Absence of phase stiffness in the quantum rotor phase glass

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We analyze here the consequence of local rotational-symmetry breaking in the quantum spin (or phase) glass state of the quantum random rotor model. By coupling the spin glass order parameter directly to a vector potential, we are able to compute whether the system is resilient (that is, possesses a phase stiffness) to a uniform rotation in the presence of random anisotropy. We show explicitly that the O(2) vector spin glass has no electromagnetic response indicative of a superconductor at mean-field and beyond, suggesting the absence of phase stiffness. This result confirms our earlier finding [Phys. Rev. Lett. **89**, 027001 (2002)] that the phase glass is metallic, due to the main contribution to the conductivity arising from fluctuations of the superconducting order parameter. In addition, our finding that the spin stiffness vanishes in the quantum rotor glass is consistent with the absence of a transverse stiffness in the Heisenberg spin glass found by Feigelman and Tsvelik [Sov. Phys. JETP **50**, 1222 (1979)].

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I. INTRODUCTION

Spin glasses are characterized by the freezing of local spins along random non-collinear directions. Because each spin points in a preferred direction, locally spin rotational symmetry is broken. Nonetheless, globally rotational symmetry is preserved because spin glasses have no net magnetization. We consider here the O(2) quantum rotor model where the exchange interactions are random. As this model is isotropic in rotor space, a global rotation of all of the rotors is an exact symmetry, even in the glass phase. Nonetheless, in the glass state, a global rotation of all of the spins around any axis generates a new state which is distinguishable from the original unrotated state. Because such uniform rotations are generated by the group SO(2), the spin glass state breaks SO(2) symmetry. All such states are energetically degenerate as a result of the inherent isotropy in rotor space. As a result of the broken SO(2) symmetry, it is reasonable to expect that a massless bosonic mode should exist.

In the strict sense, a physical system possesses a nonzero phase rigidity if upon a uniform rotation of the phase, the free energy increase is of the form

$$\Delta F = \frac{\rho_s}{2} \int d^2 r |\nabla \theta|^2, \qquad (1)$$

where ρ_s is the spin or superfluid stiffness and θ is the collective phase variable. Consequently a spin-wave mode with a dispersion $\omega = \pm ck$ would be an experimental signature of a spin stiffness consistent with Eq. (1). Experimentally, however, no such mode has ever been found in either neutron scattering or thermal measurements¹⁻⁴ on spin glasses. This failure might be attributed to that fact that overdamped modes and/or low energy excitations conspire to make ρ_s undetectable. Theoretically, in the phenomenological hydrodynamic account, Halperin and Saslow⁵ assumed that $\rho_s \neq 0$. They did caution the reader that the existence of a stiff-

ness in a spin glass is subtle and, in all likelihood, doubtful as a result of the preponderance of experimental evidence for a large density of low-energy excitations that could overdamp the spin-wave mode. This conclusion is supported by extensive numerical simulations by Walker and Walstedt⁶ who found no evidence for the characteristic ω^2 vanishing of the low-energy modes. Two microscopic calculations of the spin stiffness exist. Feigel'man and Tsvelik⁷ developed a real-time diagrammatic technique for the Heisenberg spin glass and showed explicitly that the spin stiffness vanishes. This result is particularly robust because it follows from a simple permutation symmetry of the spin correlators.⁷ Within the replica formalism of a Heisenberg spin glass Kotliar, Sompolinsky, and Zippelius^{8,9} formulated a mean-field description of the single-valley stiffness constant. This limit is relevant at sufficiently short times that the spin glass remains trapped in a single configuration. In this limit, the stiffness constant is nonzero.^{8,9} However, in the full statistical mechanical treatment of the problem in which hopping among the myriads of valleys in the energy landscape of a spin glass are allowed, the stiffness vanishes.⁸⁻¹⁰ This result implies that the spin stiffness is a transient effect approaching zero in the equilibrium or long-time limit. In this limit, a new massless mode dispersing as $\omega \propto ik^2$ emerges which leads to the vanishing of the spin stiffness, as in the real-time formalism.⁷ Hence, there is a consilience between the replica and realtime formalisms that the stiffness constant vanishes in the Heisenberg spin glass.

For quantum spin glasses, no calculation of the stiffness exists. Nonetheless, we expect the same physics to be valid. Namely, as long as the system can relax and hop among all of the configurations of the spin glass, the stiffness should vanish. For example, in quantum spin glasses, quantum tunneling among the various local minima in the spin glass landscape is permitted, thereby leading to a vanishing of the stiffness. This problem is particularly current because we have recently proposed¹¹ that the bosonic excitations arising from fluctuations of the superconducting order parameter in the glassy phase, lead to a metallic conductivity at zero temperature. In the Gaussian approximation, this conductivity $\sigma_{\rm hos}$ diverges as $1/m^4$ upon approaching the superconducting phase (m is the inverse correlation length of the superconducting fluctuations). A free energy density of the form of Eq. (1), however, leads to a superconducting response. Hence, should the phase glass itself have a well-defined stiffness, then the bosonic conductivity, though intriguing, would be irrelevant as it would be dwarfed by the infinite conductivity arising from the excitations related to the glassy order parameter. We show here explicitly that this is not the case, at least at the mean-field level. Rather than attempting to calculate the phase stiffness from the free energy, we consider the linear response regime and couple the spin glass order parameter to the appropriate vector potential. Second, we compute the role of replica symmetry breaking (RSB) on the bosonic contribution to the conductivity. We show that weak RSB does not affect the metallic character of the conductivity as $T \rightarrow 0$. Consequently, the Bose metallic phase found earlier¹¹ is robust and constitutes the only known example of a metallic phase in 2D in the presence of disorder.

