Inhomogeneous magnetoelastic states and magnetoelastic wave spectrum in a system consisting of magnetic/nonmagnetic multilayers

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A study is made of the magnetoelastic (ME) wave spectrum and ME superstructure nucleation in a system consisting of magnetic/nonmagnetic multilayers. A case of rhombic ferromagnetic layers with the hard magnetization axis $\vec{b}$ perpendicular to the layer surface is considered. We show that close to the phase transition associated with the spin reorientation in the layer plane, a ME wave with a horizontal polarization, propagating parallel to the layer plane, becomes unstable. The shear ME wave frequency and group velocity vanish for a finite value of a wave vector, and the wave becomes frozen, forming a ME domain structure localized near the layer interfaces. Existence of a new modulated phase is associated with a ME coupling of the magnetization to lattice deformation on the layer interfaces. The spectra of the surface ME in the homogeneous and modulated phases are calculated. Depending on the magnetic and nonmagnetic layer thickness and temperature, the stability regions of homogeneous collinear ($\vec{M}\|\vec{a}$) and angular phases and the ME domain structure are found, where $\vec{M}$ is the magnetization, and $\vec{a}$ is the easy orthorhombic axis direction. The calculation is extended to study the ME wave propagation in the systems consisting of two-sublattice orthoferrite/nonmagnetic multilayers.

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I. INTRODUCTION

Magnetoelastic (ME) interaction determines many properties of real magnetic crystals.[1–4] It strongly affects the quasistatic magnetization reversal, domain structure, magnetic resonance, and nonlinear dynamics of magnetics. New physical effects can arise from ME interaction. The surface Rayleigh,[5–8] Love,[9,10] and Stonely[11] waves modified by ME interaction can be damped out due to the spin wave radiation into the interior of crystal. In a ferromagnetic crystal, the linear shear volume elastic wave transforms into a surface wave if dipole-dipole and ME interactions are taken into account.[12] Types of self-localized, ME surface waves in magnets were studied in Ref. 13, the existence of which is determined entirely by ME interaction and nonlinear properties of the magnetic media. In bulk systems close to a magnetic reorientational phase transition (RPT) when the magnetic (spin) subsystem loses its stability, the energy of the ME interaction increases effectively in comparison to other types of energy, e.g., the magnetic anisotropy energy, and gives rises to a considerable deformation of the ME wave spectrum,[14–20] decrease in the sound speed, and the appearance of a gap for a spin mode[21] (the so-called ME gap). The considerable decrease in the sound speed and the important role that nonlinear processes plays close to the RPT make magnetic crystals promising materials for use in electronic devices.[17–20] The study of the spectra of ME waves in limited specimens of magnetic materials makes it possible to determine the type of soft mode involved in the RPT. In thin plates it is the flexular ME mode that is proved to be the soft mode.[22] In thick magnetic films and multilayers, a ME interaction accounts for the Néel-type domain structure formation.[2,23,24] Near the RPT induced magnetic field, the magnetic configurations and ME wave spectra of uniaxial ferromagnetic films, with the easy magnetization axis (EMA) perpendicular to their surfaces, were investigated in Refs. 25 and 26. It was shown that the ME wave frequency and group velocity vanish for a finite value of a wave vector $\vec{k}$, and the film splits into domains, the existence of which reduces the energy of the demagnetization fields in vacuum. The competition between the ME and dipole interactions leads to the drastic dependence of the wave vector and polarization of a soft ME mode involved in the RPT on the film thickness L. For thick films ($L>L_N$), it is the transverse ME wave with $\vec{k}\|\vec{M}$ and the polarization vector $\vec{p}$ perpendicular to its surface. This mode condenses into a Néel-type domain structure. For thin films ($L<L_B$), it is the transverse ME wave with $\vec{k}\perp\vec{M}$ and $\vec{p}$ parallel to the film plane. Here L_N and L_B are Bloch and Néel domain-wall thicknesses, respectively. In this case a Bloch-type domain structure is realized.

An alternative case of a magnetic film with the EMA parallel to the developed surface and supported by a bulk substrate was recently studied by Bespyatykh and Dikshtein.[27–29] They considered the specific features of the ME Love wave propagation near the RPT induced by an external magnetic field. It has been demonstrated that at a magnetic field H_C different from a field H_A of the phase transition in a bulk sample, the frequency and the group velocity of the ME Love wave, propagating along the magnetization vector in the basal film plane, vanish for the wave vector $k=k_C\neq 0$. As a result, the critical Love mode becomes unstable and transforms into a ME domain structure localized in the film and the substrate close to their interface. In this case, the formation of elastic domains results from the need of the energy reduce of the long-range elastic strain fields in the nonmagnetic substrate.

In this paper, we present a study of ME superstructure nucleation in a system consisting of magnetic/nonmagnetic

multilayers. In recent years, inhomogeneous magnetic states (domain structures) in magnetic multilayers have been intensively investigated.[30,31] The domain structure can be responsible for the giant magnetoresistance (GMR) of the magnetic multilayers.[32] A combination of the considerable dispersion and the high sensitivity of the ME wave spectrum to the external magnetic field makes such systems promising for applications to the signal processing. It is commonly supposed[30,31] that the domain structure existence has its origin in ''magnetic charges'' on the layer surfaces. We expect that for systems involving magnetic/thick nonmagnetic multilayers close to the RPT associated with the spin reorientation in the layer plane, a ME domain structure is formed. The ME domain structure is determined by the balance of the energy of the long-range elastic strain fields in the nonmagnetic layers and the energy of domain walls. The ME domain structure formation reduces the energy of the elastic strain fields in the nonmagnetic layers, which penetrate the layers to a depth of the order of the domain structure period. A boundless reduction in the domain size is hindered by an increase in the energy of the domain walls. In the other limiting case of thin nonmagnetic layers possessing the thickness compared with the penetration depth of the elastic fields into the nonmagnetic material, the system becomes uniformly deformed in the layer planes at the RPT.

II. BASIC EQUATIONS

Simple ideas may help to understand the phenomenon of ME domain structure formation in structures composed of alternating magnetic and nonmagnetic layers. Assume that the elastic deformations u_{ik} and the elastic stresses σ_{ik} in the system are small. In this case the total energy of a ferromagnet is given by[33]

$$E=\int dv\, W(\vec{M},\partial\vec{M}/\partial x_i,u_{ik}), \tag{1}$$

where $\vec{M}$ is the magnetization vector, $W=W_m+W_{\rm me}+W_e$. The contributions to W are the magnetic (W_m), magnetoelastic ($W_{\rm me}$), and elastic (W_e) energies. The elastic contribution is the quadratic form which is positively definite:

$$W_e\geqslant 0. \tag{2}$$

The density of the ME energy $W_{\rm me}$ is linear in u_{ik}.

