Quantum Hall effect in a *p*-type heterojunction with a lateral surface superlattice

V. Ya. Demikhovskii* and D. V. Khomitskiy
Nizhny Novgorod State University Gagarin Avenue 23, Nizhny Novgorod 603950, Russian Federation
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The quantization of the Hall conductance in a *p*-type heterojunction with a lateral surface superlattice is investigated. The topological properties of the four-component hole wave function are studied both in **r** and **k** spaces. A method of calculation of the Hall conductance in a two-dimensional hole gas described by the Luttinger Hamiltonian and affected by a lateral periodic potential is proposed, based on an investigation of the four-component wave-function singularities in **k** space. The effects of spin-orbit interaction, spin splitting in a magnetic field, and the asymmetric heterojunction potential are included. Deviations from the quantization rules for electron magnetic subbands of the Hofstadter "butterfly" are found, and the explanation of this effect is proposed. For the case of a strong periodic potential the mixing of magnetic subbands is taken into account, and the exchange of the Chern numbers between magnetic subbands is discussed.

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I. INTRODUCTION

Quantum states and the transport of two-dimensional (2D) Bloch electrons in a magnetic field show an extremely rich variety of physical and topological properties. The fascinating physical problems occurring here are caused by the mutual effects of the lattice periodic potential and the nonperiodic vector potential of a uniform magnetic field. It is known that the former leads to an energy band structure while the latter tends to form discrete energy levels. The parameter which plays an important role in the problem is the magnetic flux Φ penetrating the lattice elementary cell. If the flux is equal to a rational number p/q of flux quanta $\Phi_0 = 2\pi\hbar c/|e|$ where p and q are mutually prime integers, it is possible to define a new set of translations on the lattice (called magnetic translations[1,2]) for which the quasimomentum is a good quantum number. For example, if the vector potential of a uniform magnetic field B is chosen in the Landau gauge $\mathbf{A}=(0,Bx,0)$, and $\Phi/\Phi_0 = p/q$, the simplest form of magnetic translations on a square lattice with the period a is $x\rightarrow x+qna, y\rightarrow y+ma$, where n and m are integers. So, the magnetic elementary cell is q times larger in the x direction, and the corresponding magnetic Brillouin zone (MBZ) is defined as follows:

$$-\pi/qa \leqslant k_x \leqslant \pi/qa, \quad -\pi/a \leqslant k_y \leqslant \pi/a. \tag{1}$$

When the quasimomentum runs over MBZ (1), the energy varies in a band which is called a magnetic subband. When the amplitude of a periodic potential V_0 is smaller than the cyclotron energy $\hbar\omega_c$ one can neglect the influence of neighboring Landau levels and may obtain a set of p magnetic subbands arising from a single level.[3] If several electron Landau levels are taken into account, the periodic potential leads to a mixing between magnetic subbands originating from different levels.[4–6] However, the translational properties of the electron wave function remain the same both for coupled and uncoupled Landau levels. For example, one can see that, regardless of the particular form and the number of Landau levels taken into account, the electron wave function gains an additional phase under magnetic translations. The relation between the translated and initial wave functions in a magnetic field is known as the generalized Bloch conditions (or Peierls conditions) (Refs. 7 and 2)

$$\psi_{k_x k_y}(x,y,z) = \psi_{k_x k_y}(x+qa, y+a, z)\exp(-ik_x qa) \times \exp(-ik_y a)\exp(-2\pi i p y/a). \tag{2}$$

It follows from Eq. (2) that the wave function gains a phase $2\pi p$ after the circulation along the boundary of the magnetic unit cell. As a result, the magnetic field forces the periodic part $u_{\mathbf{k}}(\mathbf{r}) = \exp(-i\mathbf{kr})\psi_{\mathbf{k}}(\mathbf{r})$ of the wave function to have a $-p$ vorticity [see Eq. (14)] in the magnetic unit cell which indicates that there are at least p zeros of the wave function per each magnetic cell, as shown by Kohmoto.[8] This result has a topological nature because of its independence of the shape and amplitude of a periodic potential.

During the last years the researchers have investigated several significant theoretical and experimental features of systems where a 2D electron gas with an additional periodic potential is in the regime of the quantum Hall effect. If a single Landau level is split by a 2D periodic potential of a superlattice which has the area of an elementary cell corresponding to p/q flux quanta penetrating the cell, the spectrum transforms to the system of p magnetic subbands grouped near the unperturbed level. Here one might expect that each of the magnetic subbands gives a Hall conductance σ_H equal to e^2/ph, but this is not the case. According to Laughlin each subband must carry an integer multiple of the Hall current carried by the entire Landau level. The confirmation of this rule, which describes the quantization of the Hall conductance in periodically modulated 2D systems, has been obtained by Thouless, Kohmoto, Nightingale, and den Nijs (hereafter referred as TKNN) in their pioneering paper.[3] They studied in detail a simple quasi-1D model of a strongly anisotropic lattice for which an explicit expression for σ_H has been derived. In this paper it was shown that the Landau levels are split by a periodic potential, and that a magnetic subband spectrum consisting of p subbands is formed if the unit cell of the potential space period has an area corresponding to p/q magnetic flux quanta. Then, if the Fermi energy of the electron system falls into the rth gap of the Nth Landau level, the Hall conductance can be written as[3]

$$\sigma_H=\frac{e^2}{h}(t_r+N-1), \tag{3}$$

where t_r is an integer obtained from the Diophantine equation

$$t_r p+s_r q=r. \tag{4}$$

Equation (4) has integer solutions for some integer values of s_r where $|s_r|\leq p/2$. It was found further that the quantization of σ_H in periodically modulated systems has a topological nature. That is, the value of σ_H for a fully occupied magnetic subband is related to the number and type of the wave-function singularities in **k** space. Kohmoto showed that these singularities determine the first Chern number for a particular magnetic subband which is, in units of $-e^2/h$, exactly the Hall conductance of this subband.[8]

