Localized modes in defect-free dodecagonal quasiperiodic photonic crystals

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(Received 24 June 2003; published 27 October 2003)

The transmission property and localization of electromagnetic waves in defect free dodecagonal quasiperiodic photonic crystals (QPCs) are studied. The simulation of total energy flow and the density of states show that the dodecagonal QPC has a photonic gap for *TM* polarized electromagnetic waves. However, different from the periodic photonic crystals, electromagnetic waves with certain frequencies are localized at some special regions inside a perfect dodecagonal QPC. A corresponding experiment in the microwave region confirms the phenomenon. It is believed that the occurrence of localized modes in QPCs can be attributed to the competition between two spatial structural properties: self-similarity and nonperiodicity.

DOI: 10.1103/PhysRevB.68.165106 PACS number(s): 42.70.Qs, 41.20.Jb

Similar to the periodic potentials which have a great impact on the movement of electrons in a semiconductor, periodically arranged dielectric materials have the effect of modulating the propagation of electromagnetic waves. This kind of artificial material was named by Yablonovitch and John as a photonic crystal in 1987.[1,2] Since then, a great deal of effort has been devoted to studying the properties of the material.[3–12] The primary property of a photonic crystal is that it has photonic gaps. Light with frequencies located in the gaps cannot propagate in this material no matter what the incident direction is. The applications[13–17] of photonic crystals are mainly based on this property.

Various methods have been developed to simulate and measure the photonic band gaps and propagation of electromagnetic waves in photonic crystals.[3,4,9,18–20] In most cases the photonic gap is determined by corresponding transmission and reflection spectra or the total energy flows around the crystal. Low transmission regions in the transmission or total energy flow spectra are generally considered as gaps. As we know, the low-transmission regions of perfect periodic photonic crystals are smooth, and, when defects or disorders are introduced into them, some peaks will appear within their gaps. The relation between the defect or disorder and the defect modes[2,20–26] in periodic photonic crystals has been quite clear. Several years ago, quasiperiodic photonic crystals (QPCs) were proposed. The QPC is different in configuration from the periodic one; in particular it does not possess translational symmetry. Since then the photonic characteristics of QPCs have been studied,[27–34] and all the results show that QPCs have photonic band gaps, which are independent of the incident direction. Essentially, a QPC, however, is a kind of disordered structure. It should present certain distinctive properties for the propagation of electromagnetic wave. Therefore, we systemically examine the transmission spectra of different QPCs formed with dielectric cylinders and an air background, namely, octagonal, decagonal, and dodecagonal quasiperiodic structures, both experimentally and theoretically. The results show that all studied structures mentioned above possess spectral gaps for *TM* polarized modes and, furthermore, within the first gap of the defect free dodecagonal QPC, there are some peaks. This indicates that defect free dodecagonal quasiperiodic photonic crystals can support some localized modes.

Two approaches are used to simulate the photonic property of the dodecagonal QPC, namely, the total energy flows and the density of state. In our calculation of the total energy flows, a line-shaped source is placed within the sample. According to the multiple scattering methods,[18] the Poynting vector can be written as

$$\vec{S}(\vec{\rho}) = -\frac{c}{8\pi k_0}\mathrm{Im}[u(\vec{\rho})\nabla u^*(\vec{\rho})], \tag{1}$$

where k_0 and c is the wave vector and velocity of electromagnetic wave in vacuum, respectively. u is the electric field at position $\vec{\rho}$. Integrating the Poynting vector along a close path around the QPC, we obtain the total energy flows P;

$$P = \oint_L \vec{S}(\vec{\rho})\cdot\vec{n}\,dr. \tag{2}$$

Meanwhile, the density of states[19] (DOS) in the sample can be written as

$$\rho(\omega) = \frac{1}{s}\int\int \varepsilon(r)\rho(r,\omega)dx\,dy, \tag{3}$$

where $\rho(r,\omega) = -2\omega/\pi c^2 \mathrm{Im}[G(r,r_s,\omega)]$ is local density of states in the sample. $G(r,r_s,\omega)$ is the electromagnetic Green's function with a source at location r_s and observation point at r. ω is the angular frequency of the electromagnetic wave radiated by the line source. s is the total area of three basic cells located near the center of the sample.

