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Quantum magnetic oscillations of the chemical potential in superlattices and layered conductors

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A theory is presented for the chemical potential oscillations in superlattices in a quantizing magnetic field applied perpendicular to the layers which takes into account the electronic miniband structure attached to each Landau level. This miniband structure in the occupation below the Fermi energy reduces the amplitude of the chemical potential oscillations through a specific integral factor which depends on the density of states related to the electronic motion across the layers.

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I. INTRODUCTION

Recently a good deal of theoretical activity has been devoted to magnetoquantum oscillations of the chemical potential in view of unusual phenomena observed in magnetoquantum experiments on layered organic quasi-two-dimensional (2D) conductors. The de Haas–van Alphen[1–3] (dHvA) and Shubnikov–de Haas[4–9] (SdH) studies of the electronic system in these low-dimensional organic conductors (see Ref. 10) have shown numerous deviations from the standard Lifshitz-Kosevich theory[11] which is usually used in conventional three-dimensional (3D) metals for the analysis of magnetoquantum phenomena in Fermi-surface investigations.

In 2D conductors the Landau-level spectrum is discrete and the chemical potential jumps between successive levels each time the number of electrons in the system N is an integer multiple n of the Landau levels degeneracy such that $N=n\Phi/\Phi_0$, where Φ denotes the flux through a sample and Φ_0 the flux quantum. These jumps give rise to a sawtooth profile of the magnetization versus inverse magnetic field provided the number of electrons N in the system is fixed, as was first pointed out by Peierls as early as 1933.[12] For a 2D system with a constant chemical potential μ, the magnetization has the shape of a sawtooth with an inverted profile of the teeth (the so called "inverted sawtooth").[13,14] In a pure 2D electron gas it is impossible to have a constant chemical potential, and a special reservoir should exist for the electron states in order to pin the chemical potential.

The shape of the magnetization pattern as a function of the inverse magnetic field and the influence of the chemical potential variation has been the subject of recent theoretical publications.[15–20] However, some points remained unclear since they were in conflict with recent experiments. In particular, the observation of an inverted saw tooth magnetization in the layered organic conductor β''-(BEDT-TTF)$_2$SF$_5$CH$_2$CF$_2$SO$_3$ implies a constant chemical potential.[21] The pinning of the chemical potential in this system at the 1D open sheets of the Fermi surface remains controversial.[22] The nontrivial shape of the dHvA oscillations observed in the layered magnetic breakdown organic conductor κ-(BEDT-TTF)$_2$Cu(NCS)$_2$ is well described by the theory under the assumption of a fixed chemical potential.[23] However, the nature of the electron reservoir which pins the chemical potential in layered organic conductors remains unclear.

In the case of a 3D conventional metal, the smallness of the variation of the chemical potential is related to the free electron motion along the magnetic field characterized by the z component of the momentum k_z and the kinetic energy $\xi=\hbar^2k_z^2/2m$. The energy spectrum in that case depends on two quantum numbers, the Landau level index n and k_z, such that

$$\varepsilon_n(k_z)=\hbar\omega_c\left(n+\frac{1}{2}\right)+\frac{\hbar^2k_z^2}{2m}, \tag{1}$$

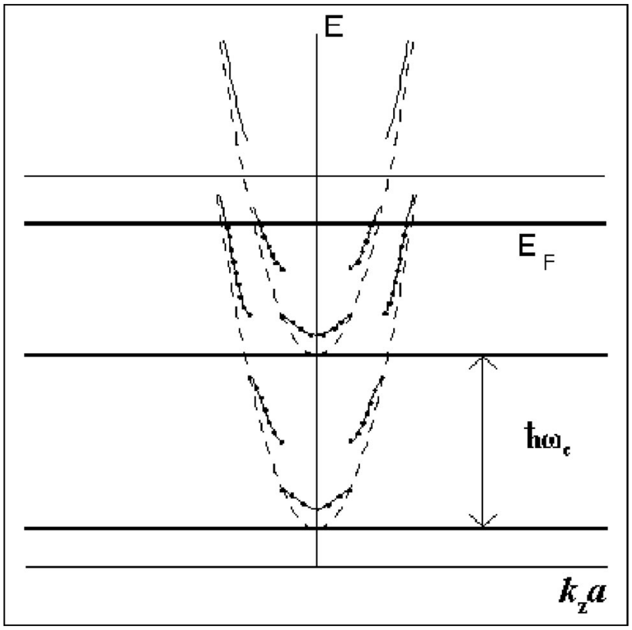

FIG. 1. The Landau energy spectrum for 2D, 3D, and superlattice systems. The 2D spectrum (solid horizontal linesis with separation $\hbar\omega_c$) is flat and show no dispersion along the k_z direction. The 3D Landau levels have parabolic dispersion (dashed lines). In superlattices, the parabolas are splitted into minibands which are populated below the Fermi level E_F (shown by dotted lines).

where $\omega_c = eB/cm$ is the cyclotron frequency and $\hbar$ the Planck's constant.

