

# Manifestation of triplet superconductivity in superconductor-ferromagnet structures

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We study proximity effects in a multilayered superconductor/ferromagnet ( $S/F$ ) structure with arbitrary relative directions of the magnetization  $\mathbf{M}$ . If the magnetizations of different layers are collinear, the superconducting condensate function induced in the  $F$  layers has only a singlet component and a triplet one with a zero projection of the total magnetic moment of the Cooper pairs on the  $\mathbf{M}$  direction. In this case the condensate penetrates the  $F$  layers over a short length  $\xi_J$  determined by the exchange energy  $J$ . If the magnetizations  $\mathbf{M}$  are not collinear, the triplet component has, in addition to the zero projection, the projections  $\pm 1$ . The latter component is even in the momentum, odd in the Matsubara frequency and penetrates the  $F$  layers over a long distance that increases with decreasing temperature and does not depend on  $J$  (the spin-orbit interaction limits this length). If the thickness of the  $F$  layers is much larger than  $\xi_J$ , the Josephson coupling between neighboring  $S$  layers is provided only by the triplet component, so that a new type of superconductivity arises in the transverse direction of the structure. The Josephson critical current is positive (negative) for the case of a positive (negative) chirality of the vector  $\mathbf{M}$ . We demonstrate that this type of the triplet condensate can be detected also by measuring the density of states in  $F/S/F$  structures.

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## I. INTRODUCTION

Multilayered superconductor/ferromagnet ( $S/F$ ) structures are under an intensive study now (for a recent review see, e.g., Ref. 1). The interest in such systems originates from a possibility to find new physical phenomena as well from the hope to construct new devices based on these structures. Although a ferromagnet  $F$  attached to a superconductor  $S$  is expected to suppress the order parameter in  $S$ , under certain conditions superconductivity and ferromagnetism may coexist and exhibit interesting phenomena.

One of them is a nonmonotonic dependence of the critical temperature  $T_c$  of the superconducting transition in  $S/F$  multilayered structures on the thickness  $d_F$  of the ferromagnetic layers. Theory of this effect has been developed in Refs. 2, and experimental results have been presented in Refs. 3.

Another interesting phenomenon is a  $\pi$  state that can be realized in  $SFS$  Josephson junctions. It was shown<sup>4</sup> that for some values of parameters (such as the temperature  $T$ , the thickness  $d_F$ , and the exchange energy  $J$ ) the lowest Josephson energy corresponds not to the zero phase difference  $\varphi$ , but to  $\varphi = \pi$  (negative Josephson critical current  $I_c$ ). Detailed theoretical studies of this effect have been presented in many papers.<sup>5,6</sup> The  $\pi$  state has been observed experimentally in Refs. 7.

Later it was discovered that the critical current  $I_c$  in Josephson junctions with ferromagnetic layers is not necessarily suppressed by the exchange interaction and it may even be enhanced. Such an enhancement of  $I_c$  has been demonstrated by the present authors on a simple model of a  $SF/II/FS$  junction, where  $I$  stands for a thin insulating layer.<sup>8</sup> It was shown for thin  $S$  and  $F$  layers that at low temperatures the critical current  $I_c$  in a  $SF/II/FS$  junction may become even larger than that in the absence of the exchange field (i.e., if the  $F$  layers are replaced by  $N$  layers, where  $N$  is a

nonmagnetic metal). More detailed calculations of  $I_c$  (for arbitrary  $S/F$  interface transmittance) for this and similar junctions have been performed later in Refs. 9.

Properties of superconductors in  $S/F$  structures may change not only due to the proximity effect but also due to the long-range magnetic interaction. A spontaneous creation of vortices caused by the magnetic interaction has been predicted in a  $S/II/F$  system ( $I$  is an insulating layer).<sup>10</sup> In most papers on  $S/F$  structures the case of collinear (parallel or antiparallel) orientations of the magnetization  $\mathbf{M}$  was considered. If the magnetization vector  $\mathbf{M}$  is not constant in space, as in a domain wall, or if the orientations of  $\mathbf{M}$  in different  $F$  layers are not collinear to each other, a qualitatively new and interesting effect occurs. For example, if a ferromagnetic wire is attached to a superconductor, a domain wall in the vicinity of the interface can generate a triplet component of the superconducting condensate<sup>11</sup> (a similar case was analyzed in a later work<sup>12</sup>).

The existence of the triplet component (TC) has far reaching consequences. It is well known that the singlet component (SC) penetrates into a ferromagnet over the length  $\xi_J = \sqrt{D_F/J}$ , where  $D_F$  is the diffusion coefficient in  $F$ . In contrast, it was shown that even for  $J \gg T$  the TC penetrates  $F$  over a much longer distance  $\xi_T = \sqrt{D_F/2\pi T}$ . This long-range penetration of the TC might lead to an increase of the conductance of the  $F$  wire if the temperature is lowered below  $T_c$ .<sup>11,12</sup>

In this paper we consider a multilayered  $S/F$  structure. Each  $F$  layer has a constant magnetization  $\mathbf{M}$  but the direction of the  $\mathbf{M}$  vector varies from layer to layer. We show that, in this case, the triplet component of the superconducting condensate is also generated and it penetrates the  $F$  layers over the long length  $\xi_T$ , which does not depend on the large exchange energy  $J$  at all.