Figure 1 shows the structure of a dodecagonal QPC. It is constructed by placing dielectric cylinders with circular cross sections in the sites of a two-dimensional dodecagonal Penrose lattice. The dodecagonal Penrose pattern is tiled by three basic cells which are a square and two kinds of rhombi whose acute angles are 30° and 60°, respectively. The cylinders in the studied QPC have the dielectric constant $\varepsilon = 8.5$ and the dielectric constant of their background is $\varepsilon_b = 1.0$. The side lengths of the square and the rhombi are both 11 mm and the volume fraction of dielectric cylinders is 25.8%. Such a structure has spectral gaps for *TM* polarized modes. It should be noted that there is another kind of dodecagonal

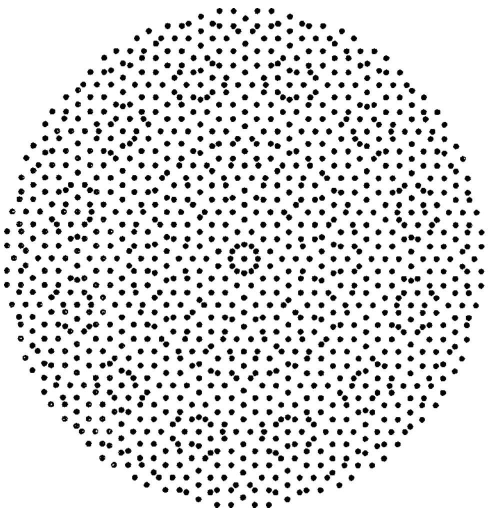

FIG. 1. Two-dimensional dodecagonal quasiperiodic photonic crystal.

lattice which is built up on the random square-triangle tiling system and studied by Zoorob *et al.*[27] and Zhang *et al.*[28]

The total energy flow and the density of states for a defect free dodecagonal QPC with total 145 cylinders are presented in Fig. 2. Both the density of states and the total energy flow spectrum exhibit that the first gap of the structure is located from 8.06 to 11.38 GHz. However, there is a quaint phenomenon. Within the gap, both spectra have three sharp peaks at the frequencies of 9.7, 10.53, and 10.87 GHz. It is quite different from the results of perfect periodic photonic crystals and defect free octagonal and decagonal QPCs where the spectra are smooth inside the gaps.

To look for where the electromagnetic waves are localized in the sample, the distributions of electric field at frequencies mentioned above are simulated. The results reveal that localizations of electromagnetic waves have indeed taken place and, as shown in Fig. 3, in the central region of the QPC, the strength of electric field is much greater than any other places. Then it decays in a length scale of one or two basic cells. The localized electromagnetic modes are quite different. For example, at 10.53 GHz the mode possesses a twelvefold symmetry, while at 10.87 GHz it is a dipole radiation pattern. It is believed that the spatial structure is a dominant factor to determine which mode can be localized.

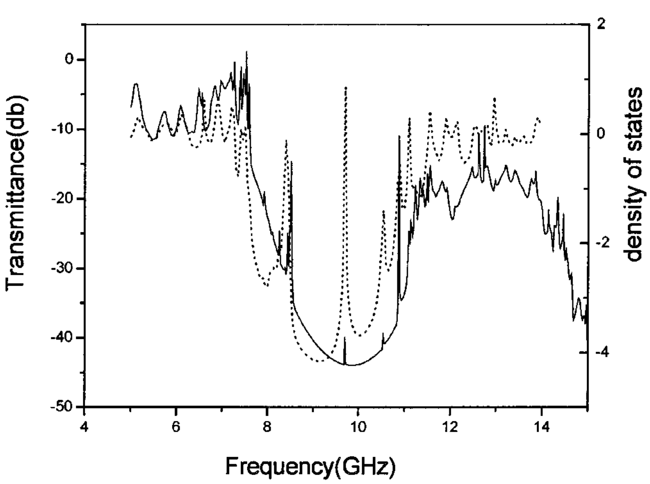

FIG. 2. Total energy flow (solid line) and the density of states (dashed line) of a dodecagonal QPC constructed with 145 cylinders.

