Magnetoexciton states in GaAs/$Ga_{0.7}Al_{0.3}As$ double quantum wells: The effects of well-width asymmetry

Carlos L. Beltrán Ríos and N. Porras-Montenegro
Departamento de Física, Universidad del Valle, A. A. 25.360, Cali, Colombia
(Received 15 May 2003; published 20 October 2003)

The binding energies for 1S-, 2$P_{\pm}$-, and 3$P_{\pm}$-like exciton states and some allowed transition energies between these states have been calculated in GaAs$Ga_{0.7}Al_{0.3}$As double quantum wells as a function of the well and barrier widths in the presence of a magnetic field applied in the growth direction. We have used the effective-mass approximation and considered the symmetric and asymmetric well widths in the double quantum well structure. We have found that the binding energy of the ground and all exciton excited states is higher in double quantum well structures, when the width of the wells are slightly different (asymmetric case), than when they are equal (symmetric case), a diference which is increased by the barrier width. Our results show that the binding energy of the exciton ground state increases with the magnetic field for all values of the well widths. We have found that the binding energy of the 2P_{+}- and 3$P_{\pm}$-like excitonic states decreases with the magnetic field, which additionally splits the binding energies of the nP ($m=\pm 1$) excited states. This behavior is similar to that observed in single quantum wells.

DOI: 10.1103/PhysRevB.68.155316 PACS number(s): 71.35.Ji, 78.66.Fd, 78.67.De

I. INTRODUCTION

In recent years the study of the double quantum well (DQW) semiconductor structures has exhibited an impressive growth. Authors have reported different theoretical and experimental studies on such systems. In particular Kamizato *et al.*[1] studied the binding energy (BE) of excitons as a function of the well and barrier widths, which determine the bidimensional character of excitons. Dignam *et al.*[2] studied the four lowest energy levels and oscillator strengths of 1s excitons in a coupled DQW in the presence of an applied static electric field. Cen and Bajaj[3] calculated the BE of excitons in symmetric DQW, under the action of a magnetic field applied in the growth direction, as a function of the interwell coupling and well size, and for different values of the magnetic field, including subband mixing effects. Kasapoglu *et al.*[4] calculated the binding energy for the ground state of the exciton in asymmetric and symmetric DQW's under the action of uniform magnetic fields applied in the growth direction, using a variational procedure for small well widths, showing basically the competition between the magnetic and the geometric confinement on the exciton BE. Mathos *et al.*[5] have studied the diamagnetic shift, of the 1S-like and 2S-like states of excitons, in symmetric DQW system using the fractional-dimensional approach. Others authors[8–13] have been focused on studying the effects of the temperature, electric field, and bosonic condensation on these systems.

While some authors[3–5] have calculated the BE of the ground and the 2s-like state of excitons in DQW with thin barrier widths, in this work we present the BE not only for the 1S-like state (the exciton ground state), but also for the 2$P_{\pm}$- and 3$P_{\pm}$-like excitonic states in a GaAs$Ga_{0.7}Al_{0.3}$As DQW as a function of the right well width, for different barrier widths, under the action of magnetic fields applied in the growth direction. Also, we present some allowed transition energies. On the basis of these results we analyze the role of the well width asymmetry on the exciton BE.

II. THEORETICAL FRAMEWORK

In Fig. 1 we show a schematic representation of a GaAs$Ga_{0.7}Al_{0.3}$As DQW. The z axis corresponds to the growth direction of the structure, the index e and h denote the electron and hole, respectively. CB and VB refer to the conduction and valence band. The magnetic field is applied in the z direction.

The Hamiltonian of the system, in dimensionless units for energy and length,[12] can be represented in separated Hamiltonians for the electron (H_e), hole (H_h), and excitonic ($H_{\rm exc}$) parts by

$$H(\rho,\phi,z_e,z_h)=H_e(z_e)+H_h(z_h)+H_{\rm exc}(\rho,\phi,z_e,z_h), \tag{1}$$

where

$$H_e(z_e)=-\frac{\mu}{m_e}\frac{\partial^2}{\partial z_e^2}+V_e(z_e), \tag{2a}$$

$$H_h(z_h)=-\frac{\mu}{m_h}\frac{\partial^2}{\partial z_h^2}+V_h(z_h), \tag{2b}$$

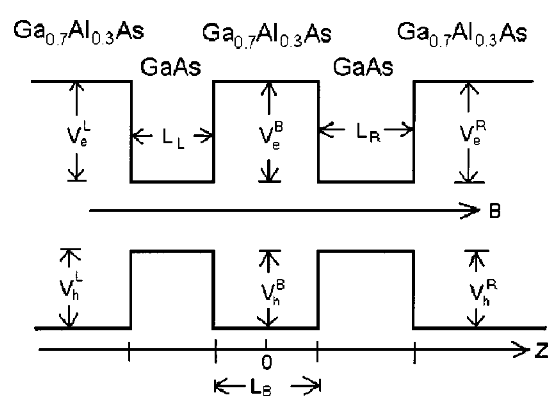

FIG. 1. Schematic representation of a GaAs$Ga_{0.7}Al_{0.3}$As DQW.