If the thickness of the  $F$  layers  $d_F$  is much larger than  $\xi_J$ , then the Josephson coupling between adjacent  $S$  layers and, therefore, superconductivity in the transverse direction, are due to the TC. In the vicinity of the  $S/F$  interface the amplitudes of the SC and TC may be comparable but, unlike the TC, the SC survives in  $F$  only over the short distance  $\xi_J$  from the  $S/F$  interface. In other words, in the multilayered  $F/S$  structures with a noncollinear magnetization orientation, a new type of superconductivity arises. The nondissipative current within the layers is due to the  $s$ -wave singlet superconductivity, whereas the transversal supercurrent across the layers is due to the  $s$ -wave, triplet superconductivity.

It is important to emphasize (see Ref. 11) that the TC in this case differs from the TC realized in the superfluid  $^3\text{He}$  and, for example, in materials like  $\text{Sr}_2\text{RuO}_4$ .<sup>13</sup> The triplet-type superconducting condensate we predict here is symmetric in momentum and therefore is insensitive to nonmagnetic impurities. It is odd in frequency and is called sometimes odd superconductivity.

This type of the pairing has been proposed by Berezinskii<sup>14</sup> in 1975 as a possible candidate for the mechanism of superfluidity in  $^3\text{He}$ . However, it turned out that another type of pairing was realized in  $^3\text{He}$ : triplet, odd in momentum  $p$  (sensitive to ordinary impurities) and even in the Matsubara frequencies  $\omega$ . Attempts to find conditions for the existence of the odd superconductivity were undertaken later in several papers in connection with the pairing mechanism in high  $T_c$  superconductors<sup>15</sup> (in Ref. 15 a singlet pairing odd in frequency and in the momentum was considered). It is also important to note that while the symmetry of the order parameter  $\Delta$  in Refs. 13–15 differs from that of the BCS order parameter, in our case  $\Delta$  is nonzero only in the  $S$  layers and is of the BCS type. It is determined by the amplitude of the singlet component. Since the triplet and singlet components are connected which each other, the TC affects  $\Delta$  in an indirect way.

Therefore the type of superconductivity analyzed in our paper complements the three known types of superconductivity:  $s$ -wave and  $d$ -wave singlet superconductivity that occur in ordinary superconductors and in high- $T_c$  superconductors, respectively and the  $p$ -wave superconductivity with triplet pairing observed in  $\text{Sr}_2\text{RuO}_4$ .

In addition, the new type of the triplet superconductivity across the  $S/F$  layers shows another interesting property related to the chirality of the magnetization  $\mathbf{M}$ . If the angle of the magnetization rotation  $2\alpha$  across the  $S_A$  layer (see Fig. 7) has the same sign as the angle of the  $\mathbf{M}$  rotation across the  $S_B$  layer, then the critical Josephson current  $I_c$  between  $S_A$  and  $S_B$  is positive. If these angles have different signs, then the critical current  $I_c$  is negative and the  $\pi$  state is realized (in this case spontaneous supercurrents arise in the structure). This negative Josephson coupling, which is caused by the TC and depends on chirality, differs from that analyzed in Ref. 4. Depending on the chirality an “effective” condensate density in the direction perpendicular to the layers may be both positive and negative.

We note that a dependence of the Josephson current on chirality has also been obtained in Ref. 16. The authors of Ref. 16 considered two magnetic superconductors  $S_m$  with

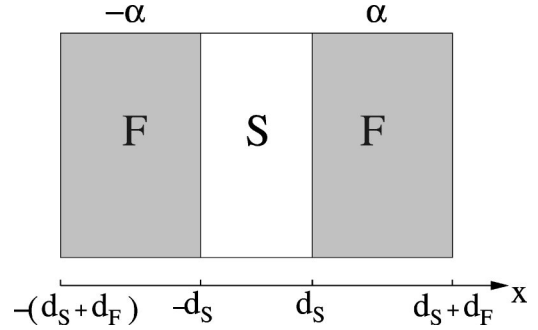


FIG. 1. The  $F/S/F$  trilayer. The magnetization vectors in the  $F$  layers make an angle  $\pm\alpha$  with the  $z$  axis, respectively.

spiral magnetization, separated by a thin insulating layer  $I$ . In the latter case the TC exists in the bulk superconductors together with the SC (and they cannot be separated), and the Josephson current depends on the chirality of the spiral structures. The main difference between our system and the system considered by the authors of Ref. 16 is that in our case only the long-range TC survives in the  $F$  layers, whereas in the  $S_mIS_m$  junction both the SC and TC exist simultaneously. Therefore in the case of a collinear alignment of  $\mathbf{M}$ , the Josephson coupling (and triplet superconductivity in the transverse direction) disappears in our system, whereas it remains in the  $S_mIS_m$  system.

Another possible detection of the TC in the  $S/F$  structures may be achieved by measuring the density of states (DOS) in a  $F/S/F$  trilayer (see Fig. 1). We will see that the long-range TC causes a measurable change of the local DOS at the outer side of the  $F$  layers even if  $d_F$  is much larger than  $\xi_J$ .

