Density-induced reorientation of the stripe at half-filled high Landau levels

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The effect of a unidirectional periodic potential on the orientation of the stripe state is studied for the two-dimensional electron system at half-filled high Landau levels. By considering a quantum well with two electric subbands, it is found that the stripe is parallel to the external potential for weak modulation and is orthogonal for strong modulation. In the intermediate range, the orientation of the stripe changes from orthogonal to parallel as the electron density is increased. This result explains the recent experiment performed by Zhu *et al.* that the anisotropy axis at half-filled high Landau levels rotates by 90° by increasing the electron density. It also supports the suggestion that the stripes are pinned by the native surface morphology at the interface of the heterojunction.

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Two-dimensional electron systems (2DES's) of the high-mobility samples in a strong magnetic field exhibit a rich variety of physical phenomena associated with the Coulomb interactions between electrons. In the lowest Landau level (LL), the electrons condense into an incompressible quantum liquid at certain rational filling factors and lead to the famous fractional quantum Hall effect (FQHE).[1–3] Recently, a new kind of many-body correlated phase which shows large anisotropy in the longitudinal resistance in the magnetotransport experiments was revealed at half-filled high Landau levels ($\nu=9/2,11/2,\ldots$).[4,5] The origin of this strange anisotropy is widely viewed as the formation of a unidirectional charge density wave (UCDW) or stripe state around these filling factors.[6,7] Even before the experimental discoveries, the UCDW was predicted theoretically by Fogler *et al.*[8] and Moessner and Chalker[9] based on Hartree-Fock (HF) discussions, where the 2D electron gas spontaneously breaks the translational symmetry. When the external field is tilted away from the sample normal, it shows that the easy transport orients orthogonal to the in-plane magnetic field.[10,11] Theoretical computations beyond the HF approximation are also consistent reasonably with the experiment results.[12–14] Some researchers proposed the existence of liquid crystalline states with stripe ordering and broken rotational symmetry.[13,15,16]

Nevertheless, the preferred orientation of the stripes in a perpendicular magnetic field remains puzzling.[17] For 2D electron systems in GaAs/AlGaAs heterostructures grown on $\langle 110\rangle$-oriented GaAs substrates, the hard transport direction is parallel to the $\langle 1\bar{1}0\rangle$ crystallographic direction while the easy direction is parallel to $\langle 110\rangle$. It is hard to believe that the crystal structure affects the orientation of the stripe. In the experiments the magnetic field is a few tesla, so the wave function of each electron in the third LL is spread over more than several hundred angstroms. The details of the crystal lattice structure will be averaged out. Fil proposed that the piezoelectric effect may play a role in determining the orientation of the stripes.[18] In a recent experiment, Willett *et al.* examined the surface morphology of high-mobility heterostructures and found that the transport is consistent with that in samples having artificially induced 1D charge modulations.[19] The native lines are orthogonal to the stripes, which, at first glance, is somehow in contrast to intuition. Several authors have studied this new effect with a periodic external potential and their results consistent with the experiment.[20,21]

Recently, Zhu *et al.* observed a density-induced interchange of anisotropy axes at half-filled high LL's.[22] They employed a tunable-density heterostructure-insulated gate-field-effect transistor to access a wide density regime and found that as the density of the 2DES is raised above 2.9×10^{11} cm^{-2}, the easy axis rotates from the $\langle 110\rangle$ direction to the $\langle 1\bar{1}0\rangle$ direction. Their result provides another way to demonstrate the pinning mechanism of the stripe phase which we will discuss in this work. We will show when take into account two electric subbands and introduce a unidirectional periodic potential to the electron system; these stripes align either parallel or perpendicular to the external potential. The stripe is parallel to the periodic potential for weak modulation whereas it perpendicular to the potential for strong modulation. For intermediate modulation, the stripes experience a rotation of 90° from parallel or orthogonal to the external potential as the density increases. Our discussion supports the suggestion that the orientation of the stripe phase is pinned by the native symmetry breaking potential at the GaAs/AlGaAs interface.

