High-temperature oscillations of Bi and $Bi_{1-x}Sb_x$ conductivity in high magnetic fields

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Special quantum oscillations of conductivity in a magnetic field [“high-temperature” oscillations (HTO)] are considered. The oscillations are periodic in the reciprocal magnetic field and are characterized by a frequency higher than that of Shubnikov–de Haas (SdH) oscillations. Results are presented of joint studies of SdH oscillations and HTO of the magnetoresistance for pure Bi and $Bi_{1-x}Sb_x$ (x=2.6 at. %) alloy in magnetic fields up to 33 T. The oscillations are measured over the temperature range 4.3 to 30 K for a variety of magnetic field directions. The high field quantum limit is realized for different ratios of spectral parameters of electrons and holes in Bi and $Bi_{1-x}Sb_x$. In all cases the last HTO minimum coincides with the SdH oscillations corresponding to the quantum limit magnetic field. Analysis of the experimental data allowed selecting one of two alternative HTO models. The oscillations of conductivity result from electron-hole transitions between the Landau levels close to the Fermi level.

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I. INTRODUCTION

Special quantum oscillations of the static conductivity of bismuth placed in a magnetic field was discovered by Bogod *et al*.[1] The oscillations are periodic in the reciprocal magnetic field and are characterized by a frequency higher than that of Shubnikov–de Haas (SdH) oscillations. In contrast to the SdH oscillations observed at temperatures $T \leqslant 4$ K, the new oscillations were registered in the temperature range $6-65$ K and were referred to as “high-temperature” oscillations (HTO’s).[2] At high temperatures, the HTO are observed for $k_B T > \hbar\omega_c$ (ω_c is the characteristic cyclotron frequency), when the SdH oscillations are exponentially small. The HTO can be studied in bismuth samples of various quality, in compensated $Bi_{1-x}Sb_x$ alloys and in noncompensated $Bi_{1-x}(Sb,Te,Sn)_x$ alloys. Thermo-emf HTO in a magnetic field have also been studied.

A correlation between the HTO frequency F^{HTO} and the charge carrier concentration was established in experiments carried out with bismuth and compensated semimetallic bismuth-based alloys.[3] In the more general case of semimetallic alloys of bismuth with different numbers of electrons and holes, it was found in Ref. 4 that $F^{\mathrm{HTO}} \propto \epsilon_F^e + \epsilon_F^h = \epsilon_0$ [ϵ_F^e and ϵ_F^h are the Fermi energies of electrons and holes, respectively, and ϵ_0 is that of the energy band overlap region; ϵ_F^e and ϵ_F^h are counted from the bottom of the electron band, E_e, and from the top of the hole band, E_h, respectively (Fig. 1)]. Thus, a characteristic feature of the HTO distinguishing them from other quantum oscillations in a magnetic field is the independence of the HTO frequency of the Fermi energy ϵ_F.

Basically, the HTO differ from the SdH oscillations in the specific temperature dependence, namely, the HTO amplitude rapidly attains its peak value at $T \approx (10-12)$ K and then decreases slowly upon heating.[2] It was found[5] that the existence of a peak on the HTO amplitude temperature dependence is determined by the relation $(\tau_{ep}^m)^{-1} \gg (\tau_i^m)^{-1}$, where $(\tau_{ep}^m)^{-1} \propto \exp^{-\Theta_m/T}$ is the frequency of the intergroup electron-hole transitions associated with inelastic scattering by acoustic phonons, and $(\tau_i^m)^{-1}$ is the frequency associated with quasielastic scattering by impurities ($\Theta_m = \hbar S q_m / k_B$, with q_m being the phonon wave vector corresponding to the separation between characteristic points on the electron and hole branches of the spectrum [L and T points in Fig. 1(a), and S is the sound velocity].

In the simplest case of a quadratic dispersion law for charge carriers the oscillation period for quasielastic scattering is given by

$$\Delta(1/B)^{HTO} = e\hbar / c m^* \epsilon_0 \tag{1.1}$$

(m^* is effective mass of charge carriers). For the case of inelastic scattering of electrons by acoustic phonons Eq. (1.1) can be rewritten in the form[5]

$$\Delta(1/B)^{\mathrm{HTO}} = e\hbar / m^* c (\epsilon_0 \pm k_B \Theta_m). \tag{1.2}$$

