Intrasubband spin-flip relaxation by one-magnon processes in $Cd_{1-x}Mn_x$ **Te quantum wells**

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The ''*s*-*d* model'' which contains a description of band electrons coupled to localized spins is used to calculate the intrasubband spin-flip lifetime due to scattering of electrons by magnons in $Cd_{1-x}Mn_xTe$ quantum well structures. We found that the low-temperature result $\tau_{\text{flip}} \approx 1$ ps agrees nicely with the low-temperature time-resolved photoluminescence measurements. Furthermore, the spin-flip lifetime broadening was found to scale with L^{-1} , *L* being the quantum well width.

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I. INTRODUCTION

During the past decade, the physics of carrier spin dynamics in GaAs/AlGaAs quantum wells (QWs) has been the focus of much interest, $1-\frac{7}{7}$ and its potential applications for ultrafast devices using a spin-dependent optical nonlinearity⁸ as well as spin-polarized electrons.⁹ Indeed, the high optical and electronic qualities of these quantum structures and their very well-known properties make them ideal candidates to explore spin-related properties. However, despite this intensive work the spin relaxation processes are still not fully understood.

Type II-VI QWs spin properties have been the focus of much less attention. However, the possibility to incorporate magnetic ions, such as manganese, opens possibilities of spin control, which, for example, have been recently used to demonstrate the spin injection from a diluted magnetic semiconductor (DMS) into a GaAs light emitting diode.¹⁰ In this emerging field of spin-dependent electronics it becomes increasingly important to unravel the spin relaxation processes, in particular for quantum structures containing magnetic ions. Furthermore, recent progress in the growth of DMSbased QW's by molecular beam epitaxy allows great flexibility when elaborating new structures. The interest in electronic spin polarization in DMS quantum well systems has grown since then^{$10-14$} fueled by the possibility of producing spin memory devices and spin transistors as well as exploiting the properties of spin coherence for quantum computation. The determination of spin-flip scattering rates between *s*-*p* electrons and the 3*d* electrons of the localized magnetic moments in Mn-based QW's is extremely important for spintronic devices, because if the spins relax too rapidly, the distances traveled by spin-polarized currents will be too short for practical applications. Photoinduced magnetization and spin-dependent absorption experiments^{15–17} in which the injection of spin-polarized carriers induces the magnetization of the magnetic ions, have confirmed the existence of spinflip process between the carrier spin and the embedded magnetic ion. Time-resolved measurements^{18,19} of the Mn magnetization in $Cd_{1-x}Mn_xTe$ due to the creation of spinpolarized photoelectrons, pointed out a strong electronenergy dependence of the spin-relaxation rate. The timeresolved measurement is particularly interesting because it addresses the question of what spin-flip scattering mechanism determines the relaxation of the injected carrier spins.

The data for the low-temperature time-dependent intensity for circularly polarized components of luminescence from an 86 Å $Cd_{1-x}Mn_xTe/Cd_{1-y}Mn_yTe$ *QW* $(x < y)$ have shown^{18,19} that for low detection energies (close to the ground sate), the response is virtually instantaneous indicating very rapid electron-spin relaxation which has been associated with spin-orbit effects in the valence band. At higher detection energies, which are thought to reflect free-carrier behavior, the spin dynamics is very slow, corresponding to recombination in the initial nonequilibrium population of free electrons and holes. The 1.88 eV detection-energy data¹⁸ give an orientation-decay time of 5 ps and suggests a strong free-carrier-ion–spin-spin-exchange relaxation mechanism. The data have also shown similar short spin-relaxation times in quantum wells containing no magnetic materials which indicates that the electron spin relaxation times are insensitive to the presence of Mn ions in the QW, a rather surprising result indeed since *s*-*d* exchange scattering should be very efficient. Latter studies 20 have revealed that these so-called nonmagnetic QW's may exhibit magneto-optical splittings, which in some instances may be larger than those from the magnetic barriers. This effect results mainly because of magnetic dilution at the interface and because of the fact that the magnetic susceptibility decreases as the Mn concentration becomes larger than 10–15 %. Therefore, in our opinion, the \arccos of Freeman and Awschalom¹⁸ and Freeman *et al.*¹⁹ in early studies must be reconsidered. More recently, Akimoto *et al.*²¹ measured electron and heavy-hole spin relaxation times in CdTe/Cd_{1-*x*}Mn_{*x*}Te QW's ($x=0.35$) by time-resolved circular dichroism. They found a steep decrease of the electron spin relaxation time τ_e as the QW width decreases, which they related to the increase of the overlap between *s* and *d* orbitals. This was the first direct hint that exchange scattering acts efficiently to flip the electron spin. Although the arguments given in those works 18,19 to explain the intriguing results seem perfectly sensible, nevertheless, the interesting diversity of results suggests a need for theoretical enquiry.