Since the Landau levels $\varepsilon_n(k_z)$ have a parabolic k_z dispersion for 3D conductors (see Fig. 1), only a small bottom fraction of the electrons from the parabola above the Fermi level drops to the next-lower Landau level with the increase of the magnetic field. At the same time numerous electrons which occupy the other parabolic branches pin the chemical potential at the Fermi level. This is in a sharp contrast to the 2D case, where the Landau levels are flat and the chemical potential jumps between the two upper populated levels with the amplitude $\hbar\omega_c$ as shown in Fig. 1. In the 3D case, the amplitude of the chemical potential oscillation, being $\hbar\omega_c\sqrt{\hbar\omega_c/E_F}$, is strongly reduced for $\hbar\omega_c \ll E_F$ (E_F is the Fermi level).

The case of superlattices or layered conductors lays between the 2D and 3D ones: the parabolas split into an alternating series of permitted and forbidden minibands whose width depends on the parameters of the superlattice. These bands are indicated by the solid lines in Fig. 1. The population of the minibands below the Fermi level E_F stabilizes the chemical potential and decreases the amplitude of its variation with the magnetic field. We show in what follows that the chemical potential oscillation amplitude in superlattices is in between the above given values $\hbar\omega_c\sqrt{\hbar\omega_c/E_F}$ and $\hbar\omega_c$ for the two extreme cases. The major aim of this paper is to give a theoretical description of the variation of the chemical potential $\mu(B)$ with the external magnetic field valid for 3D, quasi-2D (superlattices and layered systems), and 2D systems. We find that the oscillating part of the chemical potential $\mu(B)$ is proportional to the quantity $\hbar\omega_c/D(E_F)$ in a system of any dimensionality. The reduction of the chemical-potential oscillation amplitude is given by the coefficient

$$D(E_F) = \int_{-\infty}^{E_F} d\xi g(\xi), \tag{2}$$

as well as the oscillating layer factor I_p (see below) which depend on the density of states (DOS) related to the electron motion across the layers $g(\xi)$. The magnetic field B is assumed to be perpendicular to the conducting layers.

II. BASIC EQUATIONS

The point of departure in our approach is the Landau energy-spectrum which we write in the following form:

$$\varepsilon_n(\xi) = \hbar\omega_c(n+1/2) + \xi. \tag{3}$$

Here ξ is the electron energy related to the motion along the magnetic field and distributed with the DOS $g(\xi)$. The density of states of the whole system with the energy spectrum $\varepsilon_n(\xi)$ is given by

$$\rho(\varepsilon,B) = s\int_{-\infty}^{\infty} d\xi g(\xi) \sum_{n=0}^{\infty} \delta[\varepsilon - \hbar\omega_c(n+1/2) - \xi]. \tag{4}$$

The quantity $s = \Phi/\Phi_0$ is the degeneracy of the Landau level. Using the Poisson summation formula

$$\sum_{n=0}^{\infty} f(n+1/2) = \int_0^{\infty} f(x)dx + 2\,\mathrm{Re}\sum_{p=1}^{\infty} (-1)^p \times \int_0^{\infty} f(x) e^{2\pi i p x} dx. \tag{5}$$

Equation (4) can be rewritten as a sum of monotonous and oscillating terms

$$\rho(\varepsilon,B) = \rho_0(\varepsilon,B) + \tilde{\rho}(\varepsilon,B), \tag{6}$$

where the monotonous part of the density of states is given by

$$\rho_0(\varepsilon,B) = \frac{s}{\hbar\omega_c}\int_{-\infty}^{\infty} d\xi g(\xi)\,\theta(\varepsilon-\xi) = \frac{s}{\hbar\omega_c}\int_{-\infty}^{\varepsilon} d\xi g(\xi) \tag{7}$$

[$\theta(x)$ is the Heavyside step function]. The oscillating part can be written as

$$\tilde{\rho}(\varepsilon,B) = \frac{s}{\hbar\omega_c}\mathrm{Re}\sum_{p=1}^{\infty} (-1)^p \exp\left(\frac{2\pi i p \varepsilon}{\hbar\omega_c}\right) \times \int_{-\infty}^{\varepsilon} d\xi g(\xi) \exp\left(-\frac{2\pi i p \xi}{\hbar\omega_c}\right). \tag{8}$$

Since the ratio $s/\hbar\omega_c$ does not depend on the magnetic field, the quantity $\rho_0(\varepsilon,B)$ is in fact independent of B.

