Distribution function of electrons and phonons in semiconductors and semimetals in high electric and quantizing magnetic fields

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The distribution functions of electrons and phonons interacting with electrons in semiconductors and semimetals in high electric **E** and classically high and quantizing magnetic **H** fields as a result of the solution of the coupled system of equations for the density matrices of electrons and phonons is obtained. The effects of heating of electrons and phonons and their arbitrary mutual drag are taken into account. In the absence of **E** the dispersion relation of electrons is assumed to be arbitrarily spherically symmetric. The spectrum of phonons is assumed to be isotropic. The distribution functions of electrons and phonons, the amplification coefficient of phonons, and the dependence of chemical potential on E, H, electron concentration, and effective electron temperature are obtained. The nonstationary distribution function of phonons is obtained for arbitrary phonon drift velocities which enables us to consider the spontaneous and stimulated emission of phonons by hot electrons.

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I. INTRODUCTION

It is well known that a high electric field leads to some different processes in solids. It causes new qualitative changes of the quantum states of current carriers and their energy spectrum, which leads to the dependence of macroscopic behavior of solids on the applied external electric field **E**. Among examples of such a process may be the **E** dependence of the imaginary part of the dielectric permeability, connected with the possibility of intrinsic absorption of photons with energies less than the band gap in semiconductors (the Frantz-Keldish effect), the tunneling current in degenerate p-n junctions (the Esaki effect), and so on.

By assistance of high electric field, it is possible to take the semiconductors to the states far from the thermodynamical equilibrium state. Such a state takes place in piezoelectric or other types of semiconductors and in Bi under the conditions of amplification or generation of sound by drifted conduction carriers.[1,2]

Another nonequilibrium state is realized in the existence of so-called ''hot electrons'' in semiconductors. The heating of electrons leads to changes in many physical behaviors of semiconductors and semimetals.[3,4] The behavior of the cold and hot electrons in semiconductors and semimetals in the presence of high magnetic field is another important field of study. High magnetic field leads to the quantization of the orbital motion of carriers and, as a result, to the quantization of their energy spectrum.[5] Moreover, the high magnetic field leads to the change in the rate of relaxation processes in electron and phonon subsystems in semiconductors and semimetals. This enables the investigators to manage the characteristic frequencies of relaxation processes in the system.

In a quantizing magnetic field, the ratio of interelectronic collision frequency ν_{ee} to electron-phonon collision frequency ν_p decreases sharply. This takes place because of the nondegenerate statistics of carriers in ultraquantum limit, the lowest Landau level is fully occupied whereas other levels contain an exponentially lower number of carriers if the energy of carriers is much smaller than the cyclotron energy, i.e, $\hbar\Omega \gg T_e$ (here Ω is the cyclotron frequency and T_e is the effective carrier temperature). In such a situation the collision frequency between the carriers which belong to different Landau levels becomes exponentially small. The collisions between the carriers in the lowest Landau level are elastic as a result of the one-dimensional character of motion and do not contribute to intercarrier relaxation. On the other hand, the collision frequency of carriers with phonons ν_p, and phonons with electrons β_e grows with increasing magnetic field strength (H) as H^2.[6] Therefore in quantizing magnetic fields, in contrast to the classical case, we have

$$\frac{\nu_{ee}}{\nu_p} \sim \left(\frac{T_e}{\hbar\Omega}\right)^2 \exp\left(-\frac{\hbar\Omega}{T_e}\right). \tag{1}$$

Therefore, the decrease of interelectronic collisions in quantizing magnetic fields leads to a sharp decrease in the efficiency of redistribution of electron energy in quantum states, and essentially changes the form of the carrier distribution function. In connection with these facts, it seems that it may be necessary to consider the collisions between carriers in different Landau levels, such as the lowest and the next level, i.e., $N=0$ and $N=1$. This situation was considered by Calecki[7] who notes that: ''As ν_{ee} is proportional to the occupancy of the two levels, we are not surprised by the appearance of the exponential factor in its expression. In conclusion, the apriori use of an equilibrium distribution function with an effective electron temperature is rather questionable and this is well reflected in the literature on hot carriers in a quantizing magnetic field.'' Therefore, the problem of the possibility for introducing the concept of ''effective electron temperature'' in high electric and quantizing magnetic fields arises. The papers on this subject may be classified into three groups.

(1) Papers which use, without any proof, the simple concept of electron temperature as a starting point.[8–16]

(2) Papers which entirely neglect intercarrier collisions. Either the carrier concentration is too small or only the extreme quantum limit is treated. The problem of establishing a master equation by the diagonal elements of density operator has been conveniently done by Budd[17] in crossed electric and magnetic fields configuration and by Barker[18] in the longitudinal one. By solving the master equation, we can find the carrier distribution function. Then, it is necessary to determine for which conditions the concept of electron temperature is meaningful. Kazarinov and Skobov[19,20] were the first to discuss this problem in the case of crossed electric and magnetic fields. They showed that the electron distribution function has a Fermi-Dirac form with effective electron temperature. In this work, it was assumed that the distribution function of phonons is the equilibrium one at lattice temperature T, and the nonequilibrium effects related to phonons, such as the phonon heating, mutual drag of electrons and phonons, and generation and amplification of phonons by heated electrons, were neglected. Moreover, the influence of high external electric and magnetic fields on the chemical potential of electrons was neglected. Later, Yamashita and Inoue[21,22] made an attempt to reduce the range of the validity of Refs. 19–22. The situation considered in Refs. 19–22 can be realized at high lattice temperature, when the phonon-phonon collision frequency β_p is much higher than the collision frequency of phonons by electrons β_e. However, for quantization of orbital motion it is necessary to carry out the experiments at low lattice temperatures (liquid helium, or lower) when $\beta_p \rightarrow 0$. In the experimental conditions, the results of Refs. 19–22 are satisfied for drift velocities $V \ll s$. Nevertheless, as follows from the result of Refs. 19–22, when $V \ll s$ the heating of electrons is negligible. In an entirely different spirit, Kurosawa and Yamada[23,24] developed a phenomenological model describing the energy exchange process of electrons with electric field and phonons by a Brownian motion in energy space. However, their starting point was not entirely rigorous, but the method was promising (see Ref. 7). It has been improved by including both electron-electron scattering and collision broadening by Partl *et al.*[25] Calecki *et al.*[26] used the Pauli master equation derived by Budd[17] and transformed it into a Fokker-Planck equation and recovered the arguments of the random walk approach of Kurosawa and Yamada on a more rigorous ground and finally established the restrictive conditions under which the electron temperature concept may be used.

(3) Papers devoted to the effect of electron-electron collisions. Zlobin and Ziryanov[27] calculated a critical density for electrons $n_{\rm cr}$ such that for $> n_{\rm cr}$ one can use a thermal equilibrium electron distribution function at temperature T_e. Their results are valid only in the case where no more than two Landau levels are occupied. In the extreme quantum limit, where only the lowest Landau level is populated, the binary electron-electron collisions do not contribute to the redistribution of the energy between electrons, and thermalization of the electron distribution to the equilibrium one does not happen. As it seems from the foregoing discussion, the main problem is the use of the electron temperature approximation at quantizing magnetic fields. It seems that in the extreme quantum limit, the binary interelectronic collisions are ineffective and have no influence on the redistribution of electron energy. Nevertheless, an experiment[28] showed that in InSb at temperature $T=4$ K and a magnetic field of 10^4 G (i.e, in the extreme quantum limit) with a concentration of 10^{14} cm^{-3}, the distribution function of electrons is a Maxwellian one (see also Refs. 29–31). In light of these experimental facts, there were several attempts to solve this contradiction by taking into account the three-body scattering processes: electron-electron-impurity[32] or electron-electron-phonon.[33] But it is known that the probability of three-body collisions is much less than that of the binary collisions. On the basis of this fact Calecki makes the following conclusion: ''The interest of such studies lies in the fact that apparently the only real system for which three-body scattering processes determine gas kinetics is the electron gas in an ultra-quantizing magnetic field.'' As is shown experimentally,[34–36] under these conditions it is necessary to take into account the heating and mutual drag of electrons and phonons, and the generation of phonons by hot electrons.

