Crossover from random-exchange to random-field critical behavior in Ising models

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We compute the crossover exponent ϕ describing the crossover from the random-exchange to the random-field critical behavior in Ising systems. For this purpose, we consider the field-theoretical approach based on the replica method, and perform a six-loop calculation in the framework of a fixed-dimension expansion. The crossover from random-exchange to random-field critical behavior has been observed in dilute anisotropic antiferromagnets, such as $Fe_xZn_{1-x}F_2$ and $Mn_xZn_{1-x}F_2$, when applying an external magnetic field. Our result $\phi=1.42(2)$ for the crossover exponent is in good agreement with the available experimental estimates.

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The most studied experimental realizations of random-field Ising systems are dilute anisotropic antiferromagnets in a uniform magnetic field applied along the spin ordering axis.[1–3] A simple lattice model is provided by the Hamiltonian

$$\mathcal{H}=J\sum_{\langle ij\rangle} \rho_i\rho_j s_i s_j - H\sum_i \rho_i s_i , \tag{1}$$

where $J>0$, the first sum extends over all nearest-neighbor sites, $s_i=\pm 1$ are the spin variables, and ρ_i are uncorrelated quenched random variables, which are equal to one with probability p (the spin concentration) and zero with probability $1-p$ (the impurity concentration). In the absence of an external field, i.e., $H=0$, the critical behavior of dilute Ising systems (above the percolation point of the spins) belongs to the universality class of the random-exchange Ising model (REIM), which differs from the standard Ising model (see, e.g., Refs. 4,5 for recent reviews). The applied uniform field H gives rise to a different critical behavior corresponding to the universality class of the random-field Ising model (RFIM) (see, e.g., Refs. 6–8 for recent reviews on RFIM).

For $t\equiv(T-T_N)/T_N\to 0$, T_N being the Néel temperature in zero field, and $H\to 0$ the singular part of the free energy can be written as (Ref. 1)

$$\mathcal{F}=|u_t|^{2-\alpha} f(H^2|u_t|^{-\phi}), \tag{2}$$

where $u_t\approx t+a_1H^2+a_2t^2$ is the scaling field associated with the temperature, α is the specific-heat exponent of the REIM, $f(x)$ is a scaling function, and ϕ is the corresponding crossover exponent. As a consequence of the crossover scaling (2), the critical temperature in the presence of the external field H is given by

$$T_c(H)\approx T_N + cH^{2/\phi} - aH^2, \tag{3}$$

for sufficiently small H. Experimental measurements on dilute antiferromagnets yielded rather precise estimates: $\phi=1.42(3)$ obtained for $Fe_xZn_{1-x}F_2$,[9] $\phi=1.43(3)$ for $Mn_xZn_{1-x}F_2$,[10] and $\phi=1.41(5)$ for $Fe_xMg_{1-x}Cl_2$.[11]

On the theoretical side, a computation of ϕ was presented in Ref. 12 using the field-theoretical ϵ expansion. We recall that in the REIM the ϵ expansion is actually an expansion in powers of $\sqrt{\epsilon}$. The exponent ϕ was computed to $O(\epsilon)$,[12] obtaining

$$\phi/\gamma = 1 + 0.16823\epsilon^{1/2} - 0.22666\epsilon + O(\epsilon^{3/2}), \tag{4}$$

where γ is the REIM susceptibility exponent. Apart from showing that $\phi\neq\gamma$, where $\gamma\approx 1.34$ (see, e.g., Refs. 13–15), this series does not yield a reliable estimate of ϕ for three-dimensional systems. This is generically true for the $\sqrt{\epsilon}$ expansion of any critical exponent of the REIM.[4,5]

In this paper we determine the crossover exponent ϕ using an alternative field-theoretical method based on a fixed-dimension expansion in powers of appropriate zero-momentum quartic couplings, which we compute to six loops. As we shall see, the analysis of the series provides a quite precise estimate of ϕ,

$$\phi = 1.42(2), \tag{5}$$

in good agreement with experiments. This result was already anticipated in Ref. 16.

The field-theoretical approach is based on an effective Landau-Ginzburg-Wilson φ^4 Hamiltonian that can be obtained by using the replica method,[17] i.e.,

$$\mathcal{H}_{\varphi^4} = \int d^3x \left\{ \frac{1}{2}\sum_{i=1}^{N} [(\partial_\mu\varphi_i)^2 + r\varphi_i^2] + \frac{1}{4!}\sum_{i,j=1}^{N} (u_0+v_0\delta_{ij})\varphi_i^2\varphi_j^2 \right\}, \tag{6}$$

where φ_i is an N-component field. The critical behavior of the REIM is expected to be described by the stable fixed point of the Hamiltonian $\mathcal{H}_{\varphi^4}$ in the limit $N\to 0$ for $u_0<0$. The most precise field-theoretical results for the critical exponents have been obtained by analyzing the fixed-dimension expansion in powers of the zero-momentum quartic couplings u,v related to u_0,v_0 [i.e., $u=Z_u u_0/m$ and $v=Z_v v_0/m$, where $Z_{u,v}=1+O(u,v)$], which have been computed to six loops.[15,18] We refer the reader to Ref. 15 for notations and definitions.

