

Current fluctuations and electron-electron interactions in coherent conductors

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We analyze current fluctuations in mesoscopic coherent conductors in the presence of electron-electron interactions. In a wide range of parameters we obtain explicit universal dependences of the current noise on temperature, voltage, and frequency. We demonstrate that Coulomb interaction decreases the Nyquist noise. In this case the interaction correction to the noise spectrum is governed by the combination $\sum_n T_n(T_n - 1)$, where T_n is the transmission of the n th conducting mode. The effect of electron-electron interactions on the shot noise is more complicated. At sufficiently large voltages we recover two different interaction corrections entering with opposite signs. The net result is proportional to $\sum_n T_n(T_n - 1)(1 - 2T_n)$; i.e., Coulomb interaction decreases the shot noise at low transmissions and increases it at high transmissions.

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I. INTRODUCTION

Recent advances in nanotechnology enable detailed investigations of a variety of quantum effects in mesoscopic conductors. These investigations are of primary interest because of the fundamental importance of such effects as well due to the rapidly growing number of potential applications. A great deal of information is usually obtained from studying electron transport. Additional and complementary information can be extracted from investigations of fluctuation effects, such as shot noise.¹ For instance, it was demonstrated²⁻⁴ that the power spectrum of the shot noise in coherent mesoscopic conductors is expressed in terms of the parameter

$$\beta = \frac{\sum_n T_n(1 - T_n)}{\sum_n T_n}. \quad (1)$$

Here and below T_n stands for the transmission of the n th conducting channel of a coherent conductor. Thus, since transport measurements only allow one to determine the combination

$$\frac{1}{R} = \frac{2e^2}{h} \sum_n T_n, \quad (2)$$

studies of the shot noise provide additional valuable information about the transmission distribution of conducting modes.

The above results apply to the situations when the interaction between electrons can be neglected. In the presence of electron-electron interactions the Landauer conductance (2) and the I - V curve are modified in a nontrivial way. Recently it was shown⁵ that the I - V curve of a (comparatively short) coherent conductor with arbitrary transmission distribution T_n in the presence of interactions can be expressed in the form

$$R \frac{dI}{dV} = 1 - \beta f(V, T), \quad (3)$$

where f is a *universal* function evaluated in Ref. 5. This result holds in the limit of large conductances $R \ll R_q = h/e^2$ or, otherwise, at sufficiently high temperatures and/or voltages. It demonstrates that the magnitude of the interaction correction is controlled by *the same* parameter β , Eq. (1), which is already well known in the theory of shot noise. Physically this result can easily be understood since both phenomena are related to the discrete nature of the electron charge. Hence, there exists a direct link between shot noise and interaction effects in mesoscopic conductors. In the case of a single-channel conductor a similar observation was also made in Ref. 6.

It is obvious that not only the I - V curve (3) but also shot noise as well as higher moments of the current operator should be affected by electron-electron interactions. This paper is devoted to a detailed investigation of current fluctuations in mesoscopic coherent conductors in the presence of electron-electron interactions. Previously various aspects of this problem have been studied for a particular case of tunnel junctions in the Coulomb blockade regime; see, e.g., Refs. 7-10. The effect of interactions on the shot noise in two-dimensional (2D) diffusive conductors at sufficiently high temperatures was recently addressed in Ref. 11.

Here we will employ a model of a coherent conductor.^{5,12,13} Within this model we will demonstrate that interactions lead to two different corrections to the shot noise spectrum. One of these corrections scales with the parameter β , Eq. (1). This correction is *negative* for any $T_n < 1$, similarly to that found in Eq. (3) for the I - V curve. It describes (partial) suppression of the current noise due to Coulomb blockade. In addition to this correction we shall find another one, which is proportional to the parameter

$$\gamma = \frac{\sum_n T_n^2(1 - T_n)}{\sum_n T_n}. \quad (4)$$

This second correction is *positive*; i.e., for any $T_n < 1$ it leads to relative enhancement of the shot noise. The latter correc-

tion turns out to be important only at voltages exceeding both frequency and temperature and is negligible otherwise. Thus, at sufficiently high voltages two interaction corrections—negative and positive—compete, for $\gamma > \beta/2$ the second one wins and, hence, in this case overall enhancement of the shot noise by interactions is predicted.

It is important to emphasize that the above results are obtained under the assumption that the conductor is *shorter* than any inelastic relaxation length in our problem. In other words, it is assumed throughout the paper that inelastic relaxation may occur in the reservoirs but not inside the coherent conductor. In this sense our model is quite different from that employed, e.g., in Refs. 14 and 15, where diffusive conductors much *longer* than the inelastic relaxation length were considered. In the latter case strong energy relaxation allows one to establish local equilibrium described by an effective coordinate-dependent electron temperature. Such a description is not possible within our model.

We also note that our results are applicable to physical situations in which one can neglect the energy dependence of transmission values T_n . In the absence of inelastic relaxation inside the conductor this assumption embraces a large variety of (comparatively short) disordered structures. On the other hand, our approach is not sufficient for scatterers with transmissions which sharply depend on energy, such as, e.g., quantum dots in the resonant tunneling regime. A generalization of our path integral technique to the case of energy-dependent transmission amplitudes is worked out elsewhere.¹⁶ We finally remark that the effect of Coulomb interaction on shot noise in conductors with energy-dependent scattering was addressed in a numerical work.¹⁷

Our paper is organized as follows. In Sec. II we will highlight our key results. A detailed derivation of these results is then performed in Sec. III. Our main conclusions are briefly summarized in Sec. IV. Most of the technical details, such as the derivation of our effective action as well as a few other issues, are presented in Appendixes A, B, and C.

II. MODEL AND KEY RESULTS

Similarly to Refs. 5, 12, and 13 we will consider a coherent scatterer between two big reservoirs. As we have already pointed out, the scatterer will be described by an arbitrary distribution of energy-independent transmissions T_n of its conducting modes, and the corresponding Landauer conductance $1/R$ is defined in Eq. (2). The scatterer is assumed to be shorter than both dephasing and inelastic lengths. The scatterer region is characterized by an effective capacitance C and the corresponding charging energy $E_C = e^2/2C$. We further assume that the scatterer is connected to the voltage source V_x via a linear external impedance Z_S (see Fig. 1). Here we restrict ourselves to a simple case $Z_S(\omega) = R_S$. If necessary, generalization of our calculation to arbitrary $Z_S(\omega)$ can be performed in a straightforward manner. Finally, we will assume that the typical traversal time τ_{tr} —i.e., the time it takes electron to propagate inside the scatterer region—is shorter than the RC time of our system, $\tau_{tr} \lesssim RR_S C / (R + R_S)$.

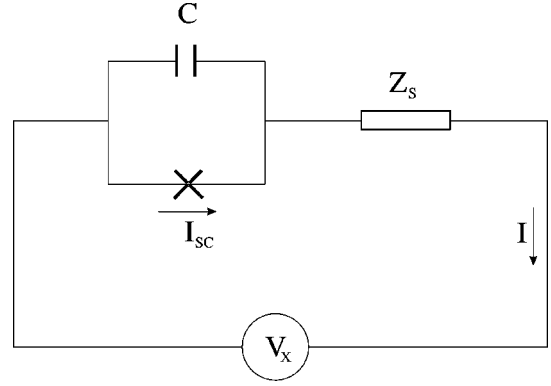


FIG. 1. The circuit under consideration. The scatterer (denoted by a cross) has a capacitance C and is connected to the voltage source V_x via an impedance Z_S .

In what follows we will investigate the current noise and evaluate the correlation function

$$S(t, t') = \frac{1}{2} \langle \hat{I}(t) \hat{I}(t') + \hat{I}(t') \hat{I}(t) \rangle - \langle \hat{I} \rangle^2, \quad (5)$$

where \hat{I} is the current operator in the circuit of Fig. 1. This correlator can be expressed in the form

$$S(t, t') = S^{ni}(t, t') + \delta S(t, t'), \quad (6)$$

where S^{ni} is the noninteracting contribution to the current noise²⁻⁴ and δS is the correction due to electron-electron interactions inside the scatterer. This correction will be evaluated in the most interesting “metallic” limit

$$g_0 = g + g_S \gg 1. \quad (7)$$

Here we introduced dimensionless conductances of the scatterer and shunt: respectively, $g = R_q/R$ and $g_S = R_q/R_S$. Equation (7) implies that at least one of these two dimensionless conductances is required to be much larger than unity.

Quite obviously, the latter correlator (5) should depend on both R and R_S . We also introduce another correlator $\tilde{S}(t, t')$ defined by the same Eq. (5) in which one should substitute the current operator across the scatterer $\hat{I} \rightarrow \hat{I}_{sc}$. The two correlators $S(t, t')$ and $\tilde{S}(t, t')$ are not independent. With the aid of the current conservation condition and performing the Fourier transformation with respect to $t - t'$, one easily finds the relation

$$\tilde{S}_\omega = \frac{R_S^2}{R_0^2} (1 + \omega^2 R_0^2 C^2) S_\omega - \frac{R_S}{R^2} (1 + \omega^2 R^2 C^2) \omega \coth \frac{\omega}{2T}, \quad (8)$$

where $R_0 = RR_S / (R + R_S)$. The second term is due the noise produced by the external resistor R_S which has to be subtracted in order to arrive at \tilde{S}_ω .

In general also the correlator \tilde{S}_ω depends on both R and R_S . However, in the limit $R_S \gg R$ the dependence on the shunt resistance is weak and can be neglected. In this case

the interaction correction to the current noise spectrum, $\delta\tilde{S}_\omega$, depends only on the properties of the scatterer. Below we will present our key results for $\delta\tilde{S}_\omega$ only in this limit. More general expressions can be found in Sec. III.