It is also well known that there are many other interesting works on quantizing magnetic fields: collision broadening effect, influence of the electric field in the Kane band structure, introduction of several valleys in a semiconductor, impact of high electric field on the screening in electron interactions, the distribution function of photoexcited electrons, and so on. But all these effects are ineffective in the extreme quantum limit. Because in this limit all electrons have the same energy, $\varepsilon = \hbar\Omega/2 = \text{const}$, for a given constant magnetic field and the scattering mechanism.

In the present paper, we show that all the problems mentioned above in high external electric and quantizing magnetic fields may be solved by taking into account the heating of electrons and phonons and their mutual drag. The possibility of using the ''effective electron temperature'' concept is based on the analysis of the electron heating in high external electric field. This leads to the problem of finding the conditions where the effective electron temperature approximation is fulfilled. It is well known that in Maxwell statistics the interelectronic collision frequency increases with increasing electron concentration. At high electron concentrations ($n > n_{\rm cr}$) the interelectronic collisions lead to a effective redistribution of the energy gained from the external electric field and to the Maxwell-Boltzmann distribution function with effective electron temperature T_e.

In a strong magnetic field the electron motion in the plane perpendicular to the magnetic field direction is quantized and Landau levels are formed, as mentioned before. The energy separation between two successive levels is $\hbar\Omega$. Hence, the separation is determined by the external magnetic field strength and effective electron mass. At sufficiently low temperatures (for example, for InSb at liquid helium temperature), the energy separation can be larger than the thermal energy, i.e., $\hbar\Omega > T$. If $\hbar\Omega \gg T$, all of the electrons are populated in the lowest Landau level and interelectronic collisions become ineffective in the energy redistribution process.

In the low electron concentration limit, the interelectronic collisions may be neglected completely. In this case, the

Maxwell-Boltzmann equilibrium distribution may be established as a result of electron-phonon collisions. Actually, in crossed electric and magnetic fields ($\mathbf{H}=H\hat{z}$, $\mathbf{E}=E\hat{x}$), the stationary state of electrons is characterized by Hall drift along the y direction with velocity $v\equiv V_H=cE/H$. In the absence of scattering the average velocity of electrons along the x direction is zero. The inclusion of scattering leads to the appearance of the conduction current. The motion of electrons along the x direction changes the equilibrium position X_0 of the center of oscillation and, since $X_0=-cp_y/eH$, it changes the p_y component of the electron momentum.

As was also shown by Kazarinov and Skobov[19,20] the effective electron temperature may be introduced if $\hbar\Omega\gg T_e$. They entirely neglected interelectronic collisions and solved the equation for the diagonal elements of density matrix for electrons directly and showed that the solution is a Maxwell-Boltzmann distribution function with effective electron temperature. In this work, it was assumed that the distribution function of phonons is the equilibrium one at lattice temperature T. Such a situation can be realized at high lattice temperature, when the phonon-phonon collision frequency β_p is much higher than the collision frequency of phonons by electrons β_e. For the quantization of orbital motion it is necessary to carry out the experiments at low temperatures of lattice (liquid helium, or lower) when $\beta_p\to 0$. As is shown experimentally,[34–36] under the conditions of ineffective interelectronic collisions in contrast to Refs. 19–22 it is necessary to take into account the heating and mutual drag of electrons and phonons, and the generation of phonons by hot electrons. In the experimental conditions, the results of Refs. 19–22 are satisfied for drift velocities $V\ll s$. Nevertheless, as follows from the result of Refs. 19–22, when $V\ll s$ the heating of electrons is negligible. Because, in the absence of phonon heating, the heating of electrons starts from the drift velocities $V>s$. The heating of phonons interacting with electrons was considered by Gurevich and Gassymov in several works.[37–42] In Ref. 37 at high external electric field, in Refs. 38 and 39 at high external electric and arbitrary nonquantized magnetic fields, in Refs. 38, 42, and by Zlobin and Zyrianov[43] at high external electric and quantizing magnetic fields are considered. In Refs. 37–42, it was shown that the effect of phonon heating leads to liquidation of the ''runaway effect,''[42] and to the explanation of some experimental results such as negative differential conductivity N- and S-type I-V characteristic in semiconductors and semimetals.[41] Moreover, it was also shown that in quantizing magnetic field the ''effective electron temperature'' concept may be used as a result of electron-phonon collisions.[40,46]

As is shown in the present paper the problem of inefficiency of interelectronic collisions in quantizing magnetic fields may be solved by taking into account the mutual drag of electrons and phonons. Actually, as was shown in our earlier investigations,[44–50] the mutual drag leads to renormalization of the cyclotron frequency of carriers as a result of the renormalization of the mass of carriers dressed by the phonons. In other words, the mutual drag leads to the formation of new quasiparticles ''electron dressed by phonons'' or ''hole dressed by phonons,'' which have the carrier charge ($\pm e$) and the phonon mass ($M\approx T/s^2$). The renormalization of the cyclotron frequency $\Omega=eH/(mc)\to\Omega^\star=eH/(Mc)=eHs^2/(T_ec)$ leads to violation of the initial condition of quantization $\hbar\Omega\equiv eH/(mc)\gg T$ or T_e and make a transition to the classical magnetic field condition $\hbar\Omega^\star<T$ or T_e. Because $\hbar\Omega^\star/\hbar\Omega=m/M\approx 10^{-28}/10^{-25}\approx 10^{-3}$ for $T_e\approx 10^{-14}$ erg, or $T_e/k_B\approx 77$ K and 10^{-2} for $T_e/k_B\approx 4$ K for the Ge or GaAs, and $\approx 10^{-4}$–$\approx 10^{-3}$ for the InSb parameters.

As is discussed above, since the initially ineffective interelectronic collisions become effective as a result of the mutual drag of electrons and phonons, it is possible to use the effective electron temperature approximation if the electron concentration is high. For low electron concentrations by solving the coupled system of equations for diagonal elements of electron and phonon density matrices at high electric and arbitrary magnetic fields (classically high and quantizing) we show that the distribution function of carriers is the Fermi one with an effective electron temperature as a result of the electron-phonon interaction and the mutual drag of electrons by phonons. Since at high electric fields the heating of electrons begins at drift velocities $V>s$, we cannot use the diffusion approximation for the phonon distribution function and we can solve the general equation for the phonon distribution function in the nondiffusion approximation at nonequilibrium and nonstationary conditions.

The mutual drag also leads to the renormalization of the energy of phonons interacting with electrons as

$$\hbar\omega_q\to\hbar\omega_q^\star\equiv\hbar\omega_q-Vq_y\equiv\hbar(sq-Vq_y).$$

It is a result of the fact that the initial equilibrium Planck distribution function of phonons becomes the drifted Planck distribution function in the mutual drag conditions. The transition $\omega_q\to\omega_q^\star\equiv\omega_q-Vq_y/\hbar$ is the well-known Doppler shift of the phonon frequency. This renormalization effect of phonon energy leads to the following situations.

