Variational description of the dimensional crossover in an array of coupled one-dimensional conductors

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Variational wave function is proposed to describe electronic properties of an array of one-dimensional conductors coupled by transverse hopping and interaction. For weak or intermediate in-chain interaction the wave function has the following structure: Tomonaga-Luttinger bosons with momentum higher than some variational quantity $\tilde{\Lambda}$ are in their ground state while other bosons (with $|k|<\tilde{\Lambda}$) form kinks — fermionlike excitations of the Tomonaga-Luttinger boson field. The nature of the ground state for these quasiparticles can be determined by solving a three-dimensional effective Hamiltonian. Since the anisotropy of the effective Hamiltonian is small the use of mean-field theory is justified. For repulsive interaction possible phases are density wave and *p*-wave superconductivity. Our method allows us to calculate the low-energy part of different electronic Green's functions. In order to do this it is sufficient to apply standard perturbation theory technique to the effective Hamiltonian. When the in-chain interaction is strong $\tilde{\Lambda}$ vanishes and no fermionic excitation is present in the system. In this regime the dynamics is described by transversally coupled Tomonaga-Luttinger bosons.

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I. INTRODUCTION

The adequate description of quasi-one-dimensional (Q1D) conductors remains an unresolved theoretical challenge. Experimentally, at low temperature such systems either occur as three-dimensional anisotropic Fermi liquids or they freeze into a three-dimensional (3D) phase with broken symmetry.[1] At high temperature their transport properties show many unusual features generally attributed to one-dimensional electron anisotropy. This crossover from one to three dimensions is a core problem of Q1D physics.

It is possible to look at the issue of the dimensional crossover from another angle. At high temperature the proper elementary excitations of the system are Tomonaga-Luttinger (TL) bosons. When the temperature is low and the interaction is weak enough the elementary excitations are fermions. Therefore, to describe the system at different energy scales one needs to explain how high-energy bosons ''cross over'' into low-energy fermions. Obviously, this is a nontrivial task.

In this paper we develop a variational approach which accomplishes this goal. To explain the structure of the variational wave function let us first consider a one-dimensional conductor described by a TL Hamiltonian. The ground state of this system is the ground state of TL bosons with all momenta k. Let's turn on the transverse hopping and couple $N_\perp$ of these conductors into a 3D array. In this situation the system will attempt to lower its ground-state energy even further by taking advantage of the transverse hopping energy. However, in order to participate in hopping the bosons have to form many-body fermionlike excitations which have a finite overlap with the physical fermion.

To accommodate for the possibility of having two types of excitations, bosonic and fermionic, we device our variational state in the following fashion. We introduce an intermediate cutoff $\tilde{\Lambda}<\Lambda$, where Λ is the cutoff of the 1D Hamiltonian. All TL bosons whose energy and momenta are high ($|k|>\tilde{\Lambda}$) remain in their ground states. The small momenta bosons ($|k|<\tilde{\Lambda}$) form fermionlike excitations which are delocalized in a transverse direction. To distinguish between the physical electrons and these fermionic excitations we refer to the latter as quasiparticles. In other words, the wave function can be factorized into two parts. The high-energy part corresponds to the ground state of $|k|>\tilde{\Lambda}$ TL bosons, and the low-energy part corresponds to the 3D anisotropic Fermi liquid composed of the quasiparticles.

The variational energy is minimized by adjusting $\tilde{\Lambda}$. The energy of quasiparticle transverse hopping is a decreasing function of $\tilde{\Lambda}$. At the same time, the in-chain energy grows when $\tilde{\Lambda}$ grows. The trade off between the transverse kinetic energy and the in-chain potential energy determines the value of $\tilde{\Lambda}$.

If the optimal value of $\tilde{\Lambda}$ is nonzero the low-energy excitations of the system are the quasiparticles. Properties of the fermionic quasiparticle state depend on the quasiparticle effective Hamiltonian. It arises naturally after high-energy bosons are ''integrated out.'' In this effective Hamiltonian the anisotropy is insignificant. Standard many-body techniques such as perturbation theory and mean-field theory can be used to calculate Green's functions and map out the quasiparticle phase diagram. Since the physical electron and the quasiparticle have finite overlap there is a direct correspondence between the broken-symmetry phases of the effective Hamiltonian and the physical system. We will show that possible phases for spinless Q1D electrons with repulsion are the charge- density wave (CDW) and the superconductivity with the Cooper pairs formed of the electrons on neighboring 1D chains.

As the in-chain interaction grows the parameter $\tilde{\Lambda}$ ap-

proaches zero. When $\tilde{\Lambda}$ vanishes the fermionic excitations cease to exist. The system is described by a 3D TL boson state. In such a regime the ground state is a CDW.

Our approach allows us to obtain several analytical results. With the help of the method it is possible to derive a formula for quasiparticle damping near the Fermi surface. Also, we evaluate transition temperatures for the CDW and superconductivity. The knowledge of these temperatures allows us to map out the phase diagram of our system. Although these quantities have been obtained using different numerical techniques[2,3] the analytical expressions have not been reported to our best knowledge.

The paper is organized as follows. In Sec. II we determine $\tilde{\Lambda}$ and derive the effective Hamiltonian for the fermions. Section III contains the evaluation of the single-particle Green's function. Different phases of the effective Hamiltonian (and the physical system) are mapped in Sec. IV. The regime where $\tilde{\Lambda}=0$ is discussed in Sec. V. We give our conclusions in Sec. VI.

II. VARIATIONAL PROCEDURE

We start our analysis by writing down the Hamiltonian for the array of coupled 1D conductors:

$$H=\int_0^L dx\mathcal{H}, \tag{1}$$

$$\mathcal{H}=\sum_i \mathcal{H}_i^{1\mathrm{D}}+\sum_{i,j}\mathcal{H}_{ij}^{\perp}, \tag{2}$$

$$\mathcal{H}_i^{1D}=iv_F(\psi_{Li}^{\dagger}\nabla\psi_{Li}-\psi_{Ri}^{\dagger}\nabla\psi_{Ri})+g\psi_{Li}^{\dagger}\psi_{Li}\psi_{Ri}^{\dagger}\psi_{Ri}, \tag{3}$$

$$\begin{aligned}\mathcal{H}_{ij}^{\perp}=&-t(i-j)\sum_{p=L,R}(\psi_{pi}^{\dagger}\psi_{pj}+\mathrm{H.c.})\\&+g_{2k_F}(i-j)(\psi_{Li}^{\dagger}\psi_{Ri}\psi_{Rj}^{\dagger}\psi_{Lj}+\mathrm{H.c.})\\&+g_0(i-j)(\psi_{Li}^{\dagger}\psi_{Li}+\psi_{Ri}^{\dagger}\psi_{Ri})(\psi_{Lj}^{\dagger}\psi_{Lj}+\psi_{Rj}^{\dagger}\psi_{Rj}),\end{aligned} \tag{4}$$