Total energy (1) is minimized for the metastable and ground states. Therefore the equilibrium distributions of magnetization $\vec{M}$ and elastic displacement $\vec{u}$ are described by the equations of state,

$$(\vec{M}\times\vec{H}_{\rm eff})=0,\quad \delta E/\delta\vec{u}=0, \tag{3}$$

where $\vec{H}_{\rm eff}=-\delta E/\delta\vec{M}$ is the effective magnetic field.

Using the second of Eqs. (3) and taking into account that W_e and $W_{\rm me}$ are, respectively, quadratic and linear in $\vec{u}$, we can get the identity

$$2W_e+W_{\rm me}=0. \tag{4}$$

It follows from Eqs. (2) and (4) that

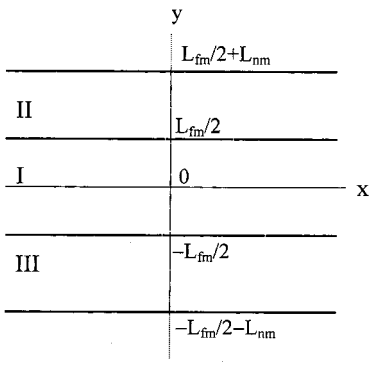

FIG. 1. Geometry of the problem. A ME wave propagates along the x axis; $\vec{a}\|\vec{e}_x$, $\vec{b}\|\vec{e}_y$, $\vec{c}\|\vec{e}_z$.

$$W_e+W_{\rm me}=W_{\rm me}/2\leqslant 0. \tag{5}$$

The equality sign in Eqs. (2) and (5) applies in the absence of elastic deformations in a system. By this is meant that the total energy of a ferromagnet can be decreased owing to the interaction of the magnetic and elastic subsystems.

Using the second equation of state (3), one can express the components of the deformation tensor u_{ik} in the terms of the magnetization, and can obtain the effective magnetic energy $E_{\rm eff}$. The additional ME contribution to the $E_{\rm eff}$, is negative. Hence, if for some reason a ferromagnet is clamped, this contribution can lead to the homogeneous state instability and ME superstructure nucleation. A similar situation should take place for a multilayered structure consisting of magnetic layers sandwiching nonmagnetic spacers.

To be specific, we shall further consider the effect of magnitostriction on the domain structure nucleation in a multilayered structure composed of rhombic ferromagnet layers sandwiching elastic nonmagnetic spacers. When the temperature is lowered, a spontaneous reorientation of the magnetization from the $\vec{c}$ axis of the orthorhombic crystal to an $\vec{a}$ axis occurs in the ac plane in certain ferromagnets (Co,Fe_3O_4) (Ref. 34), and rare-earth orthoferrites ($TmFeO_3, ErFeO_3$).[35,36,16,37–39] The RPT is due to the variation of the anisotropy constants K_1 and K_2 with temperature T. The RPTs at the temperatures T_1 and T_2 ($T_1<T_2$) corresponding to the beginning and the end of the reorientation region are second-order ones. Peculiarities of bulk[16,37–39] and Rayleigh[8] wave propagation near the RPT have been studied theoretically and experimentally in massive rare-earth orthoferrites.

We shall consider a periodic system composed of rhombic ferromagnet layers of thickness $L_{\rm fm}$ with the hard axis $\vec{b}$ normal to the basal plane ($\vec{b}\|\vec{e}_y$) near the temperature T_1. Ferromagnet layers are sandwiched nonmagnetic spacers of thickness $L_{\rm nm}$ (as shown in Fig. 1). We shall also assume that the orthorhombic axes $\vec{a}\|\vec{e}_x$ and $\vec{c}\|\vec{e}_z$. The total energy of a system is given by

$$E=E_{\rm fm}+E_{\rm nm}. \tag{6}$$

It is decomposed into the energy of the magnetic layers $E_{\rm fm}$,

$$E_{\rm fm}=\int_{V_{\rm fm}} dv\left[\frac{\alpha}{2}\frac{\partial\vec{M}}{\partial x_i}\frac{\partial\vec{M}}{\partial x_i}+w_A-\frac{\vec{H}_D\vec{M}}{2}\right.$$
$$\left.+B\left(M_iM_k-\frac{M^2}{3}\delta_{ik}\right)u_{ik}^{(\rm fm)}+\frac{1}{2}c_{iklm}^{(\rm fm)}u_{ik}^{(\rm fm)}u_{lm}^{(\rm fm)}\right], \quad (7)$$

and the elastic energy of the nonmagnetic layers $E_{\rm nm}$,

$$E_{\rm nm}=\int_{V_{\rm nm}} dv\,\frac{1}{2}c_{iklm}^{(\rm nm)}u_{ik}^{(\rm nm)}u_{lm}^{(\rm nm)}, \quad (8)$$

where $w_A=-(1/2)\beta_z M_z^2-(1/2)\beta_x M_x^2+(1/4)(b_{xx}M_x^4+2b_{xz}M_x^2M_z^2+b_{zz}M_z^4)/M^2$ is the density of the magnetic anisotropy energy; $\vec{H}_D=\nabla\Psi$ is the demagnetization field; Ψ is the demagnetization field potential; α, B, and $\hat{c}$ are the nonuniform exchange, ME, and elastic constants, respectively; and $\beta_{x,z}$ and b_{ik} are the anisotropy constants. The indices fm and nm label, respectively, the magnetic and nonmagnetic layers. The symmetry analysis of allowed contributions to the ME energy density has been performed by Callen and Callen[40] in the 1960s and later on by du Tremolet de Lacheisserie[41] for all crystal symmetries of practical interest (also see his book on magnetostriction[42]). Here, for simplicity, the ferromagnet of interest is assumed to be magnetoelastically and elastically isotropic. It is one of the simplest nontrivial model that still exhibits a ME induced microstructure. The inherent anharmonicity of the elastic lattice of the ferromagnet is neglected in Eq. (7) since it is ignorable compared to the effective anharmonicity associated with ME coupling.[17] For the same reason we ignore the nonlinear dependence of the deformation tensor on the displacement vector derivatives in magnetic and nonmagnetic layers and consider the expansion of the ME energy up to the linear terms in $u_{ik}^{(\rm fm,nm)}$. We also assume that the elastic properties of the substrate are linear and isotropic and that strain induced by the lattice mismatch between the film and the substrate brings about the renormalization of the magnetic anisotropy constant only.

