Long-range dynamics related to magnetic impurities in the two-dimensional Heisenberg antiferromagnet

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We consider a magnetic impurity in the two-dimensional Heisenberg antifferomagnet with long-range antiferromagnetic order. The present work employs three different methods: finite cluster considerations, spin-wave perturbation theory, and semiclassical nonlinear σ model. At low temperature the impurity magnetic susceptibility has a Curie term ($\propto 1/T$) and a logarithmic correction [$\propto \ln(T)$]. We calculate the Curie term and the logarithmic correction and derive related Ward identity for the impurity-spin-wave vertex.

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I. INTRODUCTION

The problem of magnetic impurities interacting with a system of strongly correlated electrons has attracted much interest recently, mainly due to the experimental discoveries of the high-T_c superconductors and new heavy fermion compounds. In the field of the high-T_c materials, the parent compounds are known to be two-dimensional (2D) antiferromagnetic (AFM) Mott-Hubbard insulators based on CuO_2 planes which are driven to a superconducting state by doping (e.g., with holes).[1,2] Even though the holes can hop, thus destroying the AFM long-range order and causing the development of superconducting pairing, the extreme limit of static holes also has a physical relevance. Systems with static holes have been also realized experimentally in cuprates.[3,4]

Several theoretical studies have addressed isolated static holes[5–9] and added spins[9–11] in 2D Heisenberg antiferromagnets with long-range AFM order. A singular logarithmic frequency behavior of the perpendicular magnetic susceptibility at zero temperature has been found in Ref. 5, see also a discussion in Ref. 12. A very interesting problem is a magnetic impurity in 2D Heisenberg antiferromagnet at O(3) quantum critical point.[8,13,14] However, this problem is out of the scope of the present paper. The low-temperature behavior of the impurity static magnetic susceptibility in a gapped system is trivial, it obeys the simple Curie law, $\delta\chi=\Omega(\Omega+1)/3T$, where Ω is the impurity spin. However, for 2D systems which possess the long-range AFM order at zero temperature, the excitation spectrum is gapless due to Goldstone spin waves and the Curie law is not obvious. A very interesting prediction[8] for such a regime is the classical Curie law, $\delta\chi=\Omega^2/3T$. The behavior is classical because of the alignment of the impurity moment with the local Neel order. This behavior has been recently confirmed in Monte Carlo simulations for 2D $S=1/2$ Heisenberg antiferromagnetic clusters with magnetic impurity.[9] Moreover, in these simulations a logarithmic correction, $\propto \ln(T)$, to the classical Curie law has been found. Both the classical Curie law and the logarithmic correction are related to the nontrivial long-range dynamics in the system. In the present work we calculate the logarithmic correction using two different methods: (i) spin-wave perturbation theory; (ii) semiclassical nonlinear σ model. In the leading $1/S$ approximation both methods give the same result. However, the results must be identical in all orders in $1/S$ and hence the comparison allows us to derive the Ward identity for the impurity-spin-wave vertex. The value of the logarithmic correction to the magnetic susceptibility is in agreement with Ref. 9.

A crossover from quantum to classical Curie law for a finite AFM cluster with impurity is discussed in Sec. II. In Sec. III we derive the impurity susceptibility using the spin-wave perturbation theory, and in Sec. IV we obtain the same result using the semiclassical nonlinear σ model and derive the Ward identity.

II. CURIE TERM

The Hamiltonian of the system under consideration is

$$H=H_0+H_{int}+H_B,$$

$$H_0=J\sum_{\langle ij\rangle}\mathbf{S}_i\cdot\mathbf{S}_j,$$

$$H_{int}=J_\perp \boldsymbol{S}_0\cdot\boldsymbol{\Omega},$$

$$H_B=-\mathbf{B}\cdot\left(\boldsymbol{\Omega}+\sum_i \boldsymbol{S}_i\right) \quad (1)$$

where $\boldsymbol{S}_i$ is spin 1/2 at the site i of square lattice with antiferromagnetic interaction ($J>0$), $\boldsymbol{\Omega}$ is the impurity spin coupled to the lattice spin at site 0, and $\boldsymbol{B}$ is magnetic field. To be specific we will assume that $J_\perp>0$, but all results are in the end independent of the sign of $J_\perp$. Consider an $L\times L$ cluster ($L\gg 1$ is even) described by the Hamiltonian H_0, so there are no impurities for the beginning. The energy spectrum of the system is known very well.[15–17] The spin of the ground state is 0, and the lowest excitations are described by rotational spectrum of the solid top (diatomic molecule with zero projection of spin on axis of the molecule, $K=0$),