Ferromagnetic ordering in *p*-doped diluted magnetic semiconductors has attracted much attention from both experimentalists²² and theorists.²³ In particular carrier induced ferromagnetism has been observed in modulation doped *p*-type $Cd_{1-x}Mn_xTe$ QW's with Curie temperatures between 1 and 5 K. 24,25 It has been proven that the antiferromagnetic coupling of the hole spin with the magnetic spin

gives rise to a ferromagnetic coupling among the localized spins. The photoluminescence studies^{24,25} on MBE-grown samples, containing a single modulation-doped, 8 nm quantum well of CdMnTe/CdMgZnTe: N, have revealed a ferromagnetic transition occurring between 1.8 and 2.5 K, depending on the Mn molar fraction. On the other hand, earlier studies on $Cd_{1-x}Mn_xTe$ QW's have also shown²⁶ a magnetically ordered low-temperature phase for $0.17 < x < 0.60$. Furthermore, Raman spectroscopy experiments^{27,28} have shown that a transition from the paramagnetic to the magnetically ordered phase is accompanied by the appearance of a new Raman feature at low temperatures. Since this excitation is associated with magnetic order it has been attributed to a magnon.

In this paper we report on a calculation for the lowtemperature scattering rate of an electron by one magnon (spin-flip) process in $Cd_{1-x}Mn_xTe$ QW structures. We argue that the ''*s*-*d* model'' allows reasonable estimates for the electron-spin lifetime due to one magnon scattering. Using this model, the low-temperature electron spin-flip relaxation time is predicted to be of the order of 1 ps for the Mn molar fraction $x=0.19$ and a density of cation sites of 2.783 $\times 10^{21}$ cm⁻³, agreeing with the experiment.^{18,19} Furthermore, we found that the low-temperature spin-flip decay rate scales with L^{-1} , *L* being the QW width.

II. HAMILTONIAN MODEL

The generic Hamiltonian for the ''*s*-*d* model'' couples electron bands $\vec{k}n$ with energy $\varepsilon(\vec{k}n)$ (independent of spin, so far) to localized spins \vec{S}_i of Mn ions ($S = 5/2$) located on Cd sublattices, at lattice sites \vec{R}_i :

$$
H = \sum_{\vec{k}n\sigma} \varepsilon(\vec{k}n)c^{+}(\vec{k}n\sigma)c(\vec{k}n\sigma)
$$

$$
-J\sum_{i}^{N_C} \sum_{m\gamma\mu} \vec{S}_{i} \cdot c^{+}(im\gamma)\vec{\tau}_{\gamma\mu}c(im\mu).
$$
 (1)

The electron bands derive from the outer atomic orbitals $\langle im \gamma \rangle$ with wave functions $\psi_m(\vec{r}-\vec{R}_i)\chi_\gamma$, where χ_γ is the spin part. The conduction electrons have a magnetic interaction with the well-localized $S = 5/2$ half filled 3*d* shell. The *s*-*d* exchange constant *J* comes from the atomic Hund rule trying to keep conduction electron spins parallel to ''core'' spins and electron bands acquire a spin splitting ΔE proportional to *J*. In Eq. (1) , N_C is the total number of cation sites, $\vec{\tau}$ is the Pauli spin matrix, and $c^+(\vec{k}n\sigma)$ and $c(\vec{k}n\sigma)$, respectively, create and destroy a conduction electron in the quasitwo-dimensional $(Q2D)$ band state *n* that is characterized by a two-dimensional Bloch wave vector \vec{k} and spin σ .

In DMS literature *J* is usually defined as a random exchange integral which takes the value $J = N_0 \alpha x$ with probability *x*. Here N_0 is the number of cation sites per unit volume and α is the s - d exchange coupling. Also, as is usual in DMS, *x* is the molar fraction of magnetic atoms to replace the Cd atoms. In this way, as we shall see below, the spin splitting will be written as $\Delta E = N_0 \alpha S x$.