So far we have not paid attention to the disorder which is known to deteriorate magnetic quantum oscillations. Impurities broaden the δ-function in Eq. (4) into a Lorentzian resulting in the appearance of the Dingle factor

$$R_D(\varepsilon,p)=\exp[-2\pi p\Gamma(\varepsilon)/\hbar\omega_c] \tag{9}$$

in Eq. (8). The imaginary part of the electron self-energy $\Gamma(\varepsilon)$ in general is a function of the energy. In the case of a conventional 3D metal it is a good approximation to take $\Gamma(\varepsilon)$ independent of the energy, i.e., $\Gamma(\varepsilon)=\Gamma=\hbar/\tau$, where τ is the electron scattering time.[13,24] In layered conductors $\Gamma(\varepsilon)$ becomes an oscillating function of the external magnetic field[25]

$$\Gamma=\frac{\Gamma(\varepsilon)}{1+2\,\mathrm{Re}\sum_{p=1}^{\infty}(-1)^p R_D(\varepsilon,p)I(p,E_F)\cos\left(\frac{2\pi p\varepsilon}{\hbar\omega_c}\right)}. \tag{10}$$

The layer factor $I(p,E_F)$, given by

$$I(p,E_F)=\int_{-\infty}^{E_F}d\xi g(\xi)\exp\left(-\frac{2\pi ip\xi}{\hbar\omega_c}\right), \tag{11}$$

is an important ingredient of the magnetic quantum oscillations in superlattices. It is an oscillating function of the inverse magnetic field $1/B$ responsible for the modulation of the dHvA (Refs. 23,26,27) as well as SdH (Ref. 25) oscillations due to the structural peculiarities of superlattices. For $p=0$, the layer factor $I(p=0,E_F)$ reduces to the coefficient $D(E_F)$ in Eq. (2).

Thus, the oscillating part of the density of states in Eq. (8) can be rewritten as

$$\tilde{\rho}(\varepsilon,B)=\frac{2s}{\hbar\omega_c}\mathrm{Re}\sum_{p=1}^{\infty}(-1)^p\exp\left(\frac{2\pi ip\varepsilon}{\hbar\omega_c}\right)R_D(\varepsilon,p)I(p,\varepsilon). \tag{12}$$

Having at hand the density of states one can calculate the thermodynamic potential in a standard fashion

$$\Omega(\mu,B,T)=-T\int_0^{\infty}\rho(\varepsilon,B)\ln\left[1+\exp\left(\frac{\mu-\varepsilon}{T}\right)\right]d\varepsilon. \tag{13}$$

Here μ is the chemical potential and T temperature. With the help of Eq. (6) the thermodynamic potential can be written as a sum of two terms $\Omega(\mu,B,T)=\Omega_0(\mu,B,T)+\tilde{\Omega}(\mu,B,T)$. The oscillating part $\tilde{\Omega}(\mu,B,T)$ determines all types of thermodynamic oscillations in an electronic system.

The chemical potential $\mu(B)$ is a solution to the equation

$$N=\left(\frac{\partial\Omega}{\partial\mu}\right)_{T,B}=\int_0^{\infty}\frac{\rho(\varepsilon,B)}{1+\exp\left(\frac{\varepsilon-\mu}{T}\right)}d\varepsilon=\text{const}. \tag{14}$$

We have to make some approximations in this equation as wells as in Eq. (12) to go ahead. We first note that because of the sharpness of the Fermi distribution function at low temperatures only energies within the vicinity of μ in the integral of Eq. (14) are essential ($\mu\gg\hbar\omega_c$ and is of the order of the E_F). Therefore the factor $I(p,\epsilon)$ can be replaced by $I(p,E_F)$ given by Eq. (11). We also, following Refs. 18–20, neglect the energy dependence in the Dingle factor approximating it by the exponent $R_D(\varepsilon,p)\approx R_D(p)=\exp(-2\pi p\Gamma/\hbar\omega_c)$. For the 3D and quasi-2D cases it is an acceptable approximation (in the 3D case, for example, it is a well established standard approach[13,24]). One could take account for small corrections to $R_D(p)$ in the dirty limit from the power series expansion of Eq. (10) as was done previously.[28] In order to simplify calculations, we do not consider these corrections here since they are irrelevant to the main point of our subject.