The plan of this paper is as follows. In the next section we make some preliminary remarks concerning the TC in  $S/F$  structures. We consider a three-layer  $FSF$  structure and calculate the condensate function in this structure. We show that the amplitude of the TC is proportional to  $\sin\alpha$  and its long-range part is an odd function of the Matsubara frequency  $\omega$  (the SC is an even function of  $\omega$ ), where  $\pm\alpha$  is the angle between the  $z$  axis and the magnetization in the right (left)  $F$  layers. We discuss properties of the TC and calculate the DOS related to it. In Sec. III we calculate the Josephson current between adjacent  $S$  layers and discuss its dependence on the chirality of the magnetization variation in the system. In Sec. IV we take into account spin-orbit interactions and study the effect of this interaction on the TC. In the conclusion we discuss the obtained results and possibilities of an experimental observation of the predicted effects. The odd triplet superconductivity in  $F/S$  structures was first predicted by the present authors in a short paper where the case of small angles  $\alpha$  and of a perfect  $F/S$  interface was considered.<sup>17</sup>

## II. THE CONDENSATE FUNCTION IN A $F/S/F$ SANDWICH

In order to get a better understanding of the properties of the superconducting condensate in the presence of the ferromagnetic layers, we consider in this section a simple case of a trilayered  $F/S/F$  structure (see Fig. 1). Generalization to a

multilayered structures is of no difficulties and will be done in the next section.

In the most general case, when the magnetization vectors  $\mathbf{M}$  of the  $F$  layers are noncollinear, the electron Green functions are  $4 \times 4$  matrices in the Nambu (particle-hole)  $\otimes$  spin space. The  $4 \times 4$  matrix Green functions have been introduced long ago<sup>18</sup> and used in other papers.<sup>19</sup> Later on they were used in Ref. 20 for a description of magnetic superconductors with a rotating magnetization.

A very convenient way for the study of proximity effects is the method of quasiclassical Green's functions.<sup>21-23</sup> Equations for the quasiclassical Green's functions have been generalized recently to the case of a nonhomogeneous exchange field (magnetization)  $\mathbf{M}$ .<sup>24</sup>

Following the notation of Ref. 6 we represent the quasiclassical Green functions in the form

$$\check{g} = g_{ss'}^{nn'} = \frac{1}{\pi} \sum_{n''} (\hat{\tau}_3)_{nn''} \int d\xi_p \langle \psi_{n''s}(t) \psi_{n's'}^\dagger(t') \rangle, \quad (1)$$

where the subscripts  $n$  and  $s$  stand for the elements in the Nambu and spin space, respectively, and  $\hat{\tau}_3$  is the Pauli matrix. The field operators  $\psi_{ns}$  are defined as  $\psi_{1s} = \psi_s$  and  $\psi_{2s} = \psi_s^\dagger$  ( $\bar{s}$  denotes the spin direction opposite to  $s$ ).

The diagonal elements of the matrix  $\check{g}$  in the Nambu space (i.e., proportional to  $\hat{\tau}_0$  and  $\hat{\tau}_3$ ) are related to the normal Green's function, while the off-diagonal elements (proportional to  $\hat{\tau}_1$  and  $\hat{\tau}_2$ ) determine the superconducting condensate function  $\check{f}$ . In the case under consideration the matrix (1) can be expanded in the Pauli matrices in the Nambu space ( $\hat{\tau}_0$  is the unit matrix):

$$\check{g} = \hat{g}_0 \hat{\tau}_0 + \hat{g}_3 \hat{\tau}_3 + \check{f}, \quad (2)$$

where the condensate function is given by

$$\check{f} = \hat{f}_1 i \hat{\tau}_1 + \hat{f}_2 i \hat{\tau}_2. \quad (3)$$

The functions  $\hat{g}_i$  and  $\hat{f}_i$  are matrices in the spin space. In the case under consideration the matrices  $\hat{f}_i$  can be represented in the form

$$\hat{f}_2(x) = f_0(x) \hat{\sigma}_0 + f_3(x) \hat{\sigma}_3, \quad (4)$$

$$\hat{f}_1(x) = f_1(x) \hat{\sigma}_1. \quad (5)$$

This follows from the equation that determines the Green's function (see below).

Let us discuss briefly properties of the condensate matrix function  $\check{f}$ . According to the definitions of the Green's functions, Eq. (1), the functions  $f_i(x)$  are related to following correlation functions:

$$\begin{aligned} f_3 &\sim \langle \psi_\uparrow \psi_\downarrow \rangle - \langle \psi_\downarrow \psi_\uparrow \rangle, \\ f_0 &\sim \langle \psi_\uparrow \psi_\downarrow \rangle + \langle \psi_\downarrow \psi_\uparrow \rangle, \\ f_1 &\sim \langle \psi_\uparrow \psi_\uparrow \rangle \sim \langle \psi_\downarrow \psi_\downarrow \rangle. \end{aligned} \quad (6)$$

The function  $f_3$  describes the SC, while the functions  $f_0$  and  $f_1$  describe the TC (see for example Ref. 25). The function  $f_0$  is proportional to the zero projection of the triplet magnetic moment of the Cooper pairs on the  $z$  axis, whereas the function  $f_1$  corresponds to the projections  $\pm 1$ .

It is important that in the absence of an exchange field  $\mathbf{J}$  (or magnetization  $\mathbf{M}$ ) acting on spins, the SC, i.e., the function  $f_3$ , exists both in the superconducting and normal (nonmagnetic) layers. If  $J$  is not equal to zero but is uniform in space and directed along the  $z$  axis, then the part  $f_0$  of the TC arises in the structure.

However, both the functions  $f_3$  and  $f_0$  decay very fast in the ferromagnet (over the length  $\xi_J$ ). The singlet component decays because a strong magnetization makes the spins of a pair be parallel to each other, thus destroying the condensate. The triplet component with the zero projection of the magnetic moment is also destroyed because it is more energetically favorable for the magnetic moment to be parallel to the magnetization.