$$E_{\mathcal{J}}=\frac{\mathcal{J}(\mathcal{J}+1)}{2I}, \quad (2)$$

where $\mathcal{J}=0,1,2\ldots$ is the spin of the state ($\mathcal{J}=0$ corresponds to the ground state), $I=L^2\chi_\perp$ is the moment of inertia of the top, and $\chi_\perp\approx 0.066/J$ is the perpendicular magnetic susceptibility.[18,19] The spectrum (2) is valid as soon as the rotation is solid, i.e., internal degrees of freedom of the top are not excited. The first internal excitation is the spin wave with wavelength $\lambda=L$ (periodic boundary condition). The energy of this excitation is

$$\Delta_{sw}=2\pi c/L, \quad c\approx\sqrt{2}J. \tag{3}$$

There are eight degenerate spin-wave excitations: $S_z=\pm 1$, x and y directions, and two excitations (cos and sin) in each direction. If we consider $T\ll\Delta_{sw}$ then only rotations (2) are important. A more accurate criterion for solid rotation is: $8\exp(-\Delta_{sw}/T)\ll 1$, i.e.,

$$T\ll T_{sw}\approx\frac{\Delta_{sw}}{\ln(8)}. \tag{4}$$

In this temperature regime magnetic susceptibility of the cluster is determined by the spectrum (2).

$$\chi_0=\frac{\partial}{\partial B}\frac{\sum_{\mathcal{J},J_z}\mathcal{J}_z e^{-(E_{\mathcal{J}}-\mathcal{J}_zB)/T}}{\sum_{\mathcal{J},J_z}e^{-(E_{\mathcal{J}}-\mathcal{J}_zB)/T}}=\frac{1}{T}\frac{\sum_{\mathcal{J},J_z}\mathcal{J}_z^2 e^{-\mathcal{J}(\mathcal{J}+1)/2IT}}{\sum_{\mathcal{J},J_z}e^{-\mathcal{J}(\mathcal{J}+1)/2IT}}$$

$$=\frac{1}{3T}\frac{\sum_{\mathcal{J}}\mathcal{J}(\mathcal{J}+1)(2\mathcal{J}+1)e^{-\mathcal{J}(\mathcal{J}+1)/2IT}}{\sum_{\mathcal{J}}(2\mathcal{J}+1)e^{-\mathcal{J}(\mathcal{J}+1)/2IT}}$$

$$=\frac{1}{3T}\frac{\sum_{\mathcal{J}}[(\mathcal{J}+1/2)^2-1/4](\mathcal{J}+1/2)e^{-(\mathcal{J}+1/2)^2/2IT}}{\sum_{\mathcal{J}}(\mathcal{J}+1/2)e^{-(\mathcal{J}+1/2)^2/2IT}}$$

$$=\frac{1}{3T}\left(\frac{\sum_{\mathcal{J}}(\mathcal{J}+1/2)^3e^{-(\mathcal{J}+1/2)^2/2IT}}{\sum_{\mathcal{J}}(\mathcal{J}+1/2)e^{-(\mathcal{J}+1/2)^2/2IT}}-\frac{1}{4}\right), \tag{5}$$

where the summation over $\mathcal{J}_z$ is performed using $\Sigma\,\mathcal{J}_z^2=\mathcal{J}(\mathcal{J}+1)(2\mathcal{J}+1)/3$, and the summation over $\mathcal{J}$ should be performed in limits $0\leqslant\mathcal{J}\leqslant\infty$. Strictly speaking total spin of the cluster is limited, $\mathcal{J}\leqslant\mathcal{J}_{max}=L^2/2$, however, because of the exponential convergence of summations in Eq. (5), and because of condition (4) we can replace $\mathcal{J}_{max}\to\infty$. If $T\ll T_{rot}=1/I$ the susceptibility (5) is zero. For the case $T_{rot}\ll T\ll T_{sw}$ let us first consider the denominator in Eq. (5). Expanding the denominator we find

$$D(IT)=\sum_{\mathcal{J}=0}^{\infty}(\mathcal{J}+1/2)e^{-(\mathcal{J}+1/2)^2/2IT}=IT+\frac{1}{24}+\frac{A}{IT}+\cdots. \tag{6}$$