Although a realistic profile of magnetic quantum wells is one with finite barriers we suppose, for the sake of simplicity that the QW barriers are infinitely high. Such an approximation is supported by a band offset calculation²⁹ in CdTe/ (Cd,Mn)Te quantum wells in which a large conduction-band offset of 400 meV [the band gap of $Cd_{1-x}Mn_xTe$ is linear with x (Ref. 18)] was deduced for a Mn molar fraction y $=0.24$ in the barrier layer with an accuracy better than 10 meV. In this model, the QW occupies the space between *z* $= -L/2$ and $z = +L/2$ and the Q2D electron energy is $\kappa(\vec{k}n) = \hbar^2 \vec{k}^2/2m^* + E_0 n^2$, where $E_0 = \hbar^2 \pi^2/2m^*L^2$ and m^* is the electron effective mass which is assumed to be constant along the growth direction. Electrons are assumed to be concentrated on the *n*th sub band of the quantum well described by the usual wave function $\phi(z)$ exp(*ikr*)/*A*^{1/2}, where $A = L_x L_y$ is the normalizing area and $\phi_n(z)$ $= \sqrt{2/L} \sin(n\pi z/L)$ is the usual envelope function.³⁰

Since the system order ferromagnetically in account of carriers we write the spin operator \tilde{S}_i in the second term of Eq. (1) in terms of rising and lowering operators of spin deviation

$$
S^+(\vec{R}_i) = \frac{1}{\sqrt{2SN_C}} \sum_{\vec{Q}} e^{i\vec{Q}\cdot\vec{R}_i} [a_{\vec{Q}} + \cdots],
$$

$$
S^-(\vec{R}_i) = \frac{1}{\sqrt{2SN_C}} \sum_{\vec{Q}} e^{-i\vec{Q}\cdot\vec{R}_i} [a_{\vec{Q}}^+ + \cdots].
$$

Here $a_{\vec{Q}}^{\dagger}$ and $a_{\vec{Q}}$ are magnon creation and destruction operators. We used the well-known representation of the spin operators in terms of a_Q^{\dagger} and a_Q^{\dagger} and left the first terms of the series. The \vec{Q} = 0 tem in lowest order gives the spin splitting (see below)³¹ ΔE , lowering the energy of bands with spin parallel to the localized spin $\vec{S}_i \approx S \hat{z}$ and raising the other bands equally. The S^{\pm} terms give rise to spin-flip scattering events and $\tilde{Q} = (\tilde{q}, q_z)$, where \tilde{q} is the in-plane magnon wave vector and q_z its component along the confinement direction $(z \text{ axis}).$

In terms of magnon creation and annihilation operators $a_{\vec{q}}^{\dagger}$, *a_{* \vec{q} *}*, the second term (interaction) in Eq. (1) after some algebraic manipulations now reads

$$
H_{s-d} = -(N_0 \alpha S x/2) \sum_{\vec{k}n\sigma} \sigma c^+(\vec{k}n\sigma) c(\vec{k}n\sigma)
$$

$$
-N_0 \alpha x \sqrt{S/2N_C} \sum_{\vec{k},\vec{q}} \sum_{n,n'} \{M_{n,n'}(q_z) c^+_{\vec{k}+\vec{q}n'} c^{\vec{k}}_{\vec{k}n\uparrow} a_{\vec{q}}
$$

+H.c.}, \t(2)

where $\sigma = +1(-1)$ for the up (down) spin state and $M_{n,n}$ ^{(q_z)} is the usual overlap integral of the electron envelope wave function $\phi(z)$ of the *n*th state in the potential well given by^{30}

$$
M_{n,n'}(q_z) = \int_{-L/2}^{+L/2} \phi_{n'}(z) e^{iq_z z} \phi_n(z) dz.
$$
 (3)

The first term of Eq. (2) gives the spin splitting of electron bands which is to be added to the first term in Eq. (1) . The second term (off diagonal) is the one responsible for electron spin-flip scattering by one magnon process. Here we considered the quantum confinement of electrons in the magnetic $Cd_{1-x}Mn_xTe$ QW and neglected any confinement of the magnons. By using a similar approach that was proposed by Zhang *et al.*³² the magnon dispersion relation is assumed to be the simple isotropic parabolic spectrum. This kind of approximation has been used successfully for many electronphonon scattering calculations in semiconductor thin films.³⁰