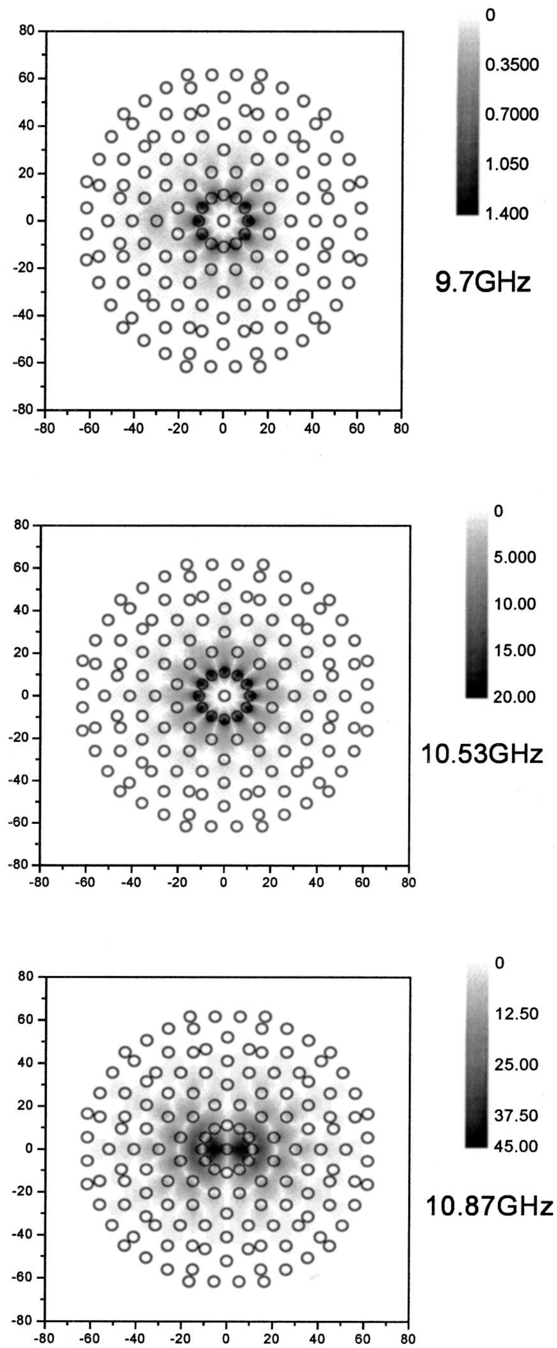

FIG. 3. The distribution of electric field at three frequencies which correspond to the peaks within the low-transmission region of Fig. 2.

In order to confirm the existence of localized modes in the defect free dodecagonal QPC, the corresponding transmittance is measured. We build a dodecagonal QPC where the side length of the square and the rhombi is 12.1 mm. 229 alumina ceramic cylinders are inserted in a polystyrene foam template. The radius of the cylinders is 3.0 mm and the di-

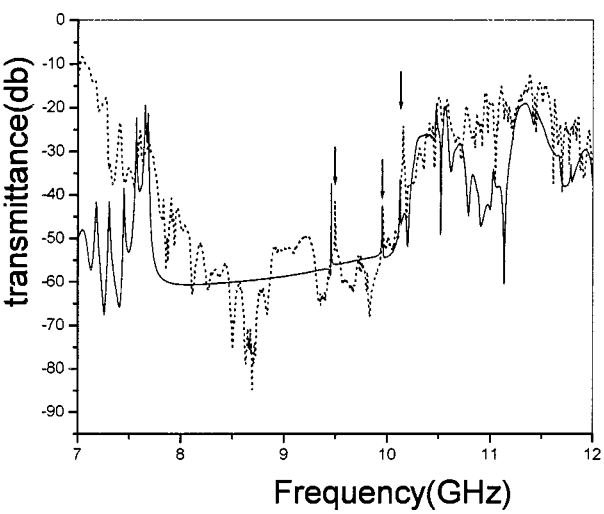

FIG. 4. The measured (dotted line) and calculated (solid line) transmittance of the defect-free dodecagonal quasiperiodic photonic crystal.