Over the years of experimental investigation of HTO several attempts have been made to treat their nature primarily by using the magnetophonon resonance predicted in Ref. 6 and first observed in Ref. 7. The magnetophonon oscillations (MPO’s) are oscillations of the galvanomagnetic and thermomagnetic effects associated with the sharply inelastic electron scattering by optical phonons when the variation in the electron energy $\Delta\epsilon = \hbar\omega_0$ is greater than the value of $k_B T$ (ω_0 being the limiting frequency of optical phonons). The resonance condition for the transverse magnetoconductivity is that the optical phonon frequency should be a multiple of the cyclotron frequency, or in other words, of the separation between Landau quantum levels, i.e.,

$$\omega_0 = M\omega_c, \tag{1.3}$$

with $M = 1,2,3\ldots$ (see transitions type 1 in Fig. 1). The period of MPO is determined by the effective mass and does not depend on charge carrier concentration $\Delta(1/B)^{\mathrm{MPO}} = e/m^* c\omega_0$. Since the conductivity related to scattering by optical phonons is proportional to $\exp(-\hbar\omega_0/k_B T)$, the contribution of inelastic scattering and the amplitudes of MPO both increase exponentially with temperature at $k_B T < \hbar\omega_0$. However, substitution in Eq. (1.3) of the real values

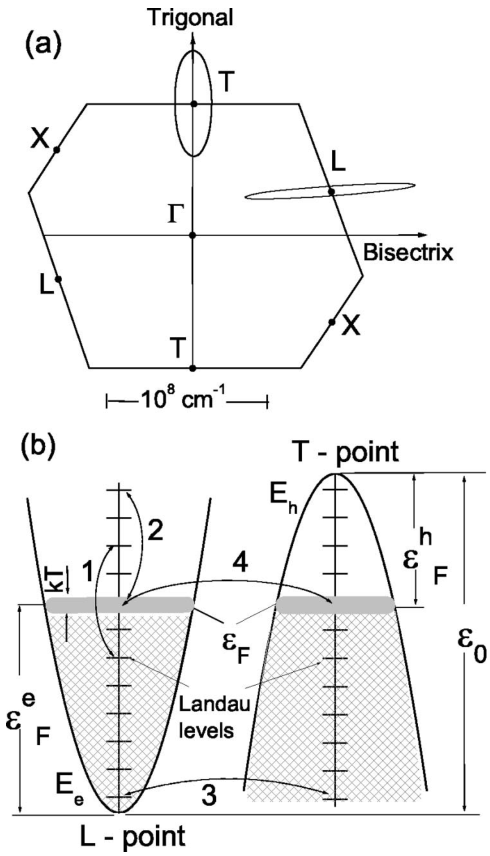

FIG. 1. (a) Location of Fermi surface of bismuth in the Brillouin zone [after Brown *et al.* (Ref. 29)]. The dimensions of the electron (L-point) and hole (T-point) ellipsoids are increased by a factor of about 5.6 relative to those of the zone. (b) Schematic diagram (energy vs wave vector) shows electron and hole Fermi surfaces. To make the figure more clear, the lowest Landau parabolas of electron and hole are shown only. Other parabols are indicated by short lines close to $k_z=0$. Intraband and interband (electron-hole) transitions for different models are represented according to various authors: 1 corresponds to Ref. 6, 2 to Ref. 10, 3 to Ref. 12, and 4 to Ref. 14.

typical of bismuth, for example, $m^*=0.063m_0$ (Ref. 8) and $\omega_0=9.2$ or 12.4 meV (Ref. 9) for $\mathbf{B}\|\mathbf{C_3}$, gives HTO periods almost twice as long as the measured ones.

The magnetophonon resonance condition for the longitudinal magnetoconductivity in the case of a degenerate electron gas takes the form[10]

$$\epsilon_F \pm \hbar\omega_0 = \hbar\omega_c(N+1/2) \tag{1.4}$$

($N=0,1,2,\ldots$), i.e., the interaction with optical phonons results in that electrons are transferred from the Fermi level to the bottom of a Landau subband (see transitions type 2 in Fig. 1). In this case the oscillation period is given by $\Delta(1/B)^{\mathrm{MPO}}=e/m^*c(\epsilon_F^e\pm\hbar\omega_0)$. This type of oscillations was observed in the experiments with HgTe and n-GaSb.[10] For anisotropic crystals the analysis of paper[10] can be formally generalized to the case of transverse magnetoresistance because each of the matrix elements $|\rho_{ik}|$ contains all the matrix elements $|\sigma_{ik}|$. Equation (1.4) can be used to describe the experimental relation between the HTO period and the magnetic field.[11] In this case, however, we arrive at a value of ω_0 higher by a factor of 2 than the limiting energy of optical phonons in bismuth.

In principle, the resonance conditions (1.3) and (1.4) can be used also in the case of interaction between charge carriers and other types of quasiparticles. Thus, for instance, an attempt was made[2] to describe quantitatively the experimentally observed evolution of HTO periods in bismuth in magnetic fields up to 5.6 T (Ref. 11) by using a resonance of the type of Eq. (1.4) and a combination of phonon spectral frequencies with limiting plasma frequencies ω_p [19.6 and 23.2 meV (Ref. 8)]. Classical calculation,[2,11] however, gives a too steep dependence $\omega_p(B)$ which does not fit the measured data.