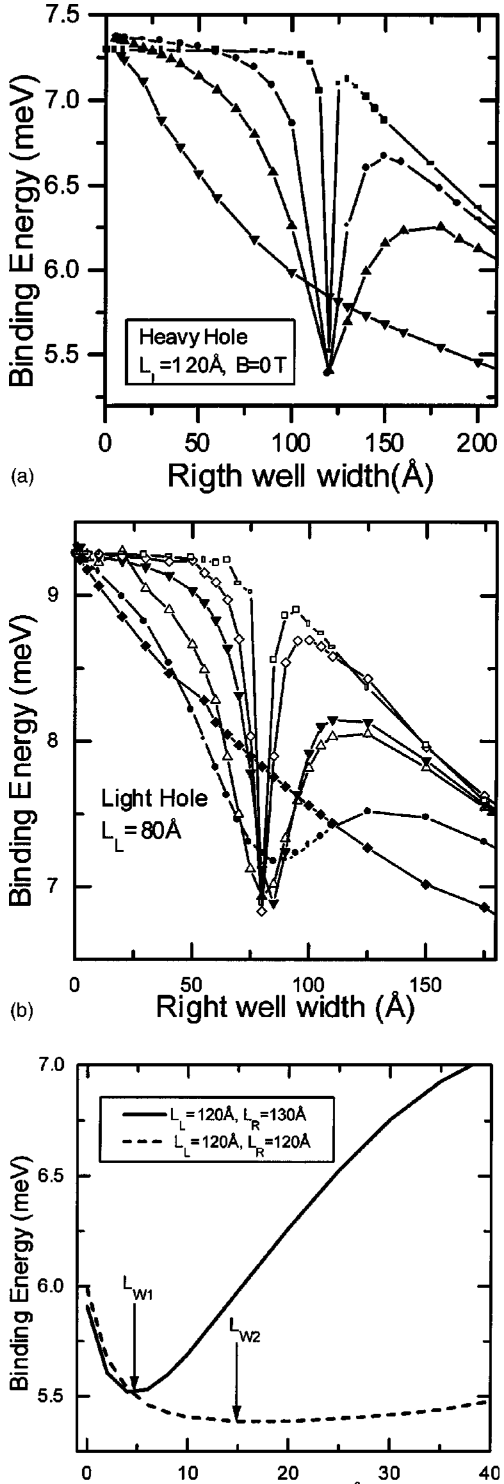

FIG. 2. (a) BE of the 1S-like heavy hole exciton state in a GaAsGa$_{0.7}$Al$_{0.3}$As DQW, with $L_L = 120$ Å, as a function of the right well width L_R, without an applied magnetic field. ▼, ▲, ●, ■ correspond to DQW's with barrier widths of 5, 10, 20, and 50 Å, respectively. (b) BE of the 1S-like light hole exciton state. ♦, ●, △, ▼, ◇, □ correspond to DQW's with barrier widths of 0, 10, 20, 30, 40, and 50 Å, respectively. (c) BE as a function of the barrier width (L_b) for both symmetric ($L_L = L_R = 120$ Å) and asymmetric ($L_L = 120$ Å, $L_R = 130$ Å) DQW's for a heavy hole exciton.