An original method for a calculation of the Hall conductance of a 2D electron gas affected by a weak periodical modulation was proposed by Usov.[9] He showed that the value of σ_H is related to the winding numbers S_m, $m=1,2,\ldots$ of the wave-function singularities in MBZ (1). The appearance of these singularities is a direct consequence of the nontrivial topology of the MBZ, and a winding number S_m is determined as the phase mismatch at the beginning and end of a circulation around the singular point $\mathbf{k}_m$. As a result, the Hall conductance of a fully occupied α-th subband is given by[9]

$$\sigma_H^{\alpha}=\frac{e^2}{h}\left[\frac{1}{p}+\frac{q}{p}\sum_m S_m\right]. \tag{5}$$

The topological features of the problem were discussed for the first time by Novikov.[10] That is, the formation of p magnetic subbands near the Landau level was treated as a fiber bundle of magnetic Bloch functions on a T^2 torus which is MBZ (1). This problem was also discussed by Avron, Seiler, and Simon using homotopy theory.[11] It should be mentioned that Simon[12] also made a connection between the topological invariant and Berry's geometrical phase factor.[13] Berry's phase expresses the Hall conductance in the spirit of the linear response Kubo formula as a 2D integral over MBZ (1).[14–16] The integrand here is called Berry's curvature, which is a **k**-dependent function. The universality of the existence of a topological invariant in the QHE was also demonstrated in systems where substrate disorder[17,18] and many-body interactions[18] are present.

Another approach to a calculation of the Hall conductance is based on the Středa formula.[19] If the Fermi level is located in the energy gap, the Hall conductance can also be given by

$$\sigma_H=ec\frac{\partial N(E)}{\partial B}, \tag{6}$$

where $N(E)$ is the number of states per unit area having an energy lower than the gap energy. Diophantine equation (4) and Středa formula (6) have been widely used for calculations of the Hall conductance of a 2D electron gas with periodical modulation,[5,20,27] even in the presence of Landau level coupling.[5] This approach has been also applied to 3D systems[21–23] where the generalization of Eq. (6) is known as the Kohmoto-Halperin-Wu formula.[23] However, the application of Středa formula (6) to the systems with multicomponent wave functions (like the hole states described by the Luttinger Hamiltonian) is not evident, and thus we shall focus on the analytical approach based on the investigation of **k**-space singularities.

During the last decade a number of experimental studies has been performed in order to investigate a 2D electron gas laterally modulated by a surface superlattice of quantum dots (antidots). Such a system is convenient for investigations of both classical effects (commensurability of the lattice periods and cyclotron radius, transition to chaos, etc.) and of the energy spectrum consisting of magnetic subbands. For example, oscillations of the longitudinal magnetoresistance have been detected under the conditions where the classical cyclotron radius $2R_c$ envelopes an integer number of antidots or where numerous reflections from one antidot occur.[24,25] The first experimental evidence of electron Landau levels split into the set of magnetic subbands was obtained by measurements of the longitudinal magnetoresistance in an n-type heterojunction with a lateral surface superlattice.[26] Then, the Hall resistance in a laterally modulated 2D electron gas was studied experimentally and the confirmation of a subband energy spectrum was found.[27] It should be mentioned that in the experiments a fundamental role is played by the effects of randomly distributed impurities. Their influence on the collision broadening and the transport scattering rate in a 2D electron gas with a periodical modulation was also studied theoretically.[28,20]

Experiments in p-type heterojunctions without periodic potential have also become possible due to the progress in technology which substantially improved the quality of p channels in GaAs/AlGaAs heterojunctions.[29–31] Thus, almost all intriguing phenomena found for 2D electron systems were also observed in 2D hole channels. The quantum states of 2D holes in a laterally modulated p-type heterojunction were also studied in our recent paper[34] together with an investigation of the magnetooptical properties. Since the transport experiments in a laterally modulated 2D hole gas have started,[32] it is now needful to derive a quantum-mechanical description of transport phenomena in such systems, and, in particular, of the quantum Hall effect.

In the current paper we present a method of calculation of the Hall conductance in a 2D hole gas affected by lateral periodic potential. The preliminary results of QHE theory in a 2D hole gas with a periodical modulation were reported earlier.[33] In Sec. II we briefly describe the magnetic hole Bloch states in a p-type heterojunction subjected to a magnetic field and affected by a lateral periodic potential. In Sec. III we generalize the method derived by Usov[9] for a calculation of the Hall conductance in a system studied in Sec. II, where the charged particle is described by a four-component eigenfunction of the Luttinger Hamiltonian. We find an unusual behavior of the Hall conductance as a function of the Fermi energy compared to the well-known dependence obtained for electron subbands of the Hofstadter "butterfly."[3]

The quantization of the Hall conductance is investigated both at weak and strong periodic potentials. In the latter case we take into account the magnetic subband mixing which leads to the exchange of the Chern numbers between magnetic subbands, changing their impact to the Hall conductance. We believe that the differences between the quantization of the Hall conductance in laterally modulated *n*- and *p*-type heterojunctions, as predicted by us, can be observed experimentally. In Sec. IV we give a summary of our results.