Recently, two alternative models were suggested to explain the HTO nature.

(1) The writers[12,13] suppose that in the case of a two-band semimetal in the temperature region $T \ll \epsilon_F/k_B$ the number of unoccupied states below the Fermi level ϵ_F (and the number of occupied states above ϵ_F) is not exponentially small but is determined by the energy levels broadening due to relaxation processes (both elastic and inelastic). Then according to Refs. 12 and 13 the HTO emerge due to electron-hole transitions near the boundaries of the energy bands in the L and T points of the Brillouin zone (see transitions type 3 in Fig. 1). When the magnetic field increases a Landau subband extremum for the hole band of the spectrum pass the bottom of the conduction band, and the collision frequency suffers a discontinuity because the electron state density is zero below the bottom of the conduction band E_e. This is also true for transitions between the Landau levels of the electron branch of the spectrum and the top of the valence band E_h. The values of E_e and E_h are the specific features of the energy spectrum and at the same time singularities of the electron-impurity scattering frequency.

(2) In papers Refs. 14 and 15 an alternative model for explanation of the HTO nature was offered. According to Refs. 14 and 15 the oscillations of conductivity result from electron-hole transitions between the Landau levels close to the Fermi level (see transitions type 4 in Fig. 1). The quantum theory of magnetoresistence in semimetals was built in Refs. 16 and 17. However, in Ref. 16 Landau quantum levels in the hole zone were not taken into account, while in Ref. 17 only the quantum limit was considered. So, the HTO are not described by these models. According to Refs. 14 and 15, the origin of HTO is as follows. As the magnetic field is varied, Landau levels of states with different signs of the effective mass are shifted in opposite directions along the energy scale. Isoenergetic or phonon absorbing (emitting) electron transitions between sharp features in the density of states are made possible near resonant values of magnetic field. Since the density of states near the Landau levels is characterized by a root singularity, the transition probability near a resonant value of the magnetic field experiences a jump of finite magnitude. This distinguishes the HTO from SdH oscillations and MPO which show, if the collisional broadening of Landau levels is neglected, a logarithmic divergence at the resonance. Another distinctive feature is due to the fact that regions with different signs of effective mass

are at different points of the Brillouin zone and the transitions between these points are accompanied by a drastic variation in the quasimomentum (of the order of reciprocal lattice parameter). Therefore, even for the interaction with acoustic phonons the scattering may be of resonant character. The necessary condition for HTO appearance in the model[14,15] is that the effective electron and hole masses should be commensurable ($km_e^* = k' m_h^*$ with integers k and k'). In this case the appropriate extrema of the electron and hole Landau subbands have a common tangent in the vicinity of the Fermi level, and hence, the electron and hole Landau levels periodically coincide in energy near ϵ_F, producing oscillations of conductivity via electron-hole transitions.

The facts described above lead to the conclusion that the origin of HTO could probably be determined with the help of experiments in which energy spectrum parameters (the Fermi energy, the energy of overlapping of the conduction and valence bands, and charge carrier concentration) can be varied over a wide range. Such experiments can be carried out for Bi and $Bi_{1-x}Sb_x$ alloys in a strong (ultraquantum, $h\omega_c > \epsilon_F^{e,h}$) magnetic field, where the energy parameters would strongly depend on the magnetic field. When a small amount of Sb is doped into Bi, the energy ϵ_0 decreases, and hence the characteristic singularities occur at lower magnetic fields. Information concerning variations in the energy spectrum can be obtained from the Shubnikov–de Haas oscillations with the help of an appropriate theoretical model of the energy spectrum.

It seems that final conclusions on the physical nature of HTO can be made after a set of measurements of the SdH oscillations and HTO in Bi and $Bi_{1-x}Sb_x$ alloys placed in strong magnetic fields (up to 30–40 T). The SdH oscillations should reach their quantum limit in magnetic fields as high as this. Under these conditions, it would be possible to select between different HTO models judging by the behavior of the oscillations, and to obtain appropriate conclusions on their nature. In this paper, we report on the results of joint measurements of the SdH oscillations and HTO in nondiagonal and diagonal components of the magnetoresistance tensor ρ_{ik} that were made on monocrystalline Bi and $Bi_{1-x}Sb_x$ (x $=2.6$ at. %) specimens at temperatures 4.3–30 K and magnetic fields up to 33 T. These experiments were aimed at obtaining information on the HTO origin from comparison of changes in the energy spectrum of Bi in the ultraquantum limit with the features shown by the HTO.