with the real-space cutoff $a=\pi/\Lambda$. The fermionic field $\psi_{pi}^{\dagger}$ creates a physical electron with the chirality $p=L(+)$ or $p=R(-)$ on chain i. Transverse interaction constants g_0 (forward scattering) and g_{2k_F} (exchange) are positive. The terms proportional to g_0 and g_{2k_F} account for the Coulomb repulsion of the electrons on different chains. It is further assumed that

$$g>g_0>g_{2k_F}. \tag{5}$$

Now we use the Abelian bosonization prescription[4]

$$\begin{aligned}\psi_p^{\dagger}(x)&=(2\pi a)^{-1/2}\eta_p e^{i\sqrt{2\pi}\varphi_p(x)}\\&=(2\pi a)^{-1/2}\eta_p e^{i\sqrt{\pi}[\Theta(x)+p\Phi(x)]}\end{aligned} \tag{6}$$

to express the electron Hamiltonian in terms of bosonic fields. In the above formula η_p are Klein factors, Θ is the TL boson field, and Φ is the dual field. The bosonized one-chain Hamiltonian is

$$\begin{aligned}\mathcal{H}^{1D}[\Theta,\Phi]=&\frac{v_F}{2}[:(\nabla\Theta)^2:+:(\nabla\Phi)^2:]\\&+\frac{g}{4\pi}[:(\nabla\Phi)^2:-:(\nabla\Theta)^2:].\end{aligned} \tag{7}$$

The symbol $:\ldots:$ denotes normal ordering of TL boson operators with respect to the non-interacting ($g=0$) ground state.

Let us introduce our main variational parameter $\tilde{\Lambda}<\Lambda$. We use it to split TL boson fields into fast ($\Lambda>||k_{\parallel}|-k_F|>\tilde{\Lambda}$, subscript ">") and slow ($||k_{\parallel}|-k_F|<\tilde{\Lambda}$, subscript "<")modes:

$$\begin{aligned}\mathcal{H}^{1D}[\Theta,\Phi]&=\mathcal{H}_<^{1D}+\mathcal{H}_>^{1D}\\&=\mathcal{H}^{1D}[\Theta_<,\Phi_<]+\mathcal{H}^{1D}[\Theta_>,\Phi_>].\end{aligned} \tag{8}$$

We define the fermionic field $\Psi_p^{\dagger}(x)$ with the help of Eq. (6) in which a is substituted by $\tilde{a}=\pi/\tilde{\Lambda}$ and $\Theta_<$ and $\Phi_<$ are placed instead of Θ and Φ. The field Ψ is our quasiparticle discussed in Introduction. Using this field we refermionize $\mathcal{H}_<^{1D}$. The result is the same as that of Eq. (3) with Ψ_p instead of ψ_p. The transverse terms (4) can be easily rewritten if one observes that the physical fermion is simply

$$\psi_p^{\dagger}=\sqrt{\tilde{a}/a}\Psi_p^{\dagger}e^{i\sqrt{\pi}(\Theta_>+p\Phi_>)}, \tag{9}$$

and that the fermionic and bosonic parts in this definition commute with each other. Therefore,

$$\begin{aligned}\mathcal{H}_{ij}^{\perp}=&-(\tilde{a}/a)t(i-j)\\&\times\sum_{p=L,R}\{\Psi_{pi}^{\dagger}\Psi_{pj}e^{i\sqrt{\pi}[(\Theta_{>i}-\Theta_{>j})+p(\Phi_{>i}-\Phi_{>j})]}+\mathrm{H.c.}\}\\&+(\tilde{a}/a)^2g_{2k_F}(i-j)[\Psi_{Li}^{\dagger}\Psi_{Ri}\Psi_{Rj}^{\dagger}\Psi_{Lj}e^{i\sqrt{4\pi}(\Phi_{>i}-\Phi_{>j})}+\mathrm{H.c.}]\\&+g_0(i-j)\Big[(\Psi_{Li}^{\dagger}\Psi_{Li}+\Psi_{Ri}^{\dagger}\Psi_{Ri})(\Psi_{Lj}^{\dagger}\Psi_{Lj}+\Psi_{Rj}^{\dagger}\Psi_{Rj})\\&+\frac{1}{\pi}\nabla\Phi_{>i}\nabla\Phi_{>j}\Big].\end{aligned} \tag{10}$$

Our variational wave function has the form

$$\begin{aligned}|\mathrm{var}\rangle&=|\{\Psi_{pi}\}\rangle\prod_j|0_{>j}\rangle\\&=|\{\Psi_{pi}\}\rangle\prod_{j,k>\tilde{\Lambda}}(2|k|/\pi\mathcal{K})^{1/2}\exp\{-|k||\Phi_{jk}|^2/\mathcal{K}\}.\end{aligned} \tag{11}$$

It is a product of some many-body state $|\{\Psi_{pi}\}\rangle$ composed of the quasiparticles Ψ_{pi} and the ground states $|0_{>j}\rangle$ of $\mathcal{H}^{1D}[\Theta_{>j},\Phi_{>j}]$.

Variational ground-state energy is found by minimizing the expression

$$E^V = N_\perp L \frac{\theta u}{4\pi}(\tilde{\Lambda}^2 - \Lambda^2) + \langle\{\Psi_{pi}\}|\left(\int_0^L dx \mathcal{H}^{\text{eff}}\right)|\{\Psi_{pi}\}\rangle, \tag{12}$$

$$\begin{aligned}\mathcal{H}^{\text{eff}} = &\sum_i iv_F(\Psi^\dagger_{Li}\partial_x\Psi_{Li} - \Psi^\dagger_{Ri}\partial_x\Psi_{Ri}) + g\Psi^\dagger_{Li}\Psi_{Li}\Psi^\dagger_{Ri}\Psi_{Ri} \\ &- \sum_{ij}\sum_{p=L,R}\tilde{t}(i-j)(\Psi^\dagger_{pi}\Psi_{pj} + \text{H.c.}) \\ &+ \sum_{ij}\tilde{g}_{2k_F}(i-j)(\Psi^\dagger_{Li}\Psi_{Ri}\Psi^\dagger_{Rj}\Psi_{Lj} + \text{H.c.}) \\ &+ g_0(i-j)(\Psi^\dagger_{Li}\Psi_{Li} + \Psi^\dagger_{Ri}\Psi_{Ri})(\Psi^\dagger_{Lj}\Psi_{Lj} + \Psi^\dagger_{Rj}\Psi_{Rj}),\end{aligned} \tag{13}$$

$$\tilde{t} = \zeta^\theta t, \quad \tilde{g}_{2k_F} = \zeta^{2\mathcal{K}-2} g_{2k_F}, \qquad \zeta = \tilde{\Lambda}/\Lambda. \tag{14}$$

The number of chains is $N_\perp$. The TL liquid parameter $\mathcal{K}$, the electron anomalous dimension θ, and boson velocity u are defined in the usual way:

$$\mathcal{K} = \sqrt{\frac{2\pi v_F - g}{2\pi v_F + g}},$$

$$\theta = \frac{1}{2}(\mathcal{K} + \mathcal{K}^{-1} - 2),$$

$$u = \frac{1}{2\pi}\sqrt{(2\pi v_F - g)(2\pi v_F + g)}. \tag{15}$$

The first term of Eq. (12) has a purely one-dimensional origin. The second term is the energy of the quasiparticle ground state.

