Disorder in fractional quantum Hall states and the gap at $\nu=5/2$

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(Received 16 June 2003; published 29 September 2003)

Theoretical results for the gaps of fractional quantum Hall states are substantially larger than experimental values determined from the activated behavior of charge transport. The disparity in the case of the enigmatic $\nu=5/2$ state is worrying as it amounts to a factor of 20–30. We argue that disorder effects are responsible for this disparity and show how intrinsic gaps can be extracted from the measured transport gaps of particle-hole symmetric states within the same Landau level. We present theoretical results for gaps at $\nu=5/2$ and 7/2, as well as at $\nu=1/3$, 2/5, 3/7, and 4/9, based on exact diagonalizations, taking account of the finite thickness of the two-dimensional electron layer and Landau level mixing effects. We find these to be consistent with the intrinsic gaps inferred from measured transport gaps. While earlier analyses [Du *et al.*, Phys. Rev. Lett. **70**, 2944 (1993)] assumed constant broadening for each sample, our results for the disorder broadening depend on the filling fraction and appear to scale with the charge of the elementary excitations of the corresponding fractional state. This result is consistent with quasiparticle mediated dissipative transport.

DOI: 10.1103/PhysRevB.68.113309 PACS number(s): 73.20.Mf, 73.43.Cd, 73.21.−b, 73.43.Lp

The gaps obtained from analyzing the activated temperature dependence of the longitudinal conductance near the center of fractional quantized Hall (FQH) plateaus in GaAs heterostructures[1,2] disagree with the values obtained from direct diagonalizations of finite systems.[3] This is the case even taking account of the softening of the Coulomb interaction between electrons resulting from the nonzero thickness of the two-dimensional (2D) electron layer and of Landau level mixing effects. The discrepancies can be around a factor of 2 in the highest mobility samples for FQH states in the lowest Landau level (LLL). For FQH states in the second Landau level the discrepancies are even larger, as much as a factor of 20 at filling fraction $\nu=5/2$ (Refs. 3–5) and a factor of 30 at $\nu=7/2$ (Ref. 6). Such large discrepancies make one wonder whether the $\nu=5/2$ state has been correctly identified.

We argue that disorder effects are responsible for these discrepancies. We show how the intrinsic gap of FQH states, which are strongly affected by disorder, can be estimated directly from measurements of the transport gaps using a simple model. These estimates are consistent with results we obtain from exact diagonalizations of finite systems provided we take account both of the nonzero width of the electron wave function in the direction perpendicular to the two-dimensional electron layer and of Landau level mixing (LLM). Our analysis also provides estimates for the reduction of the measured activation gaps relative to the disorder-free intrinsic gaps—the so-called "disorder broadening." We find that the disorder-induced gap reduction depends on the FQH state studied and is roughly proportional to the fractional charge of the corresponding elementary excitations. Our results show that, using a combination of the scaling analysis (described here) and comparisons with the results of exact diagonalizations, it will be possible to extract from measurements of activation energies reliable estimates both for the intrinsic FQH gap and for the disorder-induced gap reduction.

This work has been motivated by the observation of a transport gap for a FQH state at $\nu=7/2$ by Eisenstein *et al.*[6] and by their report of transport gaps for the $\nu=7/2$ and 5/2 FQH states of $\Delta^a(7/2)=0.07$ K and $\Delta^a(5/2)=0.31$ K. The latter is almost a factor of 3 larger than the earlier value $\Delta^a(5/2)=0.11$ K (first reported in Ref. 4 and confirmed in Ref. 5). The smaller gaps at $\nu=5/2$ were obtained for samples with electron density $n_S=2.3\times10^{11}/\mathrm{cm}^2$, whereas the most recent results are for $n_S=3\times10^{11}/\mathrm{cm}^2$ (Ref. 6). The factor of 3 difference between the new and old results for $\Delta^a(5/2)$ cannot result from Coulomb interaction effects alone, as these scale with $\sqrt{n_S}$.