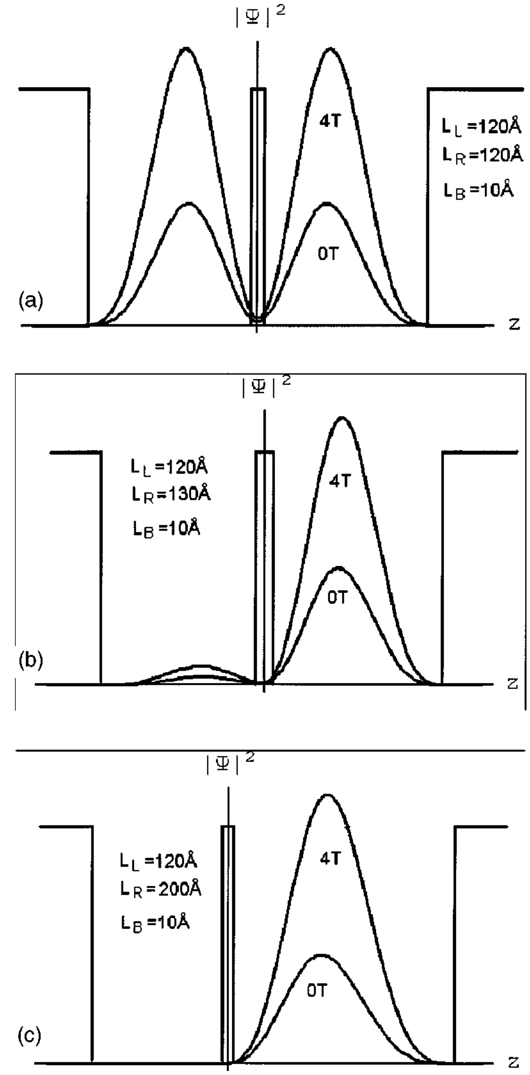

FIG. 3. Probability amplitud for the ground state of a heavy hole exciton in three different GaAsGa$_{0.7}$Al$_{0.3}$As DQW's, for two values of the applied magnetic field. (a) For a DQW with $L_R = L_L = 120$ Å, and $L_B = 10$ Å, (b) for a DQW with $L_L = 120$ Å, $L_R = 130$ Å and $L_B = 10$ Å, and (c) for a DQW with $L_L = 120$ Å, $= 200$ Å and $L_B = 10$ Å.

$$H_{\rm exc}(\rho,\phi,z_e,z_h) = -\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho}\frac{\partial^2}{\partial\phi^2}\right] + \left(\frac{m_h}{M}-\frac{m_e}{M}\right)\gamma i\frac{\partial}{\partial\phi}+\frac{1}{4}\gamma^2\rho^2-\frac{2}{r}, \tag{2c}$$

with $r = \sqrt{\rho^2 + (z_e - z_h)^2}$, $M = m_e + m_h$, m_e and m_h are the electron and hole effective mass in the z direction, μ is the effective mass in the xy plane, and $\gamma = (e/c\hbar)(a_0^2)B$, where a_0 is the effective Bohr radius. In the above equation ρ is the relative distance between the electron and the hole in the xy plane, ϕ is the azimutal coordinate, and B is the applied magnetic field.