The dynamics of a multilayered structure is described by the equations of motion

$$\frac{\partial\vec{M}}{\partial t}=-g(\vec{M}\times\vec{H}_{\rm eff}),\quad \rho^{(\rm fm,nm)}\frac{\partial^2u_i^{(\rm fm,nm)}}{\partial t^2}=\frac{\partial\sigma_{ik}^{(\rm fm,nm)}}{\partial x_k}, \quad (9)$$

and by Maxwell's equations in the magnetostatic approximation

$$\mathrm{div}(\vec{H}_D^{(\rm fm)}+4\pi\vec{M})=0,\quad \mathrm{rot}\vec{H}_D^{(\rm fm)}=0,$$
$$\mathrm{div}\,\vec{H}_D^{(\rm nm)}=0,\quad \mathrm{rot}\vec{H}_D^{(\rm nm)}=0, \quad (10)$$

where $\vec{H}_D^{(\rm fm,nm)}=\nabla\Psi^{(\rm fm,nm)}$ are the demagnetization and scattering fields, respectively; $\Psi^{(\rm fm,nm)}$ are the magnetic potentials inside magnetic and nonmagnetic layers, respectively; $\sigma_{ik}^{(\rm fm,nm)}=\delta E/\delta u_{ik}^{(\rm fm,nm)}$ is the stress tensor; $g>0$ is the gyromagnetic ratio; and $\rho^{(\rm fm,nm)}$ are the densities of magnetic and nonmagnetic layers respectively.

The above system of equations must be solved subject to the conditions that the σ_{jy} components of the elastic stress tensor, the elastic displacements, the normal component of energy flux density, the magnetic potential, and the normal component of magnetic induction are continuous at the magnetic-nonmagnetic interfaces

$$\sigma_{jy}^{(\rm fm)}=\sigma_{jy}^{(\rm nm)}\ \ (j=x,y,z),\quad \vec{u}^{(\rm fm)}=\vec{u}^{(\rm nm)},$$
$$\partial\vec{M}/\partial y=0,\quad \Psi^{(\rm fm)}=\Psi^{(\rm nm)},$$
$$\partial\Psi^{(\rm fm)}/\partial y+4\pi M_y=\partial\Psi^{(\rm nm)}/\partial y. \quad (11)$$

III. INTRINSIC DEFORMATIONS OF MAGNETIC AND NONMAGNETIC LAYERS

Intrinsic deformations of magnetic and nonmagnetic layers $u_{ik0}^{(\rm fm,nm)}$ are nonzero even in a uniformly magnetized system. To be specific, we shall evaluate the intrinsic deformations of the system in the low-temperature phase $T\leqslant T_1$. In this case, $\vec{M}\|\vec{e}_x$ and nonzero components of the tensors $u_{ik0}^{(\rm fm,nm)}$ are equal to

$$u_{xx0}^{(\rm fm)}=u_{xx0}^{(\rm nm)}=-2u_{zz0}^{(\rm fm)}=-2u_{zz0}^{(\rm nm)}$$
$$=-\frac{2BM^2L_{\rm fm}}{3(c_{11}-c_{12})(L_{\rm fm}+L_{\rm nm})}, \quad (12)$$

$$u_{yy0}^{(\rm fm)}=\frac{BM^2}{3(c_{11}-c_{12})}\left[1-\frac{c_{12}L_{\rm nm}}{c_{11}(L_{\rm fm}+L_{\rm nm})}\right],$$

$$u_{yy0}^{(\rm fm)}=\frac{BM^2c_{12}L_{\rm fm}}{3c_{11}(c_{11}-c_{12})(L_{\rm fm}+L_{\rm nm})}. \quad (13)$$

In this section, we assume for simplicity that magnetic and nonmagnetic layers have the same elastic moduli $c_{iklm}^{\rm fm}=c_{iklm}^{\rm nm}$.

The additional part of the free energy δE associated with the intrinsic deformations of magnetic and nonmagnetic layers is given by

$$\delta E=\frac{B^2M^2V}{c_{11}-c_{12}}\left[\left(1-\frac{(2c_{11}+c_{12})L_{\rm nm}}{3c_{11}(c_{11}-c_{12})(L_{\rm fm}+L_{\rm nm})}\right)(M^2-M_x^2)\right.$$
$$\left.+\frac{L_{\rm fm}}{L_{\rm fm}+L_{\rm nm}}M_z^2\right], \quad (14)$$

where V is the system volume. The intrinsic deformations increase the free energy of the system ($\delta E>0$).

It follows from Eq. (14) that the intrinsic deformations of magnetic and nonmagnetic layers give rise to renormalization of the anisotropy constants $\beta_{x,z}$. These constants will be further assumed to be renormalized. Moreover, nonzero components of the intrinsic deformations of magnetic and nonmagnetic layers [Eqs. (12) and (13)] do not contain the deformations corresponding to the order parameter and are not associated with the RPT.

IV. A SPECTRUM OF LOW-FREQUENCY ME EXCITATIONS AND A PHASE DIAGRAM OF A MULTILAYERED STRUCTURE

Among the static solutions of system (9)–(11) both homogeneous and inhomogeneous are available. We restrict our study to an analysis of the homogeneous solutions. Energy (6) is minimized for the homogeneous ground states with

$$m_x m_z (K_1 + K_2 m_z^2) = 0, \quad m_y = 0, \quad u_{ik}^{(\mathrm{fm})} = 0, \quad H_D^{(f)} = 0, \tag{15}$$

where $K_1 = \beta_x - \beta_z - b_{xx} + b_{xz}$ and $K_2 = b_{xx} - 2b_{xz} + b_{zz}$ are the effective anisotropy fields, $\vec{m} = \vec{M}/M$.

For $K_2(T) > 0$, in an infinite crystal there exist four equilibrium states,[43] the boundaries of which are the second-order phase transitions points:

$$K_1(T) \geqslant 0 \ \text{(a collinear phase } \vec{M} \| \vec{e}_x) \quad \text{for} \quad T \leqslant T_1,$$

$$-K_2(T) \leqslant K_1(T) \leqslant 0$$

$$\left(\text{angular phases } M_x = M\sqrt{1 - \frac{|K_1(T)|}{K_2}}, \right.$$

$$\left. M_z = \pm M\sqrt{\frac{|K_1(T)|}{K_2}} \right)$$

$$\text{for} \qquad T_1 \leqslant T \leqslant T_2, \tag{16}$$

$$K_1(T) \leqslant -K_2(T) \ \text{(a collinear phase } \vec{M} \| \vec{e}_z)$$

$$\text{for} \qquad T \geqslant T_2.$$

We shall further discuss the stability of the homogeneous ferromagnetic ground state with $\vec{M} \| \vec{a} \| \vec{e}_x$ relative to small ME perturbations. For this purpose we shall examine the ME wave spectrum of the multilayered structure. We start with investigation of the ME waves propagating along the magnetic field $\vec{H}$ and having the lowest threshold for the instability in T and the strongest ME coupling. In this case the forming domain walls are not magnetically charged, and the energy of the demagnetization field associated with magnetic charges is missing.