Let us define the average voltage across the scatterer $V = V_x R / (R + R_S)$ and consider first the limit of relatively small voltages. At sufficiently large temperatures and/or frequencies we find

$$\delta\tilde{S}_\omega = -\frac{2\beta E_C}{3R}, \quad \text{if } T \gg gE_C, |eV|, |\omega|, \quad (9)$$

$$\delta\tilde{S}_\omega = -\frac{\beta E_C}{R}, \quad \text{if } |\omega| \gg T, gE_C, |eV|. \quad (10)$$

At lower temperatures and frequencies we obtain

$$\delta\tilde{S}_\omega = -\frac{4\beta T}{R_q} \ln \frac{gE_C}{T}, \quad \text{if } |\omega|, |eV| \ll T \ll gE_C, \quad (11)$$

$$\delta\tilde{S}_\omega = -\frac{2\beta|\omega|}{R_q} \ln \frac{gE_C}{|\omega|}, \quad \text{if } T, |eV| \ll |\omega| \ll gE_C. \quad (12)$$

These results apply as long as either temperature or frequency exceeds an exponentially small parameter $gE_C \exp(-g/2)$. For even smaller frequencies and temperatures²⁰ we get

$$\delta\tilde{S}_\omega = -\frac{\beta\omega}{R} \coth \frac{\omega}{2T}. \quad (13)$$

Note that the above expressions could also be anticipated from the fluctuation-dissipation theorem (FDT) combined with the results.⁵ Indeed, in the limit of low voltages the current noise is described by the standard Nyquist formula. Hence, in order to satisfy the FDT one should simply substitute the effective conductance (3) into this formula. In this way one gets the interaction correction $\delta\tilde{S}_\omega$ proportional to βf . For instance, in the low-frequency limit one finds⁵ $f(0, T) \simeq E_C/3T$ for $T \gg gE_C$, $f(0, T) \simeq (2/g) \ln(gE_C/T)$ for $\exp(-g/2) \ll T/gE_C \ll 1$ and $f(0, T) \simeq 1$ for $T < gE_C \times \exp(-g/2)$. Combining these expressions with the FDT one immediately reproduces Eqs. (9), (11), and (13).

It is worth stressing that here we evaluate the current-current correlation functions directly and *do not* use the results⁵ together with the FDT. However, it is satisfactory to observe that the FDT is explicitly maintained in our calculation and the results derived here are fully consistent with those of Ref. 5.

Now let us turn to the case of relatively large voltages V where the shot noise becomes important. As was already announced, in this case the correction to the noise power spectrum is proportional to the parameter

$$\beta - 2\gamma = \frac{\sum_n T_n (1 - T_n) (1 - 2T_n)}{\sum_n T_n}. \quad (14)$$

In particular we obtain

$$\delta\tilde{S}_\omega = -\frac{2(\beta - 2\gamma)|eV|}{R_q} \ln \frac{gE_C}{|eV|}, \quad (15)$$

if $T, |\omega| \ll |eV| \ll gE_C$,

$$\delta\tilde{S}_\omega = -\frac{(\beta - 2\gamma)E_C}{R}, \quad \text{if } |eV| \gg T, gE_C, |\omega|. \quad (16)$$

We note that this correction can be either negative or positive depending on the relation between the parameters β and γ . Thus, in contrast to the limit of low voltages (Nyquist noise), one *cannot* conclude that shot noise is always reduced by interactions. This reduction occurs only for conductors with relatively low transmissions $\beta > 2\gamma$, while for systems with higher transmissions the net effect of the electron-electron interaction enhances the shot noise. In the important case of diffusive conductors one has $\beta = 1/3$, $\gamma = 2/15$ and, hence,

$$\beta - 2\gamma = \frac{1}{15}.$$

In this case the shot noise is reduced by interactions.

The above results have a transparent physical interpretation. At low voltages the power spectrum of the Nyquist noise is proportional to the system conductance $\propto \sum_n T_n$. Since in the presence of interactions the conductance acquires a correction proportional to β , the interaction correction to the Nyquist noise should scale with the same parameter (1). On the other hand, shot noise is determined by the combination $\sum_n (T_n - T_n^2)$. Accordingly, the interaction correction to the shot noise power should consist of two contributions. One of them comes from $\sum_n T_n$ and is again proportional to β . Another contribution originates from the interaction correction to $\sum_n T_n^2$ which turns out to scale as 2γ . Since these two corrections enter with opposite signs we immediately arrive at the combination (14).

We also point out that the third cumulant of the current operator for noninteracting electrons is known¹⁸ to be proportional to the parameters (1) and (14), respectively, at low and high voltages (for recent results related to the third cumulant see also Ref. 19). Following the same arguments as above we can *conjecture* that the interaction correction to the third cumulant should scale as $\beta - 2\gamma$ at low voltages, while in the limit of large voltages one can expect that this correction is governed by the combination $\beta - 6\gamma + 6\delta$, where

$$\delta = \frac{\sum_n T_n^3 (1 - T_n)}{\sum_n T_n}. \quad (17)$$

This conjecture can also be generalized to higher cumulants of the current operator.

We would like to emphasize that—although the above conjecture seems intuitively appealing—it should still be verified by means of a rigorous calculation which is beyond the scope of the present paper. In the next section we will concentrate on the current noise and will provide a detailed derivation of the results presented above.

III. EFFECTIVE ACTION AND CURRENT NOISE

Similarly to Ref. 5 we will use the effective action technique in order to evaluate the current-current correlator for the system depicted in Fig. 1. It is convenient to introduce the quantum phase variable φ which is proportional to the integral of the fluctuating voltage (see Appendix A). We will proceed within the Keldysh formalism and introduce two phase variables $\varphi_{1,2}$ related to the two branches of the Keldysh contour. Defining $\varphi^+ = (\varphi_1 + \varphi_2)/2$ and $\varphi^- = \varphi_1 - \varphi_2$ one can denote the overall phase jumps across the scatterer as $\varphi^+ + eVt$ and φ^- . Correspondingly, the phase jumps across the Ohmic shunt are $(eV_x - eV)t - \varphi^+$ and $-\varphi^-$. The symmetric current-current correlation function (5) can be expressed as follows:

$$\begin{aligned} & \frac{1}{2} \langle \hat{I}(t) \hat{I}(t') + \hat{I}(t') \hat{I}(t) \rangle \\ &= -e^2 \int \mathcal{D}\varphi^\pm \frac{\delta^2}{\delta\varphi_S^-(t) \delta\varphi_S^-(t')} e^{iS_{\text{tot}}[\varphi^\pm]}, \end{aligned} \quad (18)$$

see Appendix A for further discussion. By φ_S^\pm we denote the phase jumps over the Ohmic shunt. The variational derivatives in Eq. (18) act on the shunt part of the action. Here $S_{\text{tot}}[\varphi^\pm]$ is the total action of our system,

$$S_{\text{tot}}[\varphi^\pm] = S[\varphi^\pm] + S_S[\varphi^\pm], \quad (19)$$

where the term

$$\begin{aligned} iS_S[\varphi^\pm] &= \frac{i}{e^2 R_S} \int_0^\infty dt \varphi^-(t) \left(\frac{eV_x R_S}{R + R_S} - \dot{\varphi}^+(t) \right) \\ &\quad - \frac{1}{2e^2 R_S} \int_0^\infty dt_1 \int_0^\infty dt_2 \alpha(t_1 - t_2) \varphi^-(t_1) \varphi^-(t_2) \end{aligned} \quad (20)$$

comes from the shunt, $S[\varphi^\pm]$ is the scatterer action, and

$$\alpha(t - t') = -\frac{1}{\pi} \left(\frac{\pi T}{\sinh[\pi T(t - t')]} \right)^2. \quad (21)$$

A detailed derivation of the action $S[\varphi^\pm]$ is carried out in Appendixes A, B, and C. The main idea of this derivation is to expand $S[\varphi^\pm]$ in powers of φ^- , keeping the full nonlinearity of the corresponding terms in φ^+ . This procedure is just the quasiclassical approximation for the phase variable. It is parametrically justified under the condition (7). In Ref. 5 the action $S[\varphi^\pm]$ was evaluated up to the second order in

φ^- . This is sufficient to derive the current-voltage characteristics of the scatterer. However, in order to describe the current noise it is necessary to expand the action $S[\varphi^\pm]$ further and to retain all terms up to the third order in φ^- :

$$S[\varphi^\pm] = S^{(1)} + S^{(2)} + S^{(3)}. \quad (22)$$

This expansion is analyzed in Appendix C. We will now use these results and explicitly evaluate the current-current correlator (5).

A. Contribution of first- and second-order terms

Let us first restrict our attention to the contribution of the first- and second-order terms in Eq. (22). They read

$$\begin{aligned} & iS^{(1)}[\varphi^\pm] + iS^{(2)}[\varphi^\pm] \\ &= -\frac{i}{e^2} \int_0^\infty dt \varphi^-(t) \left[C \ddot{\varphi}^+(t) + \frac{1}{R} [\dot{\varphi}^+(t) + eV] \right] \\ &\quad - \frac{1}{2e^2 R} \int_0^\infty dt_1 \int_0^\infty dt_2 \alpha(t_1 - t_2) \\ &\quad \times \varphi^-(t_1) \varphi^-(t_2) \{ 1 - \beta + \beta \cos[eV(t_1 - t_2)] \\ &\quad + \varphi^+(t_1) - \varphi^+(t_2) \}. \end{aligned}$$

Employing Eq. (18) we obtain

$$\frac{1}{2} \langle \hat{I}(t) \hat{I}(t') + \hat{I}(t') \hat{I}(t) \rangle = \frac{\alpha(t - t')}{R_S} + e^2 \langle K(t) K(t') \rangle, \quad (23)$$

where we defined

$$K(t) = \frac{1}{e^2 R_S} \left[eV_x - eV - \dot{\varphi}^+ + i \int_0^\infty d\tilde{t} \alpha(t - \tilde{t}) \varphi^-(\tilde{t}) \right]. \quad (24)$$

Angular brackets in Eq. (23) imply averaging with the path integral:

$$\begin{aligned} \langle \dots \rangle &= \int \mathcal{D}\varphi^\pm(\dots) \exp(iS^{(1)}[\varphi^\pm] + iS^{(2)}[\varphi^\pm] \\ &\quad + iS_S[\varphi^\pm]). \end{aligned}$$