II. PHASE STIFFNESS

The starting point for our analysis is the O(2) quantum rotor model

$$H = -E_C \sum_{i} \left(\frac{\partial}{\partial \theta_i}\right)^2 - \sum_{\langle i,j \rangle} J_{ij} \cos(\theta_i - \theta_j - A_{ij}), \quad (2)$$

where $A_{ij} = (e^*/\hbar) \int_i^j \mathbf{A} \cdot d\mathbf{l}$ ($e^* = 2e$). The Josephson couplings are assumed to be random and governed by a distribution

$$P(J_{ij}) = \frac{1}{\sqrt{2\pi J^2}} \exp\left[-\frac{(J_{ij} - J_0)^2}{2J^2}\right]$$
(3)

with nonzero mean, J_0 and J the variance. When the distribution has a nonzero mean, three phases are possible: (1) disordered paramagnet, (2) quantum phase glass, and (3) superconductor. Because the existence of the spin stiffness in the spin glass can be answered with the simpler model with zero mean ($J_0=0$), we utilize this model at the outset. For a random system, the technique for treating disorder is now standard: (1) replicate the partition function, (2) perform the average over disorder, and (3) introduce the appropriate fields to decouple the interacting terms that arise. As the corresponding action has been detailed previously,^{12,13} we will provide additional steps that are necessary to determine how the electromagnetic gauge couples to the spin glass order parameter. We write the replicated partition function as

$$\overline{Z^n} = \int \mathcal{D}\theta_i \mathcal{D}J_{ij} e^{-S}, \qquad (4)$$

where the Euclidean action is given by

$$S = \int_{0}^{\beta} d\tau \Biggl\{ \sum_{i,a} \frac{1}{4E_{C}} \Biggl(\frac{\partial \theta_{i}^{a}(\tau)}{\partial \tau} \Biggr)^{2} - \sum_{a} \sum_{\langle ij \rangle} J_{ij} \cos[\theta_{i}^{a}(\tau) - \theta_{j}^{a}(\tau) - A_{ij}(\tau)] \Biggr\},$$
(5)

where the superscript *a* represents the replica index. For $J_0 = 0$, the integration over J_{ij} in Eq. (4) results in the effective action

$$S_{\text{eff}} = \int_{0}^{\beta} d\tau \sum_{i,a} \frac{1}{4E_{C}} \left(\frac{\partial \theta_{i}^{a}}{\partial \tau} \right)^{2} \\ + \frac{J^{2}}{2} \sum_{a,b} \sum_{\langle ij \rangle} \int_{0}^{\beta} \int_{0}^{\beta} d\tau d\tau' \frac{1}{4} \sum_{\alpha=\pm 1,-1} \exp(i\{\theta_{i}^{a}(\tau) - \alpha \theta_{i}^{b}(\tau') - [\theta_{j}^{a}(\tau) - \alpha \theta_{j}^{b}(\tau')] \\ - [A_{ij}(\tau) - \alpha A_{ij}(\tau')]\}) + \text{c.c.}$$
(6)

with $\alpha = +1, -1$. As a result of the sum over α , we see that the vector potential enters both symmetrically and antisymmetrically. To simplify the notation, we introduce the two-component vector

$$S^{a}(\tau) = [\cos \theta^{a}(\tau), \sin \theta^{a}(\tau)]$$
(7)

and the corresponding auxiliary field

$$Q^{ab}_{\mu\nu}(\tau,\tau') = \left\langle S^a_{\mu}(\tau) S^b_{\nu}(\tau') \right\rangle \tag{8}$$

which will be used in decoupling the action and ultimately determines the Edwards-Anderson order parameter for the quantum spin glass transition. The remaining steps involve performing the cumulant expansion and taking the continuum limit. The final action can be separated into the local and gradient parts:

$$S_{\rm eff} = S_{\rm loc} + S_{\rm gr}, \qquad (9)$$

where the local part

$$S_{\rm loc} = \int d^d x \left\{ \frac{1}{\kappa} \int d\tau \sum_a \left(r + \frac{\partial}{\partial \tau_1} \frac{\partial}{\partial \tau_2} \right) \right. \\ \left. \times Q^{aa}_{\mu\mu}(\mathbf{x}, \tau_1, \tau_2) \right|_{\tau_1 = \tau_2 = \tau} - \frac{\kappa}{3} \int d\tau_1 d\tau_2 d\tau_3 \\ \left. \times \sum_{a,b,c} \left. Q^{ab}_{\mu\nu}(\mathbf{x}, \tau_1, \tau_2) Q^{bc}_{\nu\rho}(\mathbf{x}, \tau_2, \tau_3) Q^{ca}_{\rho\mu}(\mathbf{x}, \tau_3, \tau_1) \right. \\ \left. + \frac{1}{2} \int d\tau \sum_a \left[u Q^{aa}_{\mu\nu}(\mathbf{x}, \tau, \tau) Q^{aa}_{\mu\nu}(\mathbf{x}, \tau, \tau) \right. \\ \left. + v Q^{aa}_{\mu\mu}(\mathbf{x}, \tau, \tau) Q^{aa}_{\nu\nu}(\mathbf{x}, \tau, \tau) \right] \right\} \\ \left. - \frac{y_1}{6t} \int d^d x \int d\tau_1 d\tau_2 \sum_{a,b} \left[Q^{ab}_{\mu\nu}(\mathbf{x}, \tau_1, \tau_2) \right]^4$$
(10)