On the other hand, the structure of the matrix  $\check{f}$  (the functions  $\hat{f}_i$ ) depends on the choice of the  $z$  axis. If the uniform magnetization  $\mathbf{M}$  is directed not along the  $z$  axis (but, say, along the  $x$  axis), terms like  $\hat{f}_1 i \hat{\tau}_1$  inevitably appear in the condensate function (see, for example, Ref. 20 where such a term was obtained even at  $Q=0$ , where  $Q$  is the wave vector of a spiral magnetic structure). However, the condensate component corresponding to this term penetrates the  $F$  layer over the short distance  $\xi_J$  only.

Therefore, we can conclude that the presence of terms such as  $\hat{f}_1 i \hat{\tau}_1$  in the condensate function does not necessarily mean that the TC penetrates the  $F$  layer over the long distance  $\xi_T$ . Actually, long-range effects arise only if the direction of the vector  $\mathbf{M}$  varies in space. If the magnetization has different directions in neighboring  $F$  layers, then not only  $f_0$  but also  $f_1$  arise in the system and both functions penetrate the ferromagnetic layer over a long distance  $\xi_T$ .

In order to find the Green's function  $\check{g}$ , we consider the diffusive case when the Usadel equation is applicable. This equation can be used provided the condition  $J\tau \ll 1$  is satisfied ( $\tau$  is the momentum relaxation time). Of course, this condition can hardly be satisfied for strong ferromagnets like Fe, and in this case one should use a more general Eilenberger equation for a quantitative computation. However, the Usadel equation may give qualitatively reasonable results even in this case.

The Usadel equation is a nonlinear equation for the  $4 \times 4$  matrix Green's function  $\check{g}$  and can be written as

$$\begin{aligned} D \partial_x (\check{g} \partial_x \check{g}) - |\omega| [\hat{\tau}_3 \hat{\sigma}_0, \check{g}] + iJ \operatorname{sgn} \omega \{ [\hat{\tau}_3 \hat{\sigma}_3, \check{g}] \cos \alpha(x) \\ + [\hat{\tau}_0 \hat{\sigma}_2, \check{g}] \sin \alpha(x) \} = -i[\check{\Delta}, \check{g}]. \end{aligned} \quad (7)$$

In the  $S$  layer  $D = D_S$ ,  $J = 0$ ,  $\check{\Delta} = \Delta i \hat{\tau}_2 \sigma t$  (the phase of  $\Delta$  is chosen to be zero). In the  $F$  layers  $D = D_F$ ,  $\alpha(x) = \pm \alpha$  for the right (left) layer and  $\Delta = 0$ . Eq. (7) is complemented by the boundary conditions at the  $S/F$  interface<sup>26</sup>

$$\gamma (\check{g} \partial_x \check{g})_F = (\check{g} \partial_x \check{g})_S, \quad x = \pm d_S, \quad (8)$$

$$2\gamma_b \xi_J (\check{g} \partial_x \check{g})_F = \pm [\check{g}_S, \check{g}_F], \quad x = \pm d_S, \quad (9)$$

where  $\gamma = \sigma_F / \sigma_S$ ,  $\sigma_{S,F}$  are the conductivities of the  $F$  and  $S$  layers, and  $\gamma_b = \sigma_F R_b / \xi_J$  is a coefficient characterizing the transmittance of the  $S/F$  interface with resistance per unit area  $R_b$ .

If linearized, the Usadel equation can be solved analytically rather easily. The linearization may be justified in the two limiting cases: (a)  $T$  is close to the critical temperature of the structures  $T_c^*$  (the latter can be different from the critical temperature of the bulk superconductor  $T_c$ ), and (b) the resistance of the  $S/F$  interface  $R_b$  is not small. In the latter case the condensate function in the  $S$  layer is weakly disturbed by the  $F$  film and the function  $f_3$  in Eq. (3) can be represented in the form

$$f_3(x) = f_S + \delta f_3(x), \quad |x| < d_S, \quad (10)$$

where  $f_S = \Delta / iE_\omega$  and  $E_\omega = \sqrt{\omega^2 + \Delta^2}$ . The function  $\delta f_3$  and the functions  $f_{0,1}$  are assumed to be small. In the  $F$  layers all the components of the condensate function  $\check{f}$  are small. The functions  $\hat{g}_0$  and  $\hat{g}_3$  in Eq. (2) in the superconductor are given by

$$\hat{g}_3 = \hat{\sigma}_0 (g_S + \delta g_0) + \hat{\sigma}_3 g_3, \quad (11)$$

$$\hat{g}_0 = \hat{\sigma}_2 g_2. \quad (12)$$

Here  $\tilde{g}_S = \text{sgn } \omega g_S$ ,  $g_S = |\omega| / E_\omega$ . From the normalization condition

$$\check{g}^2 = 1 \quad (13)$$

we obtain expressions relating the functions  $\delta g_0$ ,  $g_{2,3}$  to the functions  $\delta f_3$ ,  $f_{0,1}$ :

$$\delta g_0 = (f_S / \tilde{g}_S) \delta f_3, \quad g_3 = (f_S / \tilde{g}_S) f_0, \quad g_2 = (f_S / \tilde{g}_S) f_1. \quad (14)$$