Using the harmonic approximation of $\exp[i(kx - \omega t)]$ for nonuniform parts $\tilde{\vec{m}}$ and $\tilde{\vec{u}}^{(\mathrm{fm,nm})}$ of the magnetization and the elastic displacements respectively, the solutions of the dynamical equations (9)–(11) above will be sought in the Bloch function form

$$\tilde{\vec{m}}(\vec{r},t) = \vec{V}_{\omega,k,\kappa}(y)\exp[i(kx + \kappa y - \omega t)],$$

$$\tilde{\vec{u}}^{(\mathrm{fm,nm})}(\vec{r},t) = \vec{U}^{(\mathrm{fm,nm})}_{\omega,k,\kappa}(y)\exp[i(kx + \kappa y - \omega t)], \tag{17}$$

where the wave number κ lies within the first Brillouin zone $(-\pi/D \leqslant \kappa \leqslant \pi/D)$, and $D = L_{\mathrm{fm}} + L_{\mathrm{nm}}$ is the period of the multilayered structure. The functions $\vec{V}_{\omega,k,\kappa}(y)$ and $\vec{U}^{(\mathrm{fm,nm})}_{\omega,k,\kappa} \times (y)$ are periodic in y with the period D.

As shown in Ref. 29, near the RPT for ferromagnets with a sufficiently large easy-plane type anisotropy $(K_1 \leqslant 4\pi \leqslant \beta_x)$, we can neglect the effect of the demagnetization field on the ME wave spectrum and not take into account all components of the nonuniform magnetization and the elastic displacements except $\tilde{m}_z$ and $\tilde{u}_z^{(\mathrm{fm,nm})}$. In these approximations the Fourier components of the magnetization and the elastic displacements in the ferromagnetic layer I (see Fig. 1) can be written as

$$\tilde{m}_{Iz}(k,\omega) = a_1\cos(q_1 y) + a_2\cosh(q_2 y) + b_1\sin(q_1 y) + b_2\sinh(q_2 y), \tag{18}$$

$$\tilde{u}_{Iz}^{(\mathrm{fm})}(k,\omega) = \gamma_1[a_1\cos(q_1 y) + b_1\sin(q_1 y)] + \gamma_2[a_2\cosh(q_2 y) + b_2\sinh(q_2 y)], \tag{19}$$

where $\gamma_{1,2} = -i\{\Omega^2 - \beta_x[K_1 + \alpha(k^2 \pm q_{1,2}^2)]\}/(k\beta_x B)$, $\Omega = \omega/\omega_M$, and $\omega_M = gM$.

The insertion of Eqs. (18) and (19) into Eq. (9) yields the characteristic equation for q

$$[(k^2 \pm q_{1,2}^2) - \Omega^2 k_{\mathrm{fm}}^2]\{\Omega^2 - \beta_x[K_1 + \alpha(k^2 \pm q_{1,2}^2)]\} + k^2 h_{\mathrm{me}}\beta_x = 0, \tag{20}$$

where $S_t^{(\mathrm{fm})} = \sqrt{c_{44}^{(\mathrm{fm})}/\rho^{(\mathrm{fm})}}$ is the velocity of the transverse sound in the magnetic layer, $k_{\mathrm{fm}} = \omega_M/S_t^{(\mathrm{fm})}$ and $h_{\mathrm{me}} = B^2M^2/c_{44}^{(\mathrm{fm})}$. Its roots

$$q_{1,2} = [(\sqrt{Q_+^2 + 4\alpha^{-1}h_{\mathrm{me}}c_{44}^{(\mathrm{fm})2}k^2} \pm Q_-)/(2c_{44}^{(\mathrm{fm})}) \mp k^2]^{1/2}, \tag{21}$$

with $Q_\pm = \rho^{(\mathrm{fm})}\omega^2 \pm c_{44}^{(\mathrm{fm})}[\beta_x K_1 - \Omega^2]/(\alpha\beta_x)$.

The nonuniform distributions of the elastic displacements in the nonmagnetic layers II and III (see Fig. 1) can be sought in the forms

$$\tilde{u}_{IIz}^{(\mathrm{fm})}(k,\omega) = \{a_3\cosh[q_3(y + D/2)] + b_3\sinh[q_3(y + D/2)]\}\exp(-i\kappa D/2), \tag{22}$$

$$\tilde{u}_{IIIz}^{(\mathrm{fm})}(k,\omega) = \{a_3\cosh[q_3(y - D/2)] + b_3\sinh[q_3(y - D/2)]\}\exp(i\kappa D/2), \tag{23}$$

with $q_3 = \sqrt{k^2 - \omega^2/S_t^{(\mathrm{nm})2}}$ and $S_t^{(\mathrm{nm})} = \sqrt{c_{44}^{(\mathrm{nm})}/\rho^{(\mathrm{nm})}}$. The components of $\vec{m}$ and $\vec{u}$ [Eqs. (18), (19), (22), and (23)] satisfy the periodicity conditions (17).

Substituting solutions (18)–(23) into the boundary conditions (11), which are reduced to the forms

$$c_{44}^{(\mathrm{fm})}\frac{\partial\tilde{u}_z^{(\mathrm{fm})}}{\partial y} = c_{44}^{(\mathrm{nm})}\frac{\partial\tilde{u}_z^{(\mathrm{nm})}}{\partial y}, \quad \tilde{u}_z^{(\mathrm{fm})} = \tilde{u}_z^{(\mathrm{nm})}, \quad \frac{\partial\tilde{m}_z}{\partial y} = 0, \tag{24}$$

the unknown coefficients $a_{1,2,3}$ and $b_{1,2,3}$ and the dispersion equation for the ME waves can be found. In the general case the dispersion equation is rather cumbersome. It simplifies substantially for $\kappa = 0$. In this case the ME excitations in layers composed of the same material have the same phase and the lowest threshold for the instability in T. For $\kappa = 0$, the symmetric $(a_{1,2,3} \neq 0, b_{1,2,3} = 0)$ and antisymmetric

$(a_{1,2,3}=0, b_{1,2,3}\neq 0)$ ME modes are separated. The dispersion equations for the symmetric and antisymmetric ME mode, respectively, are given by

$$F_s(\omega,k)=\left[\epsilon\frac{q_1}{q_3}\left(1-\frac{\gamma_2}{\gamma_1}\right)\tanh\frac{q_2L_{\rm fm}}{2}-\frac{\gamma_2}{\gamma_1}\frac{q_1}{q_2}\tanh\frac{q_3L_{\rm nm}}{2}\right]\tan\frac{q_1L_{\rm fm}}{2}-\tanh\frac{q_2L_{\rm fm}}{2}\tanh\frac{q_3L_{\rm nm}}{2}=0, \tag{25}$$

$$F_a(\omega,k)=\epsilon\frac{q_1}{q_3}\left(1-\frac{\gamma_2}{\gamma_1}\right)\tanh\frac{q_3L_{\rm nm}}{2}-\frac{\gamma_2}{\gamma_1}\frac{q_1}{q_2}\tanh\frac{q_2L_{\rm fm}}{2}-\tan\frac{q_1L_{\rm fm}}{2}=0, \tag{26}$$

where $\epsilon=c_{44}^{(\rm fm)}/c_{44}^{(\rm nm)}$.