Observe that the parameters of the effective Hamiltonian $\tilde{t}/\tilde{\Lambda}$ and $\tilde{g}_{2k_F}$ are connected to the corresponding bare parameters as if they are subject to the renormalization-group (RG) flow in the vicinity of the TL fixed point. The explanation for this fact is quite obvious: our method of deriving the effective Hamiltonian is equivalent to the tree level RG scaling near the TL fixed point.

If the transversal interactions are small (g_0 and $\tilde{g}_{2k_F}$ both less then $\tilde{t}/\tilde{\Lambda}$) they can be neglected. In addition, we neglect corrections to the energy due to spontaneous symmetry breaking. The latter assumption works when $\theta \ll 1$. Its validity away from this point is discussed at the end of Sec. V. Under these two conditions the expression (12) becomes

$$E^V/(LN_\perp) \approx \frac{\theta u}{4\pi}(\zeta^2 - 1)\Lambda^2 - \frac{2}{\pi v_F}\zeta^{2\theta}\sum_i [t(i)]^2. \tag{16}$$

This variational energy attains its minimum at

$$\zeta = \begin{cases} (8\overline{t^2}/uv_F\Lambda^2)^{1/(2-2\theta)} & \text{if} \quad \theta < 1, \\ 0 & \text{if} \quad \theta > 1, \end{cases} \tag{17}$$

$$\overline{t^2} = \sum_i [t(i)]^2. \tag{18}$$

We have to remember, however, that value of the numerical coefficient in Eq. (17) is not accurate. This is due to the fact that the second term in Eq. (16) is calculated under assumption $\tilde{t} < \tilde{\Lambda}$. When $\tilde{\Lambda}$ gets smaller the coefficient in front of this term acquires some $\tilde{\Lambda}$ dependence. We neglect the corrections due to this dependence since they are less singular (at small $\tilde{\Lambda}$) than the second term of Eq. (16). These corrections modify the result for ζ quantitatively, therefore, it is more appropriate to write

$$\zeta \propto \left(\frac{t}{\overline{u}\Lambda}\right)^{1/(1-\theta)}, \tag{19}$$

$$\overline{u} = \sqrt{uv_F}. \tag{20}$$

Using this formula it is easy to show that

$$\tilde{t} \propto \overline{u}\tilde{\Lambda} \propto t\left(\frac{t}{\overline{u}\Lambda}\right)^{\theta/(1-\theta)}. \tag{21}$$

This means that for $\theta < 1$ the effective transverse hopping amplitude $\tilde{t}$ of the quasiparticle Ψ is of the same order as the quasiparticle longitudinal cutoff energy $v_F\tilde{\Lambda}$. Therefore, due to small anisotropy, the Hamiltonian for the quasiparticles (13) can be treated within the framework of usual mean-field theory and perturbation theory. Our calculations, in agreement with renormalization-group analysis,[5,6] show that for $\theta < 1$ there is the crossover energy scale $\tilde{t}$ above which the system is equivalent to a collection of decoupled chains while below, the transverse hopping becomes important.

Depending on the interaction and the anisotropy the region $\theta < 1$ can be further split into two parts. The transverse hopping contribution to the variational energy [second term of Eq. (16)] can be rewritten as follows:

$$\frac{2}{\pi v_F}\tilde{t}^{\,2} \propto \frac{2}{\pi v_F}t^2 \exp\left[-\frac{2\theta}{1-\theta}\ln\left(\frac{\overline{u}\Lambda}{t}\right)\right]. \tag{22}$$

If the argument of the exponential function is small the exponential can be replaced by the first few terms of the Taylor series. In such a situation the contribution of the in-chain interaction to the total energy, Eq. (16), can be calculated perturbatively. One-dimensional effects are virtually unobservable. This is the weak-coupling regime.

When the anisotropy and the in-chain interaction are strong the exponential cannot be approximated accurately by the low-order Taylor expansion. The system is in the intermediate-coupling regime now. In order to obtain a reliable answer involving such a regime it is not enough to apply finite-order perturbation theory. Our method converts the system of physical electrons with intermediate coupling into the system of quasiparticles with weak coupling. The latter can be studied by standard perturbation theory.

As a function of the bare transverse hopping amplitude t the crossover from weak coupling to intermediate coupling occurs at

$$t^* \propto \overline{u}\Lambda \exp\left(-\frac{1-\theta}{\theta}\right). \tag{23}$$

For the weak in-chain interaction $\theta \ll 1$. When this is the case it is necessary to have exponentially small transverse hopping amplitude t in order to observe nontrivial Q1D effects.

When $\theta > 1$ the effective cutoff momentum $\tilde{\Lambda}$ is zero. The quasiparticles are not formed. The system can be viewed as a collection of TL bosons weakly coupled by the transverse exchange interaction. The possibility of such a state was first pointed out by Wen.[7] It is natural to call such a regime strong coupling. Section V is reserved for discussion of strong coupling.

III. SINGLE-ELECTRON GREEN'S FUNCTION

The calculation of different propagators for a Q1D system is open for question. Our approach allows for easy evaluation of the low-energy part of Green's functions in the intermediate-coupling regime. The high-energy parts of Q1D Green's functions are believed to coincide with the Green's functions of the TL model. The latter have been discussed extensively in the literature.

The Matsubara propagator of the physical electronic field ψ_L is

$$\begin{aligned}\mathcal{G}_L(x,\mathbf{R}_\perp,\tau) = \frac{1}{\zeta}\langle \mathcal{T}\{\Psi_{Li}(x,\tau)\Psi^\dagger_{Lj}(0,0)\}\rangle_\Psi \\ \times \langle \mathcal{T}\{\exp(-i\sqrt{\pi}[\Theta_{>i}(x,\tau)+\Phi_{>i}(x,\tau)]) \\ \times \exp(i\sqrt{\pi}[\Theta_{>j}(0,0)+\Phi_{>j}(0,0)])\}\rangle_>,\end{aligned} \tag{24}$$

$$\mathbf{R}_\perp = \mathbf{R}_i - \mathbf{R}_j. \tag{25}$$

The notation $\langle \ldots \rangle_\Psi$ stands for averaging with respect to the quasiparticle ground state $|0_\Psi\rangle$. Likewise, $\langle \ldots \rangle_>$ stands for the expectation value with respect to the $|0_>\rangle$ state.