is identical to that derived previously by Read, Sachdev, and Ye^{13} and the gradient part

$$S_{gr} = \int d^{d}x \int_{0}^{\beta} d\tau_{1} d\tau_{2} \sum_{a,b} \left| \left(\nabla - \frac{ie^{*}}{\hbar} \mathbf{A}(\mathbf{x},\tau_{1}) + \frac{ie^{*}}{\hbar} \mathbf{A}(\mathbf{x},\tau_{2}) \right) Q_{-}^{ab}(\mathbf{x},\tau_{1},\tau_{2}) \right|^{2} + \int d^{d}x \int_{0}^{\beta} d\tau_{1} d\tau_{2} \sum_{a,b} \left| \left(\nabla - \frac{ie^{*}}{\hbar} \mathbf{A}(\mathbf{x},\tau_{1}) - \frac{ie^{*}}{\hbar} \mathbf{A}(\mathbf{x},\tau_{2}) \right) Q_{+}^{ab}(\mathbf{x},\tau_{1},\tau_{2}) \right|^{2}$$
(11)

in which the vector potential couples both symmetrically and asymmetrically to combinations of the Q matrices of the same parity. Using the fact that $Q_{\pm}^{ab}(\tau_1, \tau_2) \sim \langle \exp\{i[\theta_i^a(\tau) \pm \theta_i^b(\tau')]\}\rangle$, the parity combinations of the Q matrices are defined as follows:

$$Q_{\pm}^{ab}(\mathbf{x},\tau_{1},\tau_{2}) = \frac{1}{2} [Q_{11}^{ab}(\mathbf{x},\tau_{1},\tau_{2}) \mp Q_{22}^{ab}(\mathbf{x},\tau_{1},\tau_{2})] + \frac{i}{2} [Q_{12}^{ab}(\mathbf{x},\tau_{1},\tau_{2}) \pm Q_{21}^{ab}(\mathbf{x},\tau_{1},\tau_{2})].$$
(12)

It is evident that the vector potential enters in a non-time translationally invariant manner. This is a direct consequence of the fact that the *Q*-matrices themselves are a function of two independent times, not simply the difference of $\tau_1 - \tau_2$.

To calculate the conductivity, we need to focus entirely on the gradient part of the action as this is the only part that couples to the vector potential. The standard Kubo formula for the spin-glass contribution to the longitudinal conductivity takes the form

$$\sigma(i\omega_n) = -\frac{\hbar}{\omega_{n_n \to 0} n} \int d^d(\mathbf{x} - \mathbf{x}') \int_0^\beta d(\tau - \tau') \times \frac{\delta^2 \overline{Z^n}}{\delta A_x(\mathbf{x}, \tau) \, \delta A_x(\mathbf{x}', \tau')} e^{i\omega_n(\tau - \tau')}, \qquad (13)$$

where we have chosen to orient the vector potential along the x-axis. A bit lengthy variational procedure leads to the following result:

$$\sigma(i\omega_{n}) = \frac{(e^{*})^{2}}{\hbar\omega_{n}} \frac{1}{n} \sum_{a,b} \int_{0}^{\beta} d(\tau - \tau')$$

$$\times e^{i\omega_{n}(\tau - \tau')} \bigg\{ 4 \int_{0}^{\beta} d\tau_{2}(\langle |Q_{-}^{ab}(\mathbf{x}, \tau, \tau_{2})|^{2} \rangle$$

$$+ \langle |Q_{+}^{ab}(\mathbf{x}, \tau, \tau_{2})|^{2} \rangle) \delta(\tau - \tau')$$

$$+ 4(\langle |Q_{+}^{ab}(\mathbf{x}, \tau, \tau')|^{2} \rangle - \langle |Q_{-}^{ab}(\mathbf{x}, \tau, \tau')|^{2} \rangle)$$

$$- \int d^{d}(\mathbf{x} - \mathbf{x}') \langle J_{x}(\mathbf{x}, \tau) J_{x}(\mathbf{x}', \tau') \rangle \bigg\}, \quad (14)$$

where the current $J_x(\mathbf{x}, \tau)$ is defined as

$$\mathbf{J}(\mathbf{x},\tau) = \frac{ie^*}{\hbar} \sum_{ab} \sum_{\alpha=+,-} \int_0^\beta d\tau_1 [Q^{ab}_{\alpha}(\mathbf{x},\tau,\tau_1) \\ \times \nabla [Q^{ab}_{\alpha}(\mathbf{x},\tau,\tau_1)]^* - \text{c.c.}].$$
(15)