(a) For drift velocities $0<V<s$, $\hbar\omega_q^\star>0$, and tends to zero by the growing of V. In this region of drift velocities the absorption of phonons by electrons prevails.

(b) For drift velocities $V=V_H=s$, we have the case when the electron-phonon collisions are exactly elastic ($\hbar\omega_q^\star=0$), and dissipation is absent. At this point the spontaneous emission of phonons begins and current saturates. All the energy gained from external electric field is emitted as phonons.

(c) For drift velocities $V>s$, the $\hbar\omega_q^\star$ becomes negative and the absorption of phonons in the region $0<V<s$ is replaced by the emission. This region of drift velocity is the region of stimulated emission of phonons, i.e., the region of phonon generation or amplification at high external fields.

In the present paper, the behavior of semiconductors and semimetals in crossed high external electric ($\mathbf{E}=E\hat{x}$) and magnetic fields ($\mathbf{H}=H\hat{z}$) is considered. The magnetic field is assumed to be high such that the cyclotron frequency Ω is much greater than the momentum relaxation frequency of electrons ν. It is known that an electron has stationary states at high electric and magnetic fields. In Landau representation, the stationary states of electrons are characterized by

the magnetic quantum number N, the projection of momentum on the magnetic field direction p_z, and the rotation center of electrons X.

If the spectrum of electrons in the absence of electric field assumed to be isotropic and quadratic, then the energy eigenvalues in crossed electric and magnetic fields have the form[5]

$$\varepsilon_\alpha = \varepsilon_{N,p_z,X} = \hbar\Omega\left(N+\frac{1}{2}\right)+\frac{p_{z\alpha}^2}{2m_n}-eEX_\alpha+\frac{m_n v_{y\alpha}^2}{2}. \quad (2)$$

The stationary state of electrons with energy given by Eq. (2) is characterized by Hall drift along the y direction with velocity $v_y \equiv V_H = cE/H$. In the absence of scattering, the average velocity of electrons along the x direction is equal to zero. The inclusion of scattering leads to the appearance of the conductance current. The motion of electrons along the x direction changes the equilibrium position of the center of oscillation X_0 and, as a result, changes the p_y component of the electron momentum connected with them $X_0 = -cp_y/eH$. The scattering of electrons by phonons is connected with the transfer of the y component of the momentum of electron p_y to phonons, and leads to the stream of electrons along the x axis, which leads to the mutual drag of electrons and phonons. During the motion along the x axis, the electron gains energy from the field $eE(X_\alpha - X_\beta) = eEX_{\alpha\beta} \equiv eER^2 q_y/\hbar$, and makes a transition from state α to state β. Hereafter, $R=(c\hbar/eH)^{1/2}$ is the magnetic length. If this energy is greater than the emitted phonon energy $\hbar\omega_q$, electrons and phonons are heated. Thus, the presence of external electric field first leads to the mutual drag of electrons and phonons, and second if the electric field is high, we have the heating of electrons and phonons.

At high $\mathbf{E}\perp\mathbf{H}$ fields, in general in the absence of the mutual drag, the ''effective electron temperature'' approximation is not satisfied. That is a reason of why for the definition of distribution functions of electrons and phonons it is necessary to solve the coupled system of equations for the density matrix of electrons and phonons directly.

The present paper is devoted to solving the coupled system of equations for diagonal parts of electrons and phonons density matrices at high external crossed **E** and **H** fields with taking into account the heating of electrons and phonons, and their mutual drag. The problem is solved under the assumption that the spectrum of electrons in the absence of electric field is arbitrary spherically symmetric, and the spectrum of phonons is assumed to be isotropic.

The phonon generation at high external electric and magnetic fields is a nonstationary effect, i.e., it is a result of the increase in the number of phonons or their distribution function in time. Thus, for the consideration of the phonon generation, it is necessary to solve the nonstationary and nonequilibrium equation for the density matrix of phonons interacting with electrons.

II. THE SPECTRUM OF ELECTRONS IN HIGH ELECTRIC AND QUANTIZING MAGNETIC FIELDS

Let us assume that in the absence of electric field, the dispersion relation of electrons is given by

$$B(\varepsilon_{\alpha 0}) = \frac{p_z^2}{2m_n}+\varepsilon_N, \quad \varepsilon_N = \hbar\Omega\left(N+\frac{1}{2}\right). \quad (3)$$

The energy of electrons ε_α in $\mathbf{E}\perp\mathbf{H}$ case may also be written as

$$\varepsilon_\alpha = \varepsilon_{\alpha 0} - eEX_\alpha + \frac{m(\varepsilon_\alpha)v_{y\alpha}^2}{2}$$
$$= \varepsilon_{\alpha 0} - eEX_\alpha + \frac{m(\varepsilon_\alpha)}{m_n}\frac{m_n c^2}{2}\frac{E^2}{H^2}, \quad (4)$$

where m_n is the effective mass of electrons at the bottom of the conduction band and $m(\varepsilon_\alpha) = m_n[\partial B(\varepsilon_\alpha)/\partial\varepsilon_\alpha]$ is the effective electron mass. $B(\varepsilon_\alpha)=\varepsilon_\alpha$ for the parabolic and $B(\varepsilon_\alpha)=\varepsilon_\alpha[1+\varepsilon_\alpha/\varepsilon_g]$ for the two-band Kane spectrum cases. In the absence of the electric field,

$$X_\alpha = -\frac{p_{y\alpha}}{m_n\Omega}. \quad (5)$$

For the determination of the dispersion relation of electrons in the $\mathbf{E}\perp\mathbf{H}$ case, we make a transition to the reference frame which drifts together with electrons with a velocity of $V=cE/H$. It is obvious that in such a reference frame, the dispersion relation of electrons must have the form of Eq. (3), and all properties of the system must be preserved if we substitute

$$p'_{y\alpha} = p_{y\alpha} + m(\varepsilon_\alpha)V. \quad (6)$$

Then, by using Eq. (6) in Eq. (5), we may obtain X_α as

$$X_\alpha = -\frac{p_{y\alpha}}{m_n\Omega} - \frac{m(\varepsilon_\alpha)V}{m_n\Omega} = -\frac{p_{y\alpha}}{m_n\Omega} - \frac{eE}{m_n\Omega^2}\left(\frac{\partial B(\varepsilon_\alpha)}{\partial\varepsilon_\alpha}\right). \quad (7)$$

If we define $\varepsilon_{\alpha 0}$ as in Eq. (4) and substitute it into Eq. (3), we may obtain the dispersion relation of electrons in the $\mathbf{E}\perp\mathbf{H}$ case as

$$B(\varepsilon_\alpha^\star) \equiv B\left(\varepsilon_\alpha + eEX_\alpha - \frac{m(\varepsilon_\alpha)}{m_n}\frac{m_n c^2}{2}\frac{E^2}{H^2}\right)$$
$$= \frac{p_{z\alpha}^2}{2m_n} + \hbar\Omega\left(N+\frac{1}{2}\right) \quad (8)$$

or

$$\frac{p_{z\alpha}^2}{2m_n} = B(\varepsilon_\alpha^\star) - \varepsilon_N. \quad (9)$$

The X_α in Eq. (7) is determined from Eq. (5) by taking into account Eq. (6).