The bosonic part of this formula can be immediately calculated:

$$\begin{aligned}\frac{1}{\zeta}\Big\langle \mathcal{T}\{\exp(-i\sqrt{\pi}(\Theta_{>i}(x,\tau)+\Phi_{>i}(x,\tau)) \\ \times \exp(i\sqrt{\pi}(\Theta_{>j}(0,0)+\Phi_{>j}(0,0)))\}\Big\rangle_> \\ = \left(\frac{\mathcal{G}^{1D}_L}{\tilde{\mathcal{G}}^{1D}_L}\right)\delta_{ij} + \zeta^\theta(1-\delta_{ij}).\end{aligned} \tag{26}$$

Here $\mathcal{G}^{1D}_L$ ($\tilde{\mathcal{G}}^{1D}_L$) is the Matsubara Green's function of the Tomonaga-Luttinger model with the cutoff Λ ($\tilde{\Lambda}$).

Our variational wave function does not take into account correlations between $\Phi_{>i}(\Theta_{>i})$ and $\Phi_{>j}(\Theta_{>j})$ if $i \neq j$. However, the above formula is correct at least for large $\tau > 1/\overline{u}\tilde{\Lambda}$ or small frequency $\omega < \overline{u}\tilde{\Lambda}$ where those correlations are not important. In such a limit the boson part of Eq. (24) is a constant equal to ζ^θ.

Once the bosonic propagator is found it is necessary to calculate the quasiparticle Green's function. This can be done with the help of a standard diagrammatic technique. If we neglect interactions between the quasiparticles the single-electron Green's function is

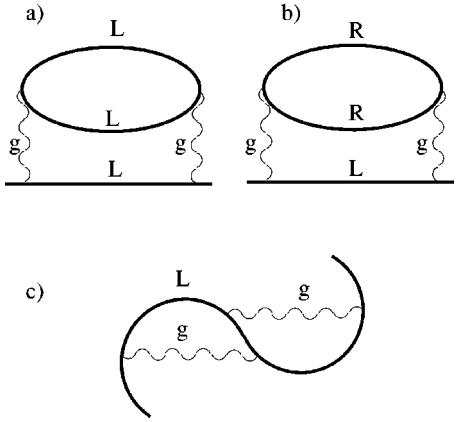

FIG. 1. Lowest-order contribution to the self-energy of the quasiparticle.

$$\mathcal{G}_L(i\omega, p_\parallel, \mathbf{p}) = \frac{\zeta^\theta}{i\omega + v_F p_\parallel - \tilde{\varepsilon}^\perp_{\mathbf{p}}}, \tag{27}$$

where the renormalized transverse kinetic energy is given by

$$\tilde{\varepsilon}^\perp_{\mathbf{p}} = -2\zeta^\theta \sum_i t(i)\cos(\mathbf{p}\cdot\mathbf{R}_i). \tag{28}$$

This result coincides with the Green's function derived by the RG.[6,2]

Our method allows to improve the above formula for the single-electron propagator by taking interaction between the quasiparticles into account. Neglecting (i) symmetry-breaking which becomes important for very small frequencies only and (ii) the transverse couplings g_0 and $\tilde{g}_{2k_F}$ [see Eq. (5)] one can identify three second-order diagrams contributing to the single-quasiparticle self-energy (Fig. 1). They are [Fig. 1(a)] scattering on the polarization bubble of the same chirality as the incoming quasiparticle, [Fig. 1(b)] scattering on the polarization bubble of the opposite chirality, and [Fig. 1(c)] the vertex correction. Figures 1(a) and 1(c) are identical in magnitude and opposite in sign. Thus, Fig. 1(b) is the only one that needs to be evaluated.

First, we calculate the quasiparticle polarization bubble $\mathcal{P}_R$,

$$\mathcal{P}_R(i\Omega, k_\parallel, \mathbf{k}) = \int_{q_\parallel \mathbf{q}} \delta(v_F q_\parallel + \tilde{\varepsilon}^\perp_{\mathbf{q}}) \frac{v_F k_\parallel + \tilde{\varepsilon}^\perp_{\mathbf{q}+\mathbf{k}} - \tilde{\varepsilon}^\perp_{\mathbf{q}}}{i\Omega - v_F k_\parallel - \tilde{\varepsilon}^\perp_{\mathbf{q}+\mathbf{k}} + \tilde{\varepsilon}^\perp_{\mathbf{q}}}, \tag{29}$$

where the notation $\int_{q_\parallel \mathbf{q}} \ldots = (2\pi)^{-3} b^2 \int dq_\parallel d^2\mathbf{q} \ldots$ is used. The symbol b denotes the transverse lattice constant. The self-energy is

$$\Sigma_L(i\omega,p_\parallel,\mathbf{p})=-T\sum_{\Omega}\int_{k_\parallel\mathbf{k}}\mathcal{G}_L(i\omega+i\Omega,p_\parallel+k_\parallel,\mathbf{p}+\mathbf{k})\mathcal{P}_R(i\Omega,k_\parallel,\mathbf{k}). \tag{30}$$

After summing over Ω, the following expression for the self-energy is derived:

$$\Sigma_L=-g^2\int_{k_\parallel\mathbf{k}}\int_{q_\parallel\mathbf{q}}(v_Fk_\parallel+\tilde{\varepsilon}^\perp_{\mathbf{k}+\mathbf{q}}-\tilde{\varepsilon}^\perp_{\mathbf{q}})\,\delta(v_Fq_\parallel+\tilde{\varepsilon}^\perp_{\mathbf{q}})\frac{n_F(v_Fk_\parallel+\tilde{\varepsilon}^\perp_{\mathbf{k}+\mathbf{q}}+\tilde{\varepsilon}^\perp_{\mathbf{q}})-n_F(v_F(k_\parallel+p_\parallel)+\tilde{\varepsilon}^\perp_{\mathbf{k}+\mathbf{p}})}{i\omega+2v_Fk_\parallel+v_Fp_\parallel-\tilde{\varepsilon}^\perp_{\mathbf{q}}+\tilde{\varepsilon}^\perp_{\mathbf{k}+\mathbf{q}}-\tilde{\varepsilon}^\perp_{\mathbf{k}+\mathbf{p}}}$$

$$+g^2\int_{k_\parallel\mathbf{k}}\int_{q_\parallel\mathbf{q}}\frac{(v_Fk_\parallel+\tilde{\varepsilon}^\perp_{\mathbf{k}+\mathbf{q}}-\tilde{\varepsilon}^\perp_{\mathbf{q}})\,\delta(v_Fq_\parallel+\tilde{\varepsilon}^\perp_{\mathbf{q}})}{\sinh\{(v_Fk_\parallel+\tilde{\varepsilon}^\perp_{\mathbf{k}+\mathbf{q}}+\tilde{\varepsilon}^\perp_{\mathbf{q}})/T\}(i\omega+2v_Fk_\parallel+v_Fp_\parallel-\tilde{\varepsilon}^\perp_{\mathbf{q}}+\tilde{\varepsilon}^\perp_{\mathbf{k}+\mathbf{q}}-\tilde{\varepsilon}^\perp_{\mathbf{k}+\mathbf{p}})}. \tag{31}$$