Rewriting the correlator $\langle KK \rangle$ as

$$\begin{aligned} \langle K(t) K(t') \rangle &= -\lim_{\eta \rightarrow 0} \int \mathcal{D}\varphi^\pm \frac{\delta^2}{\delta\eta(t) \delta\eta(t')} \\ &\quad \times \exp \left\{ i \sum_{i=1,2} S^{(i)}[\varphi^\pm] + iS_S[\varphi^\pm] \right. \\ &\quad \left. + i \int_0^\infty d\tilde{t} \eta(\tilde{t}) K(\tilde{t}) \right\} \end{aligned} \quad (25)$$

and performing a shift of $\varphi_\omega^- \rightarrow \varphi_\omega^- + \eta_\omega R_0 / R_S (1 - i\omega R_0 C)$, we obtain the expression for the Fourier-transformed noise spectrum (5):

$$\begin{aligned} \mathcal{S}_\omega = & \omega \coth \frac{\omega}{2T} \left\{ \operatorname{Re} \frac{1}{Z(\omega)} - \frac{\beta}{R\kappa\Omega} \right\} \\ & + \frac{\beta \{ \alpha(t-t') \langle \cos[eV(t-t') + \varphi^+(t) - \varphi^+(t')] \rangle \}_\omega}{R\kappa\Omega} \\ & + \frac{\beta^2}{e^2 R^2 \kappa \Omega} [\langle H(t) \rangle^2 - \langle H(t)H(t') \rangle]_\omega, \end{aligned} \quad (26)$$

where

$$\begin{aligned} H(t) = & \int_0^\infty d\tilde{t} \alpha(t-\tilde{t}) \varphi^-(\tilde{t}) \\ & \times \{ 1 - \cos[eV(t-\tilde{t}) + \varphi^+(t) - \varphi^+(\tilde{t})] \}. \end{aligned} \quad (27)$$

In Eq. (26) we also introduced the notation

$$\begin{aligned} Z(\omega) = & R_S + \frac{1}{R^{-1} - i\omega C}, \quad \Omega = 1 + \omega^2 R_0^2 C^2, \\ \kappa = & \frac{(R + R_S)^2}{R^2}. \end{aligned} \quad (28)$$

Making use of the relation (8), we arrive at the correlator $\tilde{\mathcal{S}}_\omega$,

$$\begin{aligned} \tilde{\mathcal{S}}_\omega = & \frac{1}{R} \left((1-\beta)\omega \coth \frac{\omega}{2T} + \beta \{ \alpha(t-t') \langle \cos[eV(t-t') \right. \\ & \left. + \varphi^+(t) - \varphi^+(t')] \rangle \}_\omega + \dots \right), \end{aligned} \quad (29)$$

where the ellipsis stands for the terms containing $[\langle H(t) \rangle^2 - \langle H(t)H(t') \rangle]_\omega$ in Eq. (26). We also note that exactly the same results can be obtained from the quasiclassical Langevin equation approach.^{22,23}

Within our analysis interaction effects are described by the terms containing the fluctuating variable φ^+ . If one formally sets this variable equal to zero, from Eq. (26) one immediately recovers the noninteracting result^{2,4}

$$\tilde{\mathcal{S}}_\omega^{\text{ni}} = (1-\beta) \frac{\omega}{R} \coth \frac{\omega}{2T} + \frac{\beta}{2R} \sum_{\pm} (\omega \pm eV) \coth \frac{\omega \pm eV}{2T}. \quad (30)$$

Taking the phase fluctuations into account we arrive at the expression for the interaction correction to Eq. (30). However, the corresponding expression turns out to be incomplete in two respects. First, one of the terms does not satisfy FDT. Second, the correction to Eq. (30) obtained in this way scales with the parameter β in both limits of small and large voltages. While in the former limit (Nyquist noise) this result is understandable and consistent with Ref. 5, at large voltages (shot noise) one also expects an extra contribution. Its existence can be anticipated because the shot noise is governed by the combination $\sum_n T_n (1 - T_n)$ and not simply by $\sum_n T_n$ as the Nyquist noise; see also our discussion in Sec. II.

Both these problems are remedied by taking into account the third order in φ^- contribution to the effective action. This will be demonstrated in the next subsection.

B. Corrections due to third-order terms

Following the analysis in Appendix C we identify two different contributions to the third-order term

$$S^{(3)} = S_\beta^{(3)} + S_\gamma^{(3)}.$$

The first contribution has the form

$$iS_\beta^{(3)}[\varphi^\pm] = \frac{i\beta}{6e^2 R} \int_0^\infty d\tau [\varphi^-(\tau)]^3 \dot{\varphi}^+(\tau). \quad (31)$$

Taking this term into account and repeating the above analysis we arrive at an extra contribution to the current noise in the form $-e^2\beta\delta(t-t')/2RC$; see also Appendix C. Adding this contribution to Eq. (26) and subtracting the noninteracting result (30) we arrive at the interaction correction

$$\begin{aligned} \delta S_\omega^{(\beta)} = & \frac{\beta}{R\kappa\Omega} \{ \alpha(t-t') \cos[eV(t-t')] (e^{-F(t-t')} - 1) \\ & - e^2 \delta(t-t')/2C \}_\omega. \end{aligned} \quad (32)$$

The function $F(t)$ results from averaging over the phase fluctuations

$$\begin{aligned} & \langle \cos[eV(t-t') + \varphi^+(t) - \varphi^+(t')] \rangle \\ & = \cos[eV(t-t')] e^{-F(t-t')}. \end{aligned} \quad (33)$$

This function has the form

$$\begin{aligned} F(t) = & e^2 R_0^2 \int_{-\infty}^\infty \frac{d\omega}{2\pi} \frac{1 - \cos \omega t}{\omega^2 \Omega} \left\{ \left(\frac{1}{R_0} - \frac{\beta}{R} \right) \omega \coth \frac{\omega}{2T} \right. \\ & \left. + \frac{\beta}{2R} \sum_{\pm} (\omega \pm eV) \coth \frac{\omega \pm eV}{2T} \right\}. \end{aligned} \quad (34)$$

We also note that in Eq. (32) we omitted the last term of Eq. (26) which contains averages of the function H Eq. (27). Our analysis demonstrates that these terms are small in all regimes considered below.

What remains is to evaluate the correction to the shot noise from the second contribution to $S^{(3)}$. The derivation of this contribution is presented in Appendix C. Here we only quote the result

$$\begin{aligned} iS_\gamma^{(3)}[\varphi^\pm] = & \frac{\pi i \gamma T^3}{6e^2 R} \int_0^\infty dy_1 \int_0^\infty dy_2 \\ & \times \int_0^\infty dy_3 \frac{\varphi^-(y_1) \varphi^-(y_2) \varphi^-(y_3)}{\sinh[\pi T y_{21}] \sinh[\pi T y_{32}] \sinh[\pi T y_{13}]} \\ & \times \{ \sin[eV y_{21} + \varphi^+(y_2) - \varphi^+(y_1)] \\ & + \sin[eV y_{32} + \varphi^+(y_3) - \varphi^+(y_2)] \\ & + \sin[eV y_{13} + \varphi^+(y_1) - \varphi^+(y_3)] \}, \end{aligned} \quad (35)$$

where $y_{ij} = y_i - y_j$ and the parameter γ is defined in Eq. (4).

At first glance this contribution to the effective action could be considered unimportant. This is indeed the case in several limits. For instance, at sufficiently small transmissions $\beta \gg \gamma$ the term (35) can obviously be neglected. In the limit of low voltages one can, making use of the condition (7), expand $S_\gamma^{(3)}$ in small phase fluctuations φ^\pm . Then one gets $S_\gamma^{(3)}$ proportional to the combination $(\varphi^+)^3(\varphi^-)^3$ which can be dropped as compared to other terms provided $g_0 \gg 1$. However, in the limit of large voltages the term (35) gains importance and—as we shall see—provides a significant contribution to δS_ω .

Proceeding along the lines with the above analysis we find

$$\begin{aligned} \delta S_\omega^{(\gamma)} &= \frac{2\pi^2 \gamma T^3}{g_0 R \kappa \Omega} \int_0^\infty dt \\ &\times \int_0^\infty dx \frac{(1 - e^{-x/R_0 C})(\cos[eVt] - \cos[eVx]) \cos \omega t}{\sinh[\pi T x] \sinh[\pi T t]} \\ &\times \left(\frac{1}{\sinh[\pi T(x-t)]} - \frac{1}{\sinh[\pi T(x+t)]} \right). \end{aligned} \quad (36)$$

This expression will be analyzed below in Sec. III F.

C. Relation to the FDT

Before we proceed with the analysis of the above results let us establish some useful general expressions and illustrate the relation between our approach and the FDT. Throughout this subsection we will only consider the limit of small voltages $eV \ll 1/R_0 C$ and neglect the dependence of the function $F(t)$ on V . In the spirit of the $P(E)$ theory²¹ let us define the function

$$\begin{aligned} P(E) &= \int_{-\infty}^\infty dt e^{iEt} e^{-\Phi(t)}, \\ \Phi(t) &= F(t)|_{V=0} + \frac{ie^2 R_0}{2} \operatorname{sgn}[t] (1 - e^{-|t|/R_0 C}). \end{aligned} \quad (37)$$

This function obeys the “detailed balance” symmetry $P(-E) = e^{-E/T} P(E)$ which follows from the property $\Phi(t - (i/T)) = \Phi(-t)$. Let us also introduce the function

$$\mathcal{N}_\omega = \frac{1}{4\pi R_0} \sum_{\pm} \int_{-\infty}^\infty dE \frac{E(1 + e^{-(\omega \pm eV)/T})}{1 - e^{-E/T}} P(\omega \pm eV - E) \quad (38)$$

and rewrite it in the form

$$\begin{aligned} \mathcal{N}_\omega &= \frac{1}{R_0} \int_{-\infty}^\infty dt e^{i\omega t} \left\{ -\frac{e^2}{2C} \delta(t) \right. \\ &\quad \left. + \alpha(t) e^{-F(t)} \cos(eVt) \cos \left[\frac{\pi}{g_0} (1 - e^{-t/R_0 C}) \right] \right\}. \end{aligned} \quad (39)$$

We observe that, since in the interesting for us limit $g_0 \gg 1$ the argument of $\cos[(\pi/g_0)(1 - e^{-t/R_0 C})]$ is small, with the accuracy $\sim 1/g_0^2$ one can use the function (39) in order to analyze the result (32).