II. MAGNETIC HOLE STATES IN A LATERALLY MODULATED HETEROJUNCTION

Holes are studied near the upper edge of a GaAs *p*-like valence band located at $\mathbf{k}=0$. We assume that the external magnetic field is pointed along the $\langle 001\rangle$ crystal direction which is perpendicular to the heterojunction plane (xy). The 2D holes are described in the $|J;m_J\rangle$ basis by the 4×4 Luttinger Hamiltonian where both the magnetic field and the potential of a single heterojunction $V_h(z)$ are taken into account.[29,30] In addition to $V_h(z)$ we introduce a periodic electric potential $V(x,y)$ of a lateral surface superlattice. Such superlattices are usually fabricated by electron beam lithography, and they have been extensively used in the experiments on Bloch electrons in a magnetic field.[24–27,32] The simplest form for the potential of a surface superlattice is[6]

$$V(x,y)=V_0\cos^2\frac{\pi x}{a}\cos^2\frac{\pi y}{a}. \tag{7}$$

Here a is the superlattice period and the case $V_0<0$ (>0) corresponds to the periodic electric potential generated by a quantum dot (antidot) superlattice. The Hamiltonian H_L which defines the hole magnetic Bloch states in the presence of a magnetic field with the vector potential $\mathbf{A}=(0,Bx,0)$ is obtained from the 4×4 Luttinger Hamiltonian in the following way. First, the components of the wave vector are replaced by their operator forms[29,30]:

$$k_\alpha \to \hat{k}_\alpha=-i\frac{\partial}{\partial x_\alpha}+\frac{e}{c}A_\alpha$$

where the atomic units $\hbar=m_0=1$ are used. Then, following Luttinger, the harmonic oscillator creation and annihilation operators are defined[29–31]:

$$\hat{a}^+=\frac{R}{\sqrt{2}}\hat{k}_+,\quad \hat{a}=\frac{R}{\sqrt{2}}\hat{k}_-$$

where $\hat{k}_\pm=\hat{k}_x\pm i\hat{k}_y$ and $R=[c/eB]^{1/2}$. Finally, periodic electrostatic potential (7) is introduced by adding $V(x,y)\cdot\hat{E}$ to the Hamiltonian where $\hat{E}$ is the 4×4 unit matrix. As a result, the Hamiltonian for the hole magnetic Bloch states takes the following form in the no-warping approximation[34]:

$$H_L=\begin{bmatrix} H_{11} & \bar{\gamma}\sqrt{3}(eB/c)\hat{a}^2 & \gamma_3\sqrt{6eB/c}\,k_z\hat{a} & 0\\ & H_{22} & 0 & -\gamma_3\sqrt{6eB/c}\,k_z\hat{a}\\ & & H_{33} & \bar{\gamma}\sqrt{3}(eB/c)\hat{a}^2\\ & & & H_{44}\end{bmatrix}, \tag{8}$$

where

$$H_{11}=-(\gamma_1/2-\gamma_2)k_z^2-(eB/c)\left[(\gamma_1+\gamma_2)\left(\hat{a}^+\hat{a}+\frac{1}{2}\right)+\frac{3}{2}\kappa\right]+V_h(z)+V(x,y),$$

$$H_{22}=-(\gamma_1/2+\gamma_2)k_z^2-(eB/c)\left[(\gamma_1-\gamma_2)\left(\hat{a}^+\hat{a}+\frac{1}{2}\right)-\frac{1}{2}\kappa\right]+V_h(z)+V(x,y),$$

$$H_{33}=-(\gamma_1/2+\gamma_2)k_z^2-(eB/c)\left[(\gamma_1-\gamma_2)\left(\hat{a}^+\hat{a}+\frac{1}{2}\right)+\frac{1}{2}\kappa\right]+V_h(z)+V(x,y),$$

$$H_{44}=-(\gamma_1/2-\gamma_2)k_z^2-(eB/c)\left[(\gamma_1+\gamma_2)\left(\hat{a}^+\hat{a}+\frac{1}{2}\right)-\frac{3}{2}\kappa\right]+V_h(z)+V(x,y).$$

The lower half of matrix (8) is obtained by Hermitian conjugation. In Eq. (8) e is a modulus of elementary charge, γ_1, γ_2, γ_3, and κ are the material bulk parameters which are well-known for GaAs. The hole energy is counted as negative from the upper edge of the valence band throughout the paper. In the effective mass approximation the k_z component of quasimomentum in Eq. (8) is replaced by its operator form $k_z=-i\partial/\partial z$. This substitution is performed at $B=0$ and $V(x,y)=0$ which yields an infinite set of two-fold degenerate heavy and light hole size quantization levels and eigenfunctions $c_j^\nu(z), \nu=1,2,\ldots$. The z-dependent envelope functions $C_j^\nu(z)$ at finite B can be constructed as the superpositions of zero-field functions c_j^ν.[29,30] Now let periodic potential (7) be applied, corresponding to the rational magnetic flux through the elementary cell with the area $S=a^2$:

$$\frac{BS}{\Phi_0}=\frac{BS}{2\pi\hbar c/|e|}=\frac{p}{q}. \tag{9}$$

If condition (9) is satisfied, any of four components ψ^j [see Eq. (10)] of the vector of hole envelope functions becomes a magnetic Bloch function classified by k_x and k_y quantum numbers varying in MBZ (1), and the total hole quantum state can be written as follows:

$$\Psi_{k_x,k_y}(\mathbf{r})=\psi^1_{k_xk_y}(\mathbf{r})\left|\frac{3}{2};\frac{3}{2}\right\rangle+\psi^2_{k_xk_y}(\mathbf{r})\left|\frac{3}{2};-\frac{1}{2}\right\rangle+\psi^3_{k_xk_y}(\mathbf{r})\left|\frac{3}{2};\frac{1}{2}\right\rangle+\psi^4_{k_xk_y}(\mathbf{r})\left|\frac{3}{2};-\frac{3}{2}\right\rangle. \tag{10}$$