In order to deal with the problem analytically, we assume that the electron gas is confined in a plane by a harmonic potential with characteristic frequency Ω. Before going to the details, we argue that this choice of confining potential may quantitatively correct for the problem we will deal with in this work despite the fact that the realistic confining potential in the sample is essentially a finite square well. The harmonic well is very different from the square well in its excited spectra, for the harmonic spectrum is equally gapped while that of the square well is not. However, in our work, there will be only two energy levels of confining potential to be involved and thus the unequal energy gaps between different adjacent energy level will not be a concern. Therefore, one can variationally adjust the harmonic frequency such

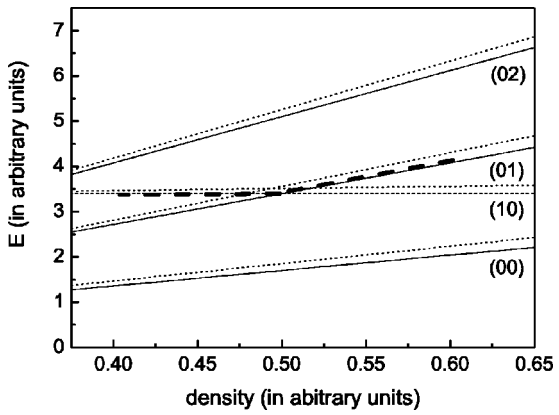

FIG. 1. A schematic description of the energy levels with two electric subbands. (mn)'s indicate the two indices of electric subbands (m) and Landau levels (n). Thin dashed lines are the corresponding Zeeman splitting. Thick dashed line is the Fermi level for $\nu=9/2$.

that the harmonic energy gap equals the gap between those two given levels in a realistic square well. In this sense, the harmonic potential may be a good approximation to a realistic potential to give a quantitative description. Such a harmonic potential has been chosen to deal with many quantum Hall systems to replace the realistic potential which is either triangular[23] or square.[6,7,24] It was also used to discuss the giant magnetoresistance induced by a parallel magnetic field.[25]

For a perpendicular magnetic field applied to the system, $\mathbf{B}=B\hat{z}$, there are two electric subbands mixing with the Landau levels, with frequencies $\omega_+=\Omega$ and $\omega_-=\omega_c$, respectively. The corresponding eigen wave functions are[26]

$$\phi_m^{\omega_+}=N_m^+ e^{-z^2/2l_+^2}H_m(z/l_+),$$

$$\phi_n^{\omega_-}=N_n^- e^{-(x-X)^2/2l_-^2}H_n((x-X)/l_-), \tag{1}$$

where $H_n(x)$ are the Hermitian polynomials and $N_n^{\pm}=1/\sqrt{2^n n!\sqrt{\pi}l_{\pm}}$ are the normalization coefficients. $l_{\pm}^2=\hbar/m_b\omega_{\pm}$. X is an integer multiple of $2\pi l_-^2/L_y$. The combined single-particle wave function is

$$\Phi_{mn}=\frac{1}{\sqrt{L_y}}e^{iXy/l^2}\phi_m^{\omega_+}\times\phi_n^{\omega_-}. \tag{2}$$

Then the energy levels of single particle states are described by a set of two indices (mn) with index m indicating the electric subbands and n the Landau levels.

Figure 1 schematically depicts the energy levels of the 2DES. Given a filling factor, e.g., $\nu=9/2$, increasing the electron density means increasing the strength of the magnetic field B. Hence there appear a series of energy level crossings since the electric subbands rarely change whereas the cyclotron frequency ω_c increases with the magnetic field. The Fermi energy is indicated by the thick dashed line. Since the single-particle state at the Fermi surface is changed from (10) on the left to (01) on the right of the crossing point as the density increases, one may expect a phase transition at the level crossing as in most circumstances.[28,29] We will show that the UCDW indeed changes from orthogonal to parallel to the periodic potential for intermediate modulations as the electron density (hence the magnetic field) increases.