Now we linearize Eq. (7) with respect to  $\delta \check{f} = i \hat{\tau}_2 (\hat{\sigma}_3 \delta f_3 + \hat{\sigma}_0 f_0) + i \hat{\tau}_1 \hat{\sigma}_1 f_1$  and obtain

$$\partial_{xx}^2 \delta \check{f} - \kappa_S^2 \delta \check{f} = 0 \quad (15)$$

in the  $S$  layer, and

$$\partial_{xx}^2 \delta \check{f} - \kappa_\omega^2 \delta \check{f} + i \kappa_J^2 \{ \hat{\tau}_0 [\hat{\sigma}_3, \delta \check{f}]_+ \cos \alpha \pm \hat{\tau}_3 [\hat{\sigma}_2, \delta \check{f}]_- \sin \alpha \} = 0 \quad (16)$$

in the  $F$  layers. Here  $\kappa_S^2 = 2E_\omega / D_S$ ,  $\kappa_\omega^2 = 2|\omega| / D_F$ ,  $\kappa_J^2 = J \text{sgn } \omega / D_F$ , and  $[A, B]_\pm = AB \pm BA$ . The signs  $\pm$  in Eq. (16) correspond to the right and left layer, respectively. The corresponding linearized boundary conditions for  $\delta \check{f}$  are

$$(\gamma g_S) \partial_x \check{f}_F = \partial_x \delta \check{f}_S, \quad (17)$$

$$\pm \gamma_b \xi_J \partial \check{f}_F = -(\check{f}_S + \delta \check{f}_S) + g_S \check{f}_F, \quad (18)$$

where  $\check{f}_S = i \hat{\tau}_2 \hat{\sigma}_3 f_S$  and the signs  $\pm$  correspond to the right and left layer respectively. Solutions for Eqs. (15) and (16) can be written as a sum of exponential functions  $\exp(\pm \kappa x)$ , where the  $\kappa$ 's are the eigenvalues of Eqs. (15) and

(16). In the  $S$  layer the equations for  $\delta f_3$ ,  $f_{0,1}$  are decoupled and there is only one eigenvalue  $\kappa = \kappa_S$ . In the  $F$  layers the equations are coupled and there are three different eigenvalues<sup>17</sup>

$$\kappa_{1,2} \equiv \kappa_\pm \approx \xi_J^{-1} (1 \pm i), \quad (19)$$

$$\kappa_3 \equiv \kappa_\omega = \sqrt{2|\omega| / D_F}. \quad (20)$$

We see from these equations that two completely different lengths  $\xi_J$  and  $\xi_T$  determine the decay of the condensate in the  $F$  layers. At all temperatures  $T < T_c^*$  the length  $\xi_T$  much exceeds  $\xi_J$  and is the same the length describing the decay of the standard singlet condensate in a normal metal.

We have assumed that  $J \gg T_c^*$ , which is realistic unless the exchange field is extremely small. In order to find analytical expressions for the functions  $f_i$  we also assume that the thicknesses of the  $S$  and  $F$  layers satisfy the conditions

$$d_S \ll \xi_S = \sqrt{D_S / 2\pi T_c^*}, \quad d_F \gg \xi_J. \quad (21)$$

In this case the solutions for Eqs. (15) and (16) have the form

$$\delta f_3(x) = a_3 \cosh(\kappa_S x), \quad (22)$$

$$f_0(x) = a_0 \cosh(\kappa_S x), \quad (23)$$

$$f_1(x) = a_1 \sinh(\kappa_S x), \quad (24)$$

in the  $S$  layer and

$$f_1(x) = b_1 \frac{\cosh \kappa_\omega (x - d_S - d_F)}{\cosh(\kappa_\omega d_F)} + \text{sgn } \omega \sin \alpha \times [-b_3 + e^{\kappa_+(x-d_S)} + b_3 - e^{-\kappa_-(x-d_S)}], \quad (25)$$

$$f_0(x) = -(\tan \alpha) b_1 \frac{\cosh \kappa_\omega (x - d_S - d_F)}{\cosh(\kappa_\omega d_F)} + \text{sgn } \omega \cos \alpha \times [-b_3 + e^{-\kappa_+(x-d_S)} + b_3 - e^{-\kappa_-(x-d_S)}], \quad (26)$$

$$f_3(x) = b_3 + e^{-\kappa_+(x-d_S)} + b_3 - e^{-\kappa_-(x-d_S)} \quad (27)$$

in the right  $F$  layer. The solutions in the left  $F$  layer can be easily obtained recalling that the function  $f_1(x)$  is odd and  $f_{0,3}(x)$  are even functions of  $x$ . From Eqs. (22)–(27) and the boundary conditions Eqs. (17) and (18) we find

$$\tilde{b}_{3\pm} = b_{3\pm} (g_S + \gamma_b \xi_J \kappa_\pm) = f_S \frac{\tilde{\kappa}_S (\tanh \Theta_S) M_\mp}{M_+ T_- + M_- T_+}, \quad (28)$$

$$\begin{aligned} \tilde{b}_1 &= b_1 (g_S + \gamma_b \xi_J \kappa_\omega \tanh \Theta_F) \\ &= -f_S \sin \alpha \frac{\tilde{\kappa}_S^2 (\tilde{\kappa}_+ - \tilde{\kappa}_-) \text{sgn } \omega}{(\cosh^2 \Theta_S) (M_+ T_- + M_- T_+)}, \end{aligned} \quad (29)$$

where  $\Theta_S = \kappa_S d_S$ ,  $\Theta_F = \kappa_\omega d_F$ ,  $\tilde{\kappa}_\pm = \kappa_\pm / (g_S + \gamma_b \xi_J \kappa_\pm)$ ,  $\tilde{\kappa} = \kappa_\omega / (g_S + \gamma_b \xi_J \kappa_\omega \tanh \Theta_F)$ ,  $\tilde{\kappa}_S = \kappa_S / (g_S \gamma)$ , and