The functional form of the electron and hole potential profiles is

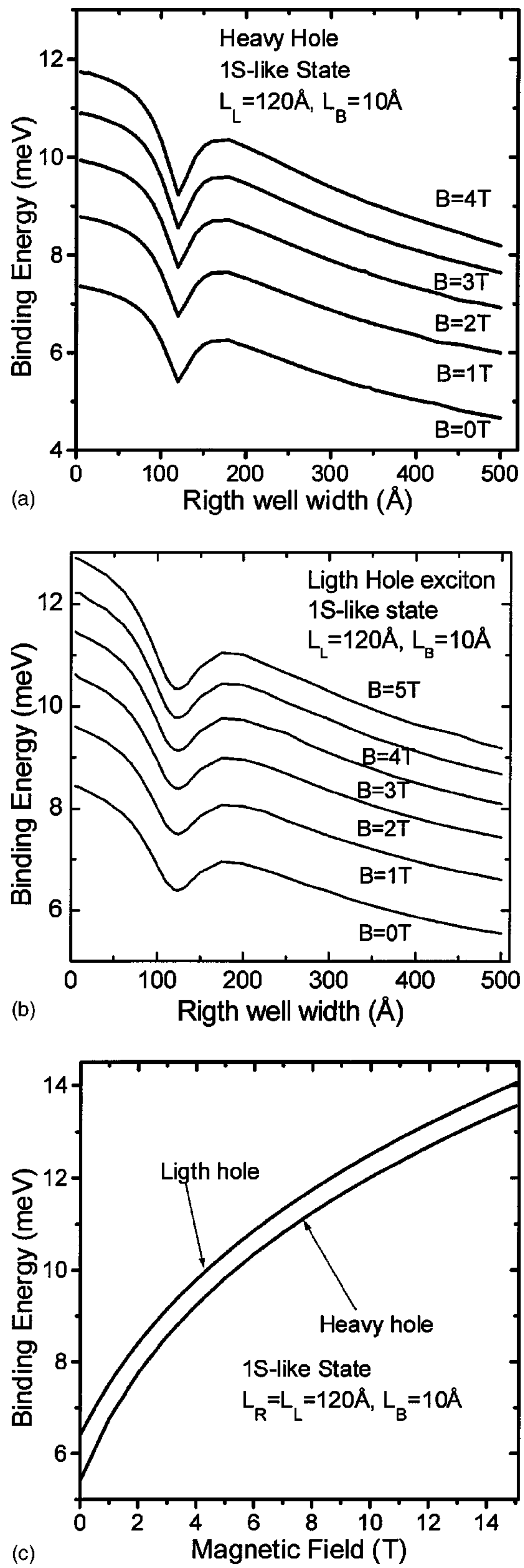

FIG. 4. BE of $1S$-like heavy (a) and light hole (b) exciton states as a function of L_R for different values of the applied magnetic field in a $GaAsGa_{0.7}Al_{0.3}As$ DQW with $L_L = 120$ Å and $L_B = 10$ Å. (c) Binding energy of the $1S$-like heavy and light hole exciton states as a function of the applied magnetic field for a symmetric DQW ($L_R = L_L = 120$ Å, $L_B = 10$ Å).

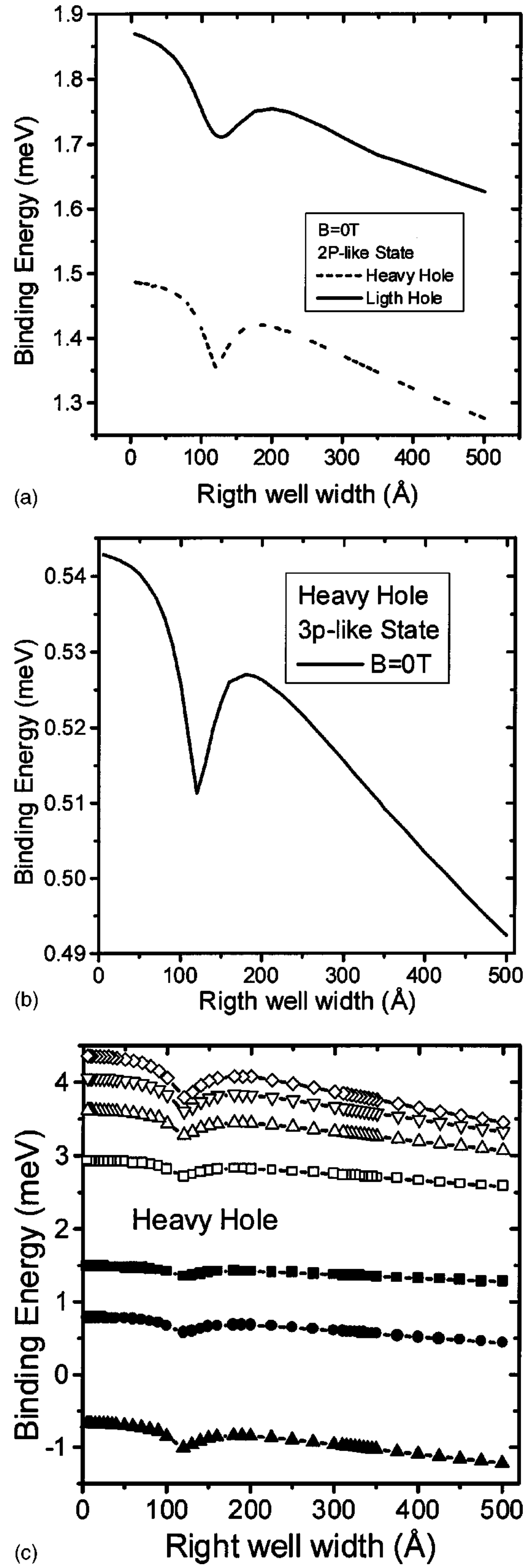

FIG. 5. BE of $2P_-$-like (a) and $3P_-$-(b) exciton states as a function of the L_R and zero magnetic field in $GaAsGa_{0.7}Al_{0.3}As$ DQW with $L_L = 120$ Å and $L_B = 10$ Å. (c) BE of the $2P_-$-like heavy hole exciton state as a function of L_R for different values of the magnetic field in a DQW with $L_L = 120$ Å, $L_B = 10$ Å.

