hamiltonian proving that the high-frequency CDT is again determined by the zeros of $J_0(z)$.

V. THIRD-ORDER CORRECTION TO THE QUASIENERGIES AND LOCALIZATION

The algebra is rather tedious but straightforward and the application of our method gives the third-order correction to the effective Hamiltonian

$$H_{eff}^{(3)}=-\frac{\Delta^3 J_0(z)}{4\omega^2}\sum_{n\neq 0}\frac{J_n^2(z)}{n^2}\sigma_3. \tag{31}$$

This is a correction to the σ_3 term of the effective Hamiltonian so, we can conjecture that even-order corrections go to the σ_1 term and the odd-order corrections go to the σ_3 term. We see again that this term is proportional to $J_0(z)$ confirming the exactness in the high-frequency limit of the occurrence of the CDT at the zeros of such a Bessel function. Such correcting terms into the effective Hamiltonian can be recognized as ac Stark shifts and Bloch-Siegert shifts which are relevant, eventually, at the resonance[28,37] where Rabi flopping also in this regime is expected with the renormalized levels $\pm(\Delta/2)J_0(z)$.

Anyhow, a further check can be obtained with the small coupling perturbation theory. Our method can be applied again but now we use the interaction picture. So, if we introduce the unitary transformation $U_I(t)=e^{-i(\Delta/2)\sigma_3 t}$, we get the transformed Hamiltonian

$$H_I=g\cos(\omega t)\sigma_1 e^{-i\Delta\sigma_3 t} \tag{32}$$

and we can built the Dyson series, out of resonance ($\Delta \neq\omega$), to second order as

$$\begin{aligned}U(t,0)=U_I(t)\Bigg[I-\frac{g}{2}\sigma_1\sigma_3\Bigg(&\frac{e^{i(\omega-\Delta)\sigma_3 t}-1}{\omega-\Delta}-\frac{e^{-i(\omega+\Delta)\sigma_3 t}-1}{\omega+\Delta}\Bigg)\\&+\frac{g^2}{4}\Bigg(\frac{\cos(2\omega t)+i\sigma_3\frac{\Delta}{\omega}\sin(2\omega t)}{\omega^2-\Delta^2}-\frac{e^{i(\omega+\Delta)\sigma_3 t}-1}{\omega^2-\Delta^2}\\&+\frac{e^{i(\omega-\Delta)\sigma_3 t}}{(\omega-\Delta)^2}-\frac{e^{-i(\omega-\Delta)\sigma_3 t}}{\omega^2-\Delta^2}+\frac{e^{i(\omega+\Delta)\sigma_3 t}}{(\omega+\Delta)^2}\\&-2\frac{\omega^2+\Delta^2}{(\omega^2-\Delta^2)^2}\Bigg)+\cdots\Bigg]e^{i(g^2/2)(\Delta/\omega^2-\Delta^2)\sigma_3 t}.\end{aligned} \tag{33}$$

A rather interesting result is that this series holds for any ratio Δ/ω different from the dual Dyson series discussed above that holds for $\Delta/\omega\ll 1$. From the effective Hamiltonian (30) and the third-order correction, we see that the coefficient of the σ_3 part is given by

$$S_3=\frac{\Delta J_0(z)}{2}-\frac{\Delta^3 J_0(z)}{4\omega^2}\sum_{n\neq 0}\frac{J_n^2(z)}{n^2} \tag{34}$$

that in the small coupling approximation $J_0(z)\to 1-(z^2/4)$, and considering just the first term of the series giving $z^2/2$, we obtain

$$S_3\approx\frac{\Delta}{2}\left(1-\frac{z^2}{4}\right)-\frac{\Delta^3}{4\omega^2}\frac{z^2}{2}+O(z^4). \tag{35}$$

From the small coupling expansion we recognize, after a simple rearranging of the terms, the σ_3 term

$$S_3'=\frac{\Delta}{2}-\frac{g^2}{2}\frac{\Delta}{\omega^2-\Delta^2} \tag{36}$$

that, in the expected limit $\frac{\Delta}{\omega}\ll 1$, is the same as S_3. Similarly, one can verify the same result at any order of the two perturbation series. This gives an *a posteriori* verification of our computations. Besides, it confirms our point that the dual Dyson series is a high-frequency series while the Dyson series holds for both the limits of high and low frequencies, but the result of the Dyson series in the high-frequency limit is contained in the dual Dyson series when the limit of small coupling $z\to 0$ is also taken. Indeed, the Dyson series and the dual Dyson series can be taken to coincide in the limits of small z and $\Delta/\omega\ll 1$, giving an interesting relationship between the transformed Hamiltonians that we used to obtain the perturbation series.

We are now in a position to discuss the CDT in the limit of small coupling $z\to 0$. To achieve our aim, we consider the analytical expression of the quasienergies obtained in Ref. 38. This gives

$$\epsilon_{\pm}=\pm\frac{\omega}{2\pi}\arccos\left[\cos\left(\pi\frac{\Delta}{\omega}\right)+\pi\frac{z^2}{4}\frac{\Delta}{\omega}\frac{\sin\left(\pi\frac{\Delta}{\omega}\right)}{1-\frac{\Delta^2}{\omega^2}}\right], \quad \mathrm{mod}(\omega) \tag{37}$$

that yields, in the limit $z\ll 1$, Eq. (36) proving again the correctness of our computations. The important point is that Eq. (37) contain the result for CDT that the quasienergies crosses for $\Delta/\omega=2n$ being n an integer and this cannot happen for the perturbative result unless we are able to resum the series. So, we can draw the conclusion that perturbation theory by Dyson series can prove to be very effective for the study of CDT for strong fields and high-frequency regime.

VI. DISCUSSION AND CONCLUSIONS

From the von Neumann–Wigner theorem[39] one has to expect that the crossings of the energy levels form a one-dimensional manifold. This result is central to fully understand CDT for the two-level system that we considered. Recent works on this question derived such a result numerically.[20,21] In this paper we have shown that unless some smart resummation technique is applied to a perturbation series, perturbation theory is helpful just to study the behavior of the model in different regimes, recovering CDT under certain conditions.

The reason for this conclusion lies on the fact that the content of the von Neumann–Wigner theorem is nonperturbative. So, if we are able to resum a perturbation series to recover a nonperturbative result, there may be a possibility to give an analytical proof of the numerical results obtained so far.

Notwithstanding such a conclusion, we have proved the power of the Dyson series in the study of two-level systems in regimes where other means may prove unsuccessful, giving explicitly the form of the Floquet modes till second order and quasienergies till third order, for a strong field and high frequency. At this stage, it appears mandatory to introduce resummation techniques in perturbation theory to complete the algorithmic approach we discussed in this paper.

Different regimes of parameter space have been analyzed by our perturbation technique. Notably, we have obtained perturbation series for the high-frequency regime $\Delta \ll \omega$ in the strong coupling approximation $g \gg \Delta$ by dual Dyson series. The Dyson series has given the small coupling expansion $g \ll \Delta$ that holds for any ratio Δ/ω but when this ratio is taken to be small, the dual Dyson series recovers the Dyson series for $g \ll \Delta$ as we have shown. These results appear very promising for the study of quantum systems in different regimes by a general way to approach computations with perturbation theory.

ACKNOWLEDGMENTS

I would like to thank Charles Creffield for suggesting to me such an interesting question.

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