Proceeding further let us rewrite Eq. (38) as

$$\mathcal{N}(\omega) = \frac{1}{2} \sum_{\pm} \coth \frac{\omega \pm eV}{2T} \mathcal{I}(\omega \pm eV), \quad (40)$$

where

$$\mathcal{I}(\omega) = \frac{1 - e^{-\omega/T}}{2\pi R_0} \int_{-\infty}^\infty \frac{dEE}{1 - e^{-E/T}} P(\omega - E). \quad (41)$$

After a simple algebra from Eq. (41) we obtain $\mathcal{I}(\omega) = \omega/R_0 + \delta\mathcal{I}(\omega)$ and

$$\delta\mathcal{I}(\omega) = \frac{2}{R_0} \int_0^\infty dt \sin(\omega t) e^{-F(t)} \alpha(t) \sin \left[\frac{\pi}{g_0} (1 - e^{-t/R_0 C}) \right]. \quad (42)$$

Comparing the above expressions with Eq. (32) we arrive at the following correction to the current noise:

$$\delta S_\omega = \frac{\beta R_0}{2R\kappa\Omega} \sum_{\pm} \coth \frac{\omega \pm eV}{2T} \delta\mathcal{I}(\omega \pm eV). \quad (43)$$

In order to illustrate the relation between our results and the FDT we notice that in the relevant limit $g_0 \gg 1$ the quantity $\delta\mathcal{I}(eV)$, Eq. (42), is defined by exactly the same time integral as the interaction correction to the I - V curve; cf. Eq. (27) of Ref. 5. In particular, in the limit of zero frequency and voltage one finds

$$\delta\tilde{S}_{\omega=0} = 2Te^2\beta \int_0^\infty t \alpha(t) e^{-F(t)} (1 - e^{-t/R_0 C}) dt. \quad (44)$$

In accordance with the FDT the combination on the right-hand side is just the interaction correction to the zero-bias conductance of a coherent scatterer⁵ multiplied by $2T$.

We will now derive the interaction correction to the current noise in several important limits.

D. High temperatures

In the limit $T \gg 1/R_0 C$ it is sufficient to evaluate the function $F(t)$ only at short times $t \lesssim 1/T$. In this limit from Eq. (34) we get

$$F(t) = \frac{e^2 t^2}{2C} \left[\left(1 - \frac{\beta R_0}{R} \right) T + \frac{\beta R_0}{2R} eV \coth \frac{eV}{2T} \right]. \quad (45)$$

Expanding $e^{-F(t)}$ in Eq. (32) to the first order in F we obtain

$$\begin{aligned} \delta S_\omega &= \frac{e^2 \beta}{2RC\kappa\Omega} \\ &\times \left\{ -1 + \left[1 + \frac{\beta R_0}{R} \left(\frac{eV}{2T} \coth \frac{eV}{2T} - 1 \right) \right] \right. \\ &\times \left. \sum_{\pm} f\left(\frac{\omega \pm eV}{2T}\right) \right\}, \end{aligned} \quad (46)$$

where the function $f(x)$ reads

$$f(x) = \frac{x \cosh x}{2 \sinh^3 x} - \frac{1}{2 \sinh^2 x}. \quad (47)$$

In the limit of small frequencies and voltages we then find

$$\delta S_\omega = -\frac{e^2 \beta}{3RC\kappa}. \quad (48)$$

At high frequencies $\omega \gg T, eV$ or large voltages $eV \gg T, \omega$ we get

$$\delta S_\omega = -\frac{e^2 \beta}{2RC\kappa\Omega}. \quad (49)$$

Both results (48) and (49) describe partial suppression of the current noise by Coulomb interaction. As we have already discussed, Eq. (48) is consistent with the results⁵ combined with the FDT, whereas Eq. (49) just corresponds to the Coulomb offset $\Delta V = -e\beta/2C$ on the I - V curve of a coherent scatterer at large voltages. For the sake of completeness we also note that in a specific limit $|\omega \pm eV| \ll T$, Eq. (46) yields a positive correction to the current noise:

$$\delta S_\omega = \frac{e^2 \beta^2 R_0 |eV|}{24CTR^2 \kappa \Omega}. \quad (50)$$

However, the magnitude of this correction is small in the parameter $\sim e^2 R_0 / (RCT) \ll 1/g_0$.

E. Low temperatures

Now let us consider the limit of low temperatures $T \ll 1/CR_0$. At low voltages $eV \ll 1/R_0C$ and times much longer than $1/R_0C$ the function $F(t)$ reads

$$F(t) \simeq \frac{2}{g_0} \ln \left(\frac{\sinh[\pi T t]}{\sinh[\pi T R_0 C]} \right). \quad (51)$$

Combining this expression with Eq. (32), in the limit of small $\omega, eV < T$ we obtain

$$S_\omega = 2T \left[\frac{1}{R+R_S} - \frac{\beta}{R\kappa} (1 - (TR_0C)^{2/g_0}) \right]. \quad (52)$$

For $T \gg g_0 E_C \exp(-g_0/2)$ the result (52) can be expanded in $2/g_0$. In this limit for the interaction correction we get

$$\delta S_\omega = -\frac{4T\beta}{g_0 R \kappa} \ln \frac{1}{R_0 C T}. \quad (53)$$

In the opposite limit of very low $T < g_0 E_C \exp(-g_0/2)$ (but still $T \gg \omega, eV$) the last term in Eq. (52) can be neglected and the interaction correction becomes

$$\delta S_\omega = -\frac{2T\beta}{R\kappa}. \quad (54)$$

In the limit $T \ll \omega, eV \ll 1/R_0C$ we can set $T=0$ in Eq. (51). Then we obtain

$$\begin{aligned} S_\omega &= |\omega| \left(\frac{1}{R+R_S} - \frac{\beta}{R\kappa} \right) \\ &+ \frac{\beta}{2R\kappa} \sum_{\pm} |\omega \pm eV| [|\omega \pm eV| R_0 C]^{2/g_0}. \end{aligned} \quad (55)$$

If both ω and V tend to zero, the last term in Eq. (55) can again be neglected and we find

$$\delta S_\omega = -\frac{|\omega|\beta}{R\kappa}. \quad (56)$$

If, however, ω and/or eV exceed the scale $g_0 E_C \times \exp(-g_0/2)$, one expands Eq. (55) in $2/g_0$ and gets

$$\delta S_\omega = -\frac{2|\omega|\beta}{g_0 R \kappa} \ln \frac{1}{|\omega| R_0 C}. \quad (57)$$

This expression applies for $\omega \gg eV$. In the opposite limit in Eq. (57) one should simply substitute eV instead of ω . Note, however, that in the latter limit the corresponding result yields only one contribution ($\delta S_\omega^{(\beta)}$) to the interaction correction. Another contribution ($\delta S_\omega^{(\gamma)}$) will be found in Sec. III F.

To complete this subsection let us find the interaction correction in the limit $\omega, eV \gg 1/R_0C$. At large voltages the dependence of $F(t)$ on V should be taken into account. Evaluating the corresponding (linear in V) correction to $F(t)$, Eq. (51), we obtain

$$\delta S_\omega = \frac{e^2 R_0^2 \beta^2 |eV|}{4\pi R^2 \kappa \Omega} \sum_{\pm} w[(\omega \pm eV) R_0 C], \quad (58)$$

where

$$w(x) = -2 - 2x \arctan x - \ln x^2 + \ln(1+x^2) + |x| \pi. \quad (59)$$

For $|\omega \pm eV| \gg 1/R_0C$ the asymptotics $w(x \gg 1) \simeq 1/3x^2$ should be used. In this case we again recover Eq. (49). If, however, $|\omega \pm eV| \leq 1/R_0C$, then the interaction correction is governed by another asymptotics $w(x \ll 1) \simeq -2 - \ln x^2$ and, hence, this correction is positive. Such an increase of the noise at $|\omega \pm eV| < 1/R_0C$ is similar to that found at higher temperatures.

F. Large voltages

Now let us evaluate the remaining correction $\delta S^{(\gamma)}$ Eq. (36). At high temperatures $T \gg 1/CR_0$ we obtain

$$\delta S_\omega^{(\gamma)} \sim \frac{e^2 \gamma}{RC\kappa\Omega} \left(\frac{eV}{T} \right)^2, \quad \text{if } \omega, eV \ll T, \quad (60)$$

$$\delta S_{\omega}^{(\gamma)} = \frac{e^2 \gamma}{RC\kappa\Omega} \theta(|eV| - |\omega|) \tanh \frac{|eV| - |\omega|}{2T},$$

if $\omega, eV \gg T$. (61)

In the limit $\omega, eV \ll T \ll 1/CR_0$ one finds

$$\delta S_{\omega}^{(\gamma)} = \frac{2\gamma(eV)^2}{3Tg_0R\kappa} \ln \frac{1}{TR_0C}, \quad (62)$$

while at higher frequencies and voltages $\omega, eV \gg T$ we derive

$$\begin{aligned} \delta S_{\omega}^{(\gamma)} &= \frac{4\gamma}{g_0R\kappa\Omega} \theta(|eV| - |\omega|) \\ &\times \left\{ \frac{\arctan[(|eV| - |\omega|)R_0C]}{R_0C} + \frac{|eV| - |\omega|}{2} \right. \\ &\left. \times \ln\{1 + [(|eV| - |\omega|)R_0C]^{-2}\} \right\}. \end{aligned} \quad (63)$$

Note that the correction $\delta S_{\omega}^{(\gamma)}$ is positive in all cases. As compared to previously obtained contribution $\delta S_{\omega}^{(\beta)}$ the correction (63) becomes important in the limit $eV \gg T, \omega$. For such voltages both corrections add up, $\delta S_{\omega} = \delta S_{\omega}^{(\beta)} + \delta S_{\omega}^{(\gamma)}$, and yield

$$\delta S_{\omega} = - \frac{2(\beta - 2\gamma)|eV|}{g_0R\kappa} \ln \frac{1}{|eV|R_0C},$$

$$\text{if } T, |\omega| \ll |eV| \ll 1/CR_0,$$

$$\delta S_{\omega} = - \frac{(\beta - 2\gamma)E_C}{R\kappa\Omega}, \quad \text{if } |eV| \gg T, 1/CR_0, |\omega|. \quad (64)$$

Finally, we present some explicit expressions for the so-called Fano factor, defined as

$$\zeta(V) = \frac{S(eV \gg T, \omega = 0)}{|eI|}. \quad (65)$$

This factor can easily be evaluated with the aid of the above results. In the regime of moderate voltages $T \ll |eV| \ll 1/CR_0$ we have

$$\zeta = \frac{R^2}{(R + R_S)^2} \left[\beta + \frac{2}{g_0} \ln \left(\frac{1}{|eV|R_0C} \right) \left(\frac{\beta^2 R}{R + R_S} + 2\gamma - \beta \right) \right], \quad (66)$$

whereas in the limit of high voltages $|eV| \gg T, 1/CR_0$ one finds

$$\zeta = \frac{R^2}{(R + R_S)^2} \left[\beta + \frac{E_C}{|eV|} \left(\frac{\beta^2 R}{R + R_S} + 2\gamma - \beta \right) \right].$$

In the diffusive case the interaction correction to the Fano factor can be both positive and negative depending on the R/R_S ratio.