II. EXPERIMENTAL PROCEDURE AND RESULTS

The measurements of SdH oscillations and HTO were made in dc magnetic fields up to 33 T on pure Bi (the room/helium temperature resistance ratio at a zero magnetic field $\simeq 220$) and $Bi_{1-x}Sb_x$ ($x = 2.6$ at. %) specimens, under the conditions when $I \| C_1$ (or C_2), $B \perp C_1$ (or C_2) (here I is the current; and C_3, C_2, and C_1 the trigonal, binary, and bisector crystallographic axes, respectively). Both diagonal and nondiagonal components of the magnetoresistance tensor ρ_{ik} were measured. All measurements were taken using low frequency (<100 Hz) ac techniques. "High temperature" oscillations were detected reliably in the first $\rho'_{ik}(B)$ or second

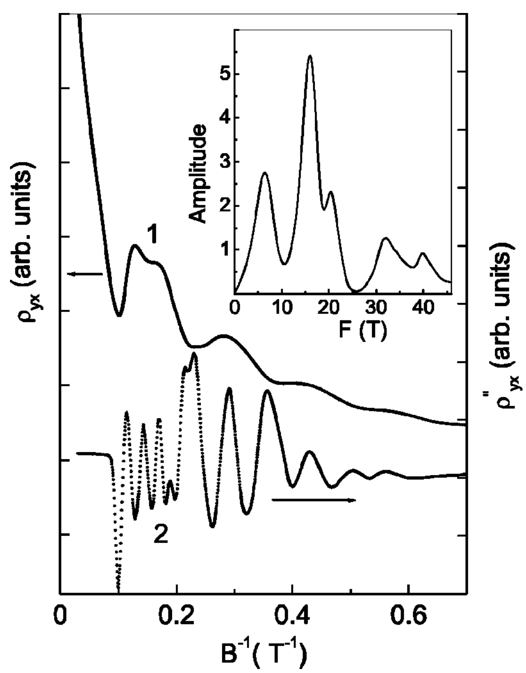

FIG. 2. Dependences of $\rho_{yx}(1/B)$ at 4.3 K (curve 1) and $\rho''_{yx}(1/B)$ at 15 K (curve 2), $\mathbf{B} \| \mathbf{C_3}$, in bismuth. In the insert: frequency spectrum of oscillations at 15 K.

$\rho''_{ik}(B)$ derivative obtained as a result of computer processing of the corresponding $\rho_{ik}(B)$ curve. Since the HTO amplitude does not exceed $0.1-1$ % of the monotonic component of $\rho_{ii}(B)$, one of the necessary conditions for measuring the magnetoresistance of HTO is high stability of temperature on the specimen (not worse than 1 mK). In our measurements, we used the following special property of the nondiagonal component ρ_{yx} of the magnetoresistance tensor which allowed us to considerably improve the sensitivity, and hence resolution of the measurements. The monotonic component of the even part ρ_{yx} vanishes when $\mathbf{B}$ is oriented strictly as $\mathbf{B} \| \mathbf{C_3}$ because of the symmetry, while the oscillating component remains unchanged.[18] It is worth noting that close to a given magnetic field direction the HTO manifest themselves directly in the dependence $\rho_{yx}(B)$. The nondiagonal magnetoresistance tensor component ρ_{zx} was measured for the orientation $\mathbf{B} \| C_2$. In this case, the monotonic component of the even part of ρ_{zx} does not vanish, but during the recording of $\rho_{zx}(B)$, the ratio of the oscillating and monotonic components increases significantly in contrast to the diagonal component ρ_{xx}. The considerable decrease in the monotonic component results in an increase of the signal-to-noise ratio. At temperatures 4 to 15 K the SdH oscillations prevail in the dependencies $\rho_{ik}(B)$, while the HTO dominate above ≈ 15 K.

The experimental results on Bi and $Bi_{1-x}Sb_x$ ($x = 2.6$ at. %) specimens for SdH oscillations (4.3 K) and HTO (15–30 K) are presented in Figs. 2–4. The SdH oscillations and the HTO were measured with the magnetic field directed along the principal crystallographic axes C_3 and C_2 or close to these axes. In these cases the quantum limit in the SdH oscillations are realized for different combinations of electron and hole energy parameters.