The translational properties of each component of the envelope function (10) in (xy) plane are the same as for the single-component electron wave function.[34] In particular, every component of Eq. (10) satisfies to the Peierls condition (2). Hence, one can write the components $\psi^j_{k_xk_y}(\mathbf{r})$ of (10) as a superposition of the Landau quantum states,[3–6,34] namely,

$$\psi^j_{k_xk_y}(\mathbf{r})=\frac{1}{La\sqrt{q}}\sum_\nu C^\nu_j(z)\sum_N\sum_{n=1}^{p} d_{j\nu Nn}(k_x,k_y)\times\sum_{l=-L/2}^{L/2}u_N\left(\frac{x-x_0-lqa-nqa/p}{\ell_H}\right)\times\exp\left(ik_x\left[lqa+\frac{nqa}{p}\right]\right)\times\exp\left(2\pi iy\frac{lp+n}{a}\right)\exp(ik_yy), \tag{11}$$

where $u_N(x)$ is a harmonic oscillator wavefunction, ℓ_H is the magnetic length and $x_0=k_y\ell_H^2$. It should be mentioned that the set of basis functions for the hole states in magnetic subbands originating from the coupled hole Landau levels has more complicated structure than those for electrons (the latter is discussed, for example, in Refs. 5 and 6). That is, in the absence of a periodic potential the four-component eigenvector of the Luttinger Hamiltonian in a single subband of size quantization ν has the form[29]

$$F^\nu_{Nk_y}=e^{ik_y}(C^\nu_1(z)u_{N-2},\quad C^\nu_2(z)u_N,\quad C^\nu_3(z)u_{N-1},\quad C^\nu_4(z)u_{N+1}). \tag{12}$$

In Eq. (12) the functions $u_N(x)$ vanishe for negative values of its index. Below we shall discuss in details the structure of expression (11).

First, we should restrict ourselves to some limited number of size quantization subbands to be taken into account. In heterojunctions with a typical hole concentration $n_h=5\times10^{11}$ cm^{-2} and a depletion-layer density $N_{dep}=10^{15}$ cm^{-3}, only the lowest hole subband of the size quantization is occupied.[29,30] Hence, in expression (11) for the hole state it seems to be relevant to consider only several subbands of size quantization neighboring the lowest one. During the investigation of hole states (11) in this section we consider the first three subbands of size quantization which correspond to two heavy-hole levels and one light-hole level. In addition, for each subband of size quantization in Eq. (11) we take into account only several Landau levels N. The basis for hole state (11) at $V(x,y)=0$ consists of the following four-component vectors:

$$e^{ik_y}[0,\ 0,\ 0,\ C^1_4(z)u_0],\quad e^{ik_y}[0,\ C^1_2(z)u_0,\ 0,\ C^1_4(z)u_1],$$
$$e^{ik_y}[C^1_1(z)u_0,\ C^1_2(z)u_2,\ C^1_3(z)u_1,\ C^1_4(z)u_3],$$
$$e^{ik_y}[0,\ 0,\ 0,\ C^2_4(z)u_0], \tag{13}$$
$$e^{ik_y}[0,\ 0,\ 0,\ C^2_4(z)u_1],\quad e^{ik_y}[C^2_1(z)u_0,\ 0,\ 0,\ C^2_4(z)u_3],$$

where the upper index $\nu=1,2$ labels the first and second subbands of size quantization for heavy holes, and the light-hole components for the second subband have been removed. It is easy to see that each term in Eq. (13) has the form of Eq. (12) with particular values of ν and N. It should be noted that the neighboring (in energy) hole levels do not, in general, correspond to a monotonic sequence of indices $N=-1,0,1,\ldots$ in Eq. (12). This is the fundamental difference between hole and electron Landau level (the latter are labeled by an increasing index). In the presence of a periodic potential $V(x,y)$ for which condition (9) is satisfied, each Landau level is split into p subbands. To define the limits for indices (ν,N) in Eq. (11), one should fix the $|J;m_J\rangle$ projection $j=1,2,3,4$ and then take the sum of jth components of all vectors in Eq. (13) with coefficients $d_{j\nu Nn}(k_x,k_y)$. The total number of nonzero components in the set of basis vectors (13) in our example was equal to 11, which is smaller than the total number of available components $4\times6=24$ because the components of Eq. (12) which had negative indices were set to zero. Hence, after substituting the total hole wave function [Eqs. (10) and (11)] into the Schrödinger equation with Hamiltonian (8) one obtains the $11p\times11p$ eigenvalue problem for the $11p$ coefficients $d_{j\nu Nn}(k_x,k_y)$ in every $11p$ hole magnetic subband.

We shall conclude this section by discussing the following property of the hole wave functions in the (xy) plane of the superlattice. In Sec. I we mentioned that the wave function of a Bloch electron has at least p zeros per magnetic cell if the magnetic flux is equal to p/q of flux quanta, which is a consequence of Peierls condition (2). It is interesting to generalize this result for a multicomponent wave function. That is, if $\theta^j_\mathbf{k}$ denotes the phase of the jth periodic part $u^j_\mathbf{k}(\mathbf{r})=\exp(-i\mathbf{kr})\psi^j_\mathbf{k}(\mathbf{r})$ of the hole component $\psi^j_\mathbf{k}(\mathbf{r})$ defined by Eq. (11), one can introduce the vorticity Γ_j for each component as follows:

$$\Gamma_j=\frac{1}{2\pi}\oint d\mathbf{l}\frac{\partial\theta^j_\mathbf{k}(x,y)}{\partial\mathbf{l}}, \tag{14}$$