III. SPIN-FLIP TRANSITION RATE

In this section we shall estimate the rate \hbar/τ_{flip} at which a single out-of-equilibrium electron in a conduction spin subband relaxes back toward equilibrium by a spin-flip-onemagnon process. This can be found using the standard "golden-rule" rate equations of Boltzmann theory.³¹

Suppose the up-spin component of state $\overline{k}n$ has population $f(kn)$. If it deviates from the equilibrium distribution function $f^0(\tilde{k}n)$, then it will evolve back toward equilibrium according to

$$
\frac{df(\vec{k}n\uparrow)}{dt} = -\frac{2\pi}{\hbar} \sum_{\vec{k}',n'} |C(\vec{k}n\uparrow;\vec{k}' = \vec{k} + \vec{Q}n'\downarrow)|^2
$$

×{F(emission) + F(absorption)}. (4)

Here

$$
F(\text{emission}) = \delta(\varepsilon - \varepsilon' - \omega)[f(1 - f')(N' + 1)
$$

$$
- N'f'(1 - f)], \tag{5}
$$

$$
F(\text{absorption}) = \delta(\varepsilon - \varepsilon' + \omega) [f(1 - f')N
$$

$$
-(N+1)f'(1 - f)]. \tag{6}
$$

In Eq. (4) $|C(\cdot \cdot \cdot)|^2 = [(\alpha N_0 x)^2 S/N_C|M_{n,n'}(q_z)|^2$ is the squared modulus of the matrix element for the process in which an electron in state $(kn\uparrow)$ is scattered to the state $(\vec{k}' = \vec{k} + \vec{Q}n' \downarrow)$ by emission of the magnon $(-\vec{Q})$ or absorption of the magnon $(+\tilde{Q})$ as represented by the interaction Hamiltonian (2) . Since we are restricted to the in-plane electron-magnon scattering only those magnons with wave vectors in this plane will interact with electrons. In this case, assuming in-plane propagation for magnons $q_z=0$ and $|M_{n,n'}(0)|^2 = 1.$

A shorthand notation is used where ε and f stand for the energy and occupancy of the electron state $(\bar{k}n \uparrow)$, ε' and f' stand for the energy and occupancy of electron state $(k$ $+Qn' \downarrow$, ω and *N* stand for the energy and occupancy of the magnon state $(+\tilde{Q})$, and ω' and *N'* stand for the energy and occupancy of the magnon state $(-\tilde{Q})$. Physically, the electron-magnon process occurring in Eq. (4) may be understood as follows. At low temperature $T>0$, depletion of an excess population toward equilibrium occurs both by emission and absorption of thermal magnons. Each process (emission or absorption) has a time-reversed process that enhances the population, namely, the "scattering-in" terms with opposite sign. In thermal equilibrium, scattering out and in occur at equal rates. This ''principle of detailed balance'' guarantees that the two parts of *F*(emission) cancel each other when the distributions *N* and *f* become the equilibrium distributions $N^{(0)}$ and f^0 , and similarly for *F*(absorption).

We now make the assumption that all electrons are in equilibrium except for the particular state $(\bar{k}n\uparrow)$ of interest, whose population (f) deviates from equilibrium (f^0) by an amount $\delta f(\vec{k}n)$. Then the rate equation (4) takes the form

$$
\frac{df(\vec{k}n\uparrow)}{dt} = -\delta f(\vec{k}n\uparrow)/\tau(\vec{k}n\uparrow),\tag{7}
$$

where

$$
\frac{1}{\tau(\vec{k}n\uparrow)} = \frac{2\,\pi(\alpha N_0 x)^2 S}{\hbar N_C} \sum_{\vec{k}',n'} \left\{ \delta(\varepsilon - \varepsilon' - \omega) [f^0(1 - f'^0) \right. \\ \times (N' + 1) - N' f'^0(1 - f^0)] + \delta(\varepsilon - \varepsilon' + \omega) \\ \times [f^0(1 - f'^0)N - (N + 1)f'^0(1 - f^0)] \right\}.
$$
 (8)