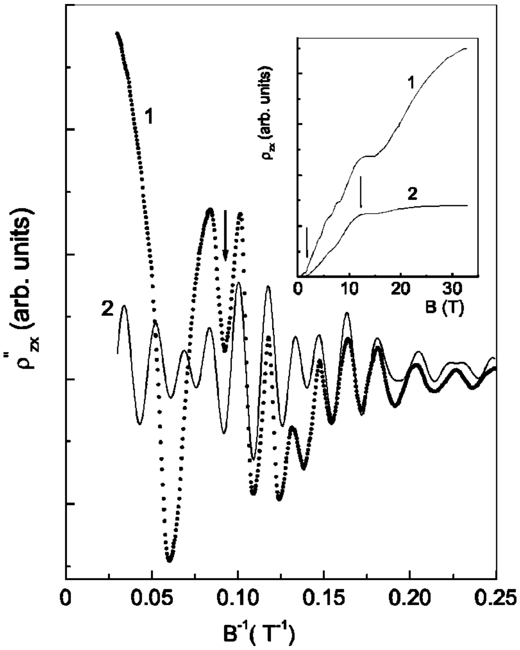

FIG. 3. "High-temperature" oscillations of resistivity $\rho''_{zx}(1/B)$ in bismuth for $\mathbf{B} \parallel C_2$, $T=20$K. Curve 1 corresponds to the experiment, and curve 2 is the result of computer simulation. The inset shows dependences of $\rho_{zx}(B)$ on magnetic field $\mathbf{B}\|\mathbf{C_2}$: curves 1 and 2 correspond to $\rho_{zx}(B)$ at 4.3 and 20 K, respectively. Transition to ultraquantum limit for light and heavy electrons are marked by the arrows.

Presented in the Fig. 2 is the $\rho_{yx}(1/B)$ dependence at 4.3 K (curve 1) and the second derivative $\rho''_{yx}(1/B)$ at 15 K (curve 2), obtained on a bismuth sample when the magnetic field was aligned with the trigonal crystallographic axis $\mathbf{B}\|\mathbf{C_3}$. The period of SdH oscillations, as a function of the inverse magnetic field, is $P_h^{\mathrm{SdH}}=0.155\ \mathrm{T}^{-1}$ (or, in terms of frequency, $F_h^{\mathrm{SdH}}=6.45$T). This value corresponds to the area of the extremal section of the Fermi surface holes.[19,20] Fourier analysis of the oscillations of curve 2 indicates the presence of SdH periods for holes $P_h^{\mathrm{SdH}}=0.155\ \mathrm{T}^{-1}(F_h^{\mathrm{SdH}}=6.45\ \mathrm{T})$ as well as HTO periods $P_1^{\mathrm{HTO}}=0.062\ \mathrm{T}^{-1}(F_1^{\mathrm{HTO}}=16\ \mathrm{T})$ and $P_2^{\mathrm{HTO}}=0.051\ \mathrm{T}^{-1}(F_2^{\mathrm{HTO}}=19.8\ \mathrm{T})$ (see the frequency spectrum of oscillations in the inset of Fig. 2). As is usually the case with bismuth, HTO are observed as a superposition of two frequencies differing by a factor 1.22.[2] The oscillation frequency also displays second harmonics of the fundamental frequencies $2F_1^{\mathrm{HTO}}=32$ T and $2F_2^{\mathrm{HTO}}=39.6$ T. The occurrence of the second HTO harmonic is easily seen in curve 2 of Fig. 2: beginning with $B\approx 4.3$ T the HTO frequency increases by a factor of 2.

The experimental dependence $\rho_{zx}(B)$ measured at 4.3 and 20 K for a magnetic field parallel to the binary axis $\mathbf{B}\|C_2$ is shown by curves 1 and 2 in the inset of Fig. 3.[21] The magnetic field values corresponding to the minima of SdH oscillations obtained in our experiments are close to those obtained in Refs. 19 and 22–24. For the given orientation of the magnetic field, oscillations from two electron ellipsoids with equal small cyclotron masses, an electron ellipsoid with heavy masses, and a hole ellipsoid with heavy masses are observed. The form of the $\rho_{zx}(B)$ dependence in a weak magnetic field is determined by the passage of Landau levels of light electrons through the Fermi level for which the quantum limit takes place near $B\approx 1.5$ T, i.e., when the level (1, -) intersects with the Fermi level. The structure of oscillations in magnetic fields stronger than 1.5 T is determined by the Landau levels of heavy holes and heavy electrons. A fragment of the dependence $\rho''_{zx}(1/B)$ is shown in Fig. 3 (curve 1). Once again, HTO are observed here as a superposition of two frequencies differing by a factor of 1.22.[2] Fourier analysis of the oscillations of curve 1 indicates the presence of HTO periods $P_1^{\mathrm{HTO}}=0.016T^{-1}(F_1^{\mathrm{HTO}}=62.5\ \mathrm{T})$ and $P_2^{\mathrm{HTO}}=0.02\ \mathrm{T}^{-1}\ (F_2^{\mathrm{HTO}}=50\ \mathrm{T})$. The frequency spectrum of oscillation also displays second harmonics of the fundamental frequencies.