The value of the anisotropy constant $K_{1c}=K_1(T=T_{1c})$, at which the collinear phase $(\vec{M}\|\vec{e}_x)$ loses its stability against the striction superstructure formation and the period of the critical symmetric mode $d_c=2\pi/k_c$, can be determined from the conditions

$$\omega_s(k_c)=0 \quad \text{and} \quad d\omega_s(k)/dk|_{k=k_c}=0, \tag{27}$$

with $\omega_s(k)$ being the frequency of the critical symmetric mode.

The two relevant length scales in terms of the elastic, anisotropy, and ME constants involved in the model can be identified. They are following: the characteristic thicknesses of the ferromagnetic $[L_c\equiv\sqrt{\alpha/(\epsilon h_{\rm me})}]$ and nonmagnetic $(L_{1c}\equiv c^2/L_{\rm fm})$ layers. The static distribution of the magnetization and elastic strain in the system depends on the relations between these parameters and the thicknesses of the ferromagnetic $(L_{\rm fm})$ and nonmagnetic $(L_{\rm nm})$ layers.

Let us analyze the solutions of Eqs. (27) and (25) above for the most interesting limiting cases. In the limiting case of thin ferromagnetic $(L_{\rm fm}\ll L_c)$ and nonmagnetic $(L_{\rm nm}\ll L_c^2/L_{\rm fm})$ layers, the dispersion equation of the lowest (soft) symmetric mode is given by

$$\omega_s^2=\omega_M^2k^2\left[\alpha k^2+K_1(T)-\frac{\epsilon h_{\rm me}L_{\rm fm}}{L_{\rm nm}+\epsilon L_{\rm fm}}\right]\times\left[\frac{k^2}{\beta_x}+\frac{h_{\rm me}\omega_M^2}{S_t^{(\rm nm)2}}\frac{\epsilon L_{\rm fm}(L_{\rm nm}+L_{\rm fm})}{(L_{\rm nm}+\epsilon L_{\rm fm})^2}\right]^{-1}. \tag{28}$$

For the typical values of the parameters used in experimental films: $M\sim 100$ G, $\alpha\sim 10^{-11}$ cm^2, $BM^2\sim 10^7$ erg/cm^3, $c_{44}\sim 10^{12}$ erg/cm^3, $\rho\sim 5$ g/cm^3, and $h_{\rm me}\sim 10^{-3}$, the critical thickness $L_c\sim 10^{-4}$ cm.

It follows from Eqs. (27) and (28) that the frequency and the group velocity of the soft mode vanish at

$$K_{1c0}(T=T_{1c0})=\epsilon h_{\rm me}L_{\rm fm}/(L_{\rm nm}+\epsilon L_{\rm fm}), \quad k_c=0, \tag{29}$$

i.e., in the periodic structure of thin magnetic and nonmagnetic layers, the ME domains are not formed, and for $T=T_{1c0}$ the RPT from the collinear phase to the uniformly magnetized angular phase takes place.

In the system consisting of the thin ferromagnetic $(L_{\rm fm}\ll L_c)$ and thick nonmagnetic $(L_{\rm nm}\gg L_c^2/L_{\rm fm})$ layers, the dispersion equation of the soft symmetric ME mode for $(k\sim L_c^2/L_{\rm fm})$ assumes the form

$$\omega_s^2=\omega_M^2\beta_x\left[K_1(T)+\alpha k^2-\frac{\epsilon h_{\rm me}kL_{\rm fm}}{2\tanh(kL_{\rm nm}/2)+\epsilon kL_{\rm fm}}\right]. \tag{30}$$

From Eqs. (27) and (30), we find the critical value of the anisotropy constant K_{1C} and the critical ME superstructure period d_c in the forms

$$K^{1c}=\frac{\epsilon h_{\rm me}}{16}\frac{L_{\rm fm}^2}{L_c^2}\left[1+4\exp\left(-\frac{L_{\rm fm}L_{\rm nm}}{4L_c^2}\right)\right], \tag{31}$$

$$d_c=8\pi\frac{L_c^2}{L_{\rm fm}}\left[1+\frac{L_{\rm fm}L_{\rm nm}}{2L_c^2}\exp\left(-\frac{L_{\rm fm}L_{\rm nm}}{4L_c^2}\right)\right]. \tag{32}$$

Then the transverse wave vectors of a single ferromagnetic layer $q_{1,2}$ and the penetration depth of the soft mode into a nonmagnetic layer $\lambda=q_3^{-1}$ can be written as

$$q_{1,2}=\sqrt{L_{\rm fm}/L_c^2}[1\mp 3L_{\rm fm}/(8L_c)], \quad \lambda\sim d_c/2\pi. \tag{33}$$

For a given $L_{\rm fm}$ the value K_{1c}, the critical period d_c, and λ decrease as the nonmagnetic layer thickness $L_{\rm nm}$ and parameter ϵ increase. When the ferromagnetic layer thickness decreases, the value K_{1c} tends to zero, i.e., ferromagnetic layers become clamped by nonmagnetic ones, and the period d_c tends to infinity.

For thick ferromagnetic $(L_{\rm fm}\gg L_c)$ and nonmagnetic $(L_{\rm nm}\gg L_c^2/L_{\rm fm})$ layers, the dispersion equation of the soft symmetric ME wave in the long wavelength limit $k\gg 1/L_{\rm fm}$ assumes the form

$$\omega_s^2=S_t^{(\rm fm)2}\left\{\left[\frac{K_1(T)-h_{\rm me}}{h_{\rm me}}+L_c^2k^2\right]k^2+\frac{\pi^2}{L_{\rm fm}^2}\left[1-\frac{2\pi\epsilon}{kL_{\rm fm}}\tanh^{-1}\left(\frac{kL_{\rm nm}}{2}\right)\right]\right\}. \tag{34}$$

Using conditions (27) and (34), we obtain

$$K^{1c}(T=T_c)=h_{\rm me}\left\{1-\frac{2\pi L_c}{L_{\rm fm}}\left[\left[1-\left(\frac{\pi\epsilon^2L_c}{L_{\rm fm}}\right)^{1/2}\right]\times\left[1+2\exp\left[-\left(\frac{\pi L_{\rm nm}^2}{L_cL_{\rm fm}}\right)^{1/2}\right]\right]\right\}, \tag{35}$$

$$d_c=\sqrt{4\pi L_cL_{\rm fm}}\left\{1+\left(\frac{9\pi\epsilon^2L_c}{16L_{\rm fm}}\right)^{1/2}\left[1+2\exp\left[-\left(\frac{\pi L_{\rm nm}^2}{L_cL_{\rm fm}}\right)^{1/2}\right]\right]+\frac{\pi\epsilon L_{\rm nm}}{2L_{\rm fm}}\exp\left[-\left(\frac{\pi L_{\rm nm}^2}{L_cL_{\rm fm}}\right)^{1/2}\right]\right\}, \tag{36}$$

