Memory effects in the magnetotransport properties of the classical Drude metal

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Memory effects are incorporated into the classical Drude model by use of a current-decay function that deviates from a simple exponential. In the hypothetical case where velocity persists after one collision but is totally lost after subsequent collisions, the magnetoresistance is negative, the Hall coefficient can occur with either sign, and both vary with magnetic field.

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INTRODUCTION

The relaxation-time approximation is pervasive in theories of the optical, transport, and magnetotransport properties of metals. It is equivalent to assuming that the current will decay in a simple exponential fashion if the driving electric field is abruptly switched off. It leads to the standard textbook result that the magnetoresistance $(\Delta\rho/\rho)$ of a free electron metal is zero, and that its Hall coefficient R should assume the classical Drude value $R_0 = -(n^*ec)^{-1}$, where n^* is the effective electron density. The author has shown[1] that there is an exceedingly simple way to go beyond the relaxation-time approximation within the Drude model by incorporating memory effects. The approach was applied initially to the infrared optical conductivity, and is extended in this Brief Report to the magnetotransport properties.

MEMORY EFFECTS

In the phenomenological approach of Ref. 1, Poisson statistics are invoked and the decay function $f(t)$ for the current is written

$$f(t) = \exp(-t/\tau)\left[1 + \sum_{n=1}^{\infty} c_n (t/\tau)^n/n!\right], \quad (1)$$

where τ is the average time between collisions, and the coefficients c_n represent the fraction of the electron's initial velocity that is retained after the nth collision. The summation in square parentheses in Eq. (1) may be regarded as persistence-of-velocity or memory effects. Alternatively, the reader may wish to disregard the heuristic interpretation of the coefficients c_n and treat Eq. (1) as merely a convenient expression for the parametrization of deviations from strict exponential decay.

Let us adopt the conventional geometry with a magnetic field H in the z direction and electric fields in the x,y directions. According to the impulse-response approach,[2] the currents j_x and j_y generated by an electric field E_x may be written

$$j_x = (n^*e^2/m)\int_0^{\infty} f(t)\cos(\omega_c t) E_x\, dt, \quad (2)$$

$$j_y = (n^*e^2/m)\int_0^{\infty} f(t)\sin(\omega_c t) E_x\, dt. \quad (3)$$

Current caused at some time in the past by an impulse $E_x\delta t$ fades away according to the decay function $f(t)$ as the electrons execute circular orbits with cyclotron frequency $\omega_c = eH/mc$. The principal components of the conductivity tensor σ_{xx} and σ_{xy} are then given by the real and imaginary parts respectively of the expression

$$\frac{\sigma_0}{(1-i\omega_c\tau)}\left[1 + \sum_{n=1}^{\infty} \frac{c_n}{(1-i\omega_c\tau)^n}\right], \quad (4)$$

where $\sigma_0 = n^*e^2\tau/m$, the standard Drude dc conductivity.

SINGLE-SCATTERING CASE

Following the earlier work,[1] let us truncate the series after a single scattering ($c_n = 0$ for $n > 1$). We are saying that memory of the electron's initial velocity persists after the first collision but is totally lost thereafter. The arbitrariness of this truncation is discussed in Ref. 1, where it is justified by its empirical success in simulating a peak in the infrared optical conductivity of certain materials. After conversion to the resistivity tensor, the magnetoresistance, $\Delta\rho/\rho[\equiv[\rho_{xx}(H) - \rho_{xx}(0)]/\rho_{xx}(0)]$, and Hall coefficient $R(\equiv(\rho_{xy}/H))$ normalized to the Drude R_0 are given by

$$\Delta\rho/\rho = \frac{-c_1^2(\omega_c\tau)^2}{(1+c_1)^2 + (\omega_c\tau)^2}, \quad (5)$$

$$R/R_0 = \frac{(1+2c_1) + (\omega_c\tau)^2}{(1+c_1)^2 + (\omega_c\tau)^2}. \quad (6)$$

The magnetic-field dependences of $\Delta\rho/\rho$ and R/R_0 generated by Eqs. (5) and (6) are illustrated in Fig. 1.