The experimental results for the $\mathrm{Bi}_{1-x}\mathrm{Sb}_x$ specimen ($x=2.6$ at. %) with the magnetic field applied close to C_3 (in the trigonal-bisector plane $\angle \mathbf{B},C_3=5$) are shown in Fig. 4. One can see the first derivatives of $\rho_{zx}(B)$ measured at 4.3, 20, and 30 K. As will be indicated below, the HTO and SdH oscillations in the quantum limit give the best correlation just in the first derivative $\rho'_{zx}(B)$: the dependence $\rho_{zx}(B)$ for 4.3 K shows no HTO, while the SdH oscillations in the curve $\rho''_{zx}(B)$ for 30 K are suppressed both by temperature and the higher frequency HTO.

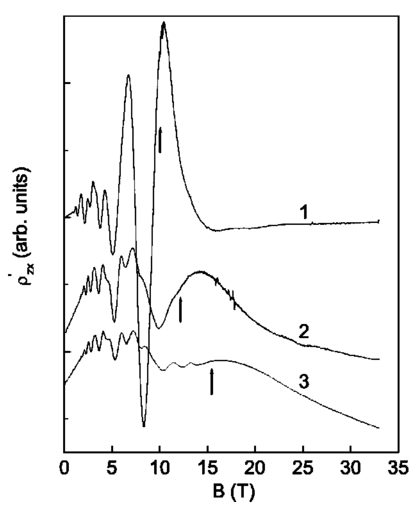

FIG. 4. The oscillations of resistivity ρ'_{zx}(B) in $\mathrm{Bi}_{1-x}Sb_x$ ($x=2.6\ at.\%$) for $\angle\mathbf{B},\ C_3\approx 5°$; curve 1, 2, and 3 measured at 4.3, 20, and 30 K, respectively.

III. DISCUSSON

We analyzed the experimental data pertaining to the SdH effect, using the results obtained in Refs. 19 and 22–24, in which variation in the position of the Fermi level and charge carrier concentration in bismuth in strong magnetic fields were calculated from resonance magnetic field values for which the Landau level energies were equal to the energy of the Fermi level. The position of the Fermi level is determined by the conditions $n_l+n_h=p$, where n_l, n_h, and p are concentrations of light electrons, heavy electrons, and holes,

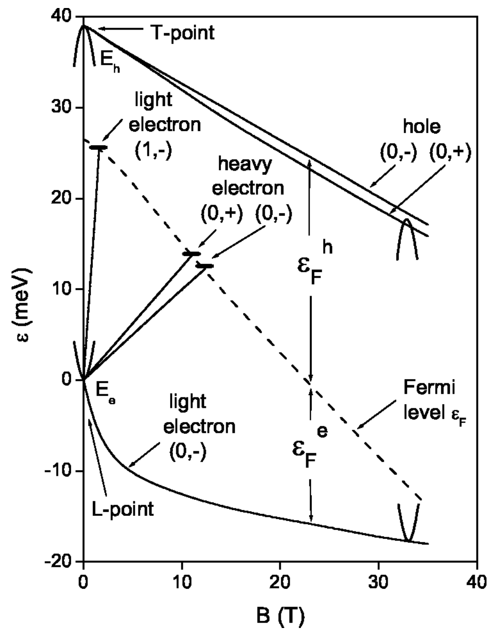

FIG. 5. Schematic diagram concerning the relations of the points E_e (the bottom of the electron band), E_h (the top of the hole band), and ϵ_F (the Fermi level) with magnetic field for **B** parallel to the binary axis [after Hiruma *et al.* (Ref. 24)].

respectively. The theoretical model which was initially proposed by Smith *et al.*[19] and later modified by Vecchi *et al.*,[22] has allowed interpreting the results on magnetooptical reflection in high magnetic fields,[22] as well as on the SdH effect.[23,24]