A FQH state at $\nu=7/2$ is expected on theoretical grounds. Its structure should be very similar to that of the $\nu=5/2$ state, as these two states are related by particle-hole conjugation symmetry, which becomes exact in the limit when LLM can be neglected. In that limit, if the energy gaps are purely controlled by the Coulomb interaction

$$E_c=e^2/\kappa\ell_0, \tag{1}$$

the intrinsic gaps $\Delta^i(\nu)$ of pure (disorder-free) systems can be written as

$$\Delta^i(\nu)=\delta(\nu)E_c. \tag{2}$$

The symbol ℓ_0 in Eq. (1) stands for the magnetic length, defined in terms of the magnetic field B, by $\ell_0=\sqrt{\hbar c/eB}$, and κ is the dielectric constant of the semiconductor material. For physically equivalent FQH states at fillings ν and ν' (those related by particle-hole conjugation symmetry), the coefficients $\delta(\nu)$ and $\delta(\nu')$ in Eq. (2) will be the same and the difference in the gap values $\Delta^i(\nu)$ and $\Delta^i(\nu')$ will reflect the difference in the Coulomb energy scale E_c at the magnetic fields B_ν and $B_{\nu'}$ at which the FQH states occur. This will happen for $\nu'=2-\nu$, and in the second Landau level, when $\delta(2+\nu)=\delta(2+(2-\nu))$, implying $\delta(5/2)=\delta(7/2)$. If, in addition, spin mixing effects can be neglected, FQH states at filling fraction ν can be mapped to states at $\nu'=1-\nu$. As an example, we expect that gaps of fractional states at $\nu=1/3,2/3,4/3$, and 5/3 will all be described by the same coefficient $\delta(1/3)$ as long as the Zeeman energy is large enough to suppress spin reversal in all these states and as long as LLM effects can be neglected.

The gap observed in the activated transport in an ideal homogeneous sample should be the intrinsic gap given by Eq. (2). In practice, the transport is found to be activated but with a gap which is smaller than the expected intrinsic gap. In the absence of any microscopic theory of the effects of impurities, previous analyses simply assumed that the intrinsic gap is reduced by a filling-factor-independent (but sample-dependent) disorder broadening of electronic states.[2] Such analyses gave intrinsic gaps which scaled as $|B_\nu - B_{1/2}|$, where B_ν is the magnetic field at which the filling is precisely ν, and implied an effective mass of the composite fermions which is independent of the filling fraction, contrary to theoretical predictions.[7]

Here we explore the implications of a much weaker assumption—namely, that the effect of disorder in any given sample is the same *only* for states which have elementary excitations carrying the same fractional charge. We define the gap reduction $\Gamma(\nu)$, as the difference between the measured activation gaps, $\Delta^a(\nu)$, and the intrinsic gap $\Delta^i(\nu) = \delta(\nu)E_c$:

$$\Delta^a(\nu) = \delta(\nu)E_c - \Gamma(\nu). \tag{3}$$

We then assume that the intrinsic gap parameter $\delta(\nu')$ and the reduction due to disorder $\Gamma(\nu')$ are the same *only* for symmetry-related states, i.e., for states with ν' in the set S_ν which is a subset of $\{\nu, 1-\nu, 1+\nu, 2-\nu\}$ (see below), so that

$$\Delta^a(\nu') \approx \delta_\nu E_c - \Gamma_\nu \quad \forall \ \nu' \in S_\nu. \tag{4}$$

Analyzing the experimental data on the basis of Eq. (4) brings the experimental estimates of the intrinsic gaps very closely into line with expectations from first-principles exact diagonalization studies. We also find that the gaps in the lowest Landau level agree to within a few percent across different samples. In the second half of the paper we test (again empirically) the validity of the ansatz (4) using the results of finite-size diagonalization studies. For states close to $\nu = 1/2$ and for the pair at $\nu = 5/2$ and 7/2, for which Landau level mixing effects are similar, we find that our weak assumption appears valid and suggests a gap reduction which, in any given sample, scales with the fractional charge of the elementary excitation.