When $T=0$ the first term can be further simplified. The Fermi distribution n_F becomes the step function. In such a situation it is possible to perform integration over $k_\parallel$ and $p_\parallel$ exactly. The second integral in the above equation appears due to the relation between the Fermi distribution n_F and the Bose distribution n_B: $n_B(\omega)+n_F(\omega)=1/\sinh(\omega/T)$. At zero temperature this integral vanishes. In the resultant $T=0$ expression for Σ_L the transverse kinetic energy $\tilde{\varepsilon}^\perp$ always enter in the combination $\tilde{\varepsilon}^\perp_{\mathbf{q}+\mathbf{k}}+\tilde{\varepsilon}^\perp_{\mathbf{k}+\mathbf{p}}-\tilde{\varepsilon}^\perp_{\mathbf{q}}$. Therefore, it is convenient to introduce the quantity

$$\nu^\perp(\varepsilon^\perp,\mathbf{p})=\int\frac{d^2\mathbf{q}d^2\mathbf{k}}{(2\pi)^4}b^4\delta(\varepsilon^\perp-\tilde{\varepsilon}^\perp_{\mathbf{q}+\mathbf{k}}-\tilde{\varepsilon}^\perp_{\mathbf{k}+\mathbf{p}}+\tilde{\varepsilon}^\perp_{\mathbf{q}}). \tag{32}$$

With this definition the self-energy can be compactly written as follows:

$$\Sigma_L(i\omega,p_\parallel,\mathbf{p})=\frac{g^2}{8\pi v_F}p_\parallel-\frac{g^2}{16\pi v_F^2}\times\int d\varepsilon^\perp\nu^\perp(\varepsilon^\perp,\mathbf{p})(i\omega-v_Fp_\parallel-\varepsilon^\perp)\times\ln\frac{4\tilde{\Lambda}^2}{\omega^2+(v_Fp_\parallel+\varepsilon^\perp)^2}. \tag{33}$$

The Green's function of the physical electron is $\zeta^\theta(i\omega+v_Fp_\parallel-\tilde{\varepsilon}^\perp-\Sigma_L)^{-1}$. Note that the logarithmic divergence of the self-energy, a hallmark of the Fermi-liquid picture breakdown in the TL model, is capped in the presence of the transverse hopping. This justifies the use of perturbation theory.

By analytically continuing Eq. (33) it is possible to calculate the retarded self-energy $\Sigma^{\rm ret}$ whose imaginary part is the quasiparticle damping:

$$\gamma=-\,\mathrm{Im}\,\Sigma^{\rm ret}=\frac{g^2}{8v_F^2}\nu^\perp(-v_Fp_\parallel,\mathbf{p})\omega^2. \tag{34}$$

The transverse density of states can be estimated as $\nu^\perp\propto 1/\tilde{t}$. This gives us $\gamma\propto(g/v_F)^2\omega^2/\tilde{t}$. On the mass shell $\omega=-v_Fp_\parallel+\tilde{\varepsilon}_{\mathbf{p}}$ the expression for γ becomes:

$$\gamma=\frac{g^2}{8}\nu^\perp(-v_Fp_F,\mathbf{p})(p_\parallel-p_F)^2\propto\frac{g^2}{\tilde{t}}(p_\parallel-p_F)^2 \tag{35}$$

where $(p_\parallel-p_F)$ is the distance from a given point $(p_\parallel,\mathbf{p})$ of the Brillouin zone to the Fermi surface $v_Fp_F=\tilde{\varepsilon}_{\mathbf{p}}$ along the x direction.

We need to issue a warning in connection to the accuracy of Σ_L. It is incorrect to think of Eq. (33) as a $\mathcal{O}(g^2)$ expression for *the physical electron* self-energy. Indeed, the physical electron Green's function (27) already contains all orders of g entering though the quasiparticle renormalization ζ^θ and renormalized transverse hopping $\tilde{\varepsilon}$. It is necessary to remember that our variational approach is uncontrollable approximation. It lacks a small parameter controlling the quality of the results. Therefore, it is not clear how accurate expression (33) is.

In Ref. 2 the self-energy was evaluated numerically for the system with infinite transverse dimensions. However, those calculations are more complicated technically and do not give an analytical answer for the self-energy.

IV. PHASE DIAGRAM

In Sec. II we derived the low-energy effective Hamiltonian for the quasiparticles. Now we apply mean-field theory to obtain the phase diagram of the effective Hamiltonian. The experimentally observable phase diagram for the physical electrons coincides exactly with that of the quasiparticles. To prove this let us calculate $\langle\psi^\dagger_{Li}\psi_{Ri}\rangle$ for $T\ll\overline{u\tilde{\Lambda}}$:

$$\langle\psi^\dagger_{Li}\psi_{Ri}\rangle=\frac{1}{\zeta}\langle\Psi^\dagger_{Li}\Psi_{Ri}\rangle_\Psi\langle e^{i\sqrt{\pi}(\Theta_{>i}+\Phi_{>i})}e^{-i\sqrt{\pi}(\Theta_{>i}-\Phi_{>i})}\rangle_>=\zeta^{\mathcal{K}-1}\langle\Psi^\dagger_{Li}\Psi_{Ri}\rangle_\Psi. \tag{36}$$

The physical CDW order parameter is proportional to the CDW expectation value of the quasiparticles. Similar formulas can be obtained for other order parameters. For example, $\langle\psi^\dagger_{Li}\psi^\dagger_{Ri}\rangle=\zeta^{1/\mathcal{K}-1}\langle\Psi^\dagger_{Li}\Psi^\dagger_{Ri}\rangle_\Psi$ and $\langle\psi^\dagger_{Li}\psi^\dagger_{Rj}\rangle=\zeta^\theta\langle\Psi^\dagger_{Li}\Psi^\dagger_{Rj}\rangle_\Psi$. Therefore, we can determine the phase diagram of Eq. (1) by mapping the phases of the Hamiltonian (13).

We consider four order parameters. One is the charge-density wave

$$\hat{\rho}_{2k_F i}=\Psi^{\dagger}_{Li}\Psi_{Ri} \tag{37}$$

and there are three types of the superconducting order:

$$\hat{\Delta}_{\pm ij}=\frac{1}{2}(\Psi^{\dagger}_{Li}\Psi^{\dagger}_{Rj}\pm\Psi^{\dagger}_{Lj}\Psi^{\dagger}_{Ri}), \tag{38}$$

$$\hat{\Delta}_{0i}=\Psi^{\dagger}_{Li}\Psi^{\dagger}_{Ri}. \tag{39}$$

The in-chain potential energy can be rewritten in terms of $\hat{\rho}$ and $\hat{\Delta}_0$ in the following manner:

$$g\Psi^{\dagger}_{Li}\Psi_{Li}\Psi^{\dagger}_{Ri}\Psi_{Ri}=-g\hat{\rho}_{2k_F i}\hat{\rho}^{\dagger}_{2k_F i}=g\hat{\Delta}^{\dagger}_{0i}\hat{\Delta}_{0i}. \tag{40}$$