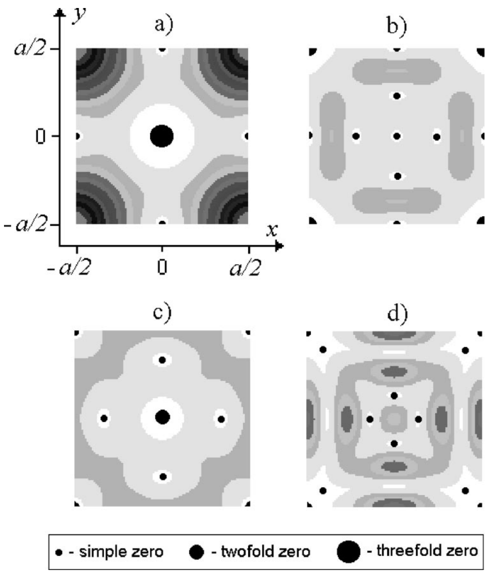

FIG. 1. Probability distributions $|\psi^j|^2$ ($j=1,2,3,4$) for the $|J;m_J\rangle$ hole envelope functions are shown in one of p nonoverlapped magnetic subbands split from the Landau level $N=2+$ at $p/q=5$ and $k_x=k_y=0$ (a)-(d). Darker areas correspond to the greater values of the probability. The positions of wave-function zeros are marked by black circles with diameters proportional to their degeneracy. The zeros which are located on the sides and in the corners of the magnetic cell are marked by the semi-circle and quarter-circle areas, respectively.

where the integration contour is taken along the boundary of the magnetic unit cell in the counterclockwise direction. It was mentioned above that condition (2) is valid for every component of the vector of hole envelope functions. So, it is not surprising that vorticity (14) has to be equal for all hole components:

$$\Gamma_j = -p, \quad j=1,2,3,4. \tag{15}$$

However, we found that the position inside the magnetic cell, the degeneracy of each zero, and the total number of zeros can be different for each of the $|J;m_J\rangle$ components. This feature can be explained by examining the particular set of basic functions in Eq. (11) for different $|J;m_J\rangle$ components. It should also be noted that the total number of zeros per magnetic cell can be greater than p due of the opposite signs of some vorticities, or that it can be smaller than p due to the appearance of multifold zeros. All of these results are shown in Fig. 1, where the probability distributions in the (xy) plane of all four hole envelope functions are plotted at $k_x=k_y=0$ in one of p nonoverlapped magnetic subbands arising from the hole Landau level $N=2+$ at the magnetic flux $p/q=5$. The zeros are shown as black circles of different sizes corresponding to their degeneracy (see the inset). One can see that different hole components have different numbers and (in general) a different degeneracy of zeros. Some of the zeros are located on the sides and in the corners of a magnetic cell, which is reflected in Fig. 1 by the semi-circle and quarter-circle areas. Again we see that despite the different positions and the number of zeros for each of the $|J;m_J\rangle$ components, the total vorticity per one magnetic cell [Eq. (15)] is equal for all components at any (k_x,k_y) in all magnetic subbands. This result reflects the topological nature of the wave-function vorticity [Eq. (14)].

III. QUANTIZATION OF HALL CONDUCTANCE

The Hall conductance σ_H of 2D electrons subjected to a periodic potential is quantized in units of e^2/h as soon as the Fermi energy E_F lays in the energy gap. A nonmonotonic dependence of σ_H on the position of E_F in such systems has been obtained by many authors,[3,5,21,22,27] and it can be qualitatively explained for the quasi-1D case as done in the TKNN paper.[3] Let the electric field be applied in the y direction. This field gives a steady change of k_y and, according to Eq. (11), a steady motion of x_0 which determines the position of the harmonic oscillator function $u_N(x-x_0)$ in the x direction. So, the Hall current appears in the x direction. However, the position of $u_N(x)$ depends not only on x_0 but, as it can be seen from Eq. (11), also on n. If at some point k_y the dominating component d_n of an eigenvector in a particular magnetic subband changes, say, from d_n to d_{n+s} where s is an integer, the corresponding oscillator function in Eq. (11) changes nonmonotonically its position in the x direction by sqa/p. In a quasi-1D model studied by TKNN,[3] such a change is provided by the proximity of 1D energy dispersion curves. As a result, various values of the Hall conductance in each of the magnetic subbands are possible, and the dependence of σ_H on the position of E_F has a nonmonotic character. The values of s for the simple parabolic spectrum can be determined from Diophantine equation (4), but it is not clear how to generalize this equation on the case of mixed hole Landau levels considered in our paper.

To calculate the Hall conductance of hole magnetic subbands, we shall generalize the topological approach developed by Kohmoto[8] and Usov[9] to the case of the complicated hole subband spectrum both at weak and at strong modulation amplitudes. Since the value of σ_H is determined by the sum of partial conductances of filled magnetic subbands, we shall first study the Hall conductance of one fully occupied magnetic subband α. In the absence of disorder and at zero temperature, its contribution to the Hall conductance is given by[3,8,9,28]

$$\sigma_H^\alpha = \frac{e^2}{\pi^2\hbar}\int \mathrm{Im}\left\langle \frac{\partial u_{\mathbf{k}}}{\partial k_y}\middle|\frac{\partial u_{\mathbf{k}}}{\partial k_x}\right\rangle d^2k, \tag{16}$$

where $u_{\mathbf{k}}=\Psi_{k_xk_y}(\mathbf{r})e^{-i\mathbf{kr}}$ is the periodic part of the Bloch function in the αth subband. The generalization of Eq. (16) for the four-component hole state [Eq. (10)] can easily be obtained by substituting wave-function (10) into Eq. (16), which gives us the following expression for σ_H:

$$\sigma_H^\alpha = \frac{e^2}{\pi^2\hbar}\sum_{j=1}^{4}\int \mathrm{Im}\left\langle \frac{\partial u_{\mathbf{k}}^j}{\partial k_y}\middle|\frac{\partial u_{\mathbf{k}}^j}{\partial k_x}\right\rangle d^2k, \tag{17}$$

where $u_{\mathbf{k}}^j=\Psi_{k_xk_y}^{(j)}(\mathbf{r})e^{-i\mathbf{kr}}$ and $\Psi_{k_xk_y}^{(j)}(\mathbf{r})$ is defined by Eq. (11). The double summation over the $|J;m_J\rangle$ basis has been reduced in Eq. (17) to an ordinary sum due to the orthogonality of the $|J;m_J\rangle$ basis functions. In this section we shall focus on the magnetic subbands originating from the lowest subband of size quantization. Thus, we can ignore the second heavy-hole subband by omitting the last three vectors in Eq. (13) containing $C_m^2(z)$, which will reduce the number of nonzero components in our case from 11 to 7. After substituting $u_{\mathbf{k}}^j$ into Eq. (16) and taking into account the orthogonality and normalization of the basis functions in Eq. (11), one may express Hall conductance (17) through the partial derivatives of the components $d_{j\nu Nn}(k_x,k_y)$ describing the quantum state. For brevity, in the following we shall replace the set of indices $(j\nu Nn)$ by a single index $n=1,\ldots,7p$ which runs sequentially all the required values. We thus get

$$\sigma_H^\alpha=\frac{e^2}{\pi^2\hbar}\int \mathrm{Im}\left[\frac{i}{2}\ell_H^2+\sum_{n=1}^{7p}\frac{\partial d_n^*}{\partial k_y}\frac{\partial d_n}{\partial k_x}\right]d^2k. \qquad (18)$$

It should be noted that the expression of the same kind as Eq. (18) has been initially derived for the Hall conductance of a magnetic subband split from a single electron Landau level.[9] Our calculations show that Eq. (18) is also valid for the case of several interacting electron or hole levels as long as the spectrum is nondegenerate. The nondegeneracy means that at any (k_x,k_y) point in MBZ (1) and for any subbands α and β the relation $\varepsilon_\alpha(k_x,k_y)\neq\varepsilon_\beta(k_x,k_y)$ holds for the energy dispersions. The difference in Eq. (18) from the single-level problem is only in the size of the matrix equation for the coefficients d_n which is now equal to $N\cdot p$ instead of p, but the orthogonality and normalization of the basis functions in Eq. (11) is of the same kind as for a single-level problem. This feature allows us to expand directly the approach proposed by Usov for the case of several interacting levels. So, we use expression (18) for calculations of the Hall conductance for magnetic subbands originating from the hole Landau levels which are coupled by the off-diagonal elements of the Luttinger Hamiltonian even in the absence of the superlattice potential.

It is evident from Eq. (18) that for a calculation of the Hall conductance one should study first the analytical properties of coefficients d_n as the functions of quasimomentum. First, one can transform the 2D integral [Eq. (18)] into a 1D contour integral. In order to simplify the integration and to reduce it to the summation of the winding numbers over the singularities (see the right side of (5)), one has to introduce the extended magnetic Brillouin zone (EMBZ) which is derived from the previously determined magnetic Brillouin zone [Eq. (1)] by extending it p/q times in the k_y direction:

$$-\pi/qa\leq k_x\leq\pi/qa, \quad -p\pi/qa\leq k_y\leq p\pi/qa. \qquad (19)$$

Similar to the case of the electron spectrum described by Usov,[9] it can be shown that the integration along the ''boundaries'' of EMBZ (19) gives no impact to the value of σ_H which is explicitly determined only by the contour integrals around the singularities (defining the winding numbers). We shall briefly repeat the outline of the derivation of this result (also see Ref. 28). One can choose a representation for which one of the components of the vector $\mathbf{d}=(d_1,\ldots d_{Np})$, say, d_1, is real. The points $\mathbf{k}_m$, $m=1,2\ldots$ where $d_1(\mathbf{k}_m)=0$

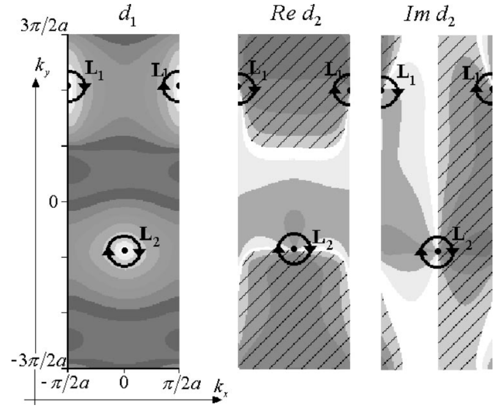

FIG. 2. Typical behaviors of the components d_1 and d_2 of the eigenvector $\mathbf{d}$ describing the hole quantum state for the magnetic flux $p/q=3/2$ [see Eq. (11)] in the representation with real d_1 (the distribution for the third component is not shown). The correspondent Hall conductivity here is equal to -1 in units e^2/h for the following parameters: the superlattice period $a=80$ nm and the amplitude of periodic potential $V_0=0.7$ meV, which provides the nonoverlapped magnetic subbands at $p/q=3/2$. Darker areas correspond to the greater values of the $d_{1,2}$ modulus, and the negative parts are shaded with lines. The contours $L_{1,2}$ around the singularities are shown together with the integration directions.