The disorder scattering in the GaAs heterostructures, for which studies of the activated transport have been reported, is thought to be due mainly to ionized donors separated from the electron gas by a spacer layer of width d, where $d \sim 800$ Å (Ref. 2). In the case of moderate to strong disorder, the system is expected to break up into regions of compressible fluid surrounded by filamentary strips of incompressible fluid, which percolate through the system and are responsible for the quantized Hall response.[9] Our assumption that the values of $\Gamma(\nu)$ in a given sample are comparable for all states ν' in S_ν is equivalent to assuming that the effect of the ionized donors on the low-lying excitations in (or at the boundary of) the incompressible strips, which are responsible for the quantized Hall response, is similar for all filling fractions in the set S_ν. In the limit $l_0/d \ll 1$, the fractionally charged elementary excitations will appear point like on the scale of the impurity potential, in which case $\Gamma(\nu)$ will depend on properties of the low-lying excitations like their charge q. These will be common to symmetry-related states in the set S_ν but different for inequivalent sets S_ν. Effects related to the internal structure of the excitations, which will result in an additional dependence of $\Gamma(\nu)$ on the ratio l_0/d, are assumed to be small. In practice this means that our ansatz (4) should hold best for states in S_ν for which the ratios l_0/d are close, for example the states at $\nu = 1/3$ and $\nu = 2/3$, but less well for the state at $\nu = 5/3$. When we compare with our results from exact diagonalizations, this is indeed what we find (see below).

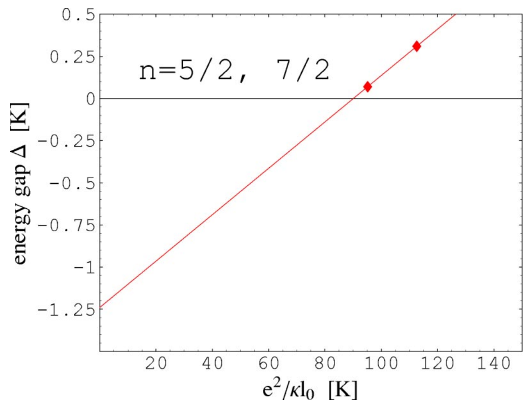

FIG. 1. The activation gaps, $\Delta^a(\nu)$ from Ref. 6 plotted against $E_c = e^2/\kappa\ell_0$ for $\nu = 5/2$ (right) and 7/2 (left). The slope of the straight line through the measured gaps [cf. Eq. (4)] yields a coefficient $\delta(5/2) = 0.014$, i.e., $\Delta_i = 0.014E_c$, and via the intercept, an estimate of the gap reduction due to disorder, $\Gamma^{est}_{5/2} \approx 1.2$ K.

In Fig. 1, we show the gap results from Ref. 6 for the $\nu = 5/2$ and 7/2 states as a function of E_c. According to Eq. (4) the slope of the straight line through the two gap values yields $\delta(5/2) \approx 0.014$. This is just $\sim 35\%$ smaller than the theoretical estimate for a spin-polarized paired state of the Moore-Read type,[10] $\delta^{th}(5/2) \approx 0.022$, which was computed without taking account of LLM effects.[3] The intercept of the straight line gives [see Eq. (4)] an estimate of the gap reduction due to disorder, $\Gamma(5/2) \approx 1.24$ K, which is only slightly less than the estimate for the intrinsic gap itself. We emphasize that the estimate for Γ for the two states is based solely on the assumption that the 7/2 and 5/2 states are particle-hole conjugates of each other and that the Coulomb interaction dominates the value of the intrinsic gap. Although, in practice, the state at $\nu = 7/2$ is likely to be more strongly affected by LLM than that at $\nu = 5/2$, the assumption of particle-hole symmetry between the two states should still be approximately valid. In the following, we show that the effects of LLM reduce the theoretical value to $\delta^{th}(5/2) \approx 0.016$, so that the discrepancy between theoretical and experimental estimates of the gap at 5/2 essentially disappears. This provides further support for the identification of the FQH state at $\nu = 5/2$ as a paired state.[11,12]