These results complete our analysis of current fluctuations in coherent conductors with electron-electron interactions.

IV. SUMMARY

Combining the standard scattering matrix approach with the effective action formalism we have analyzed the effect of electron-electron interactions on current noise in mesoscopic coherent conductors in the metallic limit (7). We have found that Coulomb interaction always leads to partial suppression of the Nyquist noise. The corresponding interaction term is proportional to the parameter β , Eq. (1), similarly to the interaction correction to the conductance.⁵ Interaction-induced suppression of both conductance and Nyquist noise has the same physical origin, and a direct relation between these two effects can easily be established with the aid of the FDT.

The effect of electron-electron interactions on the shot noise is somewhat more complicated. In this case we have recovered two different interaction corrections entering with opposite signs. One of them is negative and it is again governed by the parameter β . Another correction is positive and it is proportional to the parameter γ , Eq. (4), which depends on the transmission distribution in a different way. The net interaction correction to the shot noise scales as $\delta S_{\omega}(V) \propto 2\gamma - \beta$; i.e., it can be both negative and positive depending on the relation between β and γ . The contribution to $\delta S_{\omega}(V)$ from the n th conducting mode is *negative* provided its transmission T_n is smaller than 1/2 and it is *positive* otherwise. For coherent diffusive conductors $2\gamma - \beta = -1/15$; i.e., in this particular case electron-electron interactions tend to decrease the shot noise.

The presence of two interaction corrections to the shot noise has a transparent physical interpretation. The β correction is due to Coulomb blockade suppression of the Landauer conductance (2) while the γ correction originates from the term $-\sum_n T_n^2$ in the expression for the shot noise.²⁻⁴ The absolute value of this term is also decreased by interactions. But since it enters with a negative sign, the corresponding contribution to the noise spectrum turns out to be positive. We believe that the effect of electron-electron interactions on higher cumulants of the current operator can be described in a similar manner.

Finally, we would like to make a remark concerning possible experimental verification of our results. Our theory was developed under the same basic assumptions as those used previously in Ref. 5 for calculations of the interaction correction to the I - V curve of a disordered coherent conductor (3). In a wide range of parameters the theory⁵ predicts universal logarithmic temperature and voltage dependence of the interaction correction to the differential conductance. Exactly this dependence was measured in a variety of mesoscopic conductors, such as, e.g., diffusive metallic bridges,²⁴ metallic microstrips,²⁵ multiwalled carbon nanotubes,^{26,27} break junctions,²⁸ and nanoscale metallic contacts,²⁹ to mention a few. We believe that all these and possibly other coherent mesoscopic coherent conductors can as well be used for measurements of the current noise in order to experimentally test the predictions made in this paper.

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APPENDIX A: EFFECTIVE ACTION AND OBSERVABLES

Following Ref. 5 let us combine the effective action formalism^{30,31} with the usual Landauer scattering approach. Within the latter approach one introduces a (relatively small) scatterer which connects two bulk reservoirs. The scatterer is described by the scattering matrix. In order to include electron-electron interactions it is necessary to reckon with the many-body Hamiltonian

$$\hat{H} = \int d\mathbf{r} \hat{\Psi}^\dagger(\mathbf{r}) \left[-\frac{\nabla^2}{2m} + W(\mathbf{r}) \right] \hat{\Psi}(\mathbf{r}) + \frac{1}{2} \int d\mathbf{r} \int d\mathbf{r}' \hat{\Psi}^\dagger(\mathbf{r}) \hat{\Psi}^\dagger(\mathbf{r}') \frac{e^2}{|\mathbf{r}-\mathbf{r}'|} \Psi(\mathbf{r}') \Psi(\mathbf{r}). \quad (\text{A1})$$

Here the term $W(\mathbf{r})$ accounts for boundary and impurity potentials, external fields, etc. After the standard Hubbard-Stratonovich decoupling of the interaction term one arrives at the following path integral over an auxiliary field $V(\mathbf{r},t)$:

$$e^{-i\hat{H}t} = \frac{\int \mathcal{D}V(\mathbf{r},t') \left(\hat{\mathbf{T}} \exp \left[-i \int_0^t dt' \hat{H}_{\text{eff}}[V(\mathbf{r},t')] \right] \right) \exp \left[i \int_0^t dt' \int d\mathbf{r} [\nabla V(\mathbf{r},t')]^2 / 8\pi \right]}{\int \mathcal{D}V(\mathbf{r},t') \exp \left[i \int_0^t dt' \int d\mathbf{r} [\nabla V(\mathbf{r},t')]^2 / 8\pi \right]}. \quad (\text{A2})$$

Here $\hat{\mathbf{T}}$ is the time-ordering operator and

$$\hat{H}_{\text{eff}}[V(\mathbf{r},t)] = \int d\mathbf{r} \hat{\Psi}^\dagger(\mathbf{r}) \left[-\frac{\nabla^2}{2m} + W(\mathbf{r}) - eV(\mathbf{r},t) \right] \hat{\Psi}(\mathbf{r}). \quad (\text{A3})$$

We choose to define the electron charge as $-e$.

The time dynamics of the density matrix ρ is described by means of the evolution operator J defined on the Keldysh contour. In what follows we shall denote the field V on the upper and lower parts of this contour by $V_{1,2}$. The general expression for the density matrix reads

$$\rho(X_{1f}, X_{2f}, t_f) = \int dX_{1i} dX_{2i} J(X_{1f}, X_{2f}; X_{1i}, X_{2i}; t_f, t_i) \times \rho(X_{1i}, X_{2i}, t_i), \quad (\text{A4})$$

where X the set of relevant quantum degrees of freedom. We shall assume that the interaction with the fluctuating fields $V_{1,2}$ is turned on at a time $t_i=0$. The time t_f is supposed to be large. Making use Eq. (A2) and integrating over the fermionic degrees of freedom, we obtain

$$J = \int \mathcal{D}V_1 \mathcal{D}V_2 \exp iS[V_1, V_2], \quad (\text{A5})$$

where S is the effective action:

$$iS[V_1, V_2] = 2 \text{Tr} \ln \hat{G}_V^{-1} + i \frac{C}{2} \int_0^t dt' [V_{LR1}^2 - V_{LR2}^2]. \quad (\text{A6})$$

Here we defined $V_{LRj} = V_{Lj} - V_{Rj}$ and neglected the spatial dependence of the fields $V_{L1,2}$ and $V_{R1,2}$ inside both the left (L) and right (R) reservoirs. The term with the effective ca-

pacitance C appears in the same way as in the derivation of Ambegaokar-Eckern-Schön (AES) action,^{22,30} i.e., it comes from the terms

$$\frac{1}{8\pi} \int dt \int d\mathbf{r} [\nabla V_{1,2}(\mathbf{r},t)]^2 \quad (\text{A7})$$

describing the energy of the fluctuating fields in the scatterer region.

The Green-Keldysh matrix $G_V(X_1, X_2)$ [here $X=(\mathbf{r},t)$] obeys the 2×2 matrix equation

$$\left(i \frac{\partial}{\partial t_1} \hat{\mathbf{1}} - \hat{H}_0(\mathbf{r}_1) \hat{\mathbf{1}} + e \hat{V}(X_1) \right) \hat{G}_V(X_1, X_2) = \delta(X_1 - X_2) \hat{\sigma}_z, \quad (\text{A8})$$

where $\hat{H}_0 = (-\nabla^2/2m) + W(\mathbf{r})$, \hat{V} is a diagonal 2×2 matrix with components $\hat{V}_{ij} = V_i \delta_{ij}$, and $\hat{\sigma}_z$ is the Pauli matrix. The above equation for the Green-Keldysh function should be supplemented by the initial condition for the density matrix $\hat{\rho}(t=0) = \hat{\rho}_0$, where $\hat{\rho}_0$ is the equilibrium density matrix of noninteracting electrons. In what follows we will need the solution of Eq. (A8), which reads

$$G_{11}(t_1, t_2) = -i\theta(t_1 - t_2) \hat{U}_1(t_1, t_2) + i\hat{U}_1(t_1, 0) \hat{\rho}_0 \hat{U}_1(0, t_2),$$

$$G_{22}(t_1, t_2) = -i\theta(t_2 - t_1) \hat{U}_2(t_1, t_2) + i\hat{U}_2(t_1, 0) \hat{\rho}_0 \hat{U}_2(0, t_2),$$

$$G_{12}(t_1, t_2) = i\hat{U}_1(t_1, 0) \hat{\rho}_0 \hat{U}_2(0, t_2),$$

$$G_{21}(t_1, t_2) = -i\hat{U}_2(t_1, 0) [\hat{\mathbf{1}} - \hat{\rho}_0] \hat{U}_1(0, t_2), \quad (\text{A9})$$

where $\hat{U}_{1,2}(t_1, t_2)$ are the evolution operators

$$\hat{U}_{1,2}(t_1, t_2) = T \exp \left[-i \int_{t_2}^{t_1} dt' [\hat{H}_0 - eV_{1,2}(\mathbf{r}, t')] \right]. \quad (\text{A10})$$

One should keep in mind that in the operator products like $\hat{U}\hat{\rho}\hat{U}$ integration over intermediate spatial coordinates is implied.