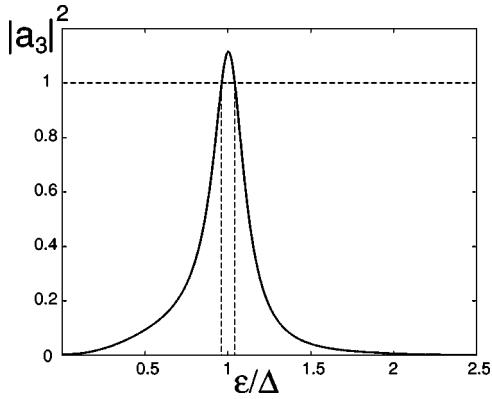


FIG. 2. Dependence of  $|a_3|^2$  on the energy  $\epsilon$ . The dashed vertical lines show the region in which our approach fails. Here  $\gamma = 0.05$ ,  $J/\Delta = 25$ ,  $d_S/\xi_\Delta = 0.4$ ,  $d_F/\xi_\Delta = 0.5$ ,  $\gamma_b = 0.5$ ,  $\alpha = \pi/4$ , and the damping factor  $\Gamma = 0.1$ . We have defined  $\xi_\Delta = \sqrt{D_S/\Delta}$ , where  $\Delta$  is the BCS order parameter.

$$\begin{aligned}
 M_\pm &= T_\pm (\tilde{\kappa}_S \coth \Theta_S + \tilde{\kappa} \tanh \Theta_F) \\
 &+ (\tan^2 \alpha) C_\pm (\tilde{\kappa}_S \tanh \Theta_S + \tilde{\kappa} \tanh \Theta_F), \\
 T_\pm &= \tilde{\kappa}_S \tanh \Theta_S + \tilde{\kappa}_\pm, \\
 C_\pm &= \tilde{\kappa}_S \coth \Theta_S + \tilde{\kappa}_\pm.
 \end{aligned}$$

The solutions presented above are valid if the correction  $\delta f_3$  to the condensate function  $f_s$  in the  $S$  layer is small (in the  $F$  layer  $\delta f_3$  is even smaller). From Eqs. (17) and (18) one can readily see that the condition

$$\delta f_3(d_S) \sim \delta f_3(0) = a_3 \cosh \Theta_S = \tilde{b}_{3+} + \tilde{b}_{3-} - f_S \ll 1 \quad (30)$$

should be satisfied. Here  $|\Theta_S| \ll 1$  is implied. Actually we have neglected the term  $\delta f_3^2$  in the normalization condition (13) assuming that  $\delta f_3^2 \ll 1$  (see Fig. 2).

The amplitude  $a_3$  of the SC depends on many parameters, such as temperature (energy),  $\gamma_b$ , etc. Therefore, the validity of our approach should be checked for every set of parameters. If we are interested in thermodynamical quantities such as the critical temperature or the Josephson current, we may set  $\omega \sim \max\{T, \Delta\}$ . When calculating the density of states the situation is different because  $f_S(\epsilon)$  has a singularity at  $\epsilon = \Delta$ , which is rounded off by a damping factor in the quasi-particle spectrum. In this case our approach breaks down near the energy  $\epsilon \sim \Delta$  (see Fig. 2), when the condition (30) is violated. It is also clear that our approach is valid provided either the temperature is close to the critical temperature  $T_c^*$  of the system or  $\gamma_b$  is not too small.

Now we discuss the properties of the obtained solutions [Eqs. (22)–(29)]. From Eqs. (27) and (28) one can see that the SC is an even function of  $\omega$  and decays sharply in the ferromagnet over the short distance  $\xi_J$ . In contrast, the amplitudes of the TC  $f_0$  and  $f_1$  are odd functions of  $\omega$  and penetrate the ferromagnet over the longer distance  $\xi_T = \sqrt{D_F/2\pi T}$ . The long-range part of TC determined by the

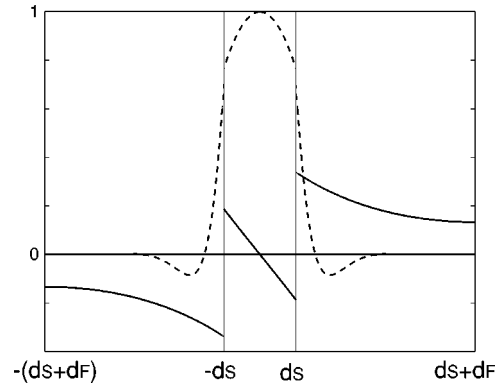


FIG. 3. The spatial dependence of  $\text{Im}(\text{SC})$  (dashed line) and the long-range part of  $\text{Re}(\text{TC})$  (solid line). We have chosen  $\gamma = 0.2$ ,  $J/T_c = 50$ ,  $\gamma_b = 0.05$ ,  $d_F\sqrt{T_c/D_S} = 2$ ,  $d_S\sqrt{T_c/D_S} = 0.4$ , and  $\alpha = \pi/4$ . The discontinuity of the TC at the  $S/F$  interface is because the short-range part is not shown in this figure.

amplitude  $b_1$  has the maximum at  $\alpha = \pi/4$ . This value of  $\alpha$  corresponds to a perpendicular orientation of the magnetizations in the  $F$  layers. For a parallel ( $\alpha = 0$ ) or antiparallel alignment of the magnetizations ( $\alpha = \pi/2$ ) this amplitude decays to zero. In Fig. 3 we plot the spatial dependence of the SC and the long-range part of the TC. We see that both amplitudes are comparable at the  $S/F$  interface but the SC decays faster than the TC.