We have also reanalyzed older results for FQH states at filling $\nu = p/(2p+1)$ and $(p+1)/(2p+1)$. In Fig. 2, we show the measured gaps taken from Refs. 1, 2, and 8 for three different very high-mobility samples at filling fractions $\nu = p/(2p+1)$, $\nu' = 1-\nu$, and $\nu' = 2-\nu$ for $p = 1$ [Fig. 2(a)], $p = 2$ [Fig. 2(b)], and $p = 3$ [Fig. 2(c)] as functions of

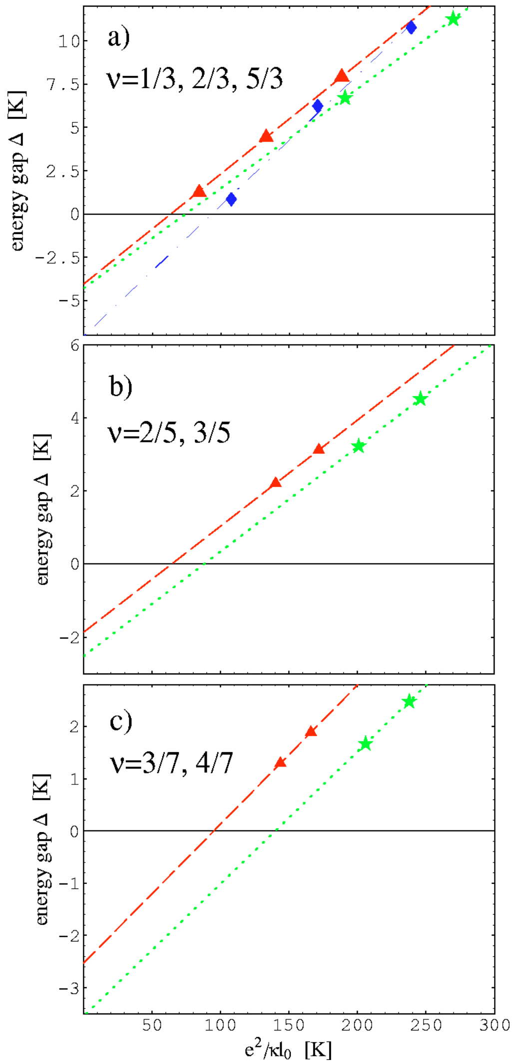

FIG. 2. The measured activation gaps $\Delta^a(\nu)$ plotted against the Coulomb energy $E_c = e^2/\kappa\ell_0$. Triangles and asterisks, respectively, refer to samples A and B in Refs. 2 and 8; diamonds represent results from Ref. 1. (a) Gaps at $\nu=1/3$, 2/3, 5/3. (b) Gaps at $\nu=2/5$, 3/5. (c) Gaps at $\nu=3/7$, 4/7. The slope of Δ^a vs E_c gives an estimate of the intrinsic gap $\Delta^i \approx \delta_\nu E_c$ of the set S_ν of symmetry-related states. Samples of differing quality lead to similar slopes δ_ν of the straight line fit, but to different intersections at $E_c=0$, which provide estimates for the gap reduction Γ_ν^{est} for that family S_ν.