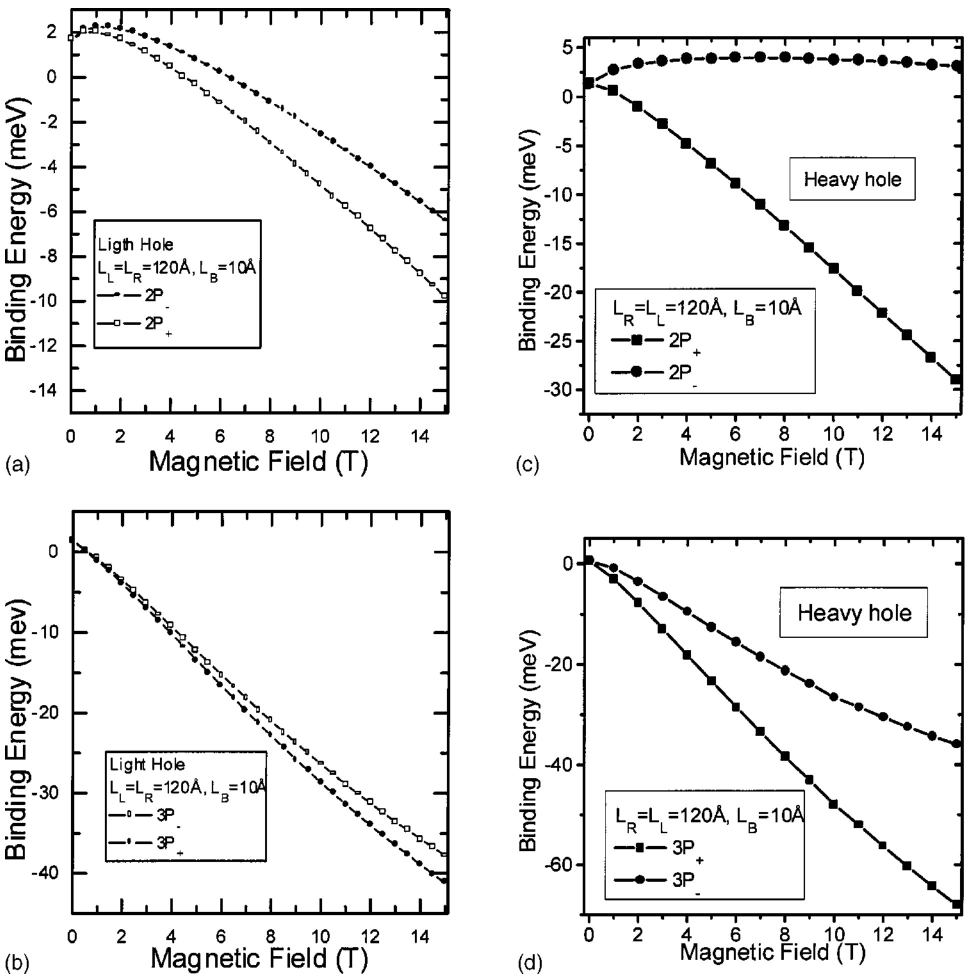

FIG. 6. BE of the $2P_{\pm}$- and $3P_{\pm}$-like exciton states as a function of the magnetic field for a symmetric $GaAsGa_{0.7}Al_{0.3}As$ DQW ($L_L=L_R=120$ Å, $L_B=10$ Å).

$$V(z)=\begin{cases} V_L^{(e,h)}, & z<-L_L-\frac{L_B}{2},\\ 0, & -L_L-\frac{L_B}{2}\leqslant z<-\frac{L_B}{2},\\ V_B^{(e,h)}, & |z|\leqslant\frac{L_B}{2},\\ 0, & \frac{L_B}{2}<z\leqslant\frac{L_B}{2}+L_R,\\ V_R^{(e,h)}, & z>\frac{L_B}{2}+L_R. \end{cases} \tag{3}$$

The trial wave function for the exciton in the (n,l,m) state is taken as

$$\Psi_{n,l,m}=f_e(z_e)f_h(z_h)\exp\left(-\frac{\gamma}{4}\rho^2\right)\rho^{|m|}\exp(-im\phi)P_{n,l}(r,\beta_{n,l,m})\exp(-\lambda_{n,l,m}r), \tag{4}$$

where $f_e(z_e)$ and $f_h(z_h)$ are the uncorrelated electron and hole wave functions and $P_{n,l}(r,\beta_{n,l,m})$ are the hidrogenic variational polynomials in which $\beta_{n,l,m}$ and $\lambda_{n,l,m}$ are variational parameters.[6]