The first two terms of the expansion have been found by reducing summation to integration using the Simpson method. The method is not sufficient to determine the constant A and we have found it using direct numerical summation, $A\approx 0.003\,645$. The numerator in Eq. (5) is

$$N(IT)=\sum_{\mathcal{J}=0}^{\infty}(\mathcal{J}+1/2)^3e^{-(\mathcal{J}+1/2)^2/2IT}=-\frac{\partial D(IT)}{\partial(1/2IT)}$$
$$=2(IT)^2-2A+\cdots. \tag{7}$$

Using Eqs. (7) and (6) we find from (5)

$$\chi_0=\frac{1}{3T}\left(2IT-\frac{1}{3}+\frac{2}{IT}\left[\frac{1}{24^2}-2A\right]+\cdots\right)$$
$$=\frac{2}{3}L^2\chi_\perp-\frac{1}{9T}\left(1-\frac{T_{rot}}{T}\left[\frac{1}{96}-12A\right]+\cdots\right). \tag{8}$$

The expansion is in integer powers of T_{rot}/T. We would like to point out that in the thermodynamic limit, $L\to\infty$, only the first term in Eq. (8)survives, $\chi_0\to\frac{2}{3}L^2\chi_\perp$. This agrees with the result known from the spin-wave theory, $\chi_0\to\frac{2}{3}L^2\chi_\perp[1+T/(2\pi\rho_s)]$, see, e.g., Ref. 17. We have not obtained the linear in temperature term, because in the thermodynamic limit the inequality (4) means that $T=0$.

Now let us put an impurity with spin $\mathbf{\Omega}$ which interacts with the cluster via H_{int}, see Eq. (1). The excitation spectrum of the cluster with impurity is slightly different from Eq. (2). Now this is a symmetric top with spin projection on axis of the top $K=\Omega$. This is like a diatomic molecule with uncompensated electron spin and strong spin-axis interaction. The rotational spectrum of such a top is[20]

$$E_{\mathcal{J}}=\frac{\mathcal{J}(\mathcal{J}+1)-2\Omega^2+\langle\mathbf{\Omega}^2\rangle}{2I}, \tag{9}$$

where $\mathcal{J}=\Omega,\Omega+1,\Omega+2,\ldots$ is total spin of the cluster, and $\langle\mathbf{\Omega}^2\rangle$ is the average value of $\mathbf{\Omega}^2$ in the intrinsic reference frame. Similar to the previous case the spin waves are not excited as soon as the inequality (4) is valid. The magnetic susceptibility of the cluster χ_1 is given by the same Eq. (5), the only difference is that summation over $\mathcal{J}$ is performed not from 0 to ∞, but from Ω to ∞. If $T\ll T_{rot}$ the cluster susceptibility is $\chi_1=\Omega(\Omega+1)/3T$. If $T_{rot}\ll T\ll T_{sw}$ evaluation of the denominator in Eq. (5) gives

$$D(IT)=\sum_{\mathcal{J}=\Omega}^{\infty}(\mathcal{J}+1/2)e^{-(\mathcal{J}+1/2)^2/2IT}=IT+\left(\frac{1}{24}-\frac{\Omega^2}{2}\right)$$
$$+\frac{1}{IT}\left(\frac{\Omega^4}{8}-\frac{\Omega^2}{16}+A\right)+\cdots. \tag{10}$$

All the terms of the expansion except of A have been found by reducing summation to integration using the Simpson method. The A term is exactly the same as in Eq. (6), $A\approx 0.003\,645$. Calculating the numerator similar to Eq. (7) and substituting all the expressions to Eq. (5) we find the following formula for the cluster susceptibility:

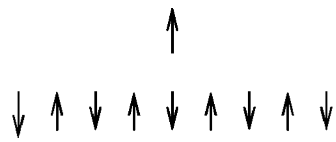

FIG. 1. Schematic picture of an AFM cluster with impurity

$$\chi_1=\frac{1}{3T}\left(2IT-\frac{1}{3}+\Omega^2+\frac{2}{IT}\left[\frac{1}{24^2}-2A+\frac{\Omega^2}{12}\right]+\cdots\right)$$
$$=\frac{2}{3}L^2\chi_\perp-\frac{1}{9T}\left([1-3\Omega^2]-\frac{T_{rot}}{T^2}\left[\frac{1}{96}-12A+\frac{\Omega^2}{2}\right]+\cdots\right). \qquad (11)$$

Hence the impurity susceptibility defined as $\chi_1-\chi_0$ reads

$$\chi_{imp}=\chi_1-\chi_0=\frac{\Omega^2}{3T}\left(1+\frac{T_{rot}}{2T}+\cdots\right). \qquad (12)$$

The expansion goes in integer powers of T_{rot}/T. The leading term in Eq. (12) agrees with Refs. 8 and 9. Plots of susceptibilities are presented in Ref. 9.

It is interesting to note that if there are n impurities with spin Ω on the same sublattice, then in Eq. (12) one shall replace $\Omega\rightarrow n\Omega$. The impurities are not independent because the cluster is rigid. In the thermodynamic limit it means that all the impurities within the correlation length are not independent.

III. LOGARITHMIC CORRECTION TO SUSCEPTIBILITY, THE SPIN-WAVE DERIVATION

The above consideration is valid for the case when the inequality (4) is valid and hence there are no *real* spin-wave excitations. However virtual spin-wave excitations are always there. In the intrinsic reference frame the cluster looks like the picture in Fig. 1.

The interaction Hamiltonian H_{int}, Eq. (1), can be rewritten in terms of spin-wave operators a and b (for a review of the spin-wave theory see, e.g., Ref. 1),

$$H_{int}=J_\perp\boldsymbol{\Omega}\cdot\boldsymbol{S}_0\rightarrow\frac{1}{2}J_\perp(\Omega_+S_{0-}+\Omega_-S_{0+})$$
$$=\frac{1}{2}J_\perp(\Omega_+b_0+\Omega_-b_0^\dagger)$$
$$=\frac{1}{2}J_\perp\sqrt{\frac{2}{L^2}}\sum_q(\Omega_+b_q+\Omega_-b_q^\dagger)$$
$$=\frac{1}{\sqrt{2L^2}}J_\perp\sum_q(\Omega_+[u_q\beta_q+v_q\alpha_{-q}^\dagger]+\Omega_-[u_q\beta_q^\dagger+v_q\alpha_{-q}])$$
$$\rightarrow\frac{1}{\sqrt{2L^2}}J_\perp\sum_q Z_\Gamma(\Omega_+[u_q\beta_q+v_q\alpha_{-q}^\dagger]+\Omega_-[u_q\beta_q^\dagger+v_q\alpha_{-q}]). \qquad (13)$$

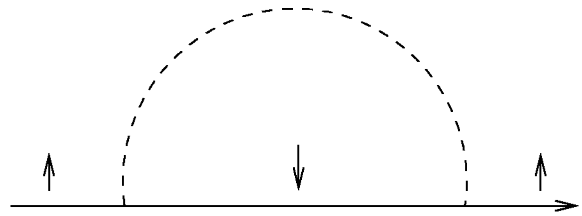

FIG. 2. One-loop correction to the impurity energy. The solid line shows the impurity and the dashed line shows virtual spin wave.

Here $\beta_q^\dagger$ and $\alpha_q^\dagger$ are creation operators for spin waves with spin projection $S_z=\pm1$, respectively, u_q and v_q are Bogoliubov parameters,

$$u_q^2=\frac{J}{\omega_q^{(0)}}+\frac{1}{2},\quad v_q^2=\frac{J}{\omega_q^{(0)}}-\frac{1}{2}, \qquad (14)$$

and $\omega_q^{(0)}=2J\sqrt{1-\gamma_q^2}\rightarrow\sqrt{2}Jq$ is the spin-wave dispersion in the leading $1/S$ approximation. In Eq. (13) we assume that $S=1/2$. In Eq. (13) the impurity-spin-wave vertex is derived in the leading $1/S$ approximation. However, at the last step we have introduced the vertex renormalization factor Z_Γ that takes into account all higher $1/S$ corrections to the vertex. Generally speaking Z_Γ depends on q, but here we consider only the small q limit, $Z_\Gamma=Z_\Gamma(q=0)$.