When the magnetic field is oriented along the binary axis C_2 and its magnitude is $B>1.5$ T, all light electrons lie at the level (0,-) (see Fig. 5). The charge carrier concentration in this case increases with the magnetic field for two reasons: (1) the linear magnetic field dependence of the density of states at the level (0,-) and (2) the downward displacement of this level on the energy scale in view of the lower spin mass of light electrons compared with the orbital mass [$\gamma=1.1$,[8] where $\gamma=\Delta(1/B)^s/\Delta(1/B)^{\mathrm{orb}}$ is the spin-to-orbital splitting ratio]. The orbital splitting is stronger for heavy electrons than the spin splitting [$\gamma=0.25$ Ref. 8], and the corresponding lower Landau level (0, -) moves upwards along the energy scale upon an increase in B, intersecting the Fermi level at $B\approx 12$ T (Fig. 5). Upon disappearance of heavy electrons, the equality $n_l=p$ holds. The effects associated with energy spectrum variations, e.g., transition to the ultraquantum limit for light electrons at ≈ 1.5 T and for heavy electrons at ≈ 12 T, are clearly pronounced on the $\rho_{zx}(B)$ dependence measured at 20 K against the background of small amplitude SdH oscillations for holes (curve 2, inset in Fig. 3). According to Ref. 24, the Fermi level ϵ_F^e for electrons decreases from 26.5 meV ($B=0$) to ≈ 5 meV ($B=33$ T), while the Fermi level ϵ_F^h for holes increases from 11.7 meV ($B=0$) to ≈ 31 meV ($B=33$ T). The charge carrier concentration increases by a greater factor than 5 and is equal to 3×10^{17} cm^{-3} ($B=0$) and 16×10^{17} cm^{-3} ($B=33$ T). The upper Landau level (0, -) for holes moves downwards on the energy scale upon an increase in B [$\gamma=0.05$ (Ref. 8)], which leads, together with a similar displacement of the lower energy level (0, -) for light electrons, to a small change in the energy of band overlapping: $\epsilon_0\approx 38\, meV(B=0)$ and ≈ 35 meV ($B=33$ T).[22]

When the magnetic field is oriented along the trigonal axis, the SdH magnetoresistance oscillations are associated with electrons and holes of identical cyclotron masses of 0.063 m_0.[20,25] As is evident from Fig. 2, the structure of $\rho_{yx}(1/B)$ at 4.3 K is dictated by the SdH oscillations of holes. As the magnetic field is increased, the electron level (0, -) follows the energy upwards because the spin mass of electrons for the given direction is greater than the orbital one $\gamma=0.53$.[8] The same is true for the edge energy of the hole band (0, -), because the orbit mass of holes is more than twice as large as the spin mass [$\gamma=2.15$ (Refs. 8 and 25)]. As a result, the quantity $\epsilon_0=\epsilon_F^e+\epsilon_F^h$ sensitive to velocities of the relative motion of the conduction band bottom and the valence band top begins to vary (to increase) considerably only in fields above 20 T.[19] The last hole minimum at $B\approx 10$ T corresponds to crossing of the Fermi surface of the level (1, +).[25] As is seen clearly from Fig. 2 the quantum limit for SdH oscillations and the last minimum HTO in Bi for $\mathbf{B}\|\mathbf{C_3}$ occur at the same value of the magnetic field $B_{\mathrm{QL}}\approx 10$ T. For a magnetic field declined by some angle from the trigonal axis the SdH oscillations of holes are determined by the heavier mass, and intersection of the Fermi surface by the level (1, +) occurs in a higher magnetic field. In this case the HTO are also observed up to this value of magnetic field.

The period of SdH oscillations for holes for the direction of $\mathbf{B}\|\mathbf{C_2}$ in fields $B>1.5$ T is 0.072 T^{-1} and is twice as large as the corresponding value in the preultraquantum range of magnetic fields. This is due to the monotonic increase in the value of ϵ_F^h with B. On the other hand, the HTO periods experience relatively small changes and remain almost the same in the preultraquantum and the ultraquantum ranges of magnetic fields (0.017 and 0.019 T^{-1}), in spite of the considerable changes in the values of ϵ_F^e, ϵ_F^h and charge carrier concentration. It seems natural to put the HTO in correspondence with the spectral parameter remaining unchanged in the given experimental situation, namely, the width of the region of band overlapping $\epsilon_0=\epsilon_F^e+\epsilon_F^h$. The conclusion of HTO frequencies being determined by the energy interval ϵ_0 is in agreement with the results of Refs. 3,4,26, and 27, where the value of ϵ_0 was varied by one way or another. Analysis of the results of our experiments, together with those obtained in Refs. 3,4,26, and 27, leads to the following conclusion. HTO periods vary in all experiments in which the energy parameters of the spectrum and ϵ_0 change simultaneously (BiSb alloys,[3] axial compression of crystals,[27] and temperature deformation of the lattice[26]) while remaining unchanged in the experiments where the energy parameters of the spectrum change while ϵ_0 is fixed [uncompensated alloys BiTe, BiSbTe, and BiSbSn (Ref. 4) and measurements in the ultra-quantum limit as in the present experiments].

Another singularity of HTO presented in Fig. 3 is worth noting. This singularity becomes more pronounced if we compare the experimental dependence $\rho''_{zx}(1/B)$ with the similar oscillatory curve simulated on a computer with the help of a band frequency filter whose limits are determined by measured HTO frequencies 50 and 62.5 T. It can be clearly seen that the experimental curve 1 coincides with the calculated curve 2 only to the value $1/B \approx 0.09\ \mathrm{T}^{-1}$ at which the last oscillation minimum for HTO (marked by a arrow in the figure) is observed. The next maximum and minimum belong to SdH oscillations for holes and, accordingly, are characterized by a greater period. It is natural to associate correspondence the vanishing of HTO with the last energy level (0, -) for heavy electrons crossing the Fermi level at $B \approx 12$ T.