IV. EVALUATION OF SPIN-FLIP LIFETIME

In order to provide an estimate of the scattering rate $1/\tau(\tilde{k}n\uparrow)$ we need to make some reasonable simplifying assumptions. First, we assume that *L* is so small that no transitions between the levels *n* can take place due to thermal agitation, or phonons. For CdTe (m^* =0.096 m_0) E_0 is about 35 meV, for $L=10$ nm, and $k_BT=0.345$ meV. That is, we consider that all the carriers are in the lowest subband *n* $= n' = 1$. This state, at low temperature, is dominantly spin up and the subband splitting energy $\Delta E = N_0 \alpha S x$ is in this case greater than the Fermi energy ε_F (note that $N_0\alpha$ =0.22 eV which gives $\Delta E = N_0 \alpha S x = 0.1$ eV for $x = 0.19$ and $\varepsilon_F = 2.5$ meV assuming¹⁸ an electron concentration 10^{11} cm⁻² and a density of states $4\times10^{10}/$ meV cm²). The condition $N_0 \alpha Sx > \varepsilon_F$ means that the first δ function in Eq. (8) (magnon emission process) does not contribute to $1/\tau(kn)$. This process is forbidden since it decreases the electron energy below the lowest energy state (ground). We also assume that the magnons are in thermal equilibrium which means that $N=N^{(0)}$. In this case Eq. (8) is simplified considerably:

$$
\frac{1}{\tau(\vec{k}n\uparrow)} \approx \frac{2\,\pi(\alpha N_0 x)^2 S}{\hbar N_C} \sum_{\vec{k}',n'} \delta(\varepsilon - \varepsilon' + \omega) f^0 (1 - f'^0) N^{(0)},\tag{9}
$$

where $f^0(\varepsilon) = {\exp[(\varepsilon - \varepsilon_F)/k_B T] + 1}^{-1}$ and $N^{(0)} = {\exp(\omega/\varepsilon)}^{-1}$ $(k_B T) - 1$ ⁻¹ are the fermion and boson distribution functions for electrons and magnons, respectively.

The low-temperature intrasubband $(n' = n)$ spin-flip scattering rate $1/\tau_{\text{flip}}$ due to the electron-magnon interaction is then given by

$$
1/\tau_{\text{flip}} = \sum_{\vec{k}, n} \tau(\vec{k}n\uparrow)^{-1} = \frac{2\pi(\alpha N_0 x)^2 SN^{(0)}}{\hbar N_C}
$$

$$
\times \sum_{\vec{k}, \vec{k}', n} \delta(\varepsilon - \varepsilon' + \omega) f^0 (1 - f'^0). \tag{10}
$$

We now perform the summation over \vec{k} and \vec{k}' by making the substitution

$$
\sum_{\vec{k},\vec{k}'} \rightarrow \frac{A}{(2\pi)^2} \int d^2k \frac{A}{(2\pi)^2} \int d^2k'
$$

into Eq. (10) , A being the QW surface area. One then obtains

$$
1/\tau_{\text{flip}} = \frac{(\alpha N_0 x)^2 A S N^{(0)}}{2 \pi \hbar N_0 L} \sum_{n} \int k dk
$$

$$
\times \int k' dk' \delta(\varepsilon - \varepsilon' + \omega) f^{0}(\varepsilon) [1 - f'^{0}(\varepsilon')] .
$$
 (11)

By solving the integral in \vec{k}' in Eq. (11) with the help of the δ function, substituting the electron distribution function by a step function, and performing the integration in k , we obtain the low-temperature spin-flip lifetime broadening of electrons in the $n=1$ ground subband, due to magnon scattering, namely,

$$
\hbar/\tau_{\text{flip}} \approx \frac{A(\alpha N_0 x)^3 S^2 D m^* N^{(0)}(T, \omega)}{4 \hbar^2 N_0 L}, \qquad (12)
$$

provided $N_0 \alpha Sx > \varepsilon_F$, E_0 . Here $D = m^*/\pi \hbar^2$ is the constant density of states at the Fermi level. The temperature comes into the spin-flip lifetime broadening via the magnon distribution function $N^{(0)}(T,\omega)$. Alternatively, if one uses the Fermi distribution function $f^0(\varepsilon) = {\exp[(\varepsilon - \varepsilon_F)/k_B T] + 1}^{-1}$ instead of the step function, we obtain

$$
\hbar/\tau_{\text{flip}} \approx \frac{\pi A (\alpha N_0 x)^2 SDm^* N^{(0)}(T, \omega)}{\hbar^2 N_0 L} \frac{k_B T}{e^{-\varepsilon_F/k_B T} + 1}.
$$
\n(13)