To use perturbation theory we will assume that $J_\perp\ll J$. The one-loop correction to the impurity energy is given by the diagram in Fig. 2. The diagram is infrared convergent because it contains the denominator, $\Delta\epsilon=\epsilon_\uparrow-\epsilon_\downarrow=J_\perp\langle S_z\rangle$, related to the flip of the impurity spin. Here $\langle S_z\rangle=\frac{1}{2}Z_s\approx0.307$ is the staggered magnetization of the lattice, and $Z_s\approx0.61$ is the renormalization factor for the staggered magnetization.[1,18,19]

The interaction of the impurity with perpendicular magnetic field is of the form

$$H_B=-B\Omega_x=-\frac{1}{2}B(\Omega_++\Omega_-). \qquad (15)$$

The leading contribution to the impurity energy related to B is given by the diagram in Fig. 3. The corresponding formula reads

$$\delta\epsilon_0=-\frac{(B/2)^2}{\Delta\epsilon}. \qquad (16)$$

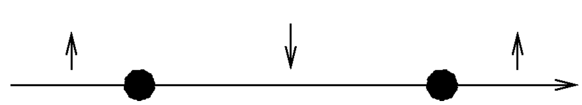

FIG. 3. Leading contribution to the impurity energy related to magnetic field. The dot denotes interaction with magnetic field.

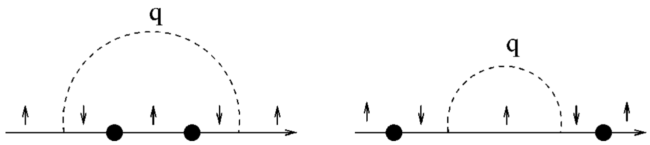

FIG. 4. One-loop spin-wave corrections to the impurity energy related to the magnetic field. The dot denotes interaction with the field.

The contribution is finite because $\Delta\epsilon$ is finite. Now let us look at one-loop corrections to $\delta\epsilon_0$ shown in Fig. 4. These corrections are infrared divergent because they contain the energy denominator without spin flip, $\epsilon_\uparrow-\epsilon_\uparrow$. The diagrams in Fig. 4 do not contain intermediate states which coincide with the initial state. In this case the Schroedinger perturbation theory formula for the fourth order energy correction is trivial[20]

$$\delta\epsilon_1=\sum_{n,m,k\neq 0}\frac{\langle 0|V|n\rangle\langle n|V|m\rangle\langle m|V|k\rangle\langle k|V|0\rangle}{(\epsilon_0-\epsilon_n)(\epsilon_0-\epsilon_m)(\epsilon_0-\epsilon_k)}, \quad (17)$$

where $V=H_{int}+H_B$ is the perturbation, see also Eqs. (13) and (15). Separating in Eq. (17) the terms corresponding to Fig. 4 we find

$$\delta\epsilon_1=-(\sqrt{2\Omega}B/2)^2\sum_q\left(\frac{\left(\frac{\sqrt{2\Omega}J_\perp Z_\Gamma}{\sqrt{2L^2}}u_q\right)^2}{(\Delta\epsilon+\omega_q)^2\omega_q}+\frac{\left(\frac{\sqrt{2\Omega}J_\perp Z_\Gamma}{\sqrt{2L^2}}v_q\right)^2}{(\Delta\epsilon)^2\omega_q}\right). \quad (18)$$

Here $\omega_q\to cq=Z_c\sqrt{2}Jq$, where $Z_c\approx 1.17$ is the spin-wave velocity renormalization due to higher $1/S$ corrections.[1,18,19] Keeping only divergent terms we find from Eq. (18) the following expression for the susceptibility:

$$\delta\chi_\perp\to\frac{J_\perp^2Z_\Gamma^2\Omega^2}{L^2(\Delta\epsilon)^2}\sum_q\frac{u_q^2+v_q^2}{\omega_q}=\frac{J_\perp^2Z_\Gamma^2\Omega^2}{Z_c(\Delta\epsilon)^2}\int\frac{1}{q^2}\frac{d^2q}{(2\pi)^2}$$
$$=\frac{Z_\Gamma^2\Omega^2}{2\pi Z_cJ\langle S_z\rangle^2}\int\frac{dq}{q}. \quad (19)$$