E_c. Samples A and B of Ref. 2 have an electron density n_S of $(1.12 \text{ and } 2.3)\times 10^{11}/\text{cm}^2$ and mobilities $\mu = (6.8 \text{ and } 12)\times 10^6 \text{cm}^2/\text{V s}$. The sample of Ref. 1 (which we will call sample C) has $n_S = 1.65\times 10^{11}/\text{cm}^2$ and a mobility $\mu = 5\times 10^6 \text{cm}^2/\text{V s}$. Gaps at $\nu=5/3$ and 4/3 were reported for sample A in Ref. 8. The dependence of the gap on total magnetic field in a tilted field experiment showed that at $\nu=5/3$ the ground and low-lying excited states were spin polarized. At $\nu=4/3$ the ground state was not spin polarized for tilt angles up to 65.1° while the excitations involved spin reversals up to even larger tilt angles. We therefore assume that only the states at $\nu=1/3$, 2/3, and 5/3 are related by symmetry and not the state at $\nu=4/3$. We show the measured gaps in untilted field as a function of E_c in Fig. 2(a). The slopes of the three straight line fits through the gaps of samples A, B, and C are quite similar, yielding estimates of $\delta_{1/3}=0.064$, 0.058, and 0.075, respectively. By contrast, the intercepts at $E_c=0$ yield estimates $\Gamma_{1/3}^{est}$ that vary by almost a factor of 2 and reflect differences in sample quality. In Figs. 2(b) and 2(c) we show the analysis of the states at $\nu=2/5$, 3/5 and $\nu=3/7$, 4/7, respectively. Again the slope of the gaps as function of E_c are very similar for the two samples A and B. They yield estimates of $\delta_{2/5}=0.029$ for both samples [Fig. 2(b)] and $\delta_{3/7}=0.027$ and 0.025 [Fig. 2(c)] for samples A and B, respectively.

We can test the ideas behind the ansatz (4) directly using the results of exact diagonalization studies. We attribute the difference between precise calculations of FQH gaps of disorder-free systems and measured gaps to the effects of disorder and then use Eq. (3) as the definition of $\Gamma(\nu)$. Such calculations must of course include the effects of the finite thickness w of the two-dimensional electron system as well as LLM. We take account of LLM within the random phase approximation for the dielectric function[14]

$$\epsilon(q,\omega) = 1 - \tilde{V}(q)\Pi(q,\omega) \tag{5}$$

and represent the electron-electron interaction by

$$U(r) = \int \frac{d^2q}{(2\pi)^2}\tilde{V}(q)\frac{e^{i\vec{q}\vec{r}}}{\epsilon(q,0)}, \tag{6}$$

where

$$\tilde{V}(q) = \frac{2\pi e^2}{\kappa q} e^{q^2w^2} erfc(qw) \tag{7}$$

is the interaction between electrons that are trapped at the interface in a Gaussian wave function of width w (Ref. 3). The polarization $\Pi(q,\omega)$ in Eq. (5) is given by

$$\Pi(q,\omega) = -\frac{m^*}{\pi\hbar^2}\sum_s \sum_{n=0}^{[\nu(s)-1]} F(\nu(s)-n)\sum_{k=n+1}^{\infty} F(k - \nu(s))\frac{(-1)^{(k-n)}(k-n)}{(\omega/\omega_c)^2+(k-n)^2} L_n^{k-n}(x) L_k^{n-k}(x) e^{-x}, \tag{8}$$

where $x=(q\ell_0)^2/2$ and Σ_s stands for the summation over spin $s=\uparrow,\downarrow$. The symbol $[x]$ denotes the largest integer $\leqslant x$. Equation (8) agrees with expression (A1) of Aleiner and Glazman,[14] which describes the spin-degenerate case $\nu(\uparrow) = \nu(\downarrow) = N$, with integer filling $2N$. The function $F(z)$ is introduced to treat the case of fractional filling and measures the filling fraction of the Landau level n, via $F(z)=z$ for $0<z<1$, $F(z)\equiv 1$ for $z\geqslant 1$ and $F(z)\equiv 0$ for $z\leqslant 0$. We have verified this method for incorporating finite width and LLM corrections at filling fraction $\nu=1/3$ where we could check that our results are consistent with those by Yoshioka.[13] Expression (6) together with Eqs. (7) and (5) lead to a modification of the electron interaction at short separation, which is controlled by the dimensionless parameter $\lambda = E_c/\hbar\omega_c$.