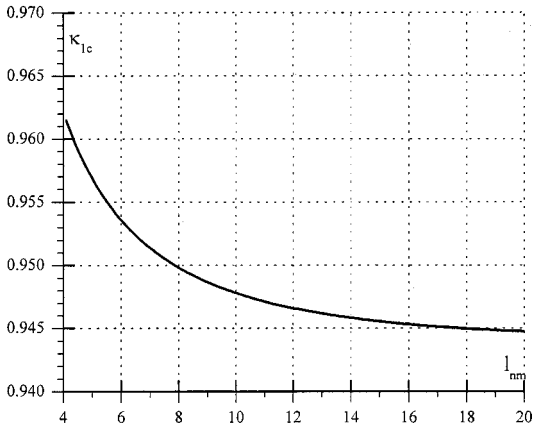

FIG. 2. Phase diagram of the system in the plane (l_{fm},κ_{1c}); $l_{fm,nm}=L_{fm,nm}/L_c$ and $\kappa_{1c}=K_{1c}/h_{me}$ are the reduced layer thickness and anisotropy constant, respectively, for $l_{nm}=2000$ and $\epsilon=1$. The solid line separates the collinear (I) and domain (II) phases.

$$q_1 \approx \pi/L_{\mathrm{fm}}, \quad q_2 \approx 1/2L_c \gg 2\pi/d_c \quad \lambda \approx d_c/(2\pi). \tag{37}$$

For $L_{\mathrm{fm}}=\mathrm{const}$, the value K_{1c}, the critical period d_c, and λ decrease as the nonmagnetic layer thickness L_{nm} increase. When the ferromagnetic layer thickness increases, the value $K_{1c}\to h_{\mathrm{me}}$, i.e., the ferromagnetic layers are nearly free, and the period d_c and λ increase. In the limiting case $L_{\mathrm{fm}}\to\infty$, the period d_c and λ tends to infinity, and in the long-wavelength approximation the ME wave spectrum is considerable deformed. For $L_{\mathrm{fm}}=\infty$, the speed of one of the magnitoacoustic waves vanishes in the RPT point, that is, the transition to the case of bulk crystal[15–20] takes place.

Figures 2 and 3 show sections of the phase diagram of the system under study $(l_{\mathrm{nm}},l_{\mathrm{fm}},\kappa_{1c})$ by the planes $l_{\mathrm{fm}}=\mathrm{const}$ and $l_{\mathrm{nm}}=\mathrm{const}$, respectively, where $l_{\mathrm{nm}}=L_{\mathrm{nm}}/L_c$ and $l_{\mathrm{fm}}=L_{\mathrm{fm}}/L_c$ are the reduced ferromagnetic and nonmagnetic layer thickness, respectively, $\kappa_{1c}=K_{1c}/h_{\mathrm{me}}$ is the reduced anisotropy constant; the dependences of d_c/L_c on l_{nm} and l_{fm} are depicted in Figs. 4 and 5, respectively.

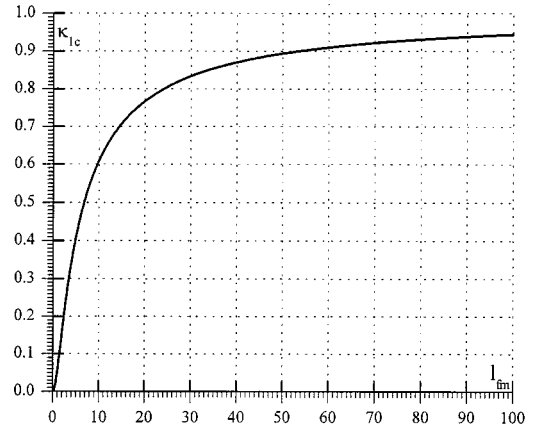

FIG. 3. Dependence of the reduced critical period of a ME superstructure d_c/L_c on l_{fm} for $l_{nm}=2000$ and $\epsilon=1$.

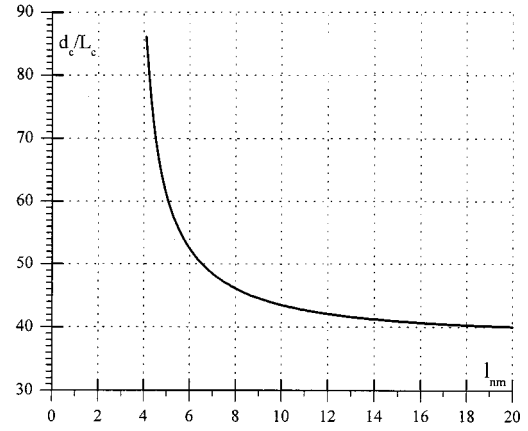

FIG. 4. Phase diagram of the system in the plane (l_{nm},κ_{1c}) for $l_{fm}=200$ and $\epsilon=1$. The solid line separates the collinear (I) and domain (II) phases.

It follows from Fig. 4 that the DS period increases with decreasing nonmagnetic layer thickness L_{nm}. Therefore for thin nonmagnetic layers $(L_{\mathrm{nm}}<L^*_{\mathrm{nm}})$, the RPT from the collinear phase to the uniformly magnetized angular phase takes place. For thick nonmagnetic layers $(L_{\mathrm{nm}}>L^*_{\mathrm{nm}}$, the RPT from the collinear phase to the ME superstructure is realized. The curve $L_{\mathrm{nm}}=L^*_{\mathrm{nm}}(L_{\mathrm{fm}})$ separating the stability regions of the angular and ME domain phases on the $(L_{\mathrm{fm}},L_{\mathrm{nm}})$-phase diagram (Fig. 6) can be determined from the condition $K_{1c0}=K_{1c}$. For $L_{\mathrm{nm}}\geqslant 1$ and $L_{\mathrm{fm}}<L_c$, the wave vector k_c of the soft critical mode tends to zero monotonically. In this case the dispersion equation (25) for $\omega=0$ is reduced to the form

$$K_1(T)+\alpha k^2 \cong (\epsilon k L_{\mathrm{fm}}/2)\tanh^{-1}(kL_{\mathrm{nm}}/2). \tag{38}$$

Using Eq. (38) we find

$$L^*_{\mathrm{nm}}=12L_c^2/L_{\mathrm{fm}}. \tag{39}$$

As seen from Fig. 6, for $L_{\mathrm{fm}}<L_c$ the curve $L^*_{\mathrm{nm}}(L_{\mathrm{fm}})$ is well described by Eq. (39). For $L_{\mathrm{fm}}<L_c$, the analytical dependence $L^*_{\mathrm{nm}}(L_{\mathrm{fm}})$ is very cumbersome.