Instead of specifying $W(\mathbf{r})$ we will describe electron transfer between the reservoirs by means of the scattering matrix formalism. This procedure is standard and further details are provided in Appendix B. In calculating the trace in Eq. (A6), we shall make an explicit integration over the longitudinal coordinates. Integration over the transverse coordinates is replaced by summing over the transmission channels of the scatterer. It is convenient to introduce the phase variables

$$\varphi^+(t) = \int_0^t dt' [eV_{LR1}(t') + eV_{LR2}(t')]/2, \quad (\text{A11})$$

$$\varphi^-(t) = \int_0^t dt' [eV_{LR1}(t') - eV_{LR2}(t')]. \quad (\text{A12})$$

Provided the dimensionless conductance is large, $g_0 \gg 1$, fluctuations of $\varphi^\pm(t)$ are strongly suppressed, so we can expand the exact action $iS[\varphi^\pm]$ in powers of φ^\pm , keeping the full nonlinearity in φ^+ . Note that the external voltage enters only in φ^+ but not in φ^- . Hence, for the system of Fig. 1 we have to impose the constraints $\sum_j \varphi_j^+(t) = eV_x t$ and $\sum_j \varphi_j^-(t) = 0$. Here the summation runs over different elements in our circuit—i.e., the scatterer and shunt.

Let us now define the current operator. The average current through the scatterer or through the shunt is defined as follows:

$$\langle \hat{I}(t) \rangle = ie \int \mathcal{D}\varphi^\pm \frac{\delta}{\delta\varphi^-(t)} e^{iS[\varphi^\pm]}. \quad (\text{A13})$$

The noise is defined analogously:

$$\begin{aligned} \frac{1}{2} \langle \hat{I}(t)\hat{I}(t') + \hat{I}(t')\hat{I}(t) \rangle &= \frac{1}{2} \text{tr}[\hat{I}(t)\hat{I}(t')\hat{\rho}_{\text{eq}} + \hat{\rho}_{\text{eq}}\hat{I}(t')\hat{I}(t)] \\ &= -\frac{e^2}{2} \int \mathcal{D}\varphi^\pm \left(\frac{\delta^2}{\delta\varphi_1^-(t)\delta\varphi_1^-(t')} + \frac{\delta^2}{\delta\varphi_2^-(t)\delta\varphi_2^-(t')} \right) e^{iS[\varphi^\pm]} \\ &= -e^2 \int \mathcal{D}\varphi^\pm \left(\frac{\delta^2}{\delta\varphi^-(t)\delta\varphi^-(t')} + \frac{1}{4} \frac{\delta^2}{\delta\varphi^+(t)\delta\varphi^+(t')} \right) e^{iS[\varphi^\pm]}. \end{aligned} \quad (\text{A14})$$

Here $\hat{\rho}_{\text{eq}}$ is the equilibrium density matrix of the whole system. Employing the properties of the action $S[-\varphi^+, -\varphi^-] = S[\varphi^+, \varphi^-]$, and $S[\varphi^+, -\varphi^-] = -S^*[\varphi^+, \varphi^-]$, one can show that the term containing $\delta^2/\delta\varphi^+(t)\delta\varphi^+(t')$ vanishes. Thus we arrive at Eq. (18).

Below we shall proceed with an explicit calculation of the action by defining the scattering states.

APPENDIX B: TRANSMISSION CHANNELS

Following the usual \hat{S} -matrix approach³² let us introduce the transmission channels. We will assume that far from the scatterer the electron propagation in transverse and longitudinal directions can be described separately. In this case the solution of the Schrödinger equation

$$-\frac{\nabla^2}{2m} \psi(\mathbf{r}) + W(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r}) \quad (\text{B1})$$

can be factorized:

$$\psi(\mathbf{r}) = \sum_n c_n \Phi_n(\mathbf{r}_\perp) \chi_n(x). \quad (\text{B2})$$

Here x is the coordinate along the lead and \mathbf{r}_\perp are the transverse coordinates. The transverse wave functions $\Phi_n(\mathbf{r}_\perp)$ satisfy the equation

$$-\frac{\nabla_\perp^2}{2m} \Phi_n(\mathbf{r}_\perp) + W(\mathbf{r}_\perp)\Phi_n(\mathbf{r}_\perp) = E_n \Phi_n(\mathbf{r}_\perp), \quad (\text{B3})$$

where the subscript n enumerates the transmission channels (we are considering only channels with $E_n < E_F$). The function $\chi_n(x)$ outside the scatterer region is defined from the equation

$$-\frac{1}{2m} \frac{d^2}{dx^2} \chi_n(x) = (E - E_n) \chi_n(x). \quad (\text{B4})$$

Since the electronic states with energies E close to the Fermi energy E_F mainly contribute, it is sufficient to describe the electron dynamics quasiclassically. We define the energy $\xi = E - E_F$ and the particle velocity in the n th channel $v_n = \sqrt{2(E_F - E_n)/m}$. Then the wave function can be expressed as

$$\chi_n(x) = e^{imv_n x} f_n^{\text{in}}(x) + e^{-imv_n x} f_n^{\text{out}}(x), \quad \text{left reservoir,}$$

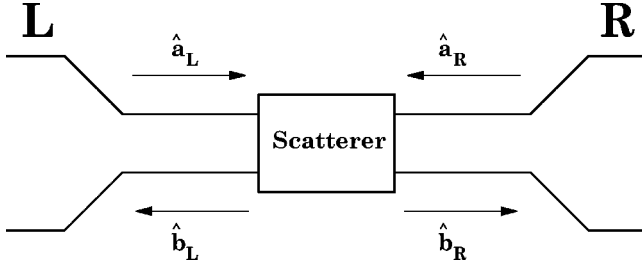


FIG. 2. Scattering states.

$$\chi_n(x) = e^{imv_n x} g_n^{out}(x) + e^{-imv_n x} g_n^{in}(x), \quad \text{right reservoir.} \quad (\text{B5})$$

In this way we have introduced the envelopes of the fast oscillating functions $\exp(\pm imv_n x)$. Consider first the left reservoir. The functions $f_n^{in}(x)$ and $f_n^{out}(x)$ satisfy the following quasiclassical equations:

$$\begin{aligned} -iv_n \frac{d}{dx} f_n^{in}(x) &= \xi f_n^{in}(x), \\ iv_n \frac{d}{dx} f_n^{out}(x) &= \xi f_n^{out}(x), \end{aligned} \quad (\text{B6})$$

with the solutions

$$f_n^{in}(x) = \frac{e^{i\xi x/v_n}}{\sqrt{v_n}}, \quad f_n^{out}(x) = \frac{e^{-i\xi x/v_n}}{\sqrt{v_n}}. \quad (\text{B7})$$

Analogously, for the right reservoir we find

$$g_m^{in}(x) = \frac{e^{-i\xi x/v_m}}{\sqrt{v_m}}, \quad g_m^{out}(x) = \frac{e^{i\xi x/v_m}}{\sqrt{v_m}}. \quad (\text{B8})$$

The eigenfunction of the whole system with the energy ξ in the left reservoir may be expressed as

$$\psi_\xi(\mathbf{r}) = \sum_n [a_{Ln} e^{imv_n x} f_n^{in}(x) + b_{Ln} e^{-imv_n x} f_n^{out}(x)] \Phi_n(\mathbf{r}_\perp), \quad (\text{B9})$$

while in the right reservoir we get

$$\psi_\xi(\mathbf{r}) = \sum_k [b_{Rk} e^{imv_k x} g_k^{out}(x) + a_{Rk} e^{-imv_k x} g_k^{in}(x)] \Phi_k(\mathbf{r}_\perp). \quad (\text{B10})$$

The amplitudes of the outgoing $b_{L,R}$ and incoming $a_{L,R}$ states (see Fig. 2) are related via the scattering matrix $\hat{S}(\xi)$:

$$\begin{pmatrix} b_{L1} \\ \vdots \\ b_{LN_L} \\ b_{R1} \\ \vdots \\ b_{RN_R} \end{pmatrix} = \hat{S}(\xi) \begin{pmatrix} a_{L1} \\ \vdots \\ a_{LN_L} \\ a_{R1} \\ \vdots \\ a_{RN_R} \end{pmatrix}. \quad (\text{B11})$$

The unitary matrix \hat{S} with dimensions $(N_L + N_R) \times (N_L + N_R)$ has the block structure

$$\hat{S}(\xi) = \begin{pmatrix} \hat{r}(\xi) & \hat{t}'(\xi) \\ \hat{t}(\xi) & \hat{r}'(\xi) \end{pmatrix}. \quad (\text{B12})$$

The diagonal blocks \hat{r} and \hat{r}' describe reflection back to the left and right reservoirs, respectively. The off-diagonal blocks describe transmission through the scatterer. Later we shall neglect the ξ dependence of \hat{S} . An appropriate generalization of our approach to the case of energy-dependent scattering will be published elsewhere.¹⁶

Let us now combine the incident $f_n^{in}(x)$ and outgoing $f_n^{out}(x)$ wave functions belonging to the same channel into one wave function $\psi_n(x)$. Namely, we assume that the scatterer is located at $x=0$, and for the left reservoir ($x < 0$) we put

$$\psi_n(y) = \begin{cases} f_n^{in}(y), & y < 0, \\ f_n^{out}(-y), & y > 0. \end{cases} \quad (\text{B13})$$

Analogously, for the right reservoir ($x > 0$) we define

$$\psi_m(y) = \begin{cases} f_m^{in}(-y), & y < 0, \\ f_m^{out}(y), & y > 0. \end{cases} \quad (\text{B14})$$

These new functions are defined in all the range $y \in [-\infty, +\infty]$ and are equal to

$$\psi_j(y) = \frac{e^{i\xi y/v_j}}{\sqrt{v_j}}. \quad (\text{B15})$$

Let us emphasize that here the index j enumerates all conducting channels, both in the left and in the right reservoirs (for convenience, we assume that the left channels are enumerated first).