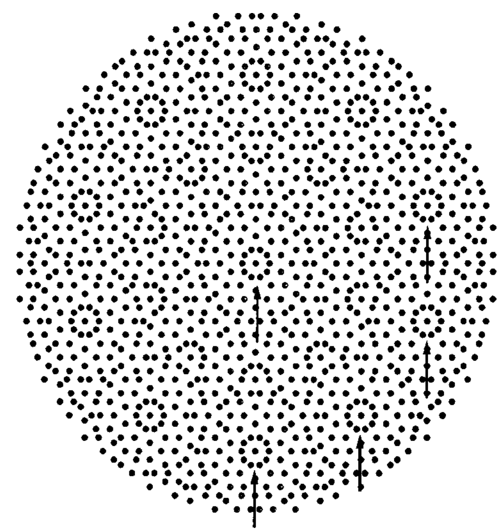

FIG. 5. Two-dimensional decagonal quasiperiodic photonic crystal. The arrows indicate the identical regions in decagonal QPC.

electric constant of the cylinders and the template are 8.9 and 1.04, respectively. The setup of the measurement is similar to that described in Ref. 29. The results are presented in Fig. 4. It is clear seen that within the low-transmission region of the measured spectrum, three peaks appear at the frequencies of 9.5, 9.95, and 10.15 GHz, as pointed out by the arrows, and the measured result agrees very well with the calculated one where the parameters are taken to be the same as the experiment. Therefore, the experiment proves that electromagnetic waves with such three frequencies are localized in the sample.

Why are there localized modes in a defect free dodecagonal QPC? A quite direct cause may be the higher dielectric density in the central dodecagon. Because the average refraction index in the surrounding area is relatively low, as shown in Fig. 1, the central zone may behave as a defect. However, if this is a dominant factor for the existence of localized modes, octagonal and decagonal quasicrystals should have similar localized modes in the central zone. We calculate the variation of the density of states as a function of dielectric fraction for the decagonal quasicrystals and find that even if the dielectric fraction reaches the maximum value 40%, i.e., the cylinders at the vertices of the center decagon contact each other, and there is not any localized mode to be observed in the decagonal quasicrystal. In addition, for the dodecagonal quasicrystal, the existence of the localized modes does not depend on the dielectric fraction. When the dielectric fraction changes, only the frequencies of the localized modes and the values of the corresponding DOS are altered. When the dielectric fraction changes from its maximum, 27%, to a very small value, 4.74%, the frequencies of the three visible peaks are changed from 9.53, 10.4, and 10.55 GHz to 14.46, 15.05, and 15.33 GHz, respectively. These facts mean that higher dielectric density in the central zone may be one of the reasons for the existence of localized modes but not the only one. Therefore, a further consideration on the specific structural property of the dodecagonal quasicrystals is necessary.

In the case of electronic quasicrystals, two major structural characteristics have been mentioned, namely, nonperiodicity and self-similarity.[35–41] The self-similarity caused by the long-range order makes the wave function of electrons extended, while the nonperiodicity may lead the wave function to be localized. Therefore, the existence of localized modes depends on the competition of these two effects. It is well known that the propagation of photons in photonic crystals is analogous to that of electrons in a typical crystal. We try to understand qualitatively the localized modes in dodecagonal photonic crystal in terms of the ideas in electronic crystal.