It is interesting that Eq. (18) is independent of $J_\perp$. We have to put some lower limit in the integral in Eq. (19). Note that the integration over q is equivalent to integration over frequency $\omega=cq$. At low temperature, when inequality (4) is valid, we have to substitute the spin-wave gap (3) as the lower frequency limit, $\omega_{min}\sim cq_{min}\sim\Delta_{sw}=2\pi c/L$. This gives a temperature-independent contribution to the impurity susceptibility,

$$\delta\chi_\perp=\frac{Z_\Gamma^2\Omega^2}{2\pi Z_cJ\langle S_z\rangle^2}\ln(L)=\frac{2\Omega^2Z_\Gamma^2}{\pi Z_cZ_s^2}\ln(L). \quad (20)$$

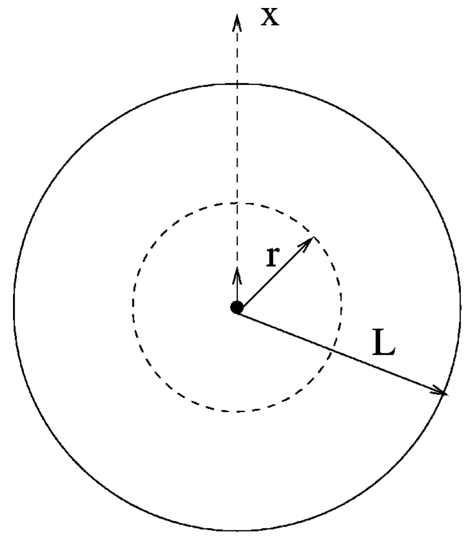

FIG. 5. An impurity (dot) in the center of a disc of radius L. The disc carries a field $\boldsymbol{n}$ described by the nonlinear σ model. Due to magnetic field $B=B_x$ the impurity spin is tilted in the x direction.

If $J\gg T\gg T_{sw}$ then the minimal frequency in Eq. (19) is of the order of minimal Matsubara frequency, $\omega_{min}\sim cq_{min}\sim T$. Hence

$$\delta\chi_\perp=\frac{2\Omega^2Z_\Gamma^2}{\pi Z_cZ_s^2J}\ln(J/T). \quad (21)$$

In the leading $1/S$ approximation one has to set $Z_\Gamma=Z_c=Z_s=1$, hence $\delta\chi_\perp\to 2\Omega^2/\pi J\ln(\cdots)$. This corresponds to the low-frequency susceptibility derived in Ref. 5. We have calculated the perpendicular susceptibility (20) and (21) in the intrinsic reference frame. The isotropic susceptibility is related to $\delta\chi_\perp$ by the standard relation, $\delta\chi=\frac{2}{3}\delta\chi_\perp$.

IV. NONLINEAR σ-MODEL DERIVATION OF THE LOG CORRECTION, AND WARD IDENTITY FOR THE IMPURITY SPIN-WAVE VERTEX

An alternative derivation of Eqs. (20) and (21) is based on the σ model. Let us consider a field $\mathbf{n}$, $|\mathbf{n}|=1$, defined on a disc of radius L. An impurity with spin Ω is in the center of the disc. The impurity spin is directed along the z axis (perpendicular to the plane) and due to the magnetic field $B=B_x$ it is tilted by angle θ in the x direction, see Fig. 5. The energy of the medium is

$$E_\sigma=\frac{1}{2}\rho_s\int(\nabla\mathbf{n})^2d^2r, \quad (22)$$

where $\rho_s=Z_\rho(J/4)$ is the spin stiffness. Here $J/4$ is the leading $1/S$ value for the stiffness, and $Z_\rho\approx 0.72$ is the renormalization factor due to higher $1/S$ corrections.[1,18,19] The field is of the form $\mathbf{n}\approx(n_x,0,1)$. Due to Eq. (22) the field n_x obeys the usual Poisson equation, the solution of the equation is