V. NUMERICAL RESULTS

Equations (12) and (13) give the spin-flip lifetime broadening of electrons by scattering with magnons in the ground subband of $Cd_{1-x}M_{x}Te$ QW's. We notice from Eqs. (12) and (13) that $\hbar / \tau_{\text{flip}}$ is proportional to the 2D constant density of states *D* and scales with L^{-1} . It then follows that by decreasing the QW width, \hbar/τ_{flip} gets large, a feature which is characteristic of low-dimensional systems. It also follows from Eqs. (12) and (13) that for $T\rightarrow 0$, the magnon contribution to the lifetime broadening is zero since in this case the magnon occupation number $N^{(0)}(0) \rightarrow 0$. It is convenient to note that in degenerate electron systems, the exponent in Eq. (13) is usually negligible in comparison with unity. In this case, $1/\tau_{\text{flip}} \propto J^2 D^2 k_B T$ which is a Korringa-like relaxation rate of the Mn ion near the two-dimensional electron gas. To get an order of magnitude estimate of the size of this spin-

FIG. 1. Intrasubband $1 \uparrow \rightarrow 1 \downarrow$ spin-flip relaxation rate [Eq. (13)] expressed in terms of $N^0/(L/a)$, as a function of magnon energy in a 100 Å wide quantum well. The solid line is for $T=1.0$ K, the dashed line is for $T=2.5$ K, the dotted line is for $T=4.0$ K, and the dash-dotted line is for $T = 5.0$ K.

flip lifetime broadening we use parameters believed to be appropriate for (Cd,Mn)Te QWsamples:^{33,34} $\alpha N_0 = 0.22$ eV,
 $m^* = 0.096m_0$, $a = 6.487$ Å, $S = 5/2$, $N_0 = 2.783$ $S = 5/2$, $N_0 = 2.783$ $\times 10^{21}$ cm⁻³ (assuming $N_0 = 4x/a^3$), $x = 0.19$, and ε_F $=2.5$ meV $=n_e/D$. (According to Ref. 18, the highest excitation powers in experiment generate approximately 10^{11} cm⁻² electrons in the quantum well and the density of states in the ground state is 4×10^{10} /meV cm⁻²). Hence, taking the above parameters into Eq. (13) considering yet that $\overline{\hbar}\omega$ = 1 meV and *T*=4 K ($N^{(0)} \approx 0.3$), and taking *L*=86 Å and $A=1$ mm², we obtain $\hbar/\tau_{\text{flip}} \approx 0.001$ eV which gives τ_{flip} 1 ps. In Fig. 1 we give a plot of $N^0(T,\omega)/(L/a)$ appearing in Eq. (13), rewritten as $1/\tau_{\text{flip}} = (1/\tau_0)$ $X/N^{(0)}(T,\omega)/(L/a)$, where $1/\tau_0 \equiv \pi A(\alpha N_0 x)^2 S D m^* k_B T /$ $\hbar^3 N_0 a$, versus the magnon energy for different values of temperature considering the case of a 10 nm wide quantum well. We notice from this plot that for different fixed temperature the spin-flip decay rate decreases as the magnon energy increases as consequence of a decrease of the magnon occupation number. Moreover, as the temperature decreases, for a fixed magnon energy, the decay rate (13) decreases, which is also associated with the decrease of the magnon occupation number. In Fig. 2 we give a plot of $N^{(0)}(T,\omega)/(L/a)$ as a function of magnon energy for *T* $=4 K$ and different quantum well width values. From this plot we see the decrease of the electron spin-flip relaxation time as the QW width decreases. Figure 3 shows a plot of $N^{(0)}(T,\omega)/(L/a)$ as a function of the inverse of the dimensionless QW width *L*/*a* for a fixed magnon energy and two different temperature values. We notice that for $T=1$ K (dashed line) the decay rate is small over the entire interval of QW widths indicating very low magnon occupation number.