To include the surface morphology at the interface of the heterostructure, we consider the effect of a periodic potential of wave vector $\vec{Q}_p$ and strength V_0 on the stripes of fundamental wave vector $\vec{Q}_s$. In realistic samples of GaAs/AlGaAs heterostructure, the wave vector of potential modulation $\vec{Q}_p$ should be in the $\langle 110\rangle$ direction.[19,30,31] Two configurations of $\vec{Q}_p$ and $\vec{Q}_s$ are considered in our work: $\vec{Q}_p\|\vec{Q}_s$ or $\vec{Q}_p\perp\vec{Q}_s$. In the orthogonal orientation, the main deformation of the stripe caused by the periodic potential is modulation of the width of the stripes. In the parallel orientation, the main deformation of the stripe is displacement of the stripes. Both deformations lower the cohesive energy of the stripe state.[21]

The Hamiltonian consists of a Coulomb interaction part H_0 and an interaction with the external potential part H_1: $H=H_0+H_1$. Here

$$H_0=\frac{1}{2L_xL_y}\sum_{\vec{q}} v(\vec{q})\rho(\vec{q})\rho(-\vec{q}), \tag{3}$$

where $v(\vec{q})=2\pi e^2/\kappa_0 q$:

$$H_1=\frac{1}{2}V_0\sum_{\vec{q}=\pm\vec{Q}_p}\rho(\vec{q})\exp(i\vec{q}\cdot\vec{r}_0), \tag{4}$$

where $\vec{r}_0$ is the origin of the potential. $\rho(\vec{q})$ is the electron density operator projected onto the upper LL. It can be written as

$$\rho(\vec{q})=\sum_X F(\vec{q})e^{-iq_xX}c_{X_+}^{\dagger}c_{X_-}, \tag{5}$$

where $X_{\pm}=X\pm q_yl^2/2$. Here $F(\vec{q})$ is computed by the state (2), which is given by

$$F(\vec{q})=e^{-q_z^2l_+^2/4-q_\parallel^2l_-^2/4}L_m(q_z^2l_+^2/2)L_n(q_\parallel^2l_-^2/2), \tag{6}$$

where $L_n(x)$ is the Laguerre polynomial. $\vec{q}_\parallel$ is the momentum in the 2DES plane.

By using the standard manipulation for the Hartree-Fock decoupling of Hamiltonian (3), we get

$$H_0^{HF}=\frac{1}{2}\sum_{\vec{q}} u_{HF}(\vec{q})\Delta(-\vec{q})\sum_X e^{-iq_xX}c_{X_+}^{\dagger}c_{X_-}. \tag{7}$$

The effective potential $u_{HF}(\vec{q})$ is explicitly written as a sum of a Hartree term (in units of $e^2/\kappa_0 l$)

$$u_H(\vec{q})=\int\frac{dq_z}{\pi l}\frac{1}{q_\parallel^2+q_z^2}[F(\vec{q})]^2 \tag{8}$$

and an exchange term

$$u_{ex}(\vec{q}) = -2\pi l^2 \int \frac{d\vec{p}}{(2\pi)^2} u_H(\vec{p}) e^{i\vec{p}\times\vec{q}l^2}. \quad (9)$$

Allowing the charge density wave (CDW) by introducing the order parameters

$$\Delta(\vec{Q}) = \frac{2\pi l^2}{L_x L_y} \sum_X e^{-iQ_x X} \langle c^\dagger_{X_+} c_{X_-} \rangle, \quad (10)$$

the cohesive energy of the electrons in the topmost LL can be obtained as

$$E_{coh} = \frac{1}{2\nu_N} \sum_{\vec{Q}\neq 0} u_{HF}(\vec{Q}) |\Delta(-\vec{Q})|^2, \quad (11)$$

where $\nu_N = 1/2$ is the filling factor at the topmost Landau level.