If the increasing of energy of electrons in electric field

$$\left(eEX_\alpha - \frac{m(\varepsilon_\alpha)c^2}{2}\frac{E^2}{H^2}\right)$$

is much less than ε_α, by expanding $B(\varepsilon_\alpha^\star)$ into series around this small parameter, we find

$$\frac{p_{z\alpha}^2}{2m_n}=B(\varepsilon_\alpha)-\varepsilon_N+\left[eEX_\alpha-\frac{m_nc^2}{2}\left(\frac{\partial B(\varepsilon_\alpha)}{\partial\varepsilon_\alpha}\right)\right]\left(\frac{\partial B(\varepsilon_\alpha)}{\partial\varepsilon_\alpha}\right) \tag{10}$$

or

$$\frac{p_{z\alpha}^2}{2m_n}=B(\varepsilon_\alpha)-\varepsilon_N+\frac{m(\varepsilon_\alpha)}{m_n}eEX_\alpha-\frac{m(\varepsilon_\alpha)}{m_n}\frac{m_nc^2}{2}\frac{E^2}{H^2}. \tag{11}$$

When $V=cE/H\ll v_{\rm cr}=s$, Eq. (10) reduces to the relation obtained earlier in Refs. 51 and 52 as a result of the solution of Schrödinger's equation for the Kane spectrum of electrons. In the present paper, Eq. (8) is obtained for arbitrary magnitude of the external electric field. This is why this expression describes a more general case than the expressions given in Refs. 51 and 52.

The linear in electric field term eEX_α in the dispersion law is usually neglected.[19–22] This leads to a loss of some effects. As we showed above, taking into account this term is equivalent to the replacement of $\hbar\omega_q$ in the argument of delta function by $\hbar\omega_q^\star=\hbar\omega_q-Vq_y=\hbar\omega_q-(cE/H)q_y$.

III. THE MAIN EQUATIONS AND THEIR SOLUTIONS

At high magnetic field ($\Omega\gg\nu$) in the Landau representation, the diagonal elements of the density matrix of electrons f_α is larger than the nondiagonal elements by (Ω/ν) times, and that is why it is enough to write and solve the equation for the diagonal elements of the density matrix of electrons. Since in the present paper we consider the space uniform case when temperature and concentration gradients are absent, it is enough to write the equations for the diagonal elements of density matrix $N(\mathbf{q},t)$ for the phonon system as well.

After averaging over the electron states with fixed energies, the system of equations for the diagonal elements of the density matrices of electrons $f_\alpha=f(\varepsilon_\alpha,t)$, and phonons $N_\mathbf{q}(t)\equiv N(\mathbf{q},t)$ have the form[6]

$$\begin{aligned}\frac{\partial f(\varepsilon,t)}{\partial t}=&\frac{2\pi}{\hbar}\sum_{\alpha,\beta,\mathbf{q}}|C_\mathbf{q}|^2|\langle\alpha|\exp(-i\mathbf{q}\cdot\mathbf{r})|\beta\rangle|^2\{\delta(\varepsilon_\beta-\varepsilon_\alpha\\&-\hbar\omega_q^\star)[(f_\beta-f_\alpha)N(\mathbf{q},t)+f_\beta(1-f_\alpha)]\\&-\delta(\varepsilon_\beta-\varepsilon_\alpha+\hbar\omega_q^\star)[(f_\alpha-f_\beta)N(\mathbf{q},t)\\&+f_\alpha(1-f_\beta)]\}\delta(\varepsilon_\alpha-\varepsilon)+I_{ee}[f]+I_{ed}[f],\end{aligned} \tag{12}$$

$$\begin{aligned}\frac{\partial N(\mathbf{q},t)}{\partial t}=&\frac{4\pi}{\hbar}\sum_{\alpha,\beta,\mathbf{q}}|C_\mathbf{q}|^2|\langle\alpha|\exp(-i\mathbf{q}\cdot\mathbf{r})|\beta\rangle|^2\\&\times\delta(\varepsilon_\beta-\varepsilon_\alpha-\hbar\omega_q^\star)[(f_\beta-f_\alpha)N(\mathbf{q},t)\\&+f_\beta(1-f_\alpha)]+I_{pp}[N(\mathbf{q})]+I_{pb}[N(\mathbf{q})],\end{aligned} \tag{13}$$

where $f(\varepsilon,t)=\Sigma_\alpha f(\varepsilon_\alpha,t)\delta(\varepsilon_\alpha-\varepsilon)$ is the average number of carriers with energy ε, i.e., their distribution function, $\hbar\omega_q^\star=\hbar\omega_q-eE(X_\beta-X_\alpha)$, $I_{ee}[f]$ and $I_{ed}[f]$ are the interelectronic and electron-defects collision integrals, respectively,

$$\begin{aligned}I_{pp}[N_\mathbf{q}]=&\beta(q)[N(\mathbf{q},t)-N(\mathbf{q},T_p)],\\I_{pb}[N_\mathbf{q}]=&\beta_b[N(\mathbf{q},t)-N(\mathbf{q},T)],\end{aligned} \tag{14}$$

are the phonon-phonon and phonon-crystal boundaries collision integrals, T_p is the temperature of the heated phonons, and T is the lattice temperature. Both phonon collision integrals are considered in the relaxation time approximation $\beta_p^{-1}=\tau_p(q)$, $\beta_b^{-1}=\tau_b$.[40] Since as a result of the mutual drag electrons and phonons scatters by each other and forms the coupled system with the same temperature T_e and drift velocity u; then their momentum and energy are transmitted to the surrounding media (e.g., liquid helium) by the collisions with the crystal boundaries. The frequencies of boundary collisions connected with momentum β_b and energy β_ε transmission were calculated for the first time by Gurevich and Gassymov[37] (see also Ref. 53).

We now consider the low electron concentration case $n<n_{\rm cr}$ when $I_{ee}\ll I_{ep}$, where I_{ep} is the collisions integral of electrons with phonons. For simplicity we neglect the contribution due to the electron-defect collisions. Since within the Born approximation under quantizing magnetic field the collision frequencies of electrons with neutral and ionized defects do not depend on the electron energy, the role of the scattering of electrons by defects may be easily taken into account in the final expressions.

Exchanging α and β in the second component of the expression under the sum in Eq. (11), we find

$$\begin{aligned}\frac{\partial f(\varepsilon,t)}{\partial t}=&\frac{2\pi}{\hbar}\sum_{\alpha,\beta,\mathbf{q}}|C_\mathbf{q}|^2|\langle\alpha|\exp(-i\mathbf{q}\cdot\mathbf{r})|\beta\rangle|^2\\&\times\{\delta(\varepsilon_\beta-\varepsilon_\alpha-\hbar\omega_q^\star)[(f_\beta-f_\alpha)N(\mathbf{q},t)\\&+f_\beta(1-f_\alpha)]\}\{\delta(\varepsilon_\alpha-\varepsilon)-\delta(\varepsilon_\beta-\varepsilon)\}.\end{aligned} \tag{15}$$

We now consider the case when the scattering of electrons by phonons is quasielastic and, therefore, changing of the energy of electrons ($\varepsilon_\alpha-\varepsilon_\beta=\hbar\omega_q-Vq_y=\hbar\omega_q^\star$) is less than the energy scale of the changing electrons distribution function. Then, expanding $(f_\beta-f_\alpha)$ and $[\delta(\varepsilon_\alpha-\varepsilon)-\delta(\varepsilon_\beta-\varepsilon)]$ into series, we find

$$f_\beta-f_\alpha=f(\varepsilon_\alpha+\hbar\omega_q^\star)-f(\varepsilon_\alpha)\approx\hbar\omega_q^\star\left(\frac{\partial f(\varepsilon_\alpha)}{\partial\varepsilon_\alpha}\right)+\cdots, \tag{16}$$

$$\begin{aligned}\{\delta(\varepsilon_\alpha-\varepsilon)-\delta(\varepsilon_\beta-\varepsilon)\}=&\delta(\varepsilon_\alpha-\varepsilon)-\delta(\varepsilon_\alpha-\varepsilon+\hbar\omega_q^\star)\\\approx&-\hbar\omega_q^\star\frac{\partial}{\partial\varepsilon_\alpha}\delta(\varepsilon_\alpha-\varepsilon)+\cdots.\end{aligned} \tag{17}$$