MAGNETORESISTANCE

Within the single-scattering case, the magnetoresistance is always negative. It varies quadratically with magnetic field in the weak-field limit, and saturates at $-c_1^2$ in the high-field limit. Negative magnetoresistance is of special interest since it is one of the signatures of weak localization, a quantum mechanical effect that arises through coherent backscattering.[3] Recent papers on the Lorentz gas[4–6] have pointed out that negative magnetoresistance can arise through classical mechanisms. The Lorentz gas is a two-dimensional model in which electrons scatter off randomly

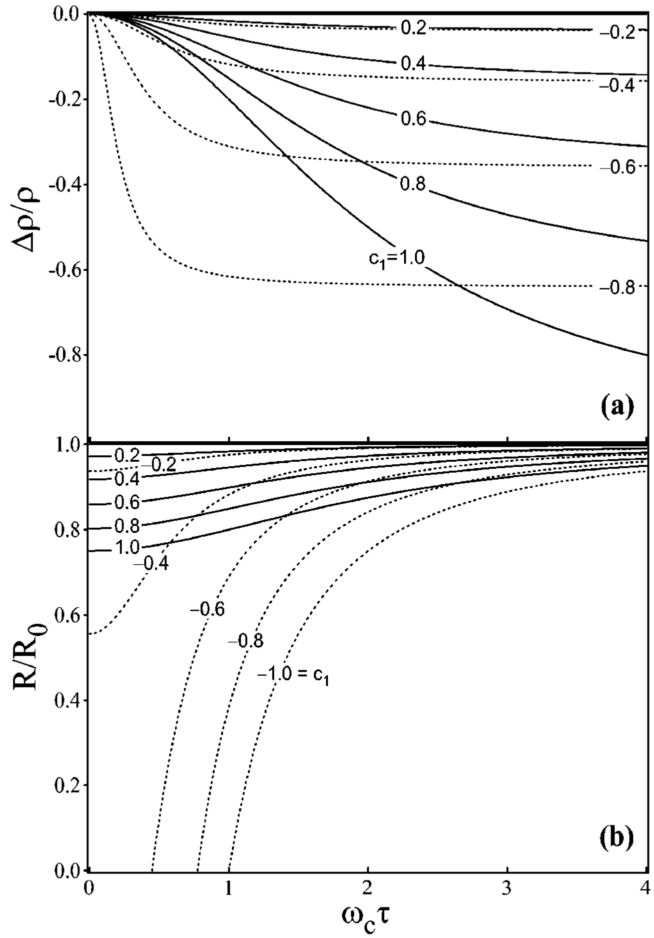

FIG. 1. Magnetic-field dependence, for various values of the persistence-of-velocity parameter c_1, of (a) the magnetoresistance $\Delta\rho/\rho$ and (b) the Hall coefficient R normalized to the Drude value R_0. Full (dotted) curves correspond to positive (negative) values of c_1. Note that $\Delta\rho/\rho$ is always negative and that R/R_0 can occur with either sign.

positioned hard disks. Memory effects within the Lorentz gas are significant and arise through random walks that return an electron to a region within a mean free path of its point of origin.[7] The curves in Fig. 1(a) bear a resemblance to those obtained by numerical simulations of the Lorentz gas,[4,5] and it would be of interest to explore this resemblance further.

HALL EFFECT

The H-dependence of the Hall coefficient within the single-scattering case is illustrated in Fig. 1(b). In the weak-field limit, $R/R_0=(1+2c_1)/(1+c_1)^2$ which, depending on the value of c_1, can occur with either sign. The behavior is striking for large negative values of c_1 corresponding to strong backscattering. For $c_1<0.5$, R/R_0 is negative for weak fields, as if the carriers were holes rather than electrons. In this case, the backscattered part of the electron's trajectory is contributing more strongly to the Hall resistivity than the original forward trajectory. In the high-field limit, R/R_0 converges on 1 regardless of the value of c_1. In this limit, the electrons execute many cyclotron orbits before scattering, and so the details of the scattering become progressively less important. Thus, for large negative values of c_1, we have dramatic behavior in which R/R_0 starts negative at zero field and, with increasing field, switches sign.

CLOSING REMARKS

It is sometimes implied that memory effects lie beyond the scope of the classical Drude model or the semiclassical Boltzmann equation. The limitation is imposed rather by the relaxation-time approximation or, equivalently, by the assumption of strict exponential current decay. When this limitation is removed, memory effects are easily incorporated into the Drude model, leading to interesting expressions for the transport properties.

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