In deriving this expression for the current, we considered the relations $Q_{+}^{ab}(\mathbf{x},\tau_2,\tau_1) = Q_{+}^{ab}(\mathbf{x},\tau_1,\tau_2)$ and $Q_{-}^{ab}(\mathbf{x},\tau_2,\tau_1) = [Q_{-}^{ab}(\mathbf{x},\tau_1,\tau_2)]^*$, that follow from the definition, Eq. (12). To evaluate the correlation functions in Eq. (14), we need to use the Fourier components of the Q fields

$$Q^{ab}_{\mu\nu}(\mathbf{x},\tau_1,\tau_2) = \int \frac{d^d \mathbf{k}}{(2\pi)^d} \frac{1}{\beta^2} \sum_{\omega_1,\omega_2} Q^{ab}_{\mu\nu}(\mathbf{k},\omega_1,\omega_2) \times e^{-i(\mathbf{k}\cdot\mathbf{x}-\omega_1\tau_1-\omega_2\tau_2)},$$
(16)

and take into account the relations between Q^{ab}_{\pm} and $Q^{ab}_{\mu\nu}$ given by Eq. (12). The general ansatz for the Fourier transformed Q matrices

$$Q_{\mu\nu}^{ab}(\mathbf{k},\omega_{1},\omega_{2}) = \beta(2\pi)^{d} \delta^{d}(\mathbf{k}) \,\delta_{\mu\nu}[\beta q^{ab} \delta_{\omega_{1},0} \delta_{\omega_{2},0} \\ + \delta^{ab} \,\delta_{\omega_{1}+\omega_{2},0} D(\omega_{1})] + \tilde{Q}_{\mu\nu}^{ab}(\mathbf{k},\omega_{1},\omega_{2})$$
(17)

consists of the spatially uniform mean-field part and the fluctuating spatial component \tilde{Q}^{ab} . In Eq. (17),

$$D(\omega) = -\left|\omega\right|/\kappa,\tag{18}$$

while the off-diagonal elements of q^{ab} constitute the ultrametric Parisi matrix¹³

$$q(s) = \begin{cases} (s/s_1)q_{\text{EA}}, & 0 < s < s_1, \\ q_{\text{EA}}, & s_1 < s < 1, \end{cases}$$

in which $s_1 = 2y_1 q_{\text{EA}} T/\kappa$, and q_{EA} is the Edwards-Anderson order parameter $(q^{aa} = q_{\text{EA}})$.

We substitute then this ansatz into Eq. (14) and obtain that $\sigma(i\omega_n)$ consists of three parts,

$$\sigma(i\omega_n) = \sigma^{(1)}(i\omega_n) + \sigma^{(2)}(i\omega_n) + \sigma^{(3)}(i\omega_n).$$
(19)

The part $\sigma^{(1)}(i\omega_n)$ contains only the contribution from the glassy order parameter q^{ab} , while $\sigma^{(2)}(i\omega_n)$ arises, once the gapless frequency-dependent field $D(\omega)$ that characterizes quantum dynamics is taken into account. The contribution $\sigma^{(3)}(i\omega_n)$ describes the role of the spatially-fluctuating part of the Q matrices in Eq. (17).

 $\sigma^{(1)}(i\omega_n)$ is given by

$$\sigma^{(1)}(i\omega_n) = \frac{16e^2}{\hbar\omega_n} \frac{4q_{\rm EA}\Delta_q}{3} \Pi(i\omega_n), \qquad (20)$$

where

$$\Pi(i\omega_n) = \int_0^\beta d\tau e^{i\omega_n\tau} [\beta\,\delta(\tau) - 1].$$
(21)

In the derivation above, we used the result $(1/n)\Sigma_{a,b}q^{ab}q^{ab} = (4/3)q_{\rm EA}\Delta_q$, where $\Delta_q = q_{\rm EA} - \int_0^1 q(s)ds$

 $=q_{\text{EA}}s_1/2$ is the broken ergodicity parameter, that vanishes linearly with temperature. Note, had we assumed that the vector potential entered in a time-translationally invariant manner, the factor of -1 in Eq. (21) would not be present. As a result, the conductivity would diverge at $\omega_n = 0$ as in a superconductor. In $\sigma^{(2)}(i\omega_n)$ we collect the terms that contain $D(\omega_n)$:

$$\sigma^{(2)}(i\omega_n) = \frac{16e^2}{\hbar\omega_n} \bigg(T\sum_{\omega_m} D^2(\omega_m) - T\sum_{\omega_m} D(\omega_m) D(\omega_m + \omega_n) - 2q_{\rm EA} D(\omega_n) \bigg).$$
(22)

The remaining term $\sigma^{(3)}(i\omega_n)$ arises from the spatially dependent part $\tilde{Q}^{ab}_{\mu\nu}(\mathbf{k},\omega_1,\omega_2)$ of the *Q* matrices. Writing the expression for the current (15) in two parts

$$\mathbf{J}_{1}(\mathbf{x},\tau) = -\frac{2e^{*}}{\hbar} \sum_{ab} \int \frac{d^{d}k}{(2\pi)^{d}} \frac{1}{\beta^{2}}$$
$$\times \sum_{\omega_{1},\omega_{2}} \mathbf{k} \left(\beta q^{ab} \delta_{\omega_{2},0} + \frac{1}{\beta} \sum_{\omega_{2}} D(\omega_{2}) \right)$$
$$\times [\tilde{Q}_{-}^{ab}(\mathbf{k},\omega_{1},\omega_{2})e^{i(\mathbf{k}\cdot\mathbf{x}-\omega_{1}\tau-\omega_{2}\tau)} + \text{c.c.}], \quad (23)$$