By using the identity $(\partial/\partial\varepsilon_\alpha)\delta(\varepsilon_\alpha-\varepsilon)\equiv(\partial/\partial\varepsilon)\delta(\varepsilon_\alpha-\varepsilon)$, we obtain

$$\frac{\partial f(\varepsilon,t)}{\partial t}=-\frac{\partial}{\partial\varepsilon}\left\{A(\varepsilon)\frac{\partial f(\varepsilon,t)}{\partial\varepsilon}+D(\varepsilon)f(\varepsilon,t)[1-f(\varepsilon,t)]\right\}, \tag{18}$$

where

$$A(\varepsilon)=\frac{2\pi}{\hbar}\sum_{\alpha,\beta,\mathbf{q}}|C_{\mathbf{q}}|^2|I_{\alpha\beta}|^2(\hbar\omega_q^\star)^2 \times\delta(\varepsilon_\beta-\varepsilon_\alpha-\hbar\omega_q^\star)N(\mathbf{q},t)\,\delta(\varepsilon_\alpha-\varepsilon), \tag{19}$$

$$D(\varepsilon)=\frac{2\pi}{\hbar}\sum_{\alpha,\beta,\mathbf{q}}|C_{\mathbf{q}}|^2|I_{\alpha\beta}|^2\hbar\omega_q^\star \times\delta(\varepsilon_\alpha-\varepsilon_\beta-\hbar\omega_q^\star)\delta(\varepsilon_\alpha-\varepsilon).$$

The stationary solution of Eq. (18) satisfying boundary condition $\lim_{\varepsilon\to\infty}f(\varepsilon)\to 0$ is

$$f(\varepsilon)=\left\{\text{const}^{-1}\exp\left(\int^{\varepsilon}\frac{d\varepsilon'}{T_e(\varepsilon')}\right)+1\right\}^{-1}, \tag{20}$$

where $T_e(\varepsilon)=A(\varepsilon)/D(\varepsilon)$ is the effective temperature of electrons which occupies the level with energy ε.

The solution of Eq. (13) is

$$N(\mathbf{q},t)=\{N(q,0)+\beta\gamma_q^{-1}\tilde{N}(q)\}\exp(\gamma_q t)-\beta\gamma_q^{-1}\tilde{N}(q), \tag{21}$$

where $\gamma_q=\beta(\mathbf{u}\cdot\mathbf{q}/\hbar\omega_q-1)$ is the increment of the generation of phonons, $\beta=\beta_e+\beta_p+\beta_b$ is the total collision frequency of phonons by the scatterers and

$$\beta_e=\frac{2\pi}{\hbar}\sum_{\alpha,\beta}|C_{\mathbf{q}}|^2|I_{\alpha,\beta}|^2(f_\beta-f_\alpha)\,\delta(\varepsilon_\beta-\varepsilon_\alpha-\hbar\omega_q^\star) \approx\frac{2\pi}{\hbar}\sum_{\alpha,\beta,\mathbf{q}}|C_{\mathbf{q}}|^2|I_{\alpha\beta}|^2(\varepsilon_\beta-\varepsilon_\alpha)\left(\frac{\partial f(\varepsilon_\alpha)}{\partial\varepsilon_\alpha}\right) \times\delta(\varepsilon_\beta-\varepsilon_\alpha-\hbar\omega_q^\star). \tag{22}$$

$N(q,0)\equiv N(q,T)$ is the initial distribution function of phonons at $t=0$ in the absence of external fields

$$u=\left(1-\frac{\beta_p}{\beta}\right)V\equiv\frac{\beta_e}{\beta}V,\quad |I_{\alpha\beta}|^2=|\langle\alpha|\exp(-i\mathbf{q}\cdot\mathbf{r})|\beta\rangle|^2. \tag{23}$$

As it follows from Eq. (21) in the $\gamma_q>0$ case, i.e., when the drift velocity of phonons u is larger than the sound velocity s, the distribution function of phonons $N(\mathbf{q},t)$ increases exponentially with time, whereas in the $\gamma_q<0$ case the solution, (21) is stationary,

$$N(\mathbf{q})=\lim_{t\to\infty}N(\mathbf{q},t)=-\beta\gamma_q^{-1}\tilde{N}(q),$$

$$\tilde{N}(q)=\gamma_e N(\mathbf{q},T_e)+\gamma_p N(q,T_p), \tag{24}$$

where $\gamma_e=\beta_e/\beta$, $\gamma_p=\beta_p/\beta$,

$$N(q,T_e)=\frac{\sum_{\alpha,\beta}|I_{\alpha\beta}|^2T_e(\varepsilon)[\partial f(\varepsilon_\alpha)/\partial\varepsilon_\alpha]\delta(\varepsilon_\beta-\varepsilon_\alpha-\hbar\omega_q^\star)}{\sum_{\alpha\beta}|I_{\alpha\beta}|^2\hbar\omega_q^\star[\partial f(\varepsilon_\alpha)/\partial\varepsilon_\alpha]\delta(\varepsilon_\beta-\varepsilon_\alpha-\hbar\omega_q^\star)}, \tag{25}$$

$$T_e=\frac{\sum_{\alpha,\beta}|I_{\alpha\beta}|^2T_e(\varepsilon)[\partial f(\varepsilon_\alpha)/\partial\varepsilon_\alpha]\delta(\varepsilon_\beta-\varepsilon_\alpha-\hbar\omega_q^\star)}{\sum_{\alpha\beta}|I_{\alpha\beta}|^2[\partial f(\varepsilon_\alpha)/\partial\varepsilon_\alpha]\delta(\varepsilon_\beta-\varepsilon_\alpha-\hbar\omega_q^\star)}, \tag{26}$$

where T_e is the effective temperature of electrons.

In case when $\hbar\omega_q^\star=0$ if $\gamma_p\to 0$, then $\mathbf{u}\cdot\mathbf{q}/\hbar\omega_q=Vq_y/\hbar\omega_q=1$ and from Eq. (18) we have $\partial f(\varepsilon,t)/\partial t=0$. At this point $f(\varepsilon,t)=\text{const}=f(\varepsilon,T_e)$, where $f(\varepsilon,T_e)$ is the distribution function of electrons at $u=s$ with a constant temperature T_e. Because, in Eq. (18) $A(\varepsilon)\sim(\hbar\omega_q^\star)^2=0$ and $D(\varepsilon)\sim(\hbar\omega_q^\star)=0$ or $f(\varepsilon,t)=\text{const}$, i.e., it does not depend on time. Since the region $\hbar\omega^\star>0$ is the region of phonon absorption, electrons make the transition to the ground state as a result of ''dressing by phonons,'' i.e., we have cooling of electrons. At this point the distribution function of phonons is nonstationary and grows by time linearly, $N(q,t)=N(q,T)+\beta tN(q,T_e)$. Indeed, $u=V=s=\text{const}$ and $dN(q,t)/dt=\beta N(q,T_e)$, does not depend on time. Namely, $P(T_e)=\Sigma_q\hbar\omega_q^\star(dN(q,t)/dt)=0$, where $P(T_e)$ is the power transferred by electrons to phonons. The point $u=s$ is the acoustical instability threshold (AIT). At this point, the stimulated emission of phonons is equal to the stimulated absorption of phonons, and we have only spontaneous emission of phonons at high external electric and magnetic fields. At this point collisions of electrons with phonons are exactly elastic, i.e., the state is nondissipative and dynamically stationary because of the power received from the electric field emitted as phonons by the process of the stimulated emission.