Similar to the usual electronic quasicrystals, two dimensional QPCs constructed on the Penrose lattice also possess nonperiodicity and self-similarity. This can be clear seen in the decagonal QPC, as shown in Fig. 5. The nonperiodicity property may have a tendency to localize the electromagnetic waves in a higher dielectric area, for example, the central dodecagon in a dodecagonal QPC and the decagons in a decagonal QPC. According to the theorem for Penrose lattices, if the distance between a selected configuration and the origin is D, we can always find the identical configuration within a distance of $2D$. This can be clear seen in decagonal QPCs. As shown by the arrows in Fig. 5, apart from the central decagon, there are many identical decagons in the sample. Therefore, when a localized mode exists in a selected configuration, an identical mode should be found within a distance of $2D$. If the nonperiodicity of the QPC is weaker, the localization lengths of the localized modes are so long that the fields of adjacent localized modes overlap with each other. Then, the neighboring localized modes will exchange their energy. It looks like that the electromagnetic waves propagate freely in the QPC, i.e., the electromagnetic wave cannot be localized. Otherwise, the electromagnetic waves with certain frequencies will be localized. Therefore, we can understand why the localized modes emerge in a defect free dodecagonal QPC and do not appear in QPCs with lower order rotational symmetry. In a decagonal QPC, as shown in Fig. 5, the dielectric density in the center zone is lower compared with that of dodecagonal one. If localized modes exist in this zone, the localization length of them will be longer. On the other hand, owing to self-similarity, other localized modes appear within a shorter distance. All the localized modes may couple together. However, in defect free dodecagonal QPC, the disorder resulted from nonperiodicity is intense. The localization length of the localized modes will be short. In addition, the distance between self-similar dodecagons is so far that the localized modes have no chance to escape from the central region of the sample.

Based on the above analysis, the reason for the localization in QPCs is clear. It may be useful for deeply understanding the propagation property of electromagnetic waves in nonperiodic photonic materials. If the effect of disorder resulting from the nonperiodicity is dominant, the electromagnetic waves with some frequencies can be localized, as in dodecagonal QPC. Inversely, in the case of octagonal and decagonal QPCs, the disorder of the spatial structure is weak and, therefore, no localized mode can appear. In general, the occurrence of localized modes in QPCs is determined by the competition between the nonperiodicity and self-similarity.

This work was supported by the Chinese National Key Basic Research Special Fund (Grant No. 2001CB6104) and the National Natural Science Foundation of China. The support of the CSTNET for computer time is also acknowledged.