We carry out a HF computation on a rectangular lattice with the wave vectors of the order parameters as $\vec{Q} = (jQ_x^0, kQ_y^0)$, where j and k are integers. Following the procedure in Refs. [21 and 27], when

$$NQ_x^0 Q_y^0 l^2 = 2M\pi, \quad (12)$$

with N and M being integers, the Landau level splits into N Hofstadter bands. The CDW state is recognized as the stripe phase when the order parameters with $\vec{Q} = \pm\vec{Q}_s$ are dominant. When $N=6$ and $M=1$ the stripe state has the lowest energy. In Ref. 21, several cases were considered for $Q_p = Q_s/k$, with $k=2,3,4,5,6$. As an example in our work, the wave vector of the external potential is typically chosen to be $Q_p = Q_s/3$. We consider two configurations in which the $\vec{Q}_p$ and $\vec{Q}_s$ are either parallel or orthogonal to each other, respectively. In the parallel orientation case, the main deformation is the displacement of the stripe, which is called the ''frequency modulation.'' In the orthogonal orientation case, the main deformation is the modulation of the stripe width, which is called the ''amplitude modulation.'' In the latter case, there is a periodic density modulation along the stripes with wave vector $\vec{Q} = \vec{Q}_p$ for weaker modulation. When the modulation becomes stronger ($V_0/\hbar\Omega \gtrsim 0.1$), the stripe looks like breaking up at the ridges of the external potential and degenerates into a rectangular CDW state.

Figure 2 shows the dependence of the cohesive energy of the electrons in the third LL on the modulation strength of the external potential V_0. The parallel orientation state and orthogonal orientation state for (mn) energy levels are denoted by ''para(mn)'' or ''orth(mn),'' respectively. When the electron density (or ω_c/Ω)rises, the Fermi level changes from the (10) state to the (01) state. Figure 2(a) is for $\omega_c/\Omega = 0.2941$ and 2(b) is for $\omega_c/\Omega = 0.8824$. Both figures show that the parallel orientation is slightly lower in energy than the orthogonal orientation for small modulation strength ($V_0/\hbar\Omega \lesssim 0.04$) whereas the orthogonal orientation dominant for large modulation strength ($V_0/\hbar\Omega \gtrsim 0.06$). Previous studies claimed that the orientation of the stripe is always perpendicular to the periodic potential.[20,21] The difference may result from the fact that their calculations did not count the width of the quantum well. We note that the two-subband

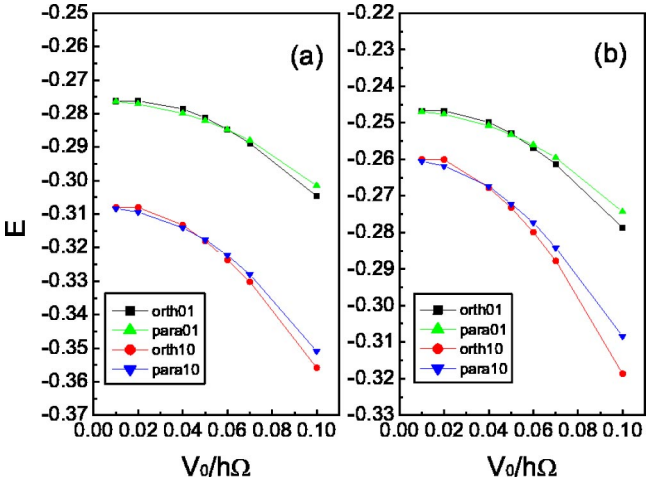

FIG. 2. The cohesive energy of the UCDW vs the modulation strength $V_0/\hbar\Omega$ of the potential. ''para'' and ''orth'' denote parallel and orthogonal orientations to the periodic potential, respectively. (a) is for $\omega_c/\Omega = 0.2941$ and (b) is for $\omega_c/\Omega = 0.8824$.

level is not the key element to whether the stripes are parallel or orthogonal to the potential. The orientation of the stripes is mainly dependent on the relative strength of the external modulation V_0 with respect to the characteristic frequency Ω. However, in the single-band model, there is no transition of orientation of the stripes as the density varies. In the two-subband levels, since the matrix elements in formula (6) are dependent on the single-particle states (2), the Hartree-Fock potentials are different at the two sides of the crossing, which the orientation transition underlies.