So far we did not give a definition of the Fermi energy E_F. We will define E_F as the chemical potential at zero magnetic field, i.e., $E_F=\mu(B=0)$. Using this definition together with Eqs. (12) and (14), and the above discussed approximation for the Dingle factor by $R(p)$, we have

$$\Delta(\mu)=\mathrm{Re}\sum_{p=1}^{\infty}\frac{(-1)^p}{\pi p}\exp\left(\frac{2\pi ip\mu}{\hbar\omega_c}\right)\frac{i\lambda p}{\sinh(\lambda p)}\times I(p,E_F)\exp\left(-\frac{2\pi p\Gamma}{\hbar\omega_c}\right), \tag{15}$$

where the function $\Delta(\mu)$ is defined by the equation

$$\Delta(\mu)=-\frac{1}{\hbar\omega_c}\int_{\mu}^{E_F}d\varepsilon\int_{-\infty}^{\varepsilon}d\xi g(\xi). \tag{16}$$

Since $\mu\approx E_F$ we can approximate $\Delta(\mu)$ as

$$\Delta(\mu)\approx\frac{1}{\hbar\omega_c}(\mu-E_F)D(E_F), \tag{17}$$

where the factor $D(E_F)$ has been introduced earlier by Eq. (2). We also used that for $\mu\gg k_BT$ holds

$$\int_{-\infty}^{\infty}\frac{e^{\xi}}{(e^{\xi}+1)^2}e^{i\alpha\xi}d\xi=\frac{\pi\alpha}{\sinh(\pi\alpha)}. \tag{18}$$

The equation for the chemical potential (16) can be written as

$$\mu=E_F+\frac{\hbar\omega_c}{D(E_F)}\mathrm{Re}\sum_{p=1}^{\infty}\frac{i(-1)^p}{\pi p}\exp\left(\frac{2\pi ip\mu}{\hbar\omega_c}\right)\times R_T(p)I(p,E_F)\exp\left(-\frac{2\pi p\Gamma}{\hbar\omega_c}\right). \tag{19}$$

This equation is the main result of the present publication. Here we introduced a standard notation for the temperature damping factor

$$R_T(p)=\frac{\lambda p}{\sinh(\lambda p)}, \tag{20}$$

with $\lambda\equiv 2\pi^2k_BT/\hbar\omega_c$.

Now one can easily single out an oscillating part of the chemical potential $\tilde{\mu}(B)\equiv\mu-E_F$ by a trivial shift of the Fermi energy. We see that the amplitude of $\tilde{\mu}(B)$ in Eq. (19) is not only much less than the Fermi energy, i.e., $\tilde{\mu}(B)\ll E_F$. It is less than $\hbar\omega_c$ because of the large factor $D(E_F)$

in the denominator and due to the smallness of the Dingle, temperature, and layer factors.

It is also possible to set up a relationship between the oscillations of the chemical potential and the magnetization, i.e., the dHvA effect, using the derivative of the oscillating part of the thermodynamical potential $\tilde{M}(B)=-\partial\tilde{\Omega}/\partial B|_{\mu,N=\mathrm{const}}$. Retrieving only the highest terms due to the derivative of the rapid oscillations of the exponent $\exp(2\pi ip\mu/\hbar\omega_c)$ in Eqs. (12) and (13) we have

$$\tilde{M}(B)=\frac{E_F}{B}\frac{s}{\hbar\omega_c}D(E_F)\tilde{\mu}(B). \tag{21}$$

The relative amplitude of the magnetization oscillations is much higher than that of the chemical potential. Because $\tilde{\mu}(B)$ is proportional to the factor $D(E_F)^{-1}$, the dHvA amplitude $\tilde{M}(B)$ is independent on $D(E_F)$. The proportionality between the magnetization $\tilde{M}(B)$ and $\tilde{\mu}(B)$ has been established previously.[18,19,29] Our more general result is obtained in terms of the density of states $g(\varepsilon)$ and is valid both for periodic and irregular superlattices for which the momentum k_z is no longer a good quantum number. Note that Eq. (21) is valid approximately only for the case of layered electron gas. The case of layered organic conductors with more complex 2D Fermi surface will be considered elsewhere.