In the presence of the fluctuating field $V(t)$, the matrix elements of the Hamiltonian in the reservoirs take the form

$$\hat{H}_{ij} = -iv_i \delta_{ij} \frac{\partial}{\partial y} - eV_i(t) \delta_{ij}, \quad (\text{B16})$$

where $V_i = V_L$ for all left channels and $V_i = V_R$ for the right channels. If at initial time t_1 the wave function was $\psi_n(t_1, y)$, then at the final time $t_2 > t_1$ it becomes

$$\psi_n(t_2, y) = e^{i[\varphi_n(t_2) - \varphi_n(t_1)]} \psi_n(t_1, y - v_n(t_2 - t_1)),$$

$$y < 0 \quad \text{or} \quad y > v_n(t_2 - t_1),$$

$$\psi_n(t_2, y) = \sum_k e^{i[\varphi_n(t_2) - \varphi_k(t_1)] - i[\varphi_n(t_2 - y/v_n) - \varphi_k(t_2 - y/v_n)]}$$

$$\times s_{nk} \sqrt{\frac{v_k}{v_n}} \psi_k \left(t_1, \frac{v_k}{v_n} y - v_k(t_2 - t_1) \right),$$

$$0 < y < v_n(t_2 - t_1). \quad (\text{B17})$$

Here s_{nk} are the matrix elements of the \hat{S} matrix and, as before, $\varphi_n(t) = \int_0^t d\tilde{t} e V_n(\tilde{t})$. On the other hand, by definition of the evolution operator we have

$$U_{nk}(t_2, t_1; y_2, y_1) = e^{i\varphi_n(t_2)} \left\{ \frac{\delta_{nk}}{v_n} \delta\left(\frac{y_2 - y_1}{v_n} - t_2 + t_1\right) + \theta(y_2) \theta(v_n(t_2 - t_1) - y_2) e^{-i\varphi_n(t_2 - y_2/v_n)} \right. \\ \left. \times [s_{nk} - \delta_{nk}] \sqrt{\frac{v_k}{v_n}} e^{i\varphi_k(t_2 - y_2/v_n)} \delta\left(\frac{v_k}{v_n} y_2 - y_1 - v_k(t_2 - t_1)\right) \right\} e^{-i\varphi_n(t_1)}.$$

It is convenient to introduce the new coordinates $\tau = y/v_n$. More precisely, instead of the wave function with the components $\psi_n(y)$ we introduce the functions $\eta_n(\tau) = \sqrt{v_n} \psi_n(y/v_n)$. The kernels of the operators will also be transformed. If the two functions are related to each other by means of a linear operator

$$\psi_n^{(2)}(y) = \sum_k \int dy' K_{nk}(y, y') \psi_k^{(1)}(y'), \quad (\text{B18})$$

then the corresponding wave functions $\eta^{(2)}$ and $\eta^{(1)}$ satisfy the following relation:

$$\eta_n^{(2)}(\tau) = \sum_k \int d\tau' \tilde{K}_{nk}(\tau, \tau') \eta_k^{(1)}(\tau'), \quad (\text{B19})$$

where

$$\tilde{K}_{nk}(\tau, \tau') = \sqrt{v_n} K_{nk}(v_n \tau, v_k \tau') \sqrt{v_k}. \quad (\text{B20})$$

In this representation the evolution operator can be simplified. We find

$$\hat{U}(t_2, t_1; \tau_2, \tau_1) \\ = \delta(\tau_2 - \tau_1 - t_2 + t_1) e^{i\hat{\varphi}(t_2)} \{ \hat{1} + \theta(\tau_2) \theta(-\tau_1) e^{-i\hat{\varphi}(t_2 - \tau_2)} \\ \times [\hat{S} - \hat{1}] e^{i\hat{\varphi}(t_1 - \tau_1)} \} e^{-i\hat{\varphi}(t_1)}. \quad (\text{B21})$$

The matrix $\hat{\varphi}$ is diagonal with respect to the channel indices $\hat{\varphi}_{ik} = \varphi_i \delta_{ik}$. We also obtain an expression for the inverse operator—i.e., the operator defined by $\int d\tau_2 \hat{U}(t_2, t_1; \tau_2, \tau_1) \hat{U}^{-1}(t_2, t_1; \tau_2, \tau_1) = \delta(\tau_3 - \tau_1)$. It reads

$$\hat{U}^{-1}(t_2, t_1; \tau_2, \tau_1) \\ = \delta(\tau_1 - \tau_2 - t_2 + t_1) e^{i\hat{\varphi}(t_1)} \{ \hat{1} + \theta(\tau_1) \theta(-\tau_2) e^{-i\hat{\varphi}(t_1 - \tau_2)} \\ \times [\hat{S}^+ - \hat{1}] e^{i\hat{\varphi}(t_2 - \tau_1)} \} e^{-i\hat{\varphi}(t_2)}. \quad (\text{B22})$$

Equations (B21) and (B22) apply for $t_2 > t_1$; in order to construct the corresponding expressions in the opposite case one should just use the property $\hat{U}(t_2, t_1; \tau_2, \tau_1) = \hat{U}^{-1}(t_1, t_2; \tau_2, \tau_1)$.

$$\psi_n(t_2, y_2) = \sum_k \int dy_1 U_{nk}(t_2, t_1; y_2, y_1) \psi_k(t_1, y_1).$$

Comparing this expression with Eqs. (B17), we find

Finally let us define the equilibrium density matrix for noninteracting electrons. It can be written in the form

$$\rho_{0,nk}(y_1, y_2) = \delta_{nk} \int \frac{dp}{2\pi} \frac{e^{ip(y_1 - y_2)}}{1 + e^{pv_n/T}} \\ = \frac{\delta_{nk}}{2} \delta(y_1 - y_2) - \frac{\delta_{nk}}{2\pi} \frac{\pi iT}{v_n \sinh\left[\frac{\pi T(y_1 - y_2)}{v_n}\right]}. \quad (\text{B23})$$

Performing the transformation (B20) we obtain

$$\hat{\rho}_0(\tau_1, \tau_2) = \frac{1}{2} \left(\delta(\tau_1 - \tau_2) - \frac{iT}{\sinh[\pi T(\tau_1 - \tau_2)]} \right) \hat{1}. \quad (\text{B24})$$

APPENDIX C: EXPANSION IN THE PHASE DIFFERENCE

We shall expand the effective action (A6) perturbatively in φ^- . The field φ^+ will be taken into account exactly in each term of this expansion. The expansion starts from the first order in φ^- , since for $\varphi^- = 0$ the contributions from the forward and backward parts of the Keldysh contour cancel each other. We get from Eq. (A8)

$$2 \text{Tr} \ln \hat{G}_V^{-1} = 2 \text{Tr} \ln \left(1 + \frac{\hat{G}_0 \hat{\varphi}^-}{2} \right). \quad (\text{C1})$$

The Green-Keldysh matrix \hat{G}_0 is evaluated for $\hat{\varphi}^- = 0$; i.e., it is defined by Eqs. (A9) with the evolution operator (B21) taken at $\hat{\varphi} = \hat{\varphi}^+$. The fluctuating field $\hat{\varphi}^-$ in Eq. (C1) is a unity matrix in Keldysh space and a diagonal matrix in the space of conducting channels. Performing the summation over the Keldysh indices we arrive at the first order in φ^- contribution to the action $iS^{(1)}$:

$$\begin{aligned}
iS^{(1)}[\varphi^\pm] = & -i \int dt \int ds \int d\tau_1 \int d\tau_2 \\
& \times \text{Tr} \left[\hat{U}(t,0;s,\tau_1) \left\{ \frac{1}{\pi} \frac{\pi iT}{\sinh[\pi T(\tau_1 - \tau_2)]} \right\} \right. \\
& \left. \times \hat{U}^{-1}(t,0;\tau_2,s) \hat{\varphi}^-(t) \right]. \quad (\text{C2})
\end{aligned}$$

For simplicity in Sec. III we have set $t_f \rightarrow \infty$. Here we will keep it finite and use the conditions $\varphi^-(0) = \varphi^-(t_f) = 0$. The δ functions contained in the \hat{U} matrices of Eq. (C2) will lead to a singularity of the form $1/\sinh[\pi T(\tau_1 - \tau_2)]$ which is cured as follows. Let us choose close but not exactly equal arguments s_1 and s_2 . Expanding the combination

$$\begin{aligned}
& -i \int dt \int d\tau_1 \int d\tau_2 \text{Tr} [\hat{U}(t,0;s_1,\tau_1) \hat{U}^{-1}(t,0;\tau_2,s_2) \hat{\varphi}^-(t)] \\
& \quad (\text{C3})
\end{aligned}$$

to the first order in $s_1 - s_2$ and multiplying the result by $1/\sinh[\pi T(s_1 - s_2)]$, we obtain

$$iS^{(1)}[\varphi^\pm] = \frac{i}{\pi} \int_0^{t_f} ds \text{Tr} \{ \hat{\varphi}^-(s) [\hat{S} \hat{\varphi}^+(s) \hat{S}^+ - \hat{\varphi}^+(s)] \}. \quad (\text{C4})$$

Making use of the condition $\text{Tr}[\hat{t}^+ \hat{t}] = \text{Tr}[\hat{t}'^+ \hat{t}']$ we get

$$iS^{(1)}[\varphi^\pm] = -\frac{i}{\pi} \text{Tr}[\hat{t}^+ \hat{t}] \int_0^{t_f} d\tau \varphi^-(\tau) \dot{\varphi}^+(\tau), \quad (\text{C5})$$

where $\varphi^\pm(\tau) = \varphi_L^\pm(\tau) - \varphi_R^\pm(\tau)$.