The long-range part of TC leads to interesting observable effects that will be discussed in the next sections. In Refs. 11 and 12 the conductance of a ferromagnetic wire attached to a superconductor was calculated. It was assumed that the  $F$  wire had a domain wall located at the  $S/F$  interface. This inhomogeneity of the magnetization induces a TC, which leads to an increase of the conductance for temperatures below  $T_c$ .

### A. Critical temperature

In this section we discuss briefly the effect of the TC on the critical temperature  $T_c^*$  of the structure. For the parallel and antiparallel alignment of the magnetizations the critical temperature of the multilayered structure  $T_c^*$  was calculated in many papers.<sup>2,27</sup> The angle dependence of the critical temperature in a  $F/S/F$  structure was analyzed in Ref. 28. However, the form of the condensate function presented in Ref. 28 is not correct because the authors started from an equation different from Eq. (7). As a result, the long-range TC was completely lost.

The equation that determines  $T_c^*$  has the form (we assume that  $d_S \ll \xi_S$ , see Refs. 2)

$$\ln \left( \frac{T_c}{T_c^*} \right) = 2\pi T_c^* \sum_{\omega=-\infty}^{\infty} \left\{ \frac{1}{\omega} - i \frac{\tilde{b}_{3p} + \tilde{b}_{3-}}{\Delta} \right\}. \quad (31)$$

We have obtained a solution for  $\tilde{b}_{3\pm}$  [Eq. (28)], assuming that  $\Delta$  is constant in space (this approximation corresponds to the so-called single-mode approximation used in many earlier works<sup>2</sup>). It is established in Ref. 27 that for some parameters this approximation gives a rough estimate for

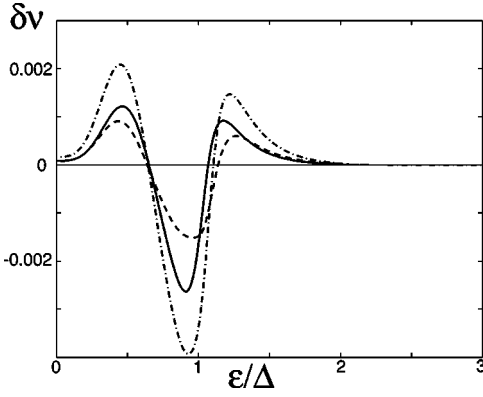


FIG. 4. The normalized DOS  $\delta\nu$  as a function of the energy for  $\alpha=3\pi/8$  (solid line),  $\alpha=\pi/8$  (dashed line), and  $\alpha=\pi/4$  (point-dashed line). Note that for  $\alpha=0,\pi/2$   $\delta\nu=0$ . We have chosen  $\gamma=0.05$ ,  $J/\Delta=25$ ,  $\gamma_b=0.5$ ,  $d_F/\xi_\Delta=0.5$ , and  $d_S/\xi_\Delta=0.4$ . Here  $\xi_\Delta=\sqrt{D_S/\Delta}$  and  $\Delta$  is the BCS order parameter.

$T_c^*$ . A careful analysis of Ref. 27 shows that  $T_c^*$  remains finite even for values of the parameters  $\gamma$ ,  $\gamma_b$ ,  $\kappa_J$ , for which other approaches predict a zero critical temperature. We will not discuss quantitatively the dependence of  $T_c^*$  on the angle  $\alpha$ . Note however that, as follows from Eqs. (28) and (31), the critical temperature  $T_c^*$  depends on  $\alpha$  and  $d_F$  even in the case when  $d_F \gg \xi_J$  (if  $\alpha \neq 0$ ). This dependence is due to the long-range part of the TC and, in order to determine it, one has, generally speaking, to go beyond the single-mode approximation. Note, however, that this dependence may be weak.

### B. Local density of states

In this section we calculate the change of the local DOS in the  $F$  layers due to the TC. It is clear that, for distances from the  $S/F$  interface larger than  $\xi_J$ , only the TC leads to a variation of the local DOS. Thus, if the thickness  $d_F$  is much larger than the  $\xi_J$  one can detect directly the presence of the TC performing measurements of the DOS at the outer side of one of the  $F$  layers. Any deviation from the normal value would be only due to the TC.