appear to be singular points for the other components d_j, $j=2,3,\ldots$, which means that d_j, $j=2,3,\ldots$ does not have a definite limit at $\mathbf{k}=\mathbf{k}_m$.[9] That is, if we write d_j as $d_j=|d_j|\exp(i\theta_j)$, only $|d_j(\mathbf{k}_m)|$ is defined while the limit of $\theta_j(\mathbf{k})$ depends on the direction approaching the point $\mathbf{k}_m$. As a result, for θ_j one obtains a phase mismatch after a circulation around $\mathbf{k}_m$ which is called (in units of 2π) a winding number S_m^j. Detailed calculations show that the winding numbers are equal for all d_j, $j=2,3,\ldots$, i.e., $S_m^j=S_m$, $j=2,3,\ldots,Np$. To be specific, let us further consider the calculation of the winding numbers for d_2. For the singular point $\mathbf{k}_m$ its winding number S_m can be calculated as an algebraic sum of rotations (modulo 2π) of the vector with the components $(\mathrm{Re}\,d_2,\mathrm{Im}\,d_2)$. Typical behaviors of d_1 and of both $\mathrm{Re}\,d_2$ and $\mathrm{Im}\,d_2$ in a magnetic subband are shown in Fig. 2, where the contours $L_{1,2}$ show the direction of integration around the singularities. The parameters here are the following: the magnetic flux $p/q=3/2$, the superlattice period $a=80$ nm, and the amplitude of periodic potential $V_0=0.7$ meV which corresponds to the case of non-overlapped magnetic subbands. It should be noted that surface superlattices with a modulation depth of the order 0.6–0.7 meV have already been used at the temperature $T=50$ mK in the experiments with the electron subband spectrum.[27] It is evident from Fig. 2 that while approaching the singular point where $d_1=0$ which is marked by the black dot, both real and imaginary parts of d_2 have different limits depending on the direction in the (k_x,k_y) plane, and thus do not have a true limit in this point. The impact of the component d_j at a singular point $\mathbf{k}_m$ to the Hall conductance is proportional to $|d_j|^2S_m^j=|d_j|^2S_m$ where S_m is the winding number for $\mathbf{k}=\mathbf{k}_m$. As a result, the summation over all components $j=1,2,\ldots,Np$ gives the impact to the Hall conductance provided by a singular point $\mathbf{k}_m$:

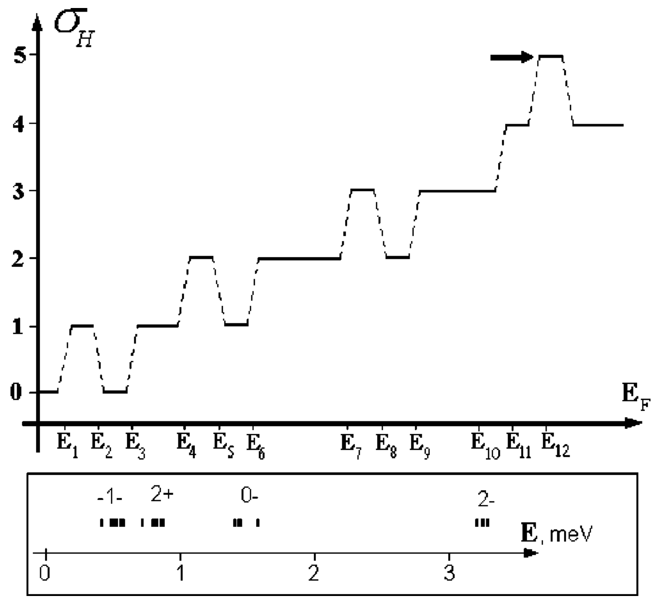

FIG. 3. Quantized values of σ_H (solid lines) as a function of the Fermi level position in energy gaps. The energies E_n schematically show the centers of the magnetic subbands, and the dashed lines serve as a guide to the eye. The arrow indicates the deviation from the quantization sequence for the Hofstadter ''butterfly'' for the electrons which appear when E_F lies in the gap between E_{11} and E_{12}. (Bottom inset) Hole energy spectrum consisting of the non-overlapped magnetic subbands originating from the four uppermost hole Landau levels with indices $N=-1,0$, and 2, and dominating spin projections $\pm$. The magnetic flux $p/q=3/2$, the superlattice period $a=80$ nm, and the amplitude of periodic potential $V_0=0.7$ meV.

$$\sum_j |d_j|^2 S_m^j = S_m \sum_j |d_j|^2 = S_m, \tag{20}$$

where we used the normalization of the vector, $\mathbf{d}=(d_1,\ldots d_{Np})$. As soon as the winding numbers are calculated, the Hall conductance of a particular magnetic subband is given by Eq. (5). By examining expression (5), one can mention that the first term in the square brackets is just the contribution of one of the *p* subbands to the Hall conductivity of a single Landau level while the remaining term in Eq. (5) appears only in the presence of a periodic potential. As a result, the expression in brackets in Eq. (5) is always an integer. It was shown by Kohmoto[8] that this integer defines the topological number, namely, the first Chern class of a vector bundle associated with the current magnetic subband.

The quantized values of σ_H as a function of the number of filled magnetic subbands (or, equivalently, of the position of the Fermi level in the energy gaps) are shown in Figs. 3 and 4 both for non-overlapped and overlapped magnetic subbands. When the amplitude V_0 of the periodic potential (7) is smaller than the distance ΔE_{12} between neighboring Landau levels, none of the subbands are overlapped (see the bottom inset in Fig. 3) and the deviations in the values of σ_H from the sequences obtained for the Hofstadter ''butterfly'' for electrons[3,27] are caused by the specific character of hole Landau states compared to the electrons. As we have seen in Sec. II, four-component hole states have structures different with those of electrons with a simple parabolic dispersion (see Ref. 34 for details). Another difference between the Hall plateaus for the electron and hole subband spectra which can be important in the experiments is the unequal plateau width for hole subbands, which reflects the complicated structure of the hole subband spectrum (see the bottom of Fig. 3). The latter is determined by the Luttinger Hamiltonian (8) where the spin-orbit interaction and the Zeeman splitting in a magnetic field are included together with the heterojunction potential, which is asymmetric in our case. It should also be noted that when the Fermi level is swept through a subband centered at E_n, the Hall conductivity interpolates smoothly between the adjacent quantized values. The interpolation is shown by the dashed lines in Figs. 3 and 4, while the quantized values are marked by the solid lines.