TABLE I. The values for the intrinsic gap δ_ν^{est} [see Eq. (2)] estimated by fitting the measured activation gaps Δ^a to the ansatz (4) for different samples, together with our theoretical values $\delta^{th}(\nu)$ for the gaps obtained from exact diagonalization studies (cf. Ref. 3) and the corresponding gap reduction $\Gamma(\nu)$ (see text). The theoretical values $\delta^{th}(\nu)$ include finite width and Landau level mixing corrections. Numbers in parentheses denote the error of the last quoted digit of δ_ν^{est}. These are calculated from the quoted experimental error (Ref. 1) or from the discretization error when extracting numerical data from experimental plots (Refs. 2 and 8). For the latter as well as for $\nu=5/2$ and 7/2, no experimental uncertainty is specified. If for the values of Δ^a of Refs. 2 and 8 similar uncertainties are assumed as specified in Ref. 1, errors for samples A and B are 4–5 times bigger than quoted.

ν	ν'	Ref	δ_ν^{est}	$\delta^{th}(\nu)$	$\delta^{th}(\nu')$	$\Gamma(\nu)$	$\Gamma(\nu')$
1/3	2/3	1	0.069(9)	0.075	0.074	7.2 K	6.4 K
1/3	5/3	1	0.077(4)	0.075	0.057	7.2 K	5.3 K
1/3	2/3	2A	0.063(2)	0.077	0.073	6.5 K	5.3 K
1/3	5/3	2,8A	0.064(1)	0.077	0.052	6.5 K	3.1 K
2/5	3/5	2A	0.029(3)	0.036	0.034	3.0 K	2.6 K
3/7	4/7	2A	0.027(5)	0.025	0.025	2.3 K	2.3 K
4/9	5/9	2A	0.013(6)	0.019	0.019	2.2 K	2.1 K
1/3	2/3	2B	0.058(2)	0.077	0.076	9.4 K	7.8 K
2/5	3/5	2B	0.029(3)	0.036	0.036	4.3 K	3.9 K
3/7	4/7	2B	0.025(3)	0.025	0.025	3.6 K	3.5 K
4/9	5/9	2B	0.007(5)	0.020	0.020	3.3 K	3.0 K
5/2	7/2	6	0.014	0.016	0.015	1.5 K	1.4 K

Here $\omega_c=eB/m^*c$ stands for the cyclotron frequency, with m^* the effective mass of the electrons.

For the particular sample of Ref. 6 we have repeated the calculation of Ref. 3 using the interaction amended for LL mixing and taking account of the nonzero thickness of the wave function. We have computed the quasiparticle and quasihole energies as described in Ref. 3 and, by extrapolating to the thermodynamic limit, we have estimated the intrinsic gaps at $\nu=5/2$ and $\nu=7/2$. The gaps are $\delta^{th}(5/2)=0.016$ and $\delta^{th}(7/2)=0.015$, and are close to the estimate $\delta_{5/2}^{est}\approx 0.14$ obtained from the experimental values of Δ^a at $\nu=5/2$ and 7/2, using Eq. (4) as discussed above. We have also calculated the finite width and LLM corrections for all the other states reported in Refs. 1,2, and 8. These results are listed in Table I. The differences between the two theoretical estimates δ^{th} for the pairs of states are generally very small reflecting very small differences in the LLM corrections for the members of each pair. With the exception of the $\nu=5/3$ case, our assumption $\delta(\nu')\approx\delta(\nu)$ for all ν in the set S_ν of symmetry-related states therefore appears reasonable. The estimates δ_ν^{est} of the gap coefficients, calculated on the basis of the simple ansatz (4) from the experimental values of the activation gap Δ^a, are consistent with the theoretical values $\delta^{th}(\nu)$ within realistic error bars (cf. caption of Table I).