Substituting $\partial N(\mathbf{q},t)/\partial t=0$ in Eq. (13), we may directly solve this equation under the boundary conditions

$$N(\mathbf{q},t)\big|_{t=0}=N(\mathbf{q},0)\equiv N(\mathbf{q},T)=\{\exp(\hbar\omega_q/T)-1\}^{-1}, \tag{27}$$

and we obtain Eq. (24). Under the considered conditions $\hbar\omega_q^\star\ll T_e,T$ from Eq. (24), we find

$$\tilde{N}(\mathbf{q})=\frac{\gamma_eT_e+\gamma_pT_p}{\hbar\omega_q}=\frac{\tilde{T}}{\hbar\omega_q}\simeq\tilde{N}(q,\tilde{T}),\quad \tilde{T}=\gamma_eT_e+\gamma_pT_p, \tag{28}$$

where $\tilde{T}$ is the temperature of the electron-phonon system coupled by the mutual drag. Therefore, the stationary solution of Eq. (13) has the form $(\mathbf{u}\cdot\mathbf{q}/\hbar\omega_q<1)$

$$N(\mathbf{q})=\frac{\tilde{N}(\mathbf{q})}{1-\mathbf{u}\cdot\mathbf{q}/\hbar\omega_q}=-\beta\gamma_q^{-1}\tilde{N}(q)\approx\frac{\tilde{T}}{\hbar\omega_q^\star}. \tag{29}$$

Let us substitute Eq. (29) in Eq. (19) and take into account the relations

$$|I_{\alpha\beta}|^2=|I_{NN'}|^2\delta_{p_{y\beta},p_y+q_y}\delta_{p_{z\beta},p_z+q_z}, \tag{30}$$

$$|I_{NN'}|^2=\left(\frac{N!}{N'!}\right)^{1/2}\exp\left(-\frac{q_\perp^2}{q_H^2}\right)\left(-\frac{q_\perp}{q_H}\right)^{N'-N}L_N^{|N'-N|}\left(\frac{q_\perp^2}{q_H^2}\right).$$

Here $q_H=\hbar R^{-1}=(m_n\varepsilon_N)^{1/2}$, $q_\perp^2=q_x^2+q_y^2$, $L_N^{|m|}$ is the Laguerre's polynomial normalized to unity, and $|m|=|N-N'|$. Choosing the cylindrical coordinate system with the axis along the magnetic field **H**

$$dq=q_\perp dq_\perp dq_z d\varphi, \quad q_y=q_\perp\sin\varphi. \tag{31}$$

As a result of the integration, we have

$$A(\varepsilon)=\tilde{T}\Phi(u/s)D(\varepsilon),$$

$$\Phi(u/s)=\varphi-2\frac{V}{u}(\varphi-1)-\left(\frac{V}{u}\right)^2(\varphi-1),$$

$$D(\varepsilon)=\frac{s\Omega}{\hbar(2\pi\hbar)^5}\sum_{N,N'}q_H^3W(q_H)\times\int_0^\infty dx\frac{x^2|I_{NN'}|^2m^2(\varepsilon)}{[B(\varepsilon+\hbar\omega_q^\star)-\varepsilon_{N'}]^{1/2}[B(\varepsilon-\hbar\omega_q^\star)-\varepsilon_N]^{1/2}}, \tag{32}$$

where $x=q_\perp/q_H$, and $W(q_H)$ is the constant part of the electron-phonon mutual interaction potential. Thus, in general, under the arbitrary degree of quantization the ratio $A(\varepsilon)/D(\varepsilon)=\tilde{T}\Phi(u/s)$ does not depend on the energy of electrons and the potential of mutual interactions with phonons. In other words, in the more general case of interaction of electrons with acoustic and optical phonons the expression

$$T_{\text{eff}}=\frac{A(\varepsilon)}{D(\varepsilon)}=\tilde{T}\Phi(u/s) \tag{33}$$

does not depend on the energy of electrons and it is the effective temperature of the interacting electron-phonon system.

By substituting Eq. (33) into Eq. (20), we find

$$f(\varepsilon)=\left\{1+\exp\left(\frac{\varepsilon-\zeta(E,H)}{T_{\text{eff}}}\right)\right\}^{-1}. \tag{34}$$

In other words, at high classic or quantizing magnetic fields the distribution function of electrons in general is a Fermi one, with effective temperature of carriers in accordance with experiments.[28,29,34–36] In quantizing magnetic field

$$T_{\text{eff}}=\tilde{T}\left\{1+\left(\frac{V}{u}-1\right)^2(\varphi_1-1)\right\}, \quad \varphi_1=\left(1-\frac{u^2}{s^2}\right)^{-1/2}. \tag{35}$$

In the classical region of strong magnetic fields ($\Omega\gg\nu$)

$$\Phi(u/s)=\left\{1-\left(\frac{V}{u}-1\right)(\varphi_2-1)+\frac{1}{3}\left(\frac{V}{s}\right)^2\right\},$$

$$\varphi_2=\frac{s}{2u}\ln\left|\frac{s+u}{s-u}\right|. \tag{36}$$

As follows from Eq. (36) in this case the distribution function of electrons is the Fermi one with effective temperature

$$T_{\text{eff}}=\tilde{T}\left\{1-\left(\frac{V}{u}-1\right)(\varphi_2-1)+\frac{1}{3}\left(\frac{V}{s}\right)^2\right\}. \tag{37}$$

The fact that the distribution function of electrons in both classical and quantum regions of magnetic fields are Fermi ones with effective temperature is a result of the independency of drift velocity of electrons $V=cE/H$ on electron energy ε.

At small values of the drift velocity of electrons $V\ll s$ from Eq. (35) for the quantizing magnetic field, we have

$$T_{\text{eff}}\approx\tilde{T}\left\{1+\frac{1}{2}\left(\frac{V}{s}\right)^2-\frac{uV}{s^2}\right\}=\tilde{T}\left\{1+\left(\gamma_p-\frac{1}{2}\right)\left(\frac{cE}{sH}\right)^2\right\}. \tag{38}$$

If $\beta_e\gg\beta_p$, i.e., $\gamma_p\to0$, then $T_{\text{eff}}=\tilde{T}\{1-\frac{1}{2}(cE/sH)^2\}$ and if $\beta_e\ll\beta_p$, $\gamma_p\to1$, then $T_{\text{eff}}=\tilde{T}\{1+\frac{1}{2}(cE/sH)^2\}$. As is seen from this expression in the strong mutual drag case $\gamma_p\to0$ at quantizing magnetic field, we have cooling of electrons by the increasing of their drift velocity (or E).

In the classic region of magnetic field, we must consider two cases: In a strong mutual drag case when $V=u$ and $\gamma_p\to0$ we have

$$T_{\text{eff}}\approx T_e\left\{1+\frac{1}{3}\left(\frac{V}{s}\right)^2\right\}. \tag{39}$$

On the other hand, in a weak mutual drag case when $\gamma_p\to1$, $s\gg V\gg u$ ($\varphi_2-1\approx-1/2$), we have

$$T_{\text{eff}}=T_p\left\{1+\frac{1}{2}\frac{V}{u}+\frac{1}{3}\left(\frac{V}{s}\right)^2\right\}\approx T_p\left\{1+\frac{1}{2}\left(\frac{V}{u}\right)\right\}\approx T_p\left\{1+\frac{1}{2}\frac{cE}{uH}\right\}.$$

If $\beta_p\gg\beta_e$, $\gamma_p\to1$, phonons are not heated, $\tilde{T}=T$, and $V\ll s$. Under these conditions from Eqs. (38) and (39) we can obtain the results of Refs. 19–22.