The exchange interaction can be expressed as

$$\begin{aligned}&\tilde{g}_{2k_F}(\Psi^{\dagger}_{Li}\Psi_{Ri}\Psi^{\dagger}_{Rj}\Psi_{Lj}+\text{H.c.})\\&=\tilde{g}_{2k_F}(\hat{\rho}_{2k_F i}\hat{\rho}^{\dagger}_{2k_F j}+\text{H.c.})\\&=2\tilde{g}_{2k_F}(\hat{\Delta}_{-ij}\hat{\Delta}^{\dagger}_{-ij}-\hat{\Delta}_{+ij}\hat{\Delta}^{\dagger}_{+ij}).\end{aligned} \tag{41}$$

Finally, a part of the transverse forward scattering, which describes the interaction between the fermions of different chiralities, is

$$\begin{aligned}&g_0(\Psi^{\dagger}_{Li}\Psi_{Li}\Psi^{\dagger}_{Rj}\Psi_{Rj}+\Psi^{\dagger}_{Ri}\Psi_{Ri}\Psi^{\dagger}_{Lj}\Psi_{Lj})\\&=2g_0(\hat{\Delta}_{+ij}\hat{\Delta}^{\dagger}_{+ij}+\hat{\Delta}_{-ij}\hat{\Delta}^{\dagger}_{-ij}).\end{aligned} \tag{42}$$

The part of the forward scattering which accounts for the interaction between the fermions of the same chirality cannot be expressed in terms of these four order parameters.

The effective coupling for the CDW is always larger than the effective coupling for the superconducting order parameter $\hat{\Delta}_+$:

$$g_{\text{CDW}}>g_{\text{sc}}, \tag{43}$$

where

$$g_{\text{CDW}}=g+z_{\perp}\tilde{g}_{2k_F}, \tag{44}$$

$$g_{\text{sc}}=\tilde{g}_{2k_F}-g_0=\zeta^{2\mathcal{K}-2}g_{2k_F}-g_0, \tag{45}$$

and $z_{\perp}$ is the coordination number for a chain. Thus, at $T=0$ the system with perfect nesting is always in the CDW phase with the $\hat{\Delta}_+$ order-parameter phase being metastable ($g_{\text{sc}}>0$) or unstable ($g_{\text{sc}}<0$). Other order parameters, $\hat{\Delta}_0$ and $\hat{\Delta}_-$, are unstable.

When external pressure is applied the amplitude t_2 for hopping to the next-to-nearest chain begins to grow and spoils the Fermi-surface nesting. This undermines stability of the CDW and drives the transition temperature to zero.[8] Indeed, in the latter reference the following simple estimate for the density wave susceptibility was obtained:

$$\chi\propto\frac{1}{2\pi v_F}\times\begin{cases}\ln(2v_F\tilde{\Lambda}/T) & \text{if} \quad T>\tilde{t}_2=\zeta^{\theta}t_2,\\ \ln(2v_F\tilde{\Lambda}/\tilde{t}_2) & \text{if} \quad T<\tilde{t}_2=\zeta^{\theta}t_2.\end{cases} \tag{46}$$

The CDW transition temperature is derived by equating $(g+z_{\perp}\tilde{g}_{2k_F})\chi$ and unity. For $\tilde{t}_2=0$ it is

$$T^{(0)}_{\text{CDW}}\propto v_F\tilde{\Lambda}\exp[-2\pi v_F/(g+z_{\perp}\tilde{g}_{2k_F})]. \tag{47}$$

If $\tilde{t}_2>0$ the transition temperature T_{CDW} becomes smaller than $T^{(0)}_{\text{CDW}}$. It vanishes when $\tilde{t}_2\propto T^{(0)}_{\text{CDW}}$. That is, exponentially small $\tilde{t}_2$ is enough to destroy the CDW.

What happens after the CDW is destroyed depends on the sign of g_{sc}. If $g_{\text{sc}}>0$ the ground state is superconducting. Otherwise, it is the Fermi liquid. We can perform the same type of analysis as above for the CDW. The superconductivity is rather insensitive to the nesting properties of the Fermi surface. The susceptibility for $\hat{\Delta}_+$ is equal to $(1/2\pi\alpha v_F)\ln(2v_F\tilde{\Lambda}/T)$, where α is a constant of order of unity. The critical temperature is found to be

$$T_c\propto v_F\tilde{\Lambda}\exp(-2\pi\alpha v_F/g_{\text{sc}}), \tag{48}$$

if $g_{\text{sc}}>0$. Even when $g_{2k_{\text{F}}}<g_0$ the effective coupling g_{sc} may be positive provided that the in-chain interaction is repulsive ($\mathcal{K}<1$) and the electron hopping anisotropy parameter $(\overline{u}\Lambda/t)$ is large:

$$\zeta^{2\mathcal{K}-2}>\frac{g_0}{g_{2k_F}}\Leftrightarrow\left(\frac{\overline{u}\Lambda}{t}\right)^{(2-2\mathcal{K})/(1-\theta)}>\frac{g_0}{g_{2k_F}}>1. \tag{49}$$

For the system in the intermediate-coupling regime this condition is likely to be satisfied.

It is interesting to note that the external pressure undermines not only the CDW but the superconductivity as well. Under growing pressure the anisotropy parameter $(\overline{u}\Lambda/t)$ decays. The superconducting transition temperature decreases as the anisotropy decreases. At pressure higher than some critical value the condition (49) is no longer satisfied. In this region the superconductivity is unstable and the ground state is the Fermi liquid.

The qualitative phase diagram is presented in Fig. 2. It shares two remarkable features with the phase diagram of the organic Q1D superconductors:[1] (i) the density wave phase and superconductivity have a common boundary; (ii) the superconducting transition temperature vanishes at high pressure.

Our order parameter $\hat{\Delta}_+$ deviates from the more common version $\hat{\Delta}_0$. The order parameter $\hat{\Delta}_+$ was proposed quite some time ago.[9] Recently, this suggestion found further support in the renormalization-group calculations of Ref. 3. The advantage of $\hat{\Delta}_+$ stems from the fact that by having two electrons of a Cooper pair on different chains we avoid increasing in-chain potential energy.

The origin of the superconducting phase in our system is an interesting question worth discussing in more detail. In a conventional BCS model the superconductivity is stable because it minimizes the potential energy of the electron-

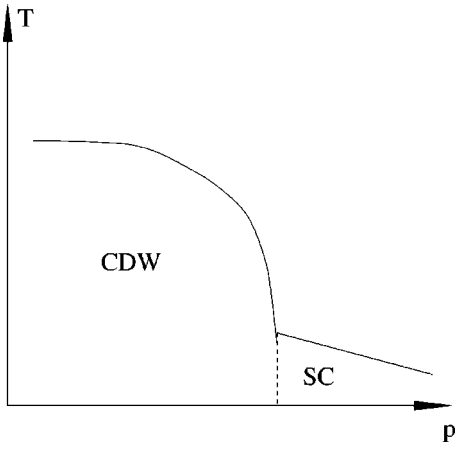

FIG. 2. Qualitative phase diagram of our model. Solid lines show second-order phase transitions into the CDW and the superconducting phase. Dashed line shows the first-order transtion between the CDW and the superconductivity.