We consider now the limiting 2D and 3D cases. In the 2D case the DOS is given by $g(\xi)=\delta(\xi)$ and the factor $D(E_F)=1$. The equation for the chemical potential then takes the known form[20]

$$\mu=E_F+\hbar\omega_c\sum_{p=1}^{\infty}\frac{(-1)^{(p+1)}}{\pi p}\sin\left(\frac{2\pi p\mu}{\hbar\omega_c}\right)\times R_T(p)\exp\left(-\frac{2\pi p\Gamma}{\hbar\omega_c}\right). \tag{22}$$

At zero temperature the factor $R_T(p)=1$ and a series for the oscillating part of the chemical potential in this equation can be calculated exactly to yield

$$\mu=E_F+\frac{\hbar\omega_c}{\pi}\arctan\left(\frac{\sin(y)}{e^b+\cos(y)}\right), \tag{23}$$

where $y\equiv 2\pi\mu/\hbar\omega_c$ with disorder parameter $b\equiv 2\pi\Gamma/\hbar\omega_c$. This equation for $\mu(B)$ was numerically investigated.[19] The variation $\tilde{\mu}(B)\equiv\mu-E_F$ has a sawtooth profile with the amplitude $\hbar\omega_c$ when $b=0$. The amplitude decreases and the teeth profile become more rounded with the increase of the disorder. But in the 3D case the amplitude of the chemical potential oscillations is much smaller. In that case

$$\xi=\frac{\hbar^2k_z^2}{2m}\quad\text{and}\quad g(\xi)=\frac{1}{2\pi\hbar}\sqrt{\frac{2m}{\xi}}, \tag{24}$$

and the factors $D(E_F)$ and $I(p,E_F)$ are given by

$$I(p,E_F)=\frac{\sqrt{2m}}{2\pi\hbar}\exp(-i\pi/4)\sqrt{\frac{\hbar\omega_c}{2p}}$$

and

$$D(E_F)=\frac{\sqrt{2mE_F}}{\pi\hbar}. \tag{25}$$

Substituting these equations into Eq. (16) we obtain the Lifshitz-Kosevich equation

$$\mu=E_F+\hbar\omega_c\sqrt{\frac{\hbar\omega_c}{E_F}}\mathrm{Re}\sum_{p=1}^{\infty}\frac{(-1)^p}{2\sqrt{2}\pi p^{3/2}}\times\exp\left(\frac{2\pi ip\mu}{\hbar\omega_c}-i\frac{\pi}{4}\right)R_T(p)\exp\left(-\frac{2\pi p\Gamma}{\hbar\omega_c}\right). \tag{26}$$

In the LK theory[11] for the magnetic oscillations in 3D conventional metals it is assumed that the chemical potential is approximately equal to the Fermi energy $\mu\approx E_F$. This follows from the inequality $\hbar\omega_c\ll E_F$, which makes the amplitude of the chemical oscillations small because $\sqrt{\hbar\omega_c/E_F}\ll 1$. In the next section we will consider the case of a superlattice.

III. CHEMICAL POTENTIAL OSCILLATIONS IN A SUPERLATTICE

The chemical potential oscillations in superlattices can be considered as an intermediate case between the above discussed 2D and 3D cases. The periodic potential in the direction perpendicular to layers splits the parabolic branches of the kinetic energy $\xi=\hbar^2k_z^2/2m$ into minibands attached to each Landau level. Electronic states in minibands below the Fermi level decrease the oscillations amplitude of the $\mu(B)$. Formally this is because the factor $D(E_F)$ in Eq. (19) is approximately equal to the number of minibands, i.e., more than unity. The layer factor is small and also damp these oscillations $I(p,E_F)\ll 1$ for $p\geqslant 1$.

The key quantity in our approach is the density of states $g(\xi)$ since it determines the factors $D(E_F)$ and $I(p,E_F)$ both for regular and irregular layer stacking. We consider in this section a simple model of a superlattice with the periodic δ potential

$$V(z)=u_0\sum_{n=-\infty}^{\infty}\delta(z-na). \tag{27}$$

The dispersion relation for this potential is well known:[30]

$$\cos(k_za)=\cos(\kappa a)-\frac{mu_0a}{\hbar^2}\frac{\sin(\kappa a)}{\kappa a}, \tag{28}$$

and relates the quasimomentum k_z with the energy ε through the quantity $\kappa(\varepsilon)=\sqrt{2m\varepsilon/\hbar^2}$. The density of states can be calculated as follows:

$$g(\varepsilon)=\frac{1}{\pi}\left|\frac{dk_za}{d\varepsilon}\right|. \tag{29}$$