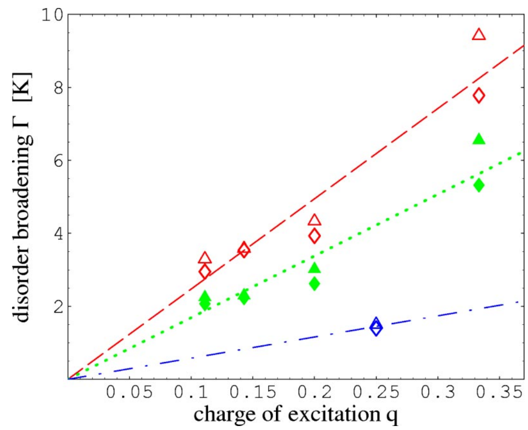

FIG. 3. Disorder broadening $\Gamma(\nu)$ for samples of different high electron mobility plotted as a function of the charge of the elementary excitations. Solid symbols refer to sample A, open triangles and diamonds to sample B (Refs. 2 and 8), and the data on the dash-dotted line represent $\nu=5/2$ and 7/2 (Ref. 6). Triangles refer to ν, diamonds to $1-\nu$.

Estimates for $\Gamma(\nu)$, based on Eq. (3), are listed in the last two columns of Table I and plotted in Fig. 3 against the charge $q=1/(2p+1)$ of the elementary excitation of the FQH state at $\nu=p/(2p+1)$ or its symmetry-related siblings. The values we obtain for $\Gamma(\nu)$ scale with the charge of the excitation in each sample. They are comparable for families of symmetry-related states as we assumed in Eq. (4), with the assumption better justified the closer the filling fractions are to $\nu=1/2$. However, in each family, $\Gamma(\nu)$ is systematically smaller for larger filling factors, with smaller differences between $\Gamma(\nu)$ and $\Gamma(1-\nu)$ when ν is close to 1/2. The largest differences are for the case of $\nu=1/3$, 2/3, and 5/3, where $\Gamma(1/3)$ is about twice $\Gamma(5/3)$. We attribute this reduction of $\Gamma(\nu)$ at larger ν to LLM and other polarization and screening effects which should increase as l_0/d increases (for the $\nu=5/3$ state in Ref. 8 $l_0/d\sim 1/3$), but which would require a microscopic model of the response of the FQH system to potential variations to quantify.

The authors acknowledge many useful discussions with B.I. Halperin and thank him for all his encouragement.

[1] R.L. Willett *et al.*, Phys. Rev. B **37**, 8476 (1988).
[2] R.R. Du *et al.*, Phys. Rev. Lett. **70**, 2944 (1993).
[3] R.H. Morf, N. d'Ambrumenil, and S. Das Sarma, Phys. Rev. B **66**, 075408 (2002).
[4] J.P. Eisenstein *et al.*, Surf. Sci. **229**, 31 (1990).
[5] W. Pan *et al.*, Phys. Rev. Lett. **83**, 3530 (1999).
[6] J.P. Eisenstein *et al.*, Phys. Rev. Lett. **88**, 076801 (2002).
[7] B.I. Halperin, P.A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993); see also Ref. 3.
[8] R.R. Du *et al.*, Phys. Rev. B **55**, 7351 (1997).
[9] F.G. Pikus and A.L. Efros, Phys. Rev. B **51**, 16 871 (1995).
[10] G. Moore and N. Read, Nucl. Phys. B **360**, 362 (1991).
[11] R.H. Morf, Phys. Rev. Lett. **80**, 1505 (1998).
[12] E.H. Rezayi and F.D.M. Haldane, Phys. Rev. Lett. **84**, 4685 (2000).
[13] D. Yoshioka, J. Phys. Soc. Jpn. **55**, 885 (1986).
[14] I.L. Aleiner and L.I. Glazman, Phys. Rev. B **52**, 11 296 (1995); C. Kallin and B.I. Halperin, *ibid.* **30**, 5655 (1984).