electron interaction. We can make this claim rigorous by considering the following derivation. BCS Hamiltonian density

$$\mathcal{H}^{\mathrm{BCS}}=\mathcal{T}+\mathcal{V}=\sum_{\sigma}\ \psi_{\sigma}^{\dagger}\left(\frac{\hat{p}^2}{2m}-\mu\right)\psi_{\sigma}-g\,\psi_{\uparrow}^{\dagger}\psi_{\downarrow}^{\dagger}\psi_{\downarrow}\psi_{\uparrow} \tag{50}$$

consists of two terms: kinetic-energy density $\mathcal{T}$ and potential-energy density $\mathcal{V}$. At zero temperature the superconducting state energy density $\mathcal{E}_{\mathrm{sc}}=\langle\mathcal{H}\rangle_{\mathrm{sc}}$ is smaller than the normal energy density $\mathcal{E}_n=\langle\mathcal{H}\rangle_n$. This condensation energy density

$$\mathcal{E}_c=\mathcal{E}_n-\mathcal{E}_{\mathrm{sc}}\propto\nu T_c^2, \tag{51}$$

$$\nu=\pi^{-2}m^2v_F,\quad v_F=\sqrt{2\mu/m} \tag{52}$$

is entirely due to depletion of interaction in the superconducting state,

$$\langle\mathcal{V}\rangle_n-\langle\mathcal{V}\rangle_{\mathrm{sc}}>0. \tag{53}$$

As for the kinetic energy it grows in the superconducting state:

$$\langle\mathcal{T}\rangle_n-\langle\mathcal{T}\rangle_{\mathrm{sc}}<0. \tag{54}$$

To prove this we use the Feynman formula which allows to calculate the ground-state expectation value of any term $c\mathcal{O}$ of the Hamiltonian density:

$$c\langle\mathcal{O}\rangle=c\frac{\partial\mathcal{E}_{\mathrm{gs}}}{\partial c}, \tag{55}$$

where $\mathcal{E}_{\mathrm{gs}}$ is the ground-state energy density. Therefore

$$\langle\mathcal{V}\rangle_n-\langle\mathcal{V}\rangle_{\mathrm{sc}}=g\frac{\partial}{\partial g}\mathcal{E}_c, \tag{56}$$

$$\langle\mathcal{T}\rangle_n-\langle\mathcal{T}\rangle_{\mathrm{sc}}=m^{-1}\frac{\partial}{\partial m^{-1}}\mathcal{E}_c, \tag{57}$$

$$\mathcal{E}_c\propto\omega_D^2\mu^{1/2}m^{3/2}\exp(-\alpha\mu^{-1/2}m^{-3/2}g^{-1}), \tag{58}$$

where ω_D is Debye frequency and $\alpha>0$ is a constant of order unity. The inequalities (53) and (54) immediately follow from the expressions above. These inequalities mean that it is the electron-electron attraction which triggers BCS superconductivity. This fact is very well known in superconductivity mean-field theory.

However, in the system with strong repulsion, such as Q1D or high-T_c materials, it is difficult to construct a mean-field superconducting phase which lowers the interaction energy. Our model for which we develop the consistent many-body approach can be used to discuss this issue beyond the mean-field approximation.

For our model it is easy to determine that the transverse forward-scattering energy is increased and the exchange energy is decreased by the superconductivity. This result is a direct consequence of Eqs. (42) and (41).

Contributions of other terms can be found with the help of the Feynman formula. The condensation energy density is of the order of $-T_c^2/v_F$. Thus, differentiating the critical temperature (48) with respect to some coupling constant of Eq. (1) we can determine how a ground-state energy contribution of a given term is modified by the presence of the superconductivity. A derivative of the critical temperature with respect to a parameter x is equal to

$$\frac{\partial}{\partial x}T_c=T_c\left(\frac{\partial}{\partial x}\ln\tilde{\Lambda}+\frac{v_F}{g_{\mathrm{sc}}}\frac{\partial}{\partial x}\ln g_{\mathrm{sc}}\right)\approx T_c\frac{v_F}{g_{\mathrm{sc}}}\frac{\partial}{\partial x}\ln g_{\mathrm{sc}}, \tag{59}$$

provided that $g_{\mathrm{sc}}\ll v_F$. Combining this result with Eq. (45) we conclude that in the superconducting state the transverse hopping energy is higher:

$$\left\langle -t\sum_{p\langle i,j\rangle}(\psi_{pi}^{\dagger}\psi_{pj}+\mathrm{H.c.})\right\rangle_n-\left\langle -t\sum_{p\langle i,j\rangle}(\psi_{pi}^{\dagger}\psi_{pj}+\mathrm{H.c.})\right\rangle_{\mathrm{sc}}$$
$$\propto t\frac{T_c^2}{g_{\mathrm{sc}}}\frac{\partial}{\partial t}\ln\left(g_{2k_F}\left(\frac{t}{\bar{u}\Lambda}\right)^{(2\mathcal{K}-2)/(1-\theta)}-g_0\right)<0 \tag{60}$$

and the in-chain potential energy is lower than in the normal state:

$$\langle g\psi_{Li}^{\dagger}\psi_{Li}\psi_{Ri}^{\dagger}\psi_{Ri}\rangle_n-\langle g\psi_{Li}^{\dagger}\psi_{Li}\psi_{Ri}^{\dagger}\psi_{Ri}\rangle_{\mathrm{sc}}$$
$$\propto g\frac{T_c^2\tilde{g}_{2k_F}}{g_{\mathrm{sc}}^2}\ln\left(\frac{t}{\bar{u}\Lambda}\right)\frac{\partial}{\partial g}\left(\frac{2\mathcal{K}-2}{1-\theta}\right)>0, \tag{61}$$

since both $\ln(t/\bar{u}\Lambda)$ and the derivative with respect to g are negative.

We have proven that in our case the superconductivity is triggered by the electron-electron repulsion. This result is quite unexpected. It has a many-body nature and cannot be obtained within a mean-field theory for Hamiltonian (1). This mechanism of superconductivity is very similar to the Kohn-Luttinger proposal. The classical Kohn-Luttinger mechanism predicts an extremely low critical temperature. In our case, however, the effective coupling constant g_{sc} is a

nonanalytical function of the bare parameters. As a consequence, our transition temperature (48) does not have to be small.

V. STRONG-COUPLING REGIME

We have seen above that if $\theta>1$ then $\tilde{\Lambda}$ is zero. This means that quasiparticles are not formed and it is more convenient to treat the system in terms of the TL boson only. The bosonized Hamiltonian (1) has the form

$$\mathcal{H}_i^{1D}=\frac{u}{2}[\mathcal{K}(\nabla\Theta_i)^2+\mathcal{K}^{-1}(\nabla\Phi_i)^2], \tag{62}$$

$$\mathcal{H}_{ij}^{\perp}=\frac{g_{2k_F}}{(2\pi)^2}\cos\sqrt{4\pi}(\Phi_i-\Phi_j). \tag{63}$$

In this formula both the transverse hopping term which is irrelevant in the RG sense and the forward-scattering term which is marginal are omitted. Their effect is small as compared with that of the strongly relevant exchange interaction, Eq. (63).