Using the dispersion relation, we have

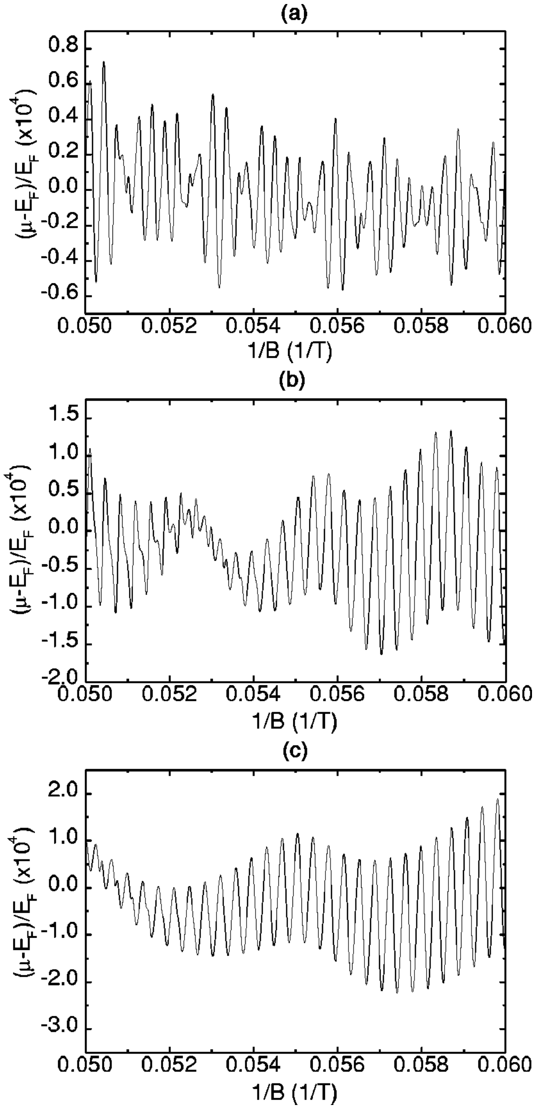

FIG. 2. The chemical potential oscillations in a superlattice as a function of the inverse magnetic field at $T=1.2$ K and $2\pi\Gamma/k_B=0.84$ K for transparency factors $\gamma=1$ (a), $\gamma=50$ (b), and $\gamma=100$.

$$g(\varepsilon)=\frac{1}{\pi\sqrt{1-f(\varepsilon)^2}}\left|\frac{df}{d\epsilon}\right|, \qquad (30)$$

with the dispersion function

$$f(\varepsilon)=\cos(\kappa a)-\gamma\frac{\sin(\kappa a)}{\kappa a} \qquad (31)$$

for the parameter $\gamma=u_0 ma/\hbar^2$ depending on the strength of the δ potential of the layered structure. The energy spectrum of a superlattice consists of alternating bands of forbidden states for $1-f(\varepsilon)^2<0$ and permitted states for $1-f(\varepsilon)^2\geqslant 0$. The band positions are determined by the transcendental equation $1-f(\varepsilon)^2=0$ which can be solved only numerically. The factors $D(E_F)$ and $I(p,E_F)$ to be used in the numerical analysis of the chemical potential $\mu(B)$ oscillations can also be calculated only numerically.

The results of this numerical analysis are shown in Figs. 2 and 3 where oscillations of the chemical potential and their Fourier transforms are presented for superlattices with three different transparencies γ of the barriers between the layers. For the superlattice parameters we have taken a layer thickness $a=10$ Å, a Fermi energy $E_F=3600$ K, and an effective electron mass $m=5m_e$ (free electron mass m_e). The choice of these parameters reflects more or less a layered organic conductor. For the Dingle parameter we have taken $2\pi\Gamma/k_B=0.84$ K. Figures 2 and 3 show the results of the chemical potential variations at $T=1.2$ K for superlattices with $\gamma=1$, 50, and 100. The γ parameter determines the transparency coefficient $T(\varepsilon)^{-1}=1+\gamma u_0/\varepsilon a$ which increases with decreasing γ (the barrier energy u_0 is proportional to γ). For the same parameter set Fig. 4 shows the result for the Lifshitz-Kosevich formula of a 3D metal.

The normalized amplitude $\tilde{\mu}(B)/E_F$ of the chemical potential oscillations in Fig. 2 equals approximately 6×10^{-5} for $\gamma=1$, 1×10^{-4} for $\gamma=50$, and 2×10^{-4} for $\gamma=100$. In the 3D case this relative amplitude is approximately 4×10^{-5} which is of the same order of magnitude as for the superlattice with the most transparent barriers corresponding to $\gamma=1$. For the limiting field values in Fig. 2, the ratio $\hbar\omega_c/E_F$ varies from 2.7×10^{-3} at 16.7 T to 3.2×10^{-3} at 20 T. Compared to the 2D case where the chemical potential jumps between the Landau levels with the relative amplitude $\hbar\omega_c/E_F$, the amplitudes of the oscillations in a superlattice description are suppressed by an additional factor $10^{-1}-10^{-2}$ depending on the value of the transparency parameter γ.