The Schrödinger equation for the exciton in a DQW is

$$H\Psi_{n,l,m}=E_{n,l,m}\Psi_{n,l,m}, \tag{5}$$

where $E_{n,l,m}$ is the energy of the exciton in the (n,l,m) state. The BE $E^b_{n,l,m}$ for the (n,l,m) excitonic state is calculated by means of

$$E^b_{n,l,m}=E^0_e+E^0_h+\gamma-E_{n,l,m}. \tag{6}$$

For computational purposes the BE is expressed in meV. The electron effective mass is $m_e=0.067m_o$, the heavy and light hole effective mass in the z direction are $m_h=0.45m_o$ and $m_l=0.082m_o$, where m_o is the bearing electron mass. The dielectric constant is $\varepsilon=12.5$, and the effective mass of the heavy and light hole in the xy plane are $\mu_h=0.04m_o$ and $\mu_l=0.051m_o$. The effective Rydberg for the heavy and light hole excitons are $Ry_h=3.47369$ meV and $Ry_l=4.44067$ meV, respectively. We have assumed the same value of these parameters for the barrier and well regions of the DQW.

III. RESULTS

In Fig. 2 (a) we show the BE for the $1S$-like excitonic state as a function of the right well width (L_R) for different values of the applied magnetic field. In this figure it is observed that when L_R increases, starting from $L_R\approx 0$ Å, the BE diminishes, but when $L_L\approx L_R$ it reaches a minimum, increases again up to a maximum at a certain value of L_R, after which the BE continues decreasing, showing the same behavior as if the exciton were confined in a single quantum well, going to the bulk limit for large values of L_R. Also, it is observed that the minimum at $L_L \approx L_R$ is reached in a smaller range of L_R for wider barrier widths. Note that this behavior is the same for both the heavy and light hole excitons. This behavior of the exciton BE in a DQW as a function of the L_R, when L_L is constant, can be understood as follows: When L_R diminishes from large values, the BE increases because the geometrical confinement is increased. When the size of the exciton is comparable with L_R the tunneling probability increases and both the electron and hole can exist separately in one or another well of the structure and consequently the BE diminishes up to get a minimum in the symmetric case ($L_R = L_L$). When L_R continues decreasing, from $L_R = L_L$ to $L_R < L_L$, the BE increases again because the exciton prefers to localize at the wider well (the left well) and to get the BE, it would have, if the structure were of a single quantum well (QW). Effectively when $L_R = 0$, the exciton BE recovers the expected value for a single QW.[6,7] In other words, we have found that the binding energy of the ground state is higher in double quantum well structures, when the width of the wells are slightly different (asymmetric case), than when they are equal (symmetric case), a difference which is increased by the barrier width. At this point it is well to stress that our results coincide quite well with those reported by Kasapoglu *et al.*[4] who only considered the study of the DQW with $L_L = 120$ Å and $L_B = 10$ Å and for $L_R \leqslant L_L = 120$ Å. As it is observed in Figs. 2(a) and (b), the BE for heavy and light hole excitons in a DQW behaves as in a single QW when the barrier width is very small. In Fig. 2(c) it is observed the behavior of the BE of a heavy hole exciton as a function of the barrier width (L_B) for both symmetric and asymmetric DQW. The BE presents a minimum value in both cases for a different value of L_B, at L_{W1} in asymmetric DQW and at L_{W2} in symmetric DQW. Also, it is observed that when $L_B = 0$, the BE in the symmetric DQW is a little greater than that one in the asymmetric DQW. This is a typical behavior of the BE in a single quantum well QW, that is, the BE diminishes with the width of the well. When L_B is increased from $L_B = 0$ the BE goes down until to reach a minimum value. L_{W1} is smaller than L_{W2} because in asymmetric DQW the exciton is more localized in the wider well and the tunneling probability diminishes faster than in the symmetric DQW, when L_B is increased. In the interval $0 < L_B < L_{W1}$ the BE of the exciton in the symmetric DQW is a little greater than in the asymmetric DQW, as expected, and it is seen that the barrier width in this range has practically no effect upon the BE. For $L_B > L_{W1}$ the BE of the asymmetric DQW is greater than the BE in symmetric DQW. This behavior of the BE is the same as that shown in Figs. 2(a) and 2(b). After the minimum, we found that the BE increases with L_B and although it is not shown, its maximum value is reached first in the asymmetric than in the symmetric DQW. These results are in good agreement with those in Refs. 1, 3, 4.

The z dependence of the probability density of the ground state of a heavy hole exciton in three different DQW's is presented for two values of the magnetic field in Fig. 3. It is observed that the symmetry of the DQW reflects itself in the symmetry of the exciton probability density as it is observed in Fig. 3(a). When the symmetry of the DQW is diminished, that is when the width of one of the quantum wells is a little different than the other [Fig. 3 (b)], the exciton probability density changes drastically, indicating that the exciton prefers to be localized at the widest well. This behavior is enhanced in Fig. 3(c), where the right well is wider than the left well in Fig. 3(b). This result is in agreement with the explanation given above for the appearance of the minimum in the BE of the $1S$-like exciton state when $L_L = L_R$.