IV. THE STATISTICS OF ELECTRONS AT HIGH MAGNETIC FIELDS

In the $\mathbf{E}\perp\mathbf{H}$ case, the chemical potential of electrons may be obtained from the normalization condition of the distribution function as

$$n_e=\int^{\infty} d\varepsilon g_H(\varepsilon) f(\varepsilon), \quad g_H(\varepsilon)=\frac{2m_n(eH/c)}{(2\pi\hbar)^2}\sum_N p_z^{-1}(\varepsilon). \tag{40}$$

For arbitrary spherical symmetric spectrum of electrons

$$n_e=\frac{2(2m_n)^{1/2}m_n\Omega}{(2\pi\hbar)^2}\sum_N \int_{\varepsilon_1^\star}^{\infty} d\varepsilon^\star [B(\varepsilon^\star)-\varepsilon_N]^{1/2} \times\left\{1+\exp\left(\frac{\varepsilon^\star-\zeta^\star(E,H)}{T_{\rm eff}}\right)\right\}^{-1}, \tag{41}$$

$$\zeta^\star(E,H)=\zeta(E,H)+eEX(\varepsilon)-\frac{m(\varepsilon)c^2}{2}\left(\frac{E^2}{H^2}\right). \tag{42}$$

The $\varepsilon_1^\star$ is determined as a solution

$$B(\varepsilon^\star)-\varepsilon_N=0. \tag{43}$$

By partial integration of Eq. (41), we find

$$n_e=\frac{4(2m_n)^{1/2}m_n\Omega}{(2\pi\hbar)^2}\sum_N \int_{\varepsilon_1^\star}^{\infty} d\varepsilon^\star [B(\varepsilon^\star)-\varepsilon_N]^{1/2} \times\left(-\frac{\partial f(\varepsilon^\star)}{\partial\varepsilon^\star}\right). \tag{44}$$

For the case of parabolic spectrum of electrons

$$\varepsilon_1^\star=\varepsilon_N=\hbar\Omega\left(N+\frac{1}{2}\right), \tag{45}$$

and for the Kane spectrum of electrons

$$\varepsilon_1^\star=-\frac{\varepsilon_g}{2}\left[1-\left(1+\frac{4\varepsilon_N}{\varepsilon_g}\right)^{1/2}\right]. \tag{46}$$

Let us now consider the case of degenerate and nondegenerate statistics of electrons separately. For strong degenerate electrons

$$\left(-\frac{\partial f_0(\varepsilon^\star)}{\partial\varepsilon^\star}\right)=\delta(\varepsilon-\zeta^\star). \tag{47}$$

With the help of this expression, we can integrate the Eq. (44) and get

$$n_e=\frac{4(2m_n)^{1/2}}{\hbar(2\pi R)^2}\sum_N\left[B(\zeta^\star)-\hbar\Omega\left(N+\frac{1}{2}\right)\right]^{1/2}. \tag{48}$$

For the parabolic spectrum of electrons

$$n_e=\frac{4(2m_n)^{1/2}}{\hbar(2\pi R)^2}\sum_N\left[\zeta^\star-\hbar\Omega\left(N+\frac{1}{2}\right)\right]^{1/2}. \tag{49}$$

For the Kane spectrum of electrons

$$n_e=\frac{4(2m_n)^{1/2}}{\hbar(2\pi R)^2}\sum_N\left[\zeta^\star\left(1+\frac{\zeta^\star}{\varepsilon_g}\right)-\hbar\Omega\left(N+\frac{1}{2}\right)\right]^{1/2}. \tag{50}$$

For ultraquantum limits $(N=N'=0)$, we have

$$n_e=\frac{4(2m_n)^{1/2}}{\hbar(2\pi R)^2}\left[B(\zeta^\star)-\frac{\hbar\Omega}{2}\right]^{1/2} \Rightarrow\left[B(\zeta^\star)-\frac{\hbar\Omega}{2}\right]=\frac{\hbar^2(2\pi R)^4 n_e^2}{16(2m_n)}. \tag{51}$$

From this relation for the parabolic spectrum of electrons we obtain the chemical potential as follows:

$$\zeta^\star(E,H)=\frac{\hbar\Omega}{2}+\frac{\pi^4\hbar^4 n_e^2}{2m_n^3\Omega^2}. \tag{52}$$

This expression is the same as Eqs. (32) and (22) in Ref. 54.

In the case of Kane spectrum of electrons

$$\zeta^\star(E,H)=-\frac{\varepsilon_g}{2}\left\{1-\left(1+\frac{2\hbar\Omega}{\varepsilon_g}+\frac{2\pi^4\hbar^2R^4n_e^2}{m_n\varepsilon_g}\right)^{1/2}\right\}. \tag{53}$$

In the case of nondegenerate electrons by taking $[B(\varepsilon^\star)-\varepsilon_N]$ out of the integral when $\varepsilon^\star=\varepsilon_1^\star(N)+T_e$, we find

$$\exp\left(\frac{\zeta^{\star\star}(E,H)}{T_e}\right)=\frac{(2\pi R)^2\hbar n_e}{4(2m_n)^{1/2}}\left\{\sum_N \exp\left(-\frac{\varepsilon_1^\star(N)}{T_e}\right) \times[B(\varepsilon_1^\star(N)+T_e)-\varepsilon_N]^{1/2}\right\}^{-1}. \tag{54}$$

As before $\varepsilon_1^\star$ is obtained from Eqs. (45) and (46). If the condition $\varepsilon_1^\star \gg T_e$ is satisfied, then by expanding $B(\varepsilon_1^\star+T_e)$ into series in Eq. (54) and by taking into account Eq. (43), we have

$$\exp\left(\frac{\zeta^{\star\star}(E,H)}{T_e}\right)\simeq\frac{(2\pi R)^2\hbar n_e}{4\sqrt{2}}T_e^{-1/2} \times\left[\sum_N \exp\left(-\frac{\varepsilon_1^\star(N)}{T_e}\right)m^{1/2}(\varepsilon_1^\star)\right]^{-1}, \tag{55}$$

where

$$m(\varepsilon_1^\star)=m_n\left(\frac{\partial B(\varepsilon^\star)}{\partial\varepsilon^\star}\right)_{\varepsilon^\star=\varepsilon_1^\star}. \tag{56}$$

Dividing Eq. (54) by itself for $E=0$, we get

$$\zeta^{\star\star}(E,H)=\zeta(H)\frac{T_e}{T}+T_e\ln\left(\frac{n_e}{n_0}\right)+T_e\ln\left[\frac{F_N(T)}{F_N(T_e)}\right], \tag{57}$$

where

$$\zeta^{\star\star}(E,H)=\zeta(E,H)-\frac{m}{m_n}\frac{m_nc^2}{2}\frac{E^2}{H^2}, \tag{58}$$

$$F_N(T_e)=\sum_N \exp\left(-\frac{\varepsilon_1^{\star}(N)}{T_e}\right)\{B(\varepsilon_1^{\star}(N)+T_e)-\varepsilon_N\}^{1/2}. \tag{59}$$

$F_N(T)$ may be obtained from Eq. (59) by replacing T_e with T.