[1] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
[2] S. John, Phys. Rev. Lett. **58**, 2486 (1987).
[3] See, for example, *Photonic Band Gaps and Localization*, edited by C. M. Soukoulis (Plenum, New York, 1993).
[4] See, for example, *Photonic Band Gap Materials, NATO Advanced Study Institute Series*, edited by C. M. Soukoulis (Kluwer, Dordrecht, 1996).
[5] H. Pier, E. Kapon, and M. Moser, Nature (London) **407**, 880 (2000).
[6] T. Ochiai and K. Sakoda, Phys. Rev. B **64**, 045108 (2001).
[7] S. Olivier, M. Rattier, H. Benisty, C. Weisbuch, C. J. Smith and R. M. De La Rue, T. F. Krauss, U. Oesterle, and R. Houdre, Phys. Rev. B **63**, 113311 (2001).
[8] S. G. Johnson, S. H. Fan, P. R. Villeneuve, J. D. Joannopoulos, and L. A. Kolodziejski, Phys. Rev. B **60**, 5751 (1999).
[9] C. J. Jin, B. Y. Cheng, Z. L. Li, D. Z. Zhang, L. M. Li, and Z. Q. Zhang, Opt. Commun. **166**, 9 (1999).
[10] Y. V. Rostovtsev, A. B. Matsko, and M. O. Scully, Phys. Rev. A **57**, 4919 (1998).
[11] N. A. Nicorovici, R. C. McPhedren, and L. C. Botten, Phys. Rev. E **52**, 1135 (1995).
[12] E. Lidorikis, M. M. Sigalas, E. N. Economou, and C. M. Soukoulis, Phys. Rev. Lett. **81**, 1405 (1998).
[13] M. Bayindir, B. Temelkuran, and E. Ozbay, Phys. Rev. Lett. **84**, 2140 (2000).
[14] O. Painter, R. K. Lee, A. Scherer, A. Yariv, J. D. O'Brien, P. D. Dapkus, and I. Kim, Science **284**, 1819 (1999).
[15] J. C. Knight, J. Broeng, T. A. Birks, and P. St. J. Russell, Science **282**, 1476 (1998).
[16] R. W. Ziolkowski and M. Tanaka, Opt. Quantum Electron. **31**, 843 (1999).
[17] S. Lan, S. Nishikawa, and O. Wada, Appl. Phys. Lett. **78**, 2101 (2001).
[18] L. M. Li and Z. Q. Zhang, Phys. Rev. B **58**, 9587 (1998).
[19] A. A. Asatryan, K. Busch, R. C. McPhedran, L. C. Botten, C. Martijn de Sterke, and N. A. Nicorovici, Phys. Rev. E **63**, 046612 (2001).
[20] S. L. McCall and P. M. Platzman, Phys. Rev. Lett. **67**, 2017 (1991).
[21] M. Agio, E. Lidorikis, and C. M. Soukoulis, J. Opt. Soc. Am. B **17**, 2037 (2000).
[22] D. S. Wiersma, P. Bartolini, A. Lagendijk, and R. Righini, Nature (London) **390**, 671 (1997).
[23] M. Bayindir, B. Temelkuran, and E. Ozbay, Phys. Rev. Lett. **84**, 2140 (2000).
[24] E. Miyai and K. Sakoda, Phys. Rev. Lett. **26**, 740 (2001).
[25] M. L. Povinelli, S. G. Johnson, S. H. Fan, and J. D. Joannopoulos, Phys. Rev. B **64**, 075313 (2001).
[26] A. R. McGurn, Phys. Lett. A **251**, 322 (1999).
[27] M. E. Zoorob, M. D. B. Charlton, G. J. Parker, J. J. Baumberg, and M. C. Netti, Nature (London) **404**, 740 (2000).
[28] X. D. Zhang, Z. Q. Zhang, and C. T. Chan, Phys. Rev. B **63**, 081105 (2001).
[29] C. J. Jin, B. Y. Cheng, B. Y. Man, Z. L. Li, and D. Z. Zhang, Phys. Rev. B **61**, 10 762 (2000).
[30] Samuel S. M. Cheng, L. M. Li, C. T. Chan, and Z. Q. Zhang, Phys. Rev. B **59**, 4091 (1999).
[31] Y. S. Chan, C. T. Chan, and Z. Y. Liu, Phys. Rev. Lett. **80**, 956 (1998).
[32] C. J. Jin, B. Y. Cheng, B. Y. Man, Z. L. Li, and D. Z. Zhang, Appl. Phys. Lett. **75**, 1848 (1999).
[33] M. Bayindir, E. Cubukcu, I. Bulu, and E. Ozbay, Phys. Rev. B **63**, 161104 (2001).
[34] M. Hase, H. Miyazaki, M. Egashira, N. Shinya, K. M. Kojima, and S. Uchida, Phys. Rev. B **66**, 214205 (2002).
[35] H. Tsunetsugu, T. Fujiwara, K. Ueda, and T. Tokihiro, Phys. Rev. B **43**, 8879 (1991).
[36] H. Tsunetsugu and K. Ueda, Phys. Rev. B **43**, 8892 (1991).
[37] R. Malfliet, Phys. Lett. A **267**, 287 (2000).
[38] M. Kohmoto, B. Sutherland, and C. Tang, Phys. Rev. B **35**, 1020 (1987).
[39] T. Odagaki and D. Nguyen, Phys. Rev. B **33**, 2184 (1986).
[40] T. C. Choy, Phys. Rev. Lett. **55**, 2915 (1985).
[41] M. Kohmoto and B. Sutherland, Phys. Rev. Lett. **56**, 2740 (1986).