The relevance of the exchange interaction indicates that at low temperature the system freezes into a state with the finite expectation value $\langle\Phi_i\rangle\neq 0$. This phase is the CDW. It can be easily proven by bosonizing the CDW order parameter: $\psi_{Li}^{\dagger}\psi_{Ri}\propto(2\pi a)^{-1}\exp(i\sqrt{4\pi}\Phi_i)$. The finite expectation value of the field Φ is inherited by the CDW order parameter.

We describe this regime with the help of our variational wave function. Since $\tilde{\Lambda}=0$ one can write the wave function in terms of the TL bosonic field only:

$$|\mathrm{var}\rangle=\prod_{k>0,i}\frac{1}{\sqrt{2\pi\sigma_k^2}}\exp\{-|\Phi_{ik}|^2/4\sigma_k^2\}. \tag{64}$$

This expression is a slight generalization of Eq. (11): in the latter equation the parameters $\sigma_k^2=\mathcal{K}/4|k|$. Here we do not fix σ_k^2. Instead, they are determined variationally. The variational energy is

$$E^V/(LN_\perp)=u\int_0^\Lambda\frac{dk}{2\pi}\left(\frac{\mathcal{K}}{8\sigma_k^2}+\frac{2k^2\sigma_k^2}{\mathcal{K}}\right)-g_{2k_F}\Lambda^2\exp\left\{-8\int_0^\Lambda dk\,\sigma_k^2\right\}. \tag{65}$$

Minimizing this energy with respect to σ_k^2 we find

$$\sigma_k^2=\frac{\mathcal{K}u}{4\sqrt{u^2k^2+\Delta_{\mathrm{CDW}}^2}}, \tag{66}$$

$$\Delta_{\mathrm{CDW}}^2=8\pi g_{2k_F}\mathcal{K}u\Lambda^2\exp\left\{-2\mathcal{K}\int_0^{u\Lambda}\frac{d\varepsilon}{\sqrt{\varepsilon^2+\Delta_{CDW}^2}}\right\}\propto g_{2k_F}u\Lambda^2\left(\frac{\Delta_{\mathrm{CDW}}}{u\Lambda}\right)^{2\mathcal{K}}. \tag{67}$$

The quantity Δ_{CDW} has the meaning of the excitation gap due to CDW order. This gap, together with the transition temperature, can be found by solving the last equation:

$$T_{\mathrm{CDW}}\propto\Delta_{\mathrm{CDW}}\propto u\Lambda\left(\frac{g_{2k_F}}{u}\right)^{1/(2-2\mathcal{K})}. \tag{68}$$

The variational energy is

$$E^V/(LN_\perp)\propto\Delta_{\mathrm{CDW}}^2/u. \tag{69}$$

These results are correct when $\mathcal{H}^{\perp}$ couples only those chains which are nearest neighbors. The next-to-nearest-neighbor coupling frustrates the CDW phase. We do not discuss the effect of the frustration in this paper.

Finally, let us discuss the crossover from a strong-to intermediate-coupling regime. Such a crossover occurs when the intermediate-coupling Fermi-liquid energy, Eq. (16), becomes equal to the strong-coupling CDW energy, Eq. (69):

$$\tilde{t}^2/v_F\propto\Delta_{\mathrm{CDW}}^2/u \quad \text{or} \quad \left(\frac{t}{u\Lambda}\right)^{1/(1-\theta)}\propto\left(\frac{g_{2k_F}}{u}\right)^{1/(2-2\mathcal{K})}. \tag{70}$$

This equation defines $\theta_c(t,g_{2k_F})<1$ at which the crossover takes place. At small $\theta<\theta_c$ the system behaves like the Fermi liquid whose properties we discussed in the previous sections. When $\theta>\theta_c$ the expression (16) is no longer applicable: the necessary requirement for smallness of the energy associated with symmetry breaking is violated. The expression (69) has to be used instead.

Figure 3 shows how the strong-coupling regime at big θ is replaced by the intermediate-coupling regime at smaller θ. The transition temperature of the the CDW Eq. (68) drops

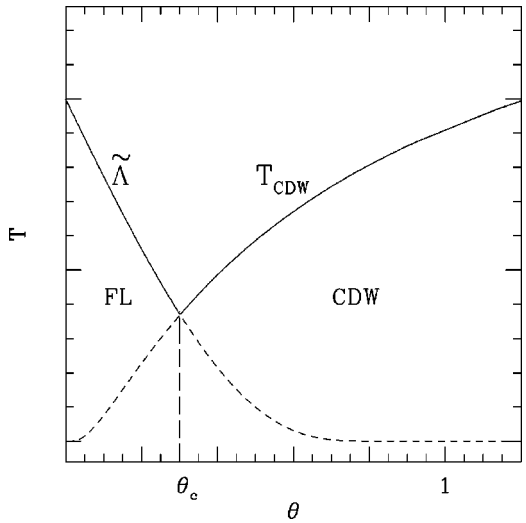

FIG. 3. The energy scale associated with transverse hopping $\tilde{\Lambda}$ decreases when θ grows. The CDW transition temperature T_{CDW} increases as θ grows. At θ_c where both energy scales are of the same order the crossover from intermediate to strong coupling occurs.

sharply and becomes exponentially small, Eq. (47), as θ gets smaller than θ_c. This diagram was discussed in Ref. 10 for a similar model.

VI. CONCLUSIONS

We propose in this paper the variational wave function for a Q1D system. The key ingredient of our procedure is the splitting of TL bosons into high-momentum and low-momentum modes. While high-momentum modes are in their ground state the low-momentum modes form quasiparticles which delocalize in the transverse directions.

Our method can be viewed as a variational implementation of the lowest-order RG scaling near a TL liquid fixed point. When the transverse hopping amplitude becomes of the order of $\overline{u}\Lambda$ the scaling must be stopped. The renormalized Hamiltonian should be treated as the Hamiltonian for the quasiparticles.

Our method gives us the possibility to compute different Green's functions beyond the RG using a standard diagrammatic technique. As an example we calculated the lowest-order self-energy for the one-particle propagator.

Depending on the strength of the in-chain interaction and the anisotropy the system may be in one of three regimes: strong, intermediate, or weak coupling. In the strong-coupling regime quasiparticles are not formed and the system is better described in terms of TL bosons. In weak- and intermediate-coupling regimes the low-lying degrees of freedom are quasiparticles. The ground state of these fermions may be either a Fermi liquid, the superconductivity, or a CDW. The phase diagram of our Q1D model looks very similar to that of organic Q1D superconductors.

Unlike the classical BCS superconducting phase, the one in our model is stabilized without any attraction between the electrons. It is similar to Kohn-Luttinger superconductivity. However, our effective coupling constant is larger than that of Kohn-Luttinger. This guarantees that the critical temperature in our model is not unacceptably small.

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