VI. COMPARISON WITH EXPERIMENTS

We shall now compare our results with the experimental data on $Cd_{1-x}Mn_xTe$ QW's. Systematic experiments have been lacking in the literature about spin-flip scattering time

FIG. 2. Intrasubband $1 \uparrow \rightarrow 1 \downarrow$ spin-flip relaxation rate [Eq. (13)] expressed in terms of $N^0/(L/a)$, as a function of magnon energy at $T=4.0$ K. The solid line is for $L=10$ Å, the dashed line is for *L* $=$ 50 Å, the dotted line is for *L* = 100 Å, and the dash-dotted line is for $L = 150 \text{ Å}.$

by one-magnon processes in $Cd_{1-x}Mn_xTe$ quantum wells. However, the data which are closest to our calculations are provided by the study of the decay of the polarization on the photoluminescence when the sample has been excited with circularly polarized light.^{18,19,34} The excitation creates spin-polarized photoelectrons. If one assumes that the photogenerated holes immediately lose their orientations, the decay of the photoluminescence polarization should reflect that of the electron spin orientation. In an 86-Å $Cd_{0.87}Mn_{0.13}Te-Cd_{0.62}Mn_{0.38}Te$ quantum well, the photoluminescence polarization (PLP) decays with a time constant
of 9 ps.¹⁸ On the other hand in an 86- \AA of 9 ps. 18 On the other hand in an 86- \AA $Cd_{0.935}Mn_{0.065}Te-Cd_{0.62}Mn_{0.38}Te$ well the PLP (Ref. 34) decays with a time constant of 3–4 ps. Our calculations, using Eqs. (12) and (13) in the approximation of infinite barriers for an 86-Å $Cd_{0.81}Mn_{0.19}Te$ QW, lead to a spin-flip relaxation time of \approx 1 ps which corresponds to an orientation decay time of ≈ 0.5 ps (the limiting value for polarized luminescence is 50% for fully polarized electrons and unpolarized holes).¹⁸ This is roughly within an order of magnitude agreement with experiment. Moreover, the data of the timeresolved magneto-optical Kerr effect in a pump-probe experiment³⁵ in modulation-doped (Cd, Mn) Te QW's indicate that the electron spin lifetime shortens as the Mn molar fraction x increases in the QW's. Our calculation predicts that the electron spin-flip lifetime shortens as the Mn content increases, agreeing with this experiment³⁵ and confirms the predominant role of electron-Mn exchange scattering $(electron-magnon scattering)$ as the main relaxation channel for the electron spin. Moreover, we found that the electron spin-flip relaxation time decreases as the quantum well width decreases agreeing with the experiment of Akimoto *et al.*²¹ in which the electron and heavy-hole spin relaxation times in $CdTe/Cd_{0.65}Mn_{0.35}Te$ have been measured by time-resolved circular dichroism.

We believe that the calculations put forward in this paper are of relevance to the fundamental understanding of the process of magnon-induced spin-flip scattering. Their applicability in realistic situations in the presence of other possible

FIG. 3. Intrasubband $1 \uparrow \rightarrow 1 \downarrow$ spin-flip relaxation rate [Eq. (13)] expressed in terms of $N^0/(L/a)$, as a function of quantum well width for two values of temperature $T=4.0$ K (solid line) and *T* $=1.0 K$ (dashed line). The magnon energy was taken fixed and equal to 1.0 meV.

processes and complications must await further, systematic experimental results.

The contribution of spin-flip processes to the relaxation time due to confined and/or interface magnons can be estimated in the same way as for bulk magnons. The only difference is the change in the magnon density of states, due to the different dispersion relation and to the low dimensionality. In fact, as reported in Ref. 36 in which spin-flip scattering in magnetic junctions was investigated, the effect of surface magnons shows up at the scale of the new coupling at the interface.

VII. SUMMARY

We have calculated the magnon contribution to the lowtemperature electron spin-flip relaxation process in CdMnTe magnetic semiconductor QW's in the approximation of infinitely high barriers. The *s*-*d* model was used to estimate the decay rate at which a single out-of-equilibrium electron in a quantum well with infinite potential barrier relax back toward equilibrium by a spin-flip one magnon process. We found the dependence of this rate on magnetic-ion concentration, magnon occupation number, temperature, and quantum well width. The result $\tau_{\text{flip}} \approx 1$ ps is in complete agreement with experiment. Furthermore, since \hbar/τ_{flip} is a function of $N_0\alpha$, the present mechanism could be useful for investigating the electron-magnon coupling constant in Mn doped CdTe QW's. The electron spin-flip scattering by the one-magnon process discussed here is expected to play an important role in the physics of spin-polarized electron transport.

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