$$n_x(r)=a\ln(L/r). \quad (23)$$

To find the constant a we have to remember that $n_x(r\sim 1)=\theta$, where θ is tilting angle of the impurity, and 1=*lattice spacing*. Therefore $a=\theta/\ln(L)$. Substituting Eq. (23) in Eq. (22) we find the elastic energy $E_\sigma=\pi\rho_s\theta^2/\ln(L)$. The total energy related to the impurity consists of the magnetic energy and the elastic energy,

$$E=-B\ \Omega\ \theta+\frac{\pi\rho_s\theta^2}{\ln(L)}. \tag{24}$$

Minimizing it with respect to θ we find $\theta=B\Omega\ln(L)/(2\pi\rho_s)$, magnetic moment $M=\Omega\theta$, and the magnetic susceptibility,

$$\delta\chi_\perp=\frac{\Omega^2}{2\pi\rho_s}\ln(L)\rightarrow\frac{2\Omega^2}{\pi Z_\rho J}\ln(L). \tag{25}$$

At the final step we have substituted $\rho_s=JZ_\rho/4$. In the leading $1/S$ approximation, $Z_\Gamma=Z_c=Z_s=Z_\rho=1$, hence Eq. (25) agrees with the spin-wave results (20) and (21). Moreover, a comparison of these equations gives a nontrivial Ward identity relating renormalization factors for the spin-wave vertex Z_Γ, the spin-wave velocity Z_c, the staggered magnetization Z_s, and the spin stiffness Z_ρ,

$$\frac{Z_\Gamma^2}{Z_cZ_s^2}=\frac{1}{Z_\rho}. \tag{26}$$

This gives the previously unknown value of Z_Γ,

$$Z_\Gamma=\sqrt{\frac{Z_cZ_s^2}{Z_\rho}}\approx 0.76. \tag{27}$$

Similar to Eq. (19), (20), and (21) typical scales (or typical momenta) are equivalent to typical frequencies, $cq\sim\omega$. Therefore, if $J\gg T>T_{sw}$ one has to substitute $\ln(J/T)$ instead of $\ln(L)$ in Eq. (25). If the external magnetic field has a nonzero frequency ω, and $\omega>T,T_{sw}$, then $\ln(L)\rightarrow\ln(J/\omega)$. Equations (20), (21), and (25) are in agreement with Ref. 5 and with recent results.[9,21]

The spin-wave approach in Sec. III assumes that $J_\perp\ll J$. There are numerous two-loop diagrams which are proportional to $[J_\perp/(2\pi)^2J^2]\ln(L)$ and even to $[J_\perp/(2\pi)^2J^2]\ln^2(L)$. Some of the diagrams which contain the logarithm squared are shown in Fig. 6. The calculation of all the diagrams is quite an involved problem. On the other hand, the semiclassical derivation based on the σ model is independent of $J_\perp/J$. The only assumption in the derivation is that the impurity magnetic moment is localized in the vicinity of the impurity. To check this assumption we have calculated the magnetic cloud around the impurity using the spin-wave theory. We have found that density of the induced magnetization $\delta\mu_z$ drops down faster than $1/r^2$, $\delta\mu_z(r)<1/r^2$. Therefore the magnetic moment of the cloud $\mu=\int\delta\mu_z(r)d^2r$ is convergent at large distances, and it means that the above assumption is valid. This implies that *all* higher order in $J_\perp$ infrared divergent diagrams must cancel out. This is a highly unusual situation and it would be interesting to check the cancellation by a direct calculation.

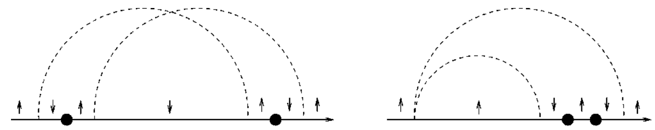

FIG. 6. Two-loop double logarithmic contributions to $\delta\chi_\perp$ proportional to $J_\perp/J^2$.

V. CONCLUSIONS

We have analyzed the excitation spectrum of a finite antiferromagnetic cluster with magnetic impurity, and considered a crossover between quantum and classical Curie law for the impurity magnetic susceptibility. We have also derived a logarithmic correction to the impurity magnetic susceptibility. Depending on the parameters this can be a logarithm of system size, temperature, or frequency of the external magnetic field. Using the results for the logarithmic correction we have derived the Ward identity for the impurity-spin-wave vertex.

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