$$\mathbf{J}_{2}(\mathbf{x},\tau) = -\frac{2e^{*}}{\hbar} \sum_{ab} \beta \sum_{\alpha=\pm} \int \frac{d^{d}k_{1}}{(2\pi)^{d}} \frac{d^{d}k_{2}}{(2\pi)^{d}} \frac{1}{\beta^{3}}$$

$$\times \sum_{\omega_{1},\omega_{2},\omega_{3}} (\mathbf{k}_{1} + \mathbf{k}_{2}) \tilde{\mathcal{Q}}_{\alpha}^{ab}(\mathbf{k}_{1},\omega_{1},\omega_{3})$$

$$\times [\tilde{\mathcal{Q}}_{\alpha}^{ab}(\mathbf{k}_{2},\omega_{2},\omega_{3})]^{*} e^{i(\mathbf{k}_{2} - \mathbf{k}_{1}) \cdot \mathbf{x}} e^{i(\omega_{1} - \omega_{2})\tau},$$
(24)

we observe that the contribution from $J_1(\mathbf{x}, \tau)$ vanishes as a result of integration over $d^d(\mathbf{x}-\mathbf{x}')$. The remaining part leads to the result that

$$\sigma^{(3)}(i\omega_n) = \frac{4(e^*)^2}{\hbar\omega_n} \frac{\beta}{n} \sum_{a,b} \sum_{\alpha=+,-} \int \frac{d^d k}{(2\pi)^d} \frac{1}{\beta^2}$$

$$\times \sum_{\omega_1,\omega_2} \left[\mathcal{G}^{ab}_{\alpha}(\mathbf{k},\omega_1,\omega_2) - 4k_x^2 \Gamma^{ab}_{\alpha}(\mathbf{k},\omega_1,\omega_2;\omega_n) + \mathcal{G}^{ab}_{\alpha}(\mathbf{k},\omega_1,\omega_2) \mathcal{G}^{ab}_{\alpha}(\mathbf{k},\omega_1,\omega_2+\omega_n) \right].$$
(25)

In Eq. (25)

$$\mathcal{G}_{\pm}^{ab}(\mathbf{k},\omega_{1},\omega_{2}) = \langle \widetilde{\mathcal{Q}}_{\pm}^{ab}(\mathbf{k},\omega_{1},\omega_{2}) \widetilde{\mathcal{Q}}_{\pm}^{ab}(-\mathbf{k},-\omega_{1},-\omega_{2}) \rangle$$
$$= \frac{1}{4} \sum_{\mu,\nu=1,2} \mathcal{G}_{\mu\nu}^{ab}(\mathbf{k},\omega_{1},\omega_{2}), \qquad (26)$$

is the exact propagator for the fluctuations of the \tilde{Q} fields. The first term is the diamagnetic contribution, while the second is paramagnetic and can be formally represented by the standard bubble diagrams¹⁴ and $\Gamma_{\alpha}^{ab}(\mathbf{k},\omega_1,\omega_2;\omega_n)$ is the corresponding vertex function.

We discuss first the contribution $\sigma^{(1)}(\omega_n)$. The explicit frequency dependence of this part is given simply by the prefactor $\Pi(i\omega_n)/\omega_n$. Should a phase stiffness exist, this prefactor would be simply proportional to $1/\omega_n$, which when analytically continued would yield the standard electromagnetic response for the conductivity of a superconductor. However, this is not the case here. The integral in Eq. (21) is simply $\beta(1 - \delta_{\omega_n,0})$ effectively removing thus the divergence at zero frequency, unlike what would be the case had we assumed that the vector potential entered the action in a time-translationally invariant manner. Note that such an expression although not analytic at $\omega_n = 0$ does not violate causality because it is, nonetheless, analytic in either the upper or lower half planes. Hence, the O(2) quantum phase glass has a vanishing stiffness in the limit $\omega_n = 0$, which of course is the physically relevant regime for the dc conductivity. It is in this limit that explorations of all available minima are possible.

To see this result more systematically, we analytically continue $\Pi(i\omega_n)$ using a Hilbert transformation. The denominator of Eq. (20) can be analytically continued trivially, $i\omega_n \rightarrow \omega + i\eta$, where η is a positive infinitesimal. We write the numerator as

$$\Pi(i\omega_n) = \int_0^\beta d\tau e^{i\omega_n\tau} \Pi(\tau),$$

$$\Pi(\tau) = \beta \,\delta(\tau) - 1 \equiv \Pi_1(\tau) - \Pi_2(\tau). \tag{27}$$

Although $\Pi_1(\tau)$ is not an analytic function, we can construct its analytical continuation using the conformal invariance condition $\delta(\tau) = \delta(\tau + \beta)$. Performing the integration over the first term in Eq. (27), we obtain that $\Pi_1(\omega) = \beta$. Because $\Pi_2(\tau) = 1$ is an analytic function, we adopt the spectral representation

$$\Pi_2(\tau) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{e^{-\tau\epsilon} \Pi_2''(\epsilon) d\epsilon}{1 - e^{-\beta\epsilon}}$$
(28)

valid for Bose systems, where $\Pi_2(\epsilon) = \Pi'_2(\epsilon) + i \Pi''_2(\epsilon)$. This representation is most convenient for constructing the analytical continuation.¹⁵ Once we know $\Pi''_2(\epsilon)$, we can obtain both the real and the imaginary parts for real frequencies using the Hilbert transformation

$$\Pi_2(\omega) = \frac{1}{\pi} \int \frac{d\epsilon \Pi_2''(\epsilon)}{\epsilon - \omega - i0^+}.$$
(29)