In Fig. 4 we present the BE of the ground state of a heavy (a) and a light (b) hole exciton in a DQW as a function of L_R for different values of the applied magnetic field. The mini-

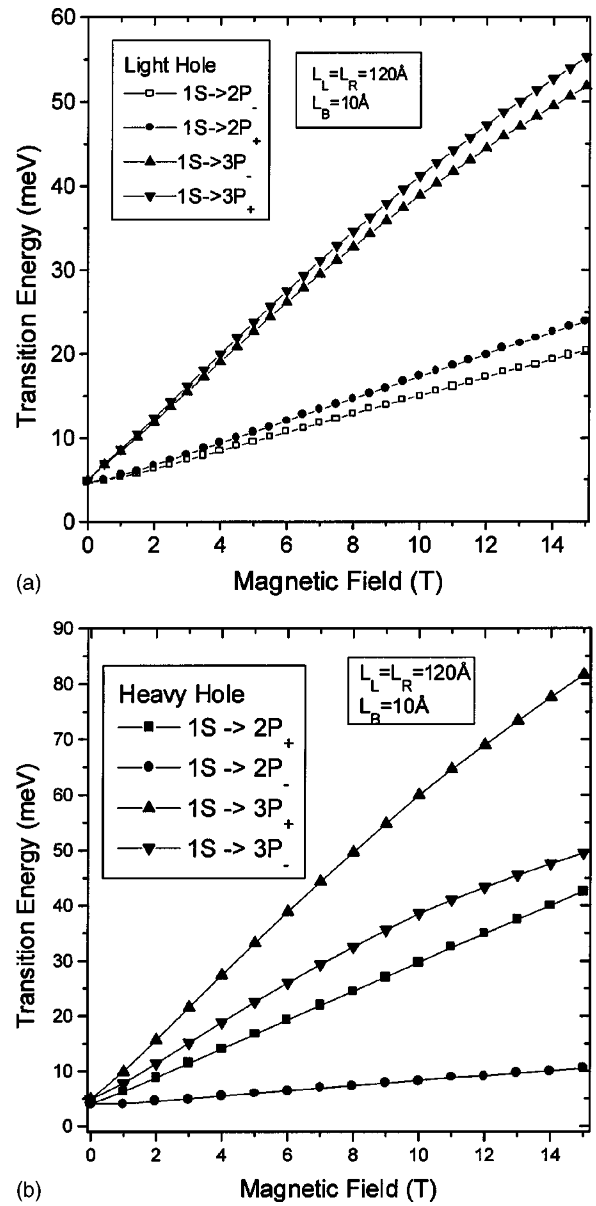

FIG. 7. $1S \rightarrow nP$ allowed transition energies for light and heavy hole excitons as a function of the applied magnetic field in a symmetric $GaAsGa_{0.7}Al_{0.3}As$ DQW ($L_L = L_R = 120$ Å, $L_B = 10$ Å).

mum of the BE at $L_R=L_L$ is present for all values of the applied magnetic field. The magnetic field applied in the growth direction certainly modifies the tunneling probability of the carriers through the barrier, spreading out the carrier wave function in the z direction, effect which combined with the preferably localization of the exciton in the wider well, as pointed out in Figs. 3(b) and 3(c), consequently increases the exciton BE. This effect is displayed in Fig. 4(c), where we show the BE of the heavy and light hole exciton ground state as a function of the applied magnetic field for a symmetric DQW with $L_R=L_L=120$ Å and $L_B=10$ Å.

In Fig. 5 we show the BE as a function of L_R and zero magnetic field for $2P$-like exciton states [Fig. 5(a)], for heavy and light hole exciton, and $3P$-like heavy hole exciton state [Fig. 5(b)]. In Fig. 5(c) we present the BE of the $2P$-like heavy hole excitonic state as a function of L_R for different values of the applied magnetic field. In these figures it is observed that the BE is smaller in these states than in the $1S$-like, although their behavior is similar with L_R. Notice that these exciton excited states also have a higher BE in asymmetric DQW than that in the symmetric DQW. It is important to mention that the BE of the $2P_+$ excited excitonic state for $B=2$ T is negative for all values of L_R. Also, it can be observed the splitting of the $2P_{\pm}$-like excitonic state due to the magnetic field. The BE is higher for the excitonic excited states with $m=-1$, than for those with $m=+1$ for all values of L_R. This behavior is similar to that observed in previous works in single QW's.[6,7]