And finally, we consider the results obtained with the $Bi_{1-x}Sb_x$ alloy for the magnetic field directed nearly along the axis C_3 (in the trigonal-bisector plane $\angle \mathbf{B}$, $C_3 \approx 5°$). As a rule, BiSb alloys display HTO only for magnetic field directions lying within a small angles ($\pm 10°$) in the vicinity of C_3.[3] Unlike bismuth, the alloy exhibits SdH oscillations of both holes and electrons. For $T = 4.3$ K the hole level $(1, +)$ intersects the Fermi surface at $B_{QL} \approx 4.5$ T; in the case of electrons $B_{\mathrm{QL}} \approx 9.5$ T (Fig. 4). It should be emphasized that as the spin splitting of electrons with the direction of **B** nearly along C_3 is two times less than the orbital one ($\gamma \approx 0.5$), the experiments demonstrated a double frequency of the SdH electron oscillations. When comparing HTO and SdH oscillations, the following factors should be taken into account. As the temperature is increased from ≈ 20 K in bismuth and ≈ 10 K in $Bi_{1-x}Sb_x$ alloys, the HTO period decreases[26] as a consequence of increase in the charge carrier concentration.[28] The increased charge carrier concentration manifests itself as a shift of the oscillations extrema toward higher magnetic fields. In Fig. 4 the points corresponding to the last minima of the SdH oscillations at three different temperatures are marked by arrows. Curve 3 measured at 30 K demonstrates clearly the correspondence between B_{QL} at this temperature and the last HTO minimum.

In the experiments under consideration the quantum limit $\hbar\omega_c^e > \epsilon_F^e$ (or $\hbar\omega_c^h > \epsilon_F^h$) at high magnetic fields is realized for different ratios between spectral parameters of electrons and holes in Bi and $Bi_{1-x}Sb_x$ alloy. While for **B** oriented near C_3 the electron Fermi energy increases and the Fermi energy of holes decreases with a growing magnetic field, for **B** oriented in the vicinity of C_2 the effect is directly opposite. In all cases the last HTO minimum coincides with the quantum limit magnetic field B_{QL} for the SdH oscillations. This behavior of HTO is typical of the oscillation mechanism proposed in Ref. 15. According to Ref. 15, the HTO are nominally SdH oscillations but with due regard for interband transitions nearby the Fermi level, followed by a change of sign of the effective mass. In this case it is reasonable that the quantum limit for SdH oscillations of both electrons and holes is bound to show itself also in HTO in the same way. It is particularly remarkable that the last minimum of HTO, which are a collective electron-hole effect, coincides with the field B_{QL} of either electrons or holes, depending on experimental conditions. At the same time, the model proposed in Ref. 12 predicts observable HTO until the condition $0.5\hbar\omega_c^h > \epsilon_F^e + \epsilon_F^h$ is fulfilled. It is evident that this condition corresponds to magnetic fields much higher than that observed in the present paper.

Thus, the results obtained favor the HTO type model given in Ref. 15. The oscillations of conductivity result from electron-hole transitions between the Landau levels close to the Fermi level. In principle, this model permits one to treat qualitatively the basic properties of HTO. The period is given as $P^{\mathrm{HTO}} = 2\pi e/c\hbar S_{kk'}$, where $S_{kk'} = kS_e + k'S_h$, $S_{e,h} = 2\pi m_{e,h}\epsilon_F^{e,h}$ are the areas of extremal sections of electron and hole sheets of the Fermi surface by a plane normal to the magnetic field. The temperature dependence of the oscillation amplitude is $\approx \exp(-2\pi^2 k_B T/\hbar\omega_{kk'})$, where $1/\omega_{kk'} = |k/\omega_c^e - k'/\omega_c^h|$, ω_c^e and ω_c^h are the cyclotron frequencies of electrons and holes. The harmonics with a maximum $\omega_{kk'}$, i.e., $km_e \approx k'm_h$, are most slowly damped with increasing temperature and, hence, make the main contribution to HTO. The low temperature "freezing out" of the phonons involved in the interband transitions results in a maximum in the temperature dependence of the HTO amplitude. The occurrence of two fundamental frequencies of HTO with a ratio of 1.22 is accounted for by the inelastic nature of the interband electron scattering by acoustic phonons. But to treat the HTO peculiarities in the ultraquantum magnetic field range, for example, the oscillation frequency doubling, requires corresponding theoretical considerations.

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