Solving Eq. (28) with $\Pi_2(\tau) = 1$ yields

$$\Pi_2''(\epsilon) = 2\pi\delta(\epsilon)\sinh\frac{\epsilon\beta}{2} = \pi\beta\epsilon\delta(\epsilon).$$
(30)

The real part is determined by the principal value

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$$\Pi_{2}^{\prime}(\omega) = \mathbb{P} \int_{-\infty}^{\infty} \frac{\beta \epsilon \delta(\epsilon) d\epsilon}{\epsilon - \omega} = \beta f(\omega) = \beta \begin{cases} 1, & \omega = 0, \\ 0, & \omega \neq 0. \end{cases}$$

Obtained in this fashion, the real and imaginary parts of $\Pi_2(\omega)$ formally satisfy the Kramers-Kronig relations. However, both are not regular functions. Hence, it is more convenient to treat the real and imaginary parts of $\Pi_2(\omega)$ as limits of two analytic functions. For example, from the regular function

$$g(\omega) = \beta \left(\frac{\eta^2}{\eta^2 + \omega^2} + i \frac{\eta \omega}{\eta^2 + \omega^2} \right), \tag{31}$$

whose real and imaginary parts satisfy the Kramers-Kronig relations, we obtain the correct limit for $\Pi_2(\omega=0)=1$ simply from $g(\omega=0)=1$, and for $\omega \neq 0$ the limiting procedure, $\lim_{\eta\to 0} g(\omega) = \Pi_2(\omega\neq 0)=0$. As a result, the limits ω $=0, \eta \rightarrow 0$ and $\eta=0, \omega \rightarrow 0$ do not commute, a fact which must be considered when we construct the $\omega=0$ conductivity. The correct order of limits is $\eta \rightarrow 0, \omega=0$. Nonetheless, the advantage of writing $\Pi_2(\omega)$ in this fashion is that for any nonzero η , the real and imaginary parts of this $g(\omega)$ obey the Kramers-Kronig relations. Combining this representation with $\Pi_1(\omega)=1$ and $i\omega_n \rightarrow \omega + i\delta$, we obtain the analytically continued form for the frequency dependence of the conductivity

$$\begin{split} \frac{\Pi(i\omega_n)}{\omega_n} &\to \beta \Biggl[\frac{\omega^2(\eta + \delta)}{(\eta^2 + \omega^2)(\eta^2 + \omega^2)} + i \frac{\omega^3 - \eta \delta \omega}{(\eta^2 + \omega^2)(\delta^2 + \omega^2)} \Biggr] \\ &= \begin{cases} 0, & \omega = 0, \lim_{\delta \to 0, \eta \to 0}, \\ i\beta/\omega, & \omega \neq 0, \lim_{\delta \to 0, \eta \to 0}. \end{cases} \end{split}$$

Recall, the correct $\omega = 0$ limit is recovered by setting $\omega = 0$ and then taking the limit, $\eta \rightarrow 0$. We find then that the contribution of $\sigma^{(1)}(\omega)$ to the conductivity is purely imaginary. The absence of the real part and, as a result, a formal violation of the Kramers-Kronig relations here is tied to the presence of the nonanalytic function $\delta(\tau)$ in Eq. (21). Such nonanalyticity at $\omega = 0$ is permissible because the requirement of causality is analyticity in either the upper or lower half planes.

To evaluate the $\omega \rightarrow 0$ limit of $\sigma^{(2)}(\omega)$ we must analytically continue the difference of the first two terms in Eq. (22). Using Eq. (18) we obtain^{16,17} that

$$\sigma^{(2)}(\omega=0) = \frac{16e^2}{\hbar} \left(\frac{2q_{\rm EA}}{\kappa} + \frac{2}{\pi\kappa^2} \int_0^{\Lambda_\omega} z \coth\frac{z}{2T} dz \right),\tag{32}$$

and is some regular function of the infrared cutoff Λ_{ω} and temperature. We see that the contribution $\sigma^{(2)}(\omega=0)$ is non-critical and metallic.

Proceeding to the third term $\sigma^{(3)}(\omega)$, we first notice that the exact calculation of the propagator $\mathcal{G}^{ab}_{\mu\nu}(\mathbf{k},\omega_1,\omega_2)$, based on the action Eq. (10) is not possible. However, at the quantum critical point in the Gaussian approximation¹³

$$\mathcal{G}_{\mu\nu}^{ab}(\mathbf{k},\omega_{1},\omega_{2}) = \frac{1}{k^{2} + |\omega_{1}| + |\omega_{2}|} \equiv \mathcal{G}_{0}(\mathbf{k},\omega_{1},\omega_{2}),$$
(33)