Figures 6(a) and 6(b) displays the BE of the $2P_{\pm}$- and $3P_{\pm}$-like light hole exciton states, and Figs. 6(c) and 6(d) the BE of the $2P_{\pm}$- and $3P_{\pm}$-like heavy hole excitonic states, respectively, as a function of the magnetic field for a symmetric DQW ($L_L=L_R=120$ Å, $L_B=10$ Å). Excepting the $2P_-$-like heavy hole exciton state, which binding energy is positive and practically does not change with the magnetic field, the BE of the $2P_+$- and $3P_{\pm}$-like heavy and ligth hole exciton states diminish with the magnetic field and are negative for a wide range of the magnetic field. The negative BE means that the energy level of these states is higher than the energy of the uncorrelated electron-hole system in their ground state in the DQW.

The $1S\rightarrow nP$ allowed transition energies for light and heavy hole excitons are displayed in Fig. 7 as function of the applied magnetic field for a symmetric DQW. It can be observed that the transition energies increase with the magnetic field and that the $1S\rightarrow nP_+$ transition energies are greater than the $1S\rightarrow nP_-$ for both light and heavy hole excitons. This behavior is similar to that found in a single QW (Refs. 6, 7) and in QWW's.[14]

IV. SUMMARY

Summing up, we have calculated the ground and some heavy and light hole exciton excited states in a $GaAsGa_{0.7}Al_{0.3}As$ double quantum well structure as a function of the well width, for different barrier widths and several values of the magnetic field applied in the growth direction. We have found that the binding energy of the ground and all exciton excited states is higher in double quantum well structures, when the width of the wells are slightly different (asymmetric case), than when they are equal (symmetric case), a difference which is increased by the barrier width. Our results show that the BE of the exciton ground state increases with the magnetic field for all the values of the well widths. Otherwise, the BE of the $2P_+$, $3P_+$, and $3P_-$ excitonic states decreases and these states are less bounded with the magnetic field. Also we have found that the magnetic field splits the binding energies of the nP ($m=\pm1$) excited states in a similar way to that observed in single QW's. We believe that the present calculation will be of importance in the quantitative understanding of future experimental works on this subject.

ACKNOWLEDGMENTS

The authors express thanks to Professors J. M. Calero and J. C. Granada for comments on the manuscript. This work was partially financed by Colciencias, the Colombian Scientific Agency, under Grant No. 1106-05-11498.

[1] T. Kamizato and M. Matsuura, Phys. Rev. B **40**, 8378 (1989).
[2] M. Dignam and J. Sipe, Phys. Rev. B **43**, 4084 (1991).
[3] J. Cen and K. K. Bajaj, Phys. Rev. B **46**, 15 280 (1992).
[4] E. Kasapoglu, H. Sari, N. Balkan, I. Sokmen, and Y. Ergun, Semicond. Sci. Technol. **15**, 219 (2000).
[5] A. Mathos-Abiague, L. E. Oliveira, and M. de Dios-Leyva, J. Phys.: Condens. Matter **12**, 5691 (2000).
[6] C. Duque, C. L. Beltrán, A. Montes, N. Porras-Montenegro, and L. E. Oliveira, Phys. Rev. B **61**, 9936 (2000).
[7] C. L. Beltrán, C. A. Duque, A. Montes, N. Porras-Montenegro, and L. E. Oliveira, Phys. Status Solidi B **220**, 137 (2000).
[8] Haiyan An, Baijun Chen, Jingyin Hou, Jiacong Shen, and Shiyong Liu, J. Phys. D **31**, 1144 (1998).
[9] Guangyou Yu, X. W. Fan, J. Y. Zhang, Z. H. Zheng, B. J. Yang, Xiaowei Zhao, Dezhen Shen, and Xianggui Kong, J. Phys. D **32**, 1506 (1999).
[10] J. Soubusta, R. Grill, P. Hl'ýdek, M. Zv'ara, L. Smricka, S. Malzer, W. Gei-elbrecht, G. H. Döhler, Phys. Rev. B **60**, 7740 (1999).
[11] V. V. Krivolapchuk, E. S. Moskalenko, and A. L. Zhmondikov, Nanotechnology **11**, 246 (2000).
[12] Francisco Vera, J. Phys.: Condens. Matter **12**, 5691 (2000).
[13] Martin Holthaus, Phys. Rev. A **64**, 011601(R) (2001).
[14] Pablo Villamil, Carlos Beltrán, and Nelson Porras-Montenegro, J. Phys.: Condens. Matter **13**, 4143 (2001).