The two major factors reducing the ratio $(\mu-E_F)/E_F$ are given by $I(p,E_F)$ and $D(E_F)=I(0,E_F)$. The factor $I(p,E_F)$ not only decreases the amplitude of the chemical potential oscillations but also adds new frequencies $\varepsilon_k mc/\hbar e$ making a more rich Fourier transform content as shown in Fig. 3. These new frequencies appear due to the band structure caused by the periodic potential of the superlattice. The factor $I(p,E_F)$ oscillates and thereby produces beats seen clearly in Fig. 2. These beats are absent in Fig. 4 for oscillations of the chemical potential in the conventional 3D Lifshitz-Kosevich case. The amplitude of the chemical potential oscillations depends only weakly on the coefficient γ. The parameter γ controls the transparency of the barriers which changes the band width but not their number below the Fermi energy. The latter depends only on the period a of the superlattice since the gaps between the minibands appear at $k_z=\pm\pi n/a$ for integer n. In our numerical calculations there are only two bands below E_F.

IV. DISCUSSION AND SUMMARY

The observation of the quantum magnetic oscillations in layered organic conductors put forward a lot of puzzles waiting for a theoretical explanation. The salt β''-(BEDT-TTF)$_2$SF$_5$CH$_2$CF$_2$SO$_3$, for example, gives evidence for an incoherent electron transport across the layers[31] and has a fixed chemical potential, unusual for the quasi-2D system. Incoherence means that the quasimomentum is not a

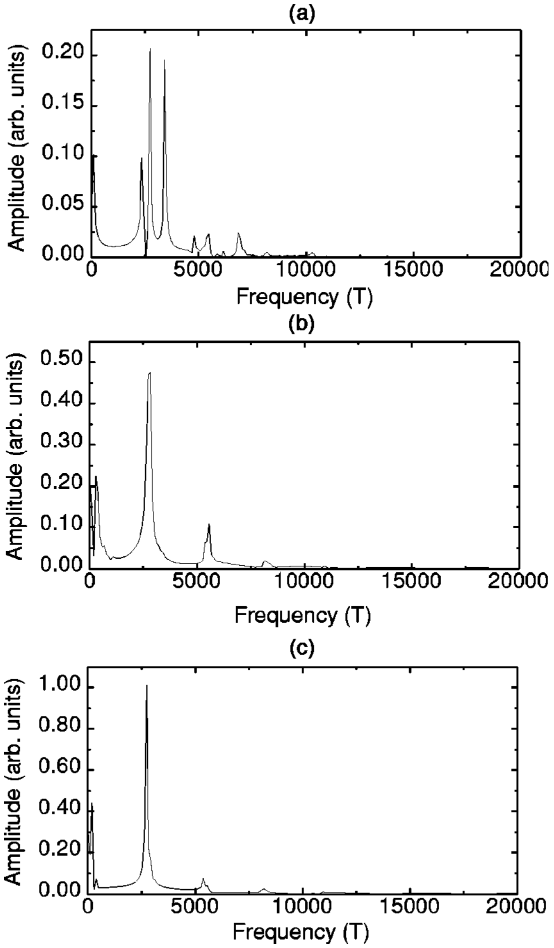

FIG. 3. The Fourier transform of the chemical potential oscillations in Fig. 2 for a superlattice with transparency factors $\gamma=1$ (a), $\gamma=50$ (b), and $\gamma=100$.

good quantum number for electrons hopping between the layers. In this connection we put forward an approach to the problem of the chemical potential dependence on the external magnetic field in superlattices and layered conductors in terms of the DOS $g(\varepsilon)$. Our main result is Eq. (19) for $\mu(B)$ which covers not only the case of a superlattice but permits a correct description of the 2D and 3D limiting cases with the appropriate density of states [see Eqs. (22),(26)]. The central physical idea is that electrons from the parabolic branches of the Landau levels, which play the role of a reservoir that pins $\mu(B)$ at the Fermi level in the 3D case, do not vanish from the balance equation for the $\mu(B)$ in the case of a superlattice.

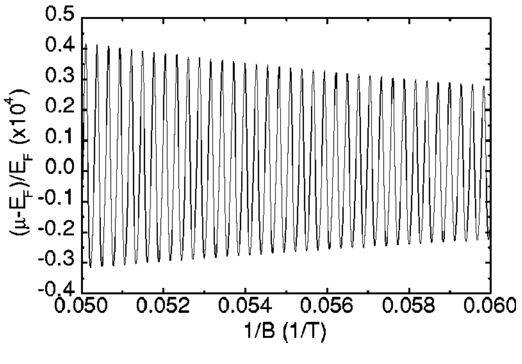

FIG. 4. The chemical potential oscillations in a 3D electron gas as a function of inverse magnetic field at $T=1.2$ K and $2\pi\Gamma/k_B=0.84$ K.