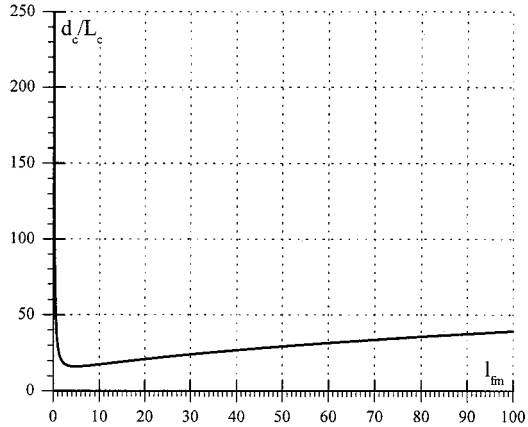

FIG. 5. Dependence of the reduced critical period of a ME superstructure d_c/L_c on l_{nm} for $l_{fm}=200$ and $\epsilon=1$.

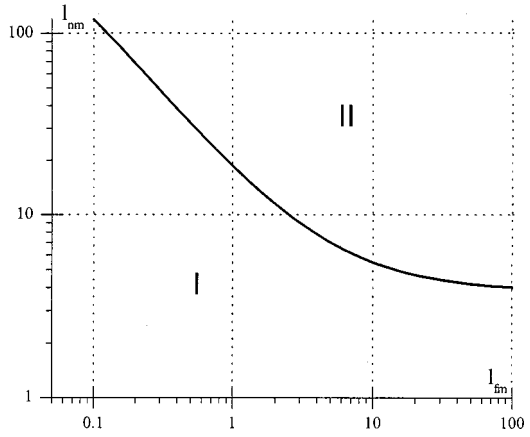

FIG. 6. Phase diagram of the system in the plane (l_{fm}, l_{nm}) for $\epsilon = 1$. The solid line separates the angular (I) and domain (II) phases.

V. SPECTRUM OF ME EXCITATIONS AND PHASE DIAGRAM OF THE SYSTEM CONSISTING OF TWO-SUBLATTICE ORTHOFERRITE/NONMAGNETIC MULTILAYERS

We shall consider a periodic system composed of rhombic orthoferrite layers of thickness $L_{\rm fm}$ with the hard axis $\vec{b}$ $(\vec{b} \| \vec{e}_y)$ normal to the basal plane near the temperature T_1. Orthoferrite layers are sandwiched nonmagnetic spacers of thickness $L_{\rm nm}$ (as shown in Fig. 1). We shall also assume that the orthorhombic axes $\vec{a} \| \vec{e}_x$ and $\vec{c} \| \vec{e}_z$. The total energy of a system is given by

$$E = \int_{V_f} dv \left\{ 2M_0 \left[\frac{1}{2} H_E m^2 - D_1 l_x m_z - D_2 l_z m_x + a^2 H_E \frac{\partial \vec{l}}{\partial x_i} \frac{\partial \vec{l}}{\partial x_i} \right] + f_A + f_{\rm me} + \frac{1}{2} c^{(f)}_{iklm} u^{(f)}_{ik} u^{(f)}_{lm} \right\} + \int_{V_s} dv \, \frac{1}{2} c^{(s)}_{iklm} u^{(s)}_{ik} u^{(s)}_{lm}, \tag{40}$$

where H_E and $D_{1,2}$ are the exchange and Morya-Dzyaloshinskii fields; $f_A = M_0[H_{A1} l_x^2 + H_{A2} l_z^2 + (H_{a1} l_x^4 + 2H_{a2} l_x^2 l_z^2 + H_{a3} l_z^4)/2]$ and $f_{\rm me} = B M_0^2 l_i l_k u^{(f)}_{ik}$ are the densities of the magnetic anisotropy and ME energy, respectively; H_A and H_a are the anisotropy fields; α is the nonuniform exchange constant; $\vec{m} = (\vec{M}_1 + \vec{M}_2)/(2M_0)$ and $\vec{l} = (\vec{M}_1 - \vec{M}_2)/(2M_0)$ are the ferromagnetic and antiferromagnetic vectors, satisfying $\vec{m}^2 + \vec{l}^2 = 1$ and $\vec{m} \cdot \vec{l} = 0$; $\vec{M}_{1,2}$ are the sublattice magnetizations $(|\vec{M}_1| = |\vec{M}_2| = M_0)$; and a is the lattice parameter.

Once again, for simplicity an orthoferrite is assumed to be ME and elastically isotropic. Besides, we shall neglect the magnetic anisotropy caused by the Morya-Dzyaloshinskii interaction $(D_2 = -D_1 = D)$. Energy (40) is minimized for the ground state with

$$\vec{m} = D(l_z \vec{e}_x - l_x \vec{e}_z)/H_E, \quad l_x l_z (K_1 + K_2 l_x^2) = 0, \tag{41}$$

where $K_1 = H_{A1} - H_{A2} + H_{a2} - H_{a3}$ and $K_2 = H_{a1} - 2H_{a2} + H_{a3}$ are the effective anisotropy fields.

For $K_2(T) > 0$, in an infinite crystal there exist four equilibrium states, the boundaries of which are the second-order phase transitions points:[37]

$$K_1(T) \geqslant 0 \quad (\vec{m} \| \vec{e}_x, \vec{l} \| \vec{e}_z) \quad \text{for} \quad T \leqslant T_1,$$

$$-K_2(T) \leqslant K_1(T) \leqslant 0 \quad \text{(two angular phases)} \quad \text{for} \quad T_1 \leqslant T \leqslant T_2, \tag{42}$$

$$K_1(T) \leqslant -K_2(T) \quad (\vec{m} \| \vec{e}_z, \vec{l} \| \vec{e}_x) \quad \text{for} \quad T_2 \leqslant T,$$

The orthoferrite dynamics is described by a system of the coupled equations for the sublattice magnetization vectors $\vec{M}_1$ and $\vec{M}_2$ and the equations for elasticity

$$\partial \vec{M}_n / \partial t = -g[\vec{M}_n \times H_n^{\rm eff}], \quad \rho^{(f,s)} \partial^2 u_i^{(f,s)} / \partial t^2 = \partial \sigma_{ik} / \partial x_k, \tag{43}$$

subject to the boundary conditions

$$\sigma^{(f)}_{iy} = 0 \quad (i = x,y,z), \quad \vec{l} \times \partial \vec{l} / \partial y = 0 \quad \text{for} \quad y = L,$$

$$\sigma^{(f)}_{iy} = \sigma^{(s)}_{iy} \quad (i = x,y,z), \quad \vec{u}_i^{(f)} = \vec{u}_i^{(s)},$$

$$\vec{l} \times \partial \vec{l} / \partial y = 0 \quad \text{for} \quad y = 0. \tag{44}$$

Here $\vec{H}_n^{\rm eff} = -\partial F / \partial \vec{M}_n$ is the effective magnetic field and $n = 1, 2$ are the magnetic sublattice indexes.