and hence is indepedent of replica and spatial indices. Substitution of this simple replica-symmetric propagator into Eq. (25) leads to the zeroth-order result for $\sigma^{(3)}(\omega)$ as a result of the replica summation. Because the renormalization group equations for the coefficients in the action (10) lead to runaway to strong coupling for $d < d_c = 8$, it is not possible to analyze the behavior of $\sigma^{(3)}(\omega=0)$ for the relevant dimensionalities. However, the structure of Eq. (25) allows us to make the conclusion that the superconducting contribution of the type $\rho_s \delta(\omega)$ is not expected. This can be proven formally by integrating by parts the diamagnetic term and employing the Ward identity. After the analytical continuation $\omega_n \rightarrow -i\omega$, we expand the ensuing expression over ω . We obtain that the zero-frequency conductivity obeys the scaling form

$$\sigma^{(3)}(\omega=0) = \frac{e^2}{\hbar} \left(\frac{T}{\hbar}\right)^{d-2} F\left(\frac{q_{\rm EA}}{T}\right),\tag{34}$$

albeit the precise form of the function F(x) and, hence, the corresponding temperature dependence cannot be determined.

We have obtained an important result that there is no real contribution to the conductivity proportional to $\rho_{c}\delta(\omega)$. The vanishing of the stiffness is tied to the nature of the vector potential coupling to the glassy order parameter. The vector potential couples in a nontime translationally invariant manner to the spin glass order parameter. If, however, the system explores only one of the myriad of configurations in the glassy landscape, a stiffness appears in agreement with the work of Kotliar et al.9 However, certainly within a single configuration, the origin of time is irrelevant. In this case the time translationally invariant coupling of the vector potential would be appropriate. But this is not the most general case. Quantum mechanically tunneling to all minima is permitted. In this case, the stiffness vanishes in agreement with the result^{7,9} on the Heisenberg spin glass that the spin stiffness is a transient and hence should vanish once tunneling between all minima is present. This result is robust and expected to hold beyond the mean-field theory. For example, the vanishing of the spin stiffness follows directly from the form of the propagator of the spatially varying part of the spin glass order parameter. From Eq. (33), we see that this propagator is strictly diffusive. Coupled with the fact the propagator at the mean-field level in the spin glass phase is independent of momentum, we deduce that the massless mode is determined entirely from the spatially fluctuating part of the order parameter and hence given by $\omega = ik^2$. The fact that the real part of the dispersion relationship vanishes implies that the spin stiffness identically vanishes. This argument reinforces our previous conclusion as well as that of others^{7,8} that the vanishing of the spin stiffness is tied to the emergence of a diffusive mode.

III. BOSONIC CONDUCTIVITY: REPLICA SYMMETRY BREAKING

Now we generalize our earlier result for the bosonic conductivity. Such a contribution arises only in the case of nonzero mean, $J_0 \neq 0$. In this case an ordered phase exists which in the O(2) case is a superconductor. Hence, in the presence of nonzero mean, a new order parameter

$$\Psi^{a}_{\mu}(\mathbf{k},\tau) = \left\langle S^{a}_{\mu}(\mathbf{k},\tau) \right\rangle \tag{35}$$

which is determined by the expectation value of the rotor spin. On the spin glass side of the phase diagram, the bosonic excitations of the superconductor develop a mass, m which is equivalent to the inverse correlation length for phase coherence. In the presence of bosonic excitations, the free energy contains the additional terms

$$\begin{split} \Delta \mathcal{F}[\Psi, Q] &= \sum_{a,\mu,k,\omega_n} (k^2 + \omega_n^2 + m^2) |\Psi_{\mu}^a(\mathbf{k},\omega_n)|^2 \\ &- \frac{1}{\kappa t} \int d^d x \int d\tau_1 d\tau_2 \sum_{a,b,\mu,\nu} \Psi_{\mu}^a(x,\tau_1) \\ &\times [\Psi_{\nu}^b(x,\tau_2)]^* Q_{\mu\nu}^{ab}(x,\tau_1,\tau_2) \\ &+ \frac{U}{2} \int d\tau \sum_{a,\mu} \{\Psi_{\mu}^a(x,\tau) [\Psi_{\mu}^a(x,\tau)]^*\}^4. \end{split}$$
(36)

At the Gaussian level, with the mean-field spin glass ansatz (17), the effective Gaussian propagator for the bosonic degrees of freedom has the form

$$\mathcal{F}_{\text{Gauss}} = \sum_{a, \mathbf{k}, \omega_n} (k^2 + \omega_n^2 + \eta |\omega_n| + m^2) |\Psi^a(\mathbf{k}, \omega_n)|^2 - \beta q \sum_{a, b, \mathbf{k}, \omega_n} \delta_{\omega_n, 0} \Psi^a(\mathbf{k}, \omega_n) [\Psi^b(\mathbf{k}, \omega_n)]^*.$$
(37)

As we have pointed out previously, the term proportional to q^{ab} in F_{Gauss} cannot be rewritten as an effective mass term because this term explicitly couples ψ fields with different replica indices. In the case of replica symmetry, that is, $q^{ab} = q_0$ for all a and b, we have shown that the resultant conductivity is nonzero and given by

$$\sigma_{\rm bos}(\omega = 0, T \to 0) = \frac{4}{3} \frac{e^2 \eta q_0}{hm^4}$$
(38)

which smoothly crosses over to $\sigma = \infty$ in the superconducting state (m=0). That the bosonic contribution to the conductivity should be nonzero is immediately obvious from the $|\omega|$ term in the action. This term arises entirely due to the glass degrees of freedom that naturally provide for dissipation to generate a metallic state.