In the ideal 2D case the Landau levels are flat and $\mu(B)$ jumps at zero temperature between the Landau levels with amplitude $\hbar\omega_c$. Nonzero temperatures as well as electron scattering decreases this amplitude. In the 3D case the amplitude of the chemical potential oscillations reduces to $\hbar\omega_c\sqrt{\hbar\omega_c/E_F}$. In superlattices the amplitude is given by $\hbar\omega_c/D(E_F)$. This value is essentially less than $\hbar\omega_c$ because the factor $D(E_F)$ may be much more than unity depending on the superlattice parameters. If, for example, the insulating barriers between the layers have a very low transparency the permitted bands have a vanishing width so that the density of states $g(\varepsilon)$ is given approximately by a sum of delta functions $\delta(\varepsilon-\varepsilon_k)$ over the set of quantized energy levels $\varepsilon_k=\hbar^2k^2/2ma^2$. The factor $D(E_F)$ in this case equals to the number of these levels below the Fermi energy which is huge for thick layers.

These points related to the miniband structure were missed in previous theories by considering only a single-band approximation with the simplest dispersion within the band

$$\varepsilon=2t\cos(k_za). \tag{32}$$

The corresponding density of states is

$$g(\varepsilon)=\frac{1}{\pi\sqrt{4t^2-\varepsilon^2}}$$

which in view of the fact that the hopping integral t is less than the Fermi energy $2t<E_F$ yields $D(E_F)=1$ and the layer factor is given by the Bessel function

$$I(p,E_F)=J_0\left(\frac{4\pi tp}{\hbar\omega_c}\right).$$

The equation for the chemical potential (20) in this approximation takes the form[20]

$$\mu=E_F+\hbar\omega_c\sum_{p=1}^{\infty}\frac{(-1)^{(p+1)}}{\pi p}\sin\left(\frac{2\pi p\mu}{\hbar\omega_c}\right)R_T(p)\times J_0\left(\frac{4\pi tp}{\hbar\omega_c}\right)\exp\left(-\frac{2\pi p\Gamma}{\hbar\omega_c}\right). \tag{33}$$

This equation permits a 2D limit $t\to0$ for which $J_0(4\pi tp/\hbar\omega_c)=1$. For large hopping integral $4\pi tp/\hbar\omega_c\gg1$ and using the asymptotic Bessel function,[19] the chemical potential has an expression similar to the Lifshitz-Kosevich result (26) but containing the pre-factor $\hbar\omega_c\sqrt{\hbar\omega_c/t}$ instead of $\hbar\omega_c\sqrt{\hbar\omega_c/E_F}$. In quasi-2D conductors E_F is determined by the electron density in layers while the t is the hopping integral between them.

In layered conductors it is assumed that electrons are nearly localized within the layers because of the small overlap of the wave functions for the neighboring layers. In our model this means negative u_0 and hyperbolic rather than trigonometric functions in the right-hand side of Eq. (21). For $\gamma \gg 1$ the dispersion can be written as in Eq. (32) with the "hopping integral" $t = (\hbar^2/ma^2)\gamma^2 \exp(-\gamma)$. In the case of a superlattice the dispersion within the minibands is more complex than given by Eq. (32).

The recent observation of an inverted sawtooth magnetization in the layered organic conductor β''-(BEDT-TTF)$_2$SF$_5$CH$_2$CF$_2$SO$_3$ (Ref. 21) gives evidence for a fixed chemical potential in this quasi-2D system. An attempt to explain this fact by the pinning of μ at the 1D open sheets of the Fermi surface in this conductor was not successful.[22] With the proposed minibands structure attached to each Landau level the amplitude of the chemical potential oscillations is suppressed. These bands comprise a reservoir of electronic states which damp the oscillations of $\mu(B)$ compared to the pure 2D case. For the given example of the superlattice model, the existence of even two minibands below the Fermi level is sufficient to suppress the amplitudes of the oscillations by a factor $10^{-1}-10^{-2}$. These minibands may be flat such that the interlayer transport may be much less that the transport within the layers or even incoherent.[31]

Due to the magnetic breakdown phenomenon in ET salts, the 2D Landau spectrum is much more complex than for the case of a layered 2D electron gas considered above.[23] It would be very interesting to extend the proposed approach towards a more realistic band structure for the ET salts.

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