For the case of parabolic spectrum of electrons $\varepsilon_1^{\star}=\varepsilon_N=\hbar\Omega(N+1/2)$ and we have

$$\zeta^{\star\star}(E,H)=\zeta(H)\frac{T_e}{T}+T_e\ln\left(\frac{n_e}{n_0}\right)-\frac{T_e}{2}\ln\left(\frac{T_e}{T}\right)+T_e\ln\left[\frac{\sinh(\hbar\Omega/2T_e)}{\sinh(\hbar\Omega/2T)}\right]. \tag{60}$$

Therefore, in the case of parabolic spectrum of electrons we finally find the chemical potential

$$\zeta(E,H)=\zeta(H)\frac{T_e}{T}-eEX+\frac{m_nc^2}{2}\frac{E^2}{H^2}+T_e\ln\left(\frac{n_e}{n_0}\right)-\frac{T_e}{2}\ln\left(\frac{T_e}{T}\right)+T_e\ln\left[\frac{\sinh(\hbar\Omega/2T_e)}{\sinh(\hbar\Omega/2T)}\right]. \tag{61}$$

As follows from Eq. (61) in an external electric field if the concentration of electrons is increased ($n_e>n$), then the chemical potential of electrons must also increase.

If we have full ionization of small impurity centers, then $n_e=n_0=$const Thus, from Eq. (61), we may obtain

$$\zeta(E,H)-\frac{\hbar\Omega}{2}=\frac{T_e}{T}\left[\zeta(H)-\frac{\hbar\Omega}{2}\right]+\frac{m_nc^2}{2}\frac{E^2}{H^2}-\frac{T_e}{2}\ln\left(\frac{T_e}{T}\right)+T_e\ln\left[\frac{1-\exp(\hbar\Omega/2T_e)}{1-\exp(\hbar\Omega/2T)}\right]. \tag{62}$$

In quantizing magnetic fields $\hbar\Omega>T_e,T$ and, for this reason, the last term in Eq. (62) may be presented as $(T_e/T)(\hbar\Omega/2)$. In this case

$$\zeta(E,H)-\frac{\hbar\Omega}{2}=\frac{T_e}{T}[\zeta(H)-\hbar\Omega]+\frac{m_nc^2}{2}\frac{E^2}{H^2}-\frac{T_e}{2}\ln\left(\frac{T_e}{T}\right). \tag{63}$$

For the weak electric fields $T_e=T$,

$$\zeta(E,H)=\zeta(H)-\frac{\hbar\Omega}{2}+\frac{m_nc^2}{2}\frac{E^2}{H^2}. \tag{64}$$

As follows from Eq. (64), the expression for $\zeta(E,H)$ differs from the expression given in Ref. 54 by the factor $(m_nc^2/2)(E^2/H^2)$, which is connected with the Hall drift of electrons. In the case of heating of electrons at external electric field the main contribution to the free energy is obtained by the expression $\zeta(H)(T_e/T)$, i.e., in high external electric field the chemical potential of electrons in common case increases linearly by T_e/T.

For the calculation of the statistical behavior of electrons in external fields such as magnetic susceptibility, heat capacity, etc., it is necessary to know the dependence of the chemical potential on the intensity of external electric and magnetic fields. The magnetic susceptibility of hot electrons in the case of high concentration of electrons was investigated earlier.[55]

V. DISCUSSIONS

In the present work, it is shown that under the conditions of arbitrary degree of quantization and for the interaction of electrons with both the acoustic and optical phonons, the distribution function of electrons has the form of a Fermi distribution function with effective electron temperature. This result is obtained by taking into account the heating of electron and phonons and their arbitrary mutual drag under the conditions where the usual ''effective temperature approximation'' for electrons is not satisfied, i.e., $\nu_{ee}\ll\nu_{ep}$. The distribution functions of electrons and phonons are obtained as a result of the solution of coupled systems of equations for the density matrices of interacting electrons and phonons for arbitrary heating and drift velocities of phonons. It is shown that if the drift velocity of phonons (or the common drift velocity of the coupled system which is formed by the mutual drag of electrons and phonons) u is smaller than the sound velocity s, then the distribution function of phonons is stationary and has the shifted Planck's distribution function form with effective temperature of phonons. In the region of drift velocities $u\geqslant s$ the distribution function of phonons is nonstationary. If $u>s$, the distribution function of phonons grows with time exponentially, i.e., we have generation or amplification of phonons in external electric field. The amplification coefficient of phonons is given by $\Gamma_q=(\beta/s)[\mathbf{u}\cdot\mathbf{q}/\hbar\omega_q-1]$ and the generation coefficient (or increment of grow) is given by $\beta[\mathbf{u}\cdot\mathbf{q}/\hbar\omega_q-1]$. At the $u=s$ point the distribution function of phonons grows with time linearly. This point is the point of the acoustical instability threshold. At this point the spontaneous emission of phonons takes place.

The region of drift velocities $0<u<s$ is the region of absorption of phonons, the region of drift velocities $u>s$ is the region of stimulated emission of phonons, and the point $u=s$ is the point of spontaneous emission of phonons. In general, $u=(\beta_e/\beta)V<V$ and the point of AIT ($u=s$) do not coincide with the point where the Cherenkov's emission of phonons $V=s$ starts. However, only in the case of strong mutual drag $\beta=\beta_e$ (or $\beta_p=0$), these two threshold coincide, i.e., $u=V=S$.

As was mentioned above, the main problem of the theory of hot electrons in high electric and quantizing magnetic fields is the possibility of using the approximation of ''effective temperature'' of carriers. The main difficulty connected with this problem is that for the nondegenerate statistics in the quantum limit all electrons occupy only the lowest Landau level and interelectronic collisions become ineffective if

$\hbar\Omega \gg T, T_e$. As is shown in this paper, under the condition of ultraquantum limit ($\hbar\Omega \gg T$), as a result of the renormalization of the mass of "carriers dressed by phonons" ($m \to M = T/s^2$ or T_e/s^2), the cyclotron frequency decreases ($\Omega^\star = eHs^2/Tc \ll \Omega$), and the transition from ultraquantum limit to the classic region of the magnetic field takes place. Thus, under the same H and n electrons occupy many Landau levels and the interelectronic collisions become effective and the "effective temperature" approximation may be used if $n > n_{cr}$. Moreover, in the low electron concentration case ($n < n_{cr}$), the direct solution of the coupled system of equations for electrons and phonons density matrices shows that in high external electric and classically high or quantizing magnetic fields the distribution function of electrons is the Fermi-Dirac or Maxwell-Boltzmann distribution function with effective electron temperature. In earlier investigations,[19–22] this problem was considered by taking into account only the heating of electrons, and was acceptable only in the case of high lattice temperatures and drift velocities $V \ll s$. However, under these conditions the heating of electrons is negligible because under these conditions it starts from the drift velocities $V \geqslant s$. In contrast to Refs. 19–22, in the present paper we consider the case of arbitrary heating of electrons and phonons and their arbitrary mutual drag. Moreover, the region of drift velocities, u or V, larger or smaller than the sound velocity s is considered. In Refs. 19–22, in the dispersion relation the term which is linear in the electric field intensity was neglected. It leads to a loss of some effects. As is shown in the present paper, considering this term is equivalent to renormalization of phonon energy in the argument of the δ function as $\hbar\omega^\star = \hbar\omega - Vq_y = \hbar\omega_q - cEq_y/H$. In other words, it leads to the change of the initial Planck's distribution function and consider the Doppler's shift in the phonon frequency in the drifted reference frame. Moreover, this term also considers the transition from the absorption to emission regime by increasing the drift velocity of electrons V.

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