In the presence of a spatially uncorrelated random field $h(x)$ with zero average and variance h^2, one may still apply the replica method. Averaging the replicated random-field term

$$\int d^3x h(x) \sum_i \varphi_i(x) \tag{7}$$

over the Gaussian distribution of the field, one obtains a new term proportional to

$$h^2 \int d^3x \sum_{i,j} \varphi_i(x) \varphi_j(x) \tag{8}$$

that must be added to the Hamiltonian $\mathcal{H}_{\varphi^4}$.[12] This term causes the crossover from the REIM to the RFIM. Thus, the REIM-to-RFIM crossover exponent is determined by the $N \to 0$ limit of the renormalization-group (RG) dimension y_T of the quadratic operator

$$T_{ij} = \varphi_i \varphi_j, \quad i \neq j. \tag{9}$$

In order to evaluate y_T and the corresponding crossover exponent $\phi = y_T \nu$, we define a related RG function Z_T from the one-particle irreducible two-point function $\Gamma_T^{(2)}$ with an insertion of the operator T_{ij}, i.e.,

$$\Gamma_T^{(2)}(0)_{ij,kl} = Z_T^{-1} A_{ijkl}, \tag{10}$$

where $A_{ijkl} = \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}$ with $i \neq j$, so that $Z_T(0) = 1$. Then, we compute the RG function

$$\eta_T(u,v) = \left.\frac{\partial \ln Z_T}{\partial \ln m}\right|_{u_0,v_0} = \beta_u \frac{\partial \ln Z_T}{\partial u} + \beta_v \frac{\partial \ln Z_T}{\partial v}, \tag{11}$$

where β_u and β_v are the β functions. The exponent η_T is given by the value of $\eta_T(u,v)$ at $u = u^*$ and $v = v^*$, where (u^*, v^*) is the REIM stable fixed point. Finally, the REIM-to-RFIM crossover exponent is obtained by using the RG scaling relation

$$\phi = (2 + \eta_T - \eta)\nu = \gamma + \eta_T \nu, \tag{12}$$

where γ, ν, and η are REIM critical exponents.

We computed the function $\Gamma^{(T,2)}(0)$ to six loops. The calculation is rather cumbersome, since it requires the evaluation of 563 Feynman diagrams. We handled it with a symbolic manipulation program, which generates the diagrams and computes the symmetry and group factors of each of them. We used the numerical results compiled in Ref. 19 for the integrals associated with each diagram. The resulting six-loop series of $\eta_T(u,v)$ is

$$\begin{aligned}
\eta_T(\bar{u},\bar{v}) = & -\tfrac{1}{4}\bar{u} + \tfrac{1}{16}\bar{u}^2 + \tfrac{1}{18}\bar{u}\bar{v} - 0.0357\,673\bar{u}^3 \\
& - 0.048\,324\bar{u}^2\bar{v} - 0.0042\,1548\bar{u}\bar{v}^2 \\
& + 0.034\,374\,8\bar{u}^4 + 0.076\,261\,6\bar{u}^3\bar{v} \\
& + 0.041\,694\,3\bar{u}^2\bar{v}^2 + 0.008\,961\,16\bar{u}\bar{v}^3 \\
& - 0.040\,895\,8\bar{u}^5 - 0.121\,377\bar{u}^4\bar{v} - 0.104\,778\bar{u}^3\bar{v}^2 \\
& - 0.037\,371\,2\bar{u}^2\bar{v}^3 - 0.005\,270\,15\bar{u}\bar{v}^4 \\
& + 0.059\,704\,8\bar{u}^6 + 0.227\,662\bar{u}^5\bar{v} + 0.287\,108\bar{u}^4\bar{v}^2 \\
& + 0.172\,31\bar{u}^3\bar{v}^3 + 0.055\,212\,4\bar{u}^2\bar{v}^4 \\
& + 0.007\,596\,54\bar{u}\bar{v}^5 + \cdots,
\end{aligned} \tag{13}$$

where $\bar{u}$ and $\bar{v}$ are the rescaled couplings $\bar{u} = u/(6\pi)$ and $\bar{v} = 3v/(16\pi)$, and the dots indicate higher-order terms. In our analysis we also considered the series $\phi(\bar{u},\bar{v})$ corresponding to ϕ, which can be obtained by using Eq. (12) and the series of the RG functions $\gamma(\bar{u},\bar{v})$ and $\nu(\bar{u},\bar{v})$ corresponding to the critical exponents γ and ν (cf. Ref. 15).

In order to obtain an estimate of η_T and ϕ, the corresponding six-loop series must be resummed and then evaluated at the fixed-point values, u^* and v^*. Although the perturbative expansion is not Borel summable,[20] various resummation schemes have been proposed and employed, obtaining rather reliable results for the critical exponents (see, e.g., Refs. 4,5). We analyzed $\eta_T(\bar{u},\bar{v})$ and $\phi(\bar{u},\bar{v})$ by using the resummation methods outlined in Ref. 15 and the estimates $u^* = -18.6(3)$ and $v^* = 43.3(2)$ obtained by Monte Carlo simulations in Ref. 13, which turn out to be more precise than those obtained from the zeros of the β functions computed to six loops.

We obtained $\eta_T = 0.095(30)$ from the analysis of $\eta_T(\bar{u},\bar{v})$, $\phi = 1.43(3)$ from $\phi(\bar{u},\bar{v})$, and $\phi = 1.42(2)$ from $1/\phi(\bar{u},\bar{v})$. The errors are related to the spread of the results obtained by using different resummation methods and different resummation parameters within each method (see Ref. 15 for details). Using the available estimates of the critical exponents (e.g., $\gamma = 1.342(6)$, $\nu = 0.683(3)$, and $\eta = 0.035(2)$ from Monte Carlo simulations,[13,14] $\gamma = 1.330(17)$, $\nu = 0.678(10)$, and $\eta = 0.030(3)$ from the analysis of the six-loop field-theoretical expansions[15]), the above-reported result for η_T and the RG scaling relation (12) give $\phi = 1.41(2)$. From these results we arrive at the final estimate reported in Eq. (5), which is in good agreement with the available experimental results obtained for various uniaxial anisotropic antiferromagnets.

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