Figure 3 shows the anisotropy energy E_a versus ω_c/Ω or electron density (in arbitrary units). E_a is the energy difference between the parallel orientation and the orthogonal ori-

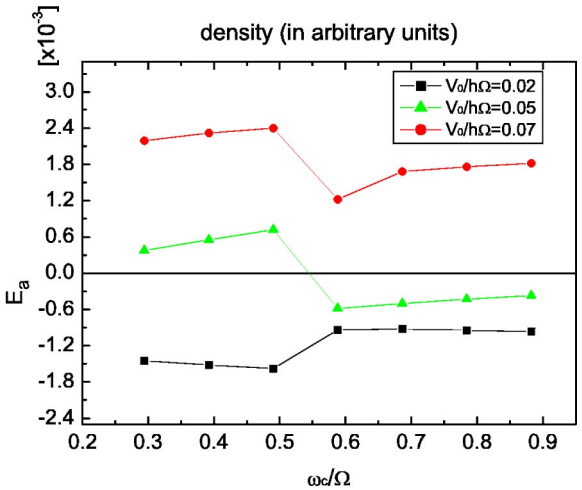

FIG. 3. Anisotropy energy E_a vs ω_c/Ω or electron density (in arbitrary units). E_a is the energy difference between the parallel orientation and the orthogonal orientation. E_a is definitely negative for $V_0/\hbar\Omega = 0.02$ whereas definitely positive for $V_0/\hbar\Omega = 0.07$. For $V_0/\hbar\Omega = 0.05$, E_a changes from positive to negative as ω_c/Ω (or electron density) increases, implying a phase transition from orthogonal orientation to parallel orientation.

entation. We depict three curves for three typical values of $V_0/\hbar\Omega$. We find that E_a is definitely negative for $V_0/\hbar\Omega \lesssim 0.04$, indicating that the parallel orientation is favored. E_a is definitely positive for $V_0/\hbar\Omega \gtrsim 0.06$, indicating that the orthogonal orientation is favored. For the curve $V_0/\hbar\Omega = 0.05$, E_a changes from positive to negative as ω_c/Ω (or electron density) increases, implying a phase transition from orthogonal orientation to parallel orientation. This result coincides with the recent experimental observation by Zhu *et al*. that the anisotropy axes at half-filled high Landau levels in the two-dimensional electron system rotate by 90° by increasing the electron density.[22] It should be noted that in previous studied samples with electron densities inside the transition region of Ref. 22, the easy direction is always parallel to the $\langle 110 \rangle$ direction.[10,11,19,30] This variety of experimental results may originate from the sensitive dependence of the stripe orientation on the roughness of the surface morphology at the interface of the heterostructure. In our calculations, the parallel phase exists only for rather weak modulation ($V_0/\hbar\Omega \lesssim 0.04$). Specifically, anisotropy axis rotation takes place only in a limited range of modulation strength and width of the quantum well ($0.04 \lesssim V_0/\hbar\Omega \lesssim 0.06$). Beyond this range, no reorientation transition can be observed.

In Fig. 1 there are more energy crossings as ω_c/Ω (or electron density) further increases or decreases. For lower density, the single-particle state will be the (02) state. The stripe will experience an additional orientation interchange, which may be observed in experiment by sweeping a larger electron density regime. We emphasize that complex transport behavior takes place only in a two-subband quantum well at half-filled Landau levels. Given a filling factor, increasing the electron density means increasing the magnetic field, which leads to energy level crossings. Reference 22 suggested that the squeeze of the electron wave function and harder press against the interface may be the origin of the reorientation of the stripes. We have checked in our computations that no such transition can take place in a single-band model. Our result provides support to the explanation that the pinning mechanism is the native surface morphology at the interface of the GaAs/AlGaAs heterostructure.

In summary, we have studied the effect of a unidirectional periodic potential on the orientation of the UCDW state. By considering two electric subbands of a wide quantum well, it is found that the stripe is parallel to the potential for weak modulation and is perpendicular for strong modulation. For intermediate modulation, the orientation can be either parallel or perpendicular to the potential. When the electron density increases, the stripes experience a rotation of 90° from parallel or orthogonal to the external potential. The result is consistent with a recent experimental observation. Our discussions may help to discern the pinning mechanism of stripes at half-filled high Landau levels.[31]

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