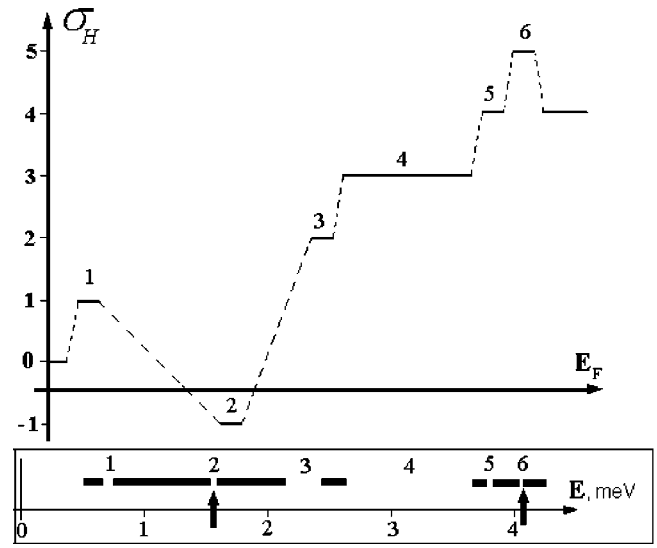

FIG. 4. Quantized values of σ_H (solid lines) in the energy gaps between overlapped hole magnetic subbands are shown as a function of the Fermi level position. (Bottom inset) Overlapped hole magnetic subbands originating from the same hole Landau levels as in Fig. 3 but split by a stronger periodic potential with the amplitude $V_0=3$ meV. The numbers label energy gaps.

If the amplitude V_0 is increased, it was found that the neighboring magnetic subbands arising from the different hole Landau levels can touch each other at some point in the MBZ. This touch means that a degeneracy of the spectrum has occurred, and the application of expression (5) is invalid. However, one can use this approach at higher values of V_0 when some of magnetic subbands overlap but the spectrum appears to be nondegenerate in the entire MBZ (1). An example of such a spectrum for $V_0=3$ meV is shown in the bottom inset of Fig. 4. One can see that the number and maximum width of gaps in Fig. 4 have decreased with respect to the system of nonoverlapped subbands in Fig. 3 which will reduce the number and the maximum width of Hall plateaus. For convenience, in Fig. 4 we label the remaining gaps and the corresponding Hall plateaus by numbers. The calculation of σ_H in every gap in Fig. 4 has been performed by summarizing the impacts [Eqs. (18)] of all subbands below this gap even if they are overlapped. It should be stressed that such an approach is possible as long as the spectrum is nondegenerate, which is true in our case. Again the dashed line in Fig. 4 serves as a guide to the eye. We note also that in a real experiment a nonmonotonic behavior of σ_H can be seen between the gaps 1 and 2, or 2 and 3 in Fig. 4, which will reflect the existence of several overlapped subbands in these intervals. Also note that the quantized sequence for σ_H shown in Fig. 4 differs both from the case of a nonoverlapped subband spectrum (see Fig. 3) and from the case of coupled electron Landau levels.[5] By examining the structure of Figs. 3 and 4 we find that in our case the differences between the quantization of σ_H shown in Figs. 3 and 4 are provided by only two changes in σ_H^α for subbands $\alpha=4$ and 8 (see Fig. 3). Detailed calculations have shown that these two subbands are degenerated at some intermediate values of V_0 which are greater than in Fig. 3 but lower than in Fig. 4. According to the topological point of view,[8,10] the subband touches have caused an exchange of the Chern classes $\Delta c = \pm q$ between these subbands where $q=2$ in our examples. It can be easily seen that such an exchange (-2 for subband 4 and $+2$ for subband 8) exactly transforms the quantization shown in Fig. 3 to the dependence in Fig. 4. One should also mention that for the mixed hole subbands shown in Fig. 4 a negative value of σ_H exists which corresponds to the Hall current directed opposite to the classical drift velocity. Such a behavior of the Hall conductance in a complicated subband energy spectrum was also obtained in several earlier papers[3,20,21] for simple electron spectra. We hope that the qualitatively novel effects in the quantization of the Hall conductance in a high-quality laterally modulated *p*-type heterojunction with a nonparabolic spectrum, which have been discussed in this paper, can be detected in transport experiments.

IV. SUMMARY

We calculated the Hall conductance in a 2D hole gas affected by a lateral periodic potential of a surface quantum dot (antidot) superlattice. Our method is a generalization of the approach derived by Kohmoto[8] and Usov[9] on the case of a system where the charged particle is described by a four-component eigenfunction of the Luttinger Hamiltonian. This generalization allowed us to study the QHE in a laterally modulated system where the effects of spin-orbit interaction, spin splitting in a magnetic field, and an asymmetric heterojunction potential are present. We found a specific behavior of the Hall conductance as a function of the Fermi energy for hole subbands compared to the well-known dependence for the Hofstadter "butterfly." The quantization law for the Hall conductance was investigated both at weak and strong modulation depths. In the latter case the magnetic subband mixing was taken into account, which leads to the exchange of the Chern numbers between magnetic subbands, changing their impact to the Hall conductance.

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*Electronic address: demi@phys.unn.ru

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