Consider now the contribution to the action of the second order in φ^- . It is defined as

$$iS^{(2)}[\varphi^\pm] = -\text{Tr}[\hat{G}_{12} \hat{\varphi}^- \hat{G}_{21} \hat{\varphi}^-]. \quad (\text{C6})$$

After a straightforward algebra one obtains

$$\begin{aligned}
iS^{(2)}[\varphi^\pm] = & - \int_0^{t_f} d\tau_1 \int_0^{t_f} d\tau_2 \rho_0(\tau_2 - \tau_1) \rho_0^*(\tau_1 - \tau_2) \\
& \times \text{Tr} \{ e^{-i[\hat{\varphi}^+(\tau_1) - \hat{\varphi}^+(\tau_2)]} [\hat{S}^+ \hat{\varphi}^-(\tau_1) \hat{S} - \hat{\varphi}^-(\tau_1)] \\
& \times e^{i[\hat{\varphi}^+(\tau_1) - \hat{\varphi}^+(\tau_2)]} [\hat{S}^+ \hat{\varphi}^-(\tau_2) \hat{S} - \hat{\varphi}^-(\tau_2)] \}. \quad (\text{C7})
\end{aligned}$$

Taking into account the block structure of the \hat{S} matrix, we find

$$\begin{aligned}
iS^{(2)}[\varphi^\pm] = & -\frac{1}{4\pi} \int_0^{t_f} d\tau_1 \int_0^{t_f} d\tau_2 \alpha(\tau_1 - \tau_2) \{ [\text{Tr}(\hat{t}'^+ \hat{t}')]^2 \\
& + \text{Tr}(\hat{t}^+ \hat{t})^2 \} \varphi^-(\tau_1) \varphi^-(\tau_2) + 2 \text{Tr}[\hat{r}' \hat{r}'^+ \hat{t}^+ \hat{t}] \\
& \times \cos[\varphi^+(\tau_1) - \varphi^+(\tau_2)] \varphi^-(\tau_1) \varphi^-(\tau_2) \}, \quad (\text{C8})
\end{aligned}$$

where $\alpha(\tau)$ is defined in Eq. (21). Introducing the parameter $\beta = \text{Tr}[\hat{r}' \hat{r}'^+ \hat{t}^+ \hat{t}] / \text{Tr}[\hat{t}^+ \hat{t}]$ we rewrite Eq. (C8) in a more compact form

$$\begin{aligned}
iS^{(2)}[\varphi^\pm] = & -\frac{\text{Tr}[\hat{t}^+ \hat{t}]}{2\pi} \int_0^{t_f} d\tau_1 \int_0^{t_f} d\tau_2 \alpha(\tau_1 - \tau_2) \\
& \times \varphi^-(\tau_1) \varphi^-(\tau_2) \{ 1 - \beta + \beta \\
& \times \cos[\varphi^+(\tau_1) - \varphi^+(\tau_2)] \}. \quad (\text{C9})
\end{aligned}$$

We now proceed to the third-order contribution $iS^{(3)}$ to the effective action. It reads

$$\begin{aligned}
iS^{(3)} = & \frac{i}{2} \text{Tr} \left[\int_0^{t_f} dt_1 \int_0^{t_f} dt_2 \int_0^{t_f} dt_3 \theta(t_1 - t_2) \right. \\
& \left. \times \theta(t_3 - t_2) \hat{\mathcal{F}}(t_1) \hat{\mathcal{F}}(t_2) \hat{\mathcal{F}}(t_3) \hat{\rho}_0^{as} \right] \\
& + \frac{1}{12} \text{Tr}[(\hat{G}_{12} \hat{\varphi}^- + \hat{G}_{21} \hat{\varphi}^-)^3]. \quad (\text{C10})
\end{aligned}$$

Here we used the notation

$$\hat{\rho}_0^{as}(\tau_1 - \tau_2) = -\frac{iT}{2 \sinh[\pi T(\tau_1 - \tau_2)]} \hat{1},$$

$$\hat{\mathcal{F}}(t) = \hat{U}^{-1}(t,0) \hat{\varphi}^-(t) \hat{U}(t,0). \quad (\text{C11})$$

We obtain two terms from Eq. (C10). The first one is

$$iS_\beta^{(3)}[\varphi^\pm] = \frac{i\beta}{6\pi} \text{Tr}[\hat{t}^+ \hat{t}] \int_0^{t_f} d\tau [\varphi^-(\tau)]^3 \dot{\varphi}^+(\tau). \quad (\text{C12})$$

In deriving this result we employed the same—although somewhat more involved—regularization procedure as for the first-order contribution $S^{(1)}$. This procedure allows us to determine the correct overall prefactor in Eq. (C12). One can then verify that the resulting effective action satisfies the requirements of the FDT.

The second term, coming from Eq. (C10), has the form

$$\begin{aligned}
iS_\gamma^{(3)}[\varphi^\pm] = & \frac{4}{3} \text{Tr}[(\hat{t}^+ \hat{t})^2 \hat{r}' \hat{r}'^+] \int_0^{t_f} dy_1 \int_0^{t_f} dy_2 \int_0^{t_f} dy_3 \\
& \times \rho_0^{as}(y_2 - y_1) \rho_0^{as}(y_3 - y_2) \rho_0^{as}(y_1 - y_3) \\
& \times \varphi^-(y_1) \varphi^-(y_2) \varphi^-(y_3) \{ \sin[\varphi^+(y_2) \\
& - \varphi^+(y_1)] + \sin[\varphi^+(y_3) - \varphi^+(y_2)] \\
& + \sin[\varphi^+(y_1) - \varphi^+(y_3)] \}. \quad (\text{C13})
\end{aligned}$$

Defining the parameter $\gamma = \text{Tr}[(\hat{t}^+ \hat{t})^2 \hat{r}' \hat{r}'^+] / \text{Tr}[\hat{t}^+ \hat{t}]$ and shifting the phase φ^+ by eV we obtain Eq. (35). Collecting now all four contributions (C5), (C9), (C12), and (C13) we arrive at the final result for the effective action:

$$S = S^{(1)} + S^{(2)} + S_{\beta}^{(3)} + S_{\gamma}^{(3)}. \quad (\text{C14})$$

This action is valid up to the third order in φ^- , and the variable φ^+ is treated exactly in each of the terms in Eq. (C14).

It is instructive to compare our results with the AES action²² derived for tunnel junctions ($\beta \rightarrow 1$) to all orders in φ^{\pm} . Rewriting the action²² in our notation together with the capacitive term one has

$$\begin{aligned} iS_{AES} = & \frac{4i}{e^2 R} \int_0^{t_f} dt_1 \int_0^{t_f} dt_2 \alpha_I(t_1 - t_2) \theta(t_1 - t_2) \\ & \times \sin[\varphi^+(t_1) - \varphi^+(t_2)] \sin \frac{\varphi^-(t_1)}{2} \cos \frac{\varphi^-(t_2)}{2} \\ & - \frac{2}{e^2 R} \int_0^{t_f} dt_1 \int_0^{t_f} dt_2 \alpha(t_1 - t_2) \\ & \times \sin \frac{\varphi^-(t_1)}{2} \sin \frac{\varphi^-(t_2)}{2} \cos[\varphi^+(t_1) - \varphi^+(t_2)] \\ & - \frac{i}{e^2} \int_0^{t_f} dt C \ddot{\varphi}^+ \varphi^-. \end{aligned} \quad (\text{C15})$$

Here we denoted $\alpha_I(t_1 - t_2) = \delta'(t_1 - t_2)$. This δ function should be understood as a smeared one.

Let us expand Eq. (C15) in φ^- and compare with our results order by order. The first-order terms are exactly the same for both models. The difference between the models shows up in the second-order terms; for our model the parameter β appears explicitly in the second-order contribution (C9). In the limit $\beta \rightarrow 1$ this expression reduces to that obtained from Eq. (C15). Expanding the action (C15) to the third order in φ^- one only recovers the term of the form (C12) with $\beta = 1$, while another term (C13) cannot be recovered. Contributions of this nature are not contained in the AES action at all since they are proportional to higher orders of the channel transmission T_n .

It is worthwhile pointing out that a formally exact representation for the effective action of a coherent scatterer (all orders in T_n and all orders in φ^{\pm}) can also be derived.^{12,13,30,33} However, this formal expression turns out to be quite complicated to deal with in the situation addressed here. For $g = R_q/R \gg 1$ and provided instanton effects^{12,13} can be neglected all necessary information is equally contained in a much simpler form of the effective action derived in the present paper.

We also note that there exists a simple relation between the action derived here and the cumulant generating function describing the full counting statistics of the charge transport in noninteracting coherent conductors.³⁴ This relation can be established if one neglects fluctuations of the phase variable—i.e., sets $\varphi^+ = eVt$ and chooses φ^- to be time independent. By identifying $\varphi^- = -\lambda$ and expanding the generating function $\ln[\chi(\lambda)]$ [defined in Eq. (37) of Ref. 34] in powers of λ one arrives at the following identity:

$$\ln[\chi(\lambda)] = \frac{i}{2} S[eVt, -\lambda].$$

It is easy to prove that this relation is exact—i.e., it holds to all orders in λ .

Finally, let us use the AES action (C15) in order to illustrate the importance of the third order in φ^- terms for the calculation of the current-current correlation functions. Applying Eq. (A14) one gets a contribution stemming from the double differentiation of the term with α_I in the action (C15):

$$\begin{aligned} \delta S(t, t') = & - \left\langle \frac{i}{R} \theta(t - t') \alpha_I(t - t') \sin[\varphi^+(t) \right. \\ & \left. - \varphi^+(t')] \sin \frac{\varphi^-(t) - \varphi^-(t')}{2} \right\rangle + (t \leftrightarrow t'). \end{aligned} \quad (\text{C16})$$

Evaluating this average with the aid of path integrals one has to keep all nonlinear terms in the preexponent. However, the dependence of the cosine term on φ^+ in the action in the exponent can be neglected provided $g \gg 1$. Applying the identity $\sin[\varphi^+(t) - \varphi^+(t')] = \sum_{\nu=\pm} \nu e^{i\nu[\varphi^+(t) - \varphi^+(t')]/2i}$ we arrive at the following integral:

$$\begin{aligned} \int \mathcal{D}\varphi^+ \exp \left\{ i\nu[\varphi^+(t) - \varphi^+(t')] \right. \\ \left. - \frac{i}{e^2} \int_0^{\infty} d\tilde{t} \varphi^+(\tilde{t}) \left[C \ddot{\varphi}^-(\tilde{t}) - \frac{\dot{\varphi}^-(\tilde{t})}{R} \right] \right\}, \end{aligned} \quad (\text{C17})$$

which yields

$$\begin{aligned} \varphi^-(\tilde{t}) = & \frac{2\pi}{g} \nu [\theta(t - \tilde{t})(1 - e^{-(t-\tilde{t})/RC}) \\ & - \theta(t' - \tilde{t})(1 - e^{-(t'-\tilde{t})/RC})]. \end{aligned} \quad (\text{C18})$$

As was expected, $\varphi^-(\tilde{t})$ is indeed small for $g \gg 1$. Combining the above expressions we arrive at the contribution

$$\delta S(t, t') = -e^2 \delta(t - t') / 2RC.$$

The same contribution multiplied by the factor β was derived in Sec. III B from the term $S_{\beta}^{(3)}$, Eq. (31). The above analysis makes the significance of the third order in φ^- terms in the action particularly transparent: The kernel $\alpha_I(t)$ introduces the time derivative of $e^{-t/RC}$ which compensates for an extra smallness $\sim 1/g$.

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