We now generalize this result to include replica symmetry breaking. Application of the Kubo formula in this case results in a conductivity

$$\sigma(i\omega_n) = \frac{2(e^*)^2}{n\hbar\omega_n} T \sum_{a,b,\omega_m} \int \frac{d^2k}{(2\pi)^2} [G_{ab}^{(0)}(\mathbf{k},\omega_m) \delta_{ab} - 2k_x^2 G_{ab}^{(0)}(\mathbf{k},\omega_m) G_{ab}^{(0)}(\mathbf{k},\omega_m+\omega_n)], \quad (39)$$

that depends entirely on the Gaussian propagator for the ψ fields. To evaluate this quantity, we need to invert Eq. (37). This calculation is difficult to perform for the general type of RSB. However, it can be readily done using the rules developed by Mezard and Parisi¹⁸ for inverting an ultrametric matrix having a one-step RSB:

$$q(s) = \begin{cases} q_0, & s < s_c, \\ q_1, & s_c < s < 1. \end{cases}$$

To apply the inversion formula detailed in Appendix II of Ref. 12, it is expedient to make the following definitions:

$$g = \frac{1}{k^2 + \eta |\omega_n| + m^2},$$

$$\tilde{g} = \frac{1}{(k^2 + \eta |\omega_n| + \Sigma_m)(k^2 + \eta |\omega_n| + m^2)},$$

$$\Sigma_m = m^2 + \beta \Sigma_1, \quad \Sigma_1 = s_c(q_1 - q_0). \quad (40)$$

Application of the inversion formula¹⁸ results in the diagonal

$$\tilde{G} = g + \beta \delta_{\omega_{n,0}} \Sigma_1 \tilde{g} \frac{1 - s_c}{s_c} + \beta q_0 \delta_{\omega_{n,0}} g^2$$
(41)

and the off-diagonal elements

$$G(s) = \beta q_0 \delta_{\omega_{n,0}} + \beta \delta_{\omega_{n,0}} \Sigma_1 \tilde{g} \frac{\theta(s-s_c)}{s_c}$$
(42)

of the propagator. In this representation of the Parisi matrices on the interval [0,1], the replica indices are absent. Nonetheless, a well-defined formula¹⁸

$$\frac{1}{n}\operatorname{Tr} AB = \tilde{a}\tilde{b} - \int_0^1 ds a(s)b(s)$$
(43)

exists for taking the trace of a product of two ultrametric matrices A and B, where \tilde{a} and \tilde{b} are the diagonal elements of A and B, respectively, and a and b are the corresponding off-diagonal elements in the continuous representation. Let us consider here only a simple case of the weak RSB $\beta \Sigma_1 \ll m^2$, assuming that $s_c \sim T$. Substitution of Eqs. (41) and (42) into Eq. (39), and expanding over $\beta \Sigma_1/m^2$ results in the following correction to the static conductivity due to the replica symmetry breaking

$$\delta\sigma^{\text{RSB}} = \frac{2(e*)^2}{h} \eta \Sigma_1 \frac{1-s_c}{s_c} \int_0^\infty \frac{x \, dx}{(x+m^2)^4}$$
$$= \frac{4}{3} \frac{e^2}{hm^4} \eta (q_1 - q_0) (1-s_c). \tag{44}$$

Combining this with Eq. (38), we obtain

$$\sigma_{\text{bosons}}^{\text{tot}} = \frac{4}{3} \frac{e^2 \eta}{hm^4} [q_1 + (q_0 - q_1)s_c]$$
(45)

as our total contribution for the bosonic conductivity. If $s_c = 1$, we recover our previous replica symmetric result. For the quantum O(2) spin glass, however, $s_c \propto T$, and hence, the correction with s_c vanishes at T=0. Setting $s_c=0$ requires that $q(s)=q_1$. Hence, replica symmetry breaking adds a simple benign constant to the conductivity which smoothly crosses over to the replica symmetric result. While the matrix inversion beyond one-step replica symmetry breaking is arduous, we suspect that in this case as well replica symmetry breaking does not lead to any spurious contribution to the conductivity. Our primary reason for this belief is the fact that quantum mechanically replica symmetry breaking vanishes in the $T \rightarrow 0$ limit.

IV. SUMMARY

We have considered here two separate questions: (1) does the O(2) vector spin glass have a nonvanishing phase stiffness and (2) what is the role of replica symmetry breaking in the bosonic contribution to the conductivity. If the answer to the first question were yes, then the answer to the second would be irrelevant as the overall conductivity would be infinite. As we have demonstrated clearly, the spin glass order parameter does not provide a superconducting contribution to the conductivity at mean field and beyond. Our calculation of the phase stiffness seems to be the first based on a direct coupling of the vector potential to the spin glass order parameter which does not assume time translational invariance at the beginning. The physical mechanism underlying the vanishing of the spin stiffness appears to be the exploration of all configuration minima as a result of quantum tunneling. In addition, we have found that replica symmetry breaking provides a small correction to bosonic conductivity. Hence, the bosonic metallic state we have found here is robust and represents a clear example of a metallic state in the presence of disorder in two dimensions.

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