In an orthoferrite of a sufficient high Neel temperature[16] $H_E \gg H_A, H_a, D$. For this case, we note

$$m \ll l, \quad l^2 = 1 - m^2 \approx 1. \tag{45}$$

In the first approximation in the parameters H_A/H_E, H_a/H_E, $D \ll 1$, Eqs. (43) become

$$\Box \vec{l} \times \vec{l} = g^2 \left[D^2 l_y (l_x \vec{e}_z - l_z \vec{e}_x) + \frac{H_E}{2M_0} \vec{l} \times \frac{\partial (f_A + f_{\rm me})}{\partial \vec{l}} \right], \tag{46}$$

where $\Box \equiv \partial^2/\partial t^2 - V_s^2 \nabla^2$, $V_s = g\sqrt{\alpha M_0 H_E} \equiv g a H_E$ is the characteristic velocity of spin wave.

The spin-wave spectrum of an orthoferrite contains two branches. We restrict our consideration to the soft ''quasiferromagnetic'' branch of the spectrum corresponding to the vibration of $\vec{l}$ in the easy plane *ac* of an orthoferrite and do not take into account the high-frequency branch corresponding to vibrations involving a departure of $\vec{l}$ from the basis plane and the relative high activation energy $(\omega \sim g\sqrt{H_E(H_{A2} + H_{a3})})$. Therefore, we assume $l_y = 0$ everywhere. Then the equations of motion for l_x and $u_z^{(f,s)}$ are reduced to the forms

$$l_x \Box l_z - l_z \Box l_x = g^2 H_E [(K_1 + K_2 l_x^2) l_x l_z + BM_0 (l_z^2 - l_x^2) \partial u_z^{(f)} / \partial x], \quad (47)$$

$$\left[c_{44}^{(\mathrm{fm})} \mu \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) - \rho \frac{\partial^2}{\partial t^2} \right] u_z^{(f)} + BM_0^2 \frac{\partial (l_x l_z)}{\partial z} = 0, \quad (48)$$

$$\left[c_{44}^{(\mathrm{nm})} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) - \rho \frac{\partial^2}{\partial t^2} \right] u_z^{(s)} = 0, \quad l_z = \sqrt{1 - l_x^2} \approx 1 - l_x^2/2. \quad (49)$$

Once again, we assume that $\rho^{(\mathrm{fm})} = \rho^{(\mathrm{nm})} = \rho$.

It follows from Eqs. (47)–(49) that Eqs. (12)–(14) and (17)–(39) derived in Secs. III and IV also apply to an orthoferrite provided the following substitutions are made:

$$m_z \to l_x, \quad K_1 \to K_1 / M_0, \quad \beta_x M \to H_E, \quad \omega_M \to g M_0, \quad \alpha \to 2 H_E a^2, \quad M \to M_0. \quad (50)$$

VI. CONCLUSION

We have studied the ME wave spectrum and the ME superstructure nucleation in the system consisting of rhombic magnetic/nonmagnetic multilayers close to the RPT associated with the spin reorientation in the magnetic layer plane. The phase diagram of the system in the space $\{K_1(T), L_{\mathrm{fm}}, L_{\mathrm{nm}}\}$ has been constructed, and the stability regions of ME DS, the existence of which is associated with the ME coupling of the magnetization to the lattice deformation on the layer interfaces, have been determined.

In studies of domain structure formation in multilayered systems, we ignore the exchange interaction between magnetic layers. This approximation holds for a system of dielectric multilayers in the case that the nonmagnetic layer thickness exceeds several lattice parameters. For the case of the conducting nonmagnetic layers, the exchange interaction between magnetic layers can be ferromagnetic or antiferromagnetic depending on the nonmagnetic layer thickness. Hence, the results above are appropriate only in the case of sufficiently thick nonmagnetic layers as we can neglect the exchange interaction between magnetic layers.

Of special interest are the multilayer structures consisting of the conducting layers that demonstrate the GMR.[32] Then the ME formation will cause the GMR to become enhanced because of the spin-dependent scattering of conduction electrons occurring at the domain walls.

The complexity of the multilayer systems is due to two sources of the strain fields; one being the long-elastic fields, which appear at the RPT, the other being the intrinsic deformations of magnetic and nonmagnetic layers (see Sec. III) and the lattice mismatch, which gives rise to so called ''misfit stresses,'' in the collinear and angular phases. For the RPT under study, the order parameter is the magnetization component M_z (or the elastic deformation u_{xz}). If the symmetry of the intrinsic and misfit deformations is distinct from the order parameter symmetry, the effect of the weak lattice mismatch and intrinsic deformations gives rise to a renormalization of the anisotropy constants. A strong lattice mismatch and intrinsic deformations may result in elastic multidomain state formation in the collinear phase. If the ME domain structure period is small compared with the period of the elastic domains, the latter ones can break into ME domains at the RPT. Otherwise a calculation of the inhomogeneous state will be very complicated, and is beyond the scope of the present paper.

For ultrathin magnetic films (multilayers), recent experiments indicated that the role of epitaxial misfit between films (multilayers) and substrate material for film stress and ME properties cannot be extrapolated from the respective bulk behavior (see Refs. 47 and 48 and Refs. 42 and 49 for a review). Ultrathin films (multilayers) are rigidly bonded by a substrate and are not free to change their length due to magnitostriction. Instead, ME stresses evolve, and additional so-called surface corrections to the ME coupling have to be considered. Therefore, bulk ME properties are not applied for ultrathin magnetic films (multilayers), and for calculating the anisotropy constant K_{1C} and the critical ME superstructure period d_c, ME data have to be measured for the system of interest.

It is pertinent to note that for thick[50–54] and ultrathin[55,56] magnetic films with the EMA perpendicular to its surface, some attempts aimed at an identification of the soft mode and eventually of what determines the characteristic size of the ensuing domain structure close to temperature- or thickness-driven reorientations have been undertaken. Here, the familiar ideas have been applied under more complicated circumstances. The ME DS formation is expected to occur to the wide class of the ferromagnetic, ferrimagnetic, and antiferromagnetic/nonmagnetic multilayers close to the RPT induced by an external magnetic field and a temperature variation and associated with the spin reorentation in the magnetic layer plane.

In conclusion, it is well known that a RPT is a particular case of ferroelastic (FE) transition. A characteristic feature of such a PT is the linear relationship between the order parameter and macroscopic lattice deformations. Many features of a FE PT are of a general nature and are independent of the microscopic realization of the order parameters. For example, striction DS formation in the proper FE films on the elastic substrate close to the FE PT was studied theoretically in Refs. 44–46 and 57. The FE DS was observed in thin films of proper FE.[58,59] The topological PT between a single and a multidomain states in FE films, in order to accommodate misfit stresses, which appear at the FE PT or exist in the high-symmetric phase owing to a mismatch between the lattice parameters of the FE film and the substrate, was predicted by Roytburd and co-workers[44,60] and observed in Ref. 61. In view of this, we expect that the results above may be extended to a study of striction DS formation in ferroelastic/nonferroelastic multilayer structures close to the FE PT.

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