We calculate the local DOS at  $x=d_S+d_F$ . The expression for the normalized DOS is (we ignore the difference in the DOS for the up and down spin directions. This approximation is consistent with the quasiclassical assumption that  $J \ll \epsilon_F$ , where  $\epsilon_F$  is the Fermi energy),

$$\tilde{\nu} = \frac{\nu}{\nu_0} = \frac{1}{8} \text{Tr}(\hat{\tau}_3 \hat{\sigma}_0) \langle \check{g}^R - \check{g}^A \rangle, \quad (32)$$

where  $\nu_0$  is the DOS in the normal states; thus  $\tilde{\nu} = 1 + \delta\nu$  ( $\delta\nu$  is a correction due to the proximity effect). As it was mentioned before, in the case  $d_F \gg \xi_J$  only the TC [i.e., the functions  $f_0(x)$  and  $f_1(x)$ ] contributes to the DOS. From the normalization condition, Eqs. (13) and (32), we obtain

$$\delta\nu = \frac{1}{2} \text{Re} \frac{(b_1^R)^2}{\cos^2 \alpha \cosh^2 \Theta_F^R}, \quad (33)$$

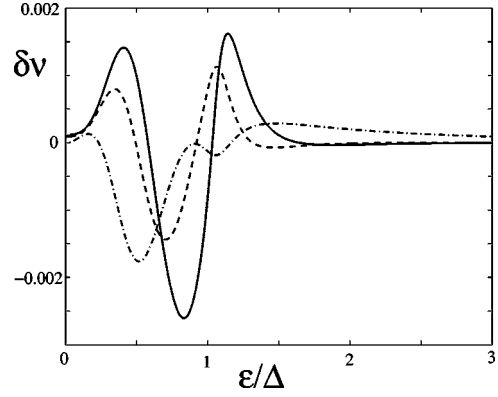


FIG. 5. The normalized DOS  $\delta\nu$  as a function of the energy for  $d_F/\xi_\Delta=0.8$  (solid line),  $d_F/\xi_\Delta=1.2$  (dashed line). The dotted-dashed line shows the contribution to the DOS from the SC ( $f_3$ ). The latter is multiplied by a factor of 100. We have chosen  $\alpha=\pi/4$ . All other parameters are the same as in Fig. 4.

where  $\Theta_F^R = \sqrt{-2i\epsilon/D_F d_F}$  and  $b_1^R$  is the amplitude of the retarded Green's function in Eqs. (25) and (26). It is obtained from  $b_1$  by replacing  $\omega$  by  $-i\epsilon$ . In Figs. 4, 5, and 6 we plot the dependence of  $\delta\nu$  on  $\epsilon$  for different  $\alpha$ ,  $d_F$ , and  $\gamma_b$ , respectively. For the range of parameters chosen in these plots the function  $|a_3(\epsilon)|^2$  has the shape shown in Fig. 2. Thus, our approach is valid almost for all energies and fails only in a very narrow region close to  $\epsilon=\Delta$ . In order to avoid singularities in  $f_S^R$  we have taken into account a finite damping factor  $\Gamma=0.1$  in the expression for  $f_S^R$ :

$$f_S^R = \frac{\Delta}{\sqrt{(\epsilon+i\Gamma)^2 - \Delta^2}}. \quad (34)$$

As follows from Eq. (29)  $\delta\nu$  is zero for  $\alpha=0,\pi/2$ . The largest change in the DOS is achieved when  $\alpha=\pi/4$  (perpendicular orientation of magnetizations in the  $F$  layers). We see that the correction to the DOS is small but observable. Kontos *et al.* presented in Ref. 29 measurements of  $\delta\nu$  in thin  $F$  layers (few nanometers). The order of magnitude of the observed  $\delta\nu$  ( $\sim 10^{-3}$ ) is the same as the presented in Figs.

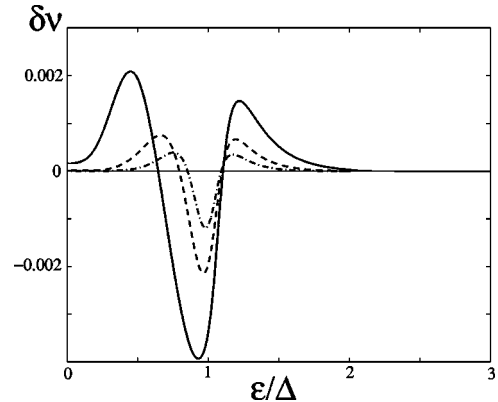


FIG. 6. The normalized DOS  $\delta\nu$  as a function of the energy for  $\gamma_b=0.5$  (solid line),  $\gamma_b=1$  (dashed line), and  $\gamma_b=1.5$  (dotted-dashed line). We have chosen  $d_F/\xi_\Delta=0.5$ . All other parameters are the same as in Fig. 5.

4–6. However, in Ref. 29 the variation of the DOS was caused by the penetration of the SC into the  $F$  layer over the short distance  $\xi_J$ . In our case such a variation can be observed in much thicker  $F$  layers ( $d_F \sim \xi_T = \sqrt{D_F/2\pi T_c^*} \gg \xi_J$ ).

It is interesting to compare our result for the  $FSF$  structure with noncollinear magnetization with corresponding results for  $NSN$  structures ( $N$  is a normal layer). At first glance, the behavior of the odd triplet condensate in the ferromagnet is very similar to that of the conventional singlet condensate in a normal metal. In both cases the amplitude of the condensate decays exponentially with the length  $\xi_T$  [Eq. (20)]. However, there is an essential difference. In the  $N$  layer an energy gap is induced due to the singlet condensate. The value of the energy gap is determined by  $\min\{\Delta, D_N/(\sigma_N d_N R_b)\}$ .<sup>30</sup> In contrast, no subgap appears in the ferromagnet due to the triplet odd condensate considered here, although the TC penetrates over the entire  $F$  layer provided its thickness  $d_F$  is not very large,  $d_F \leq \xi_T$ . The main reason for the absence of a subgap  $\epsilon_{sg}$  in the  $FSF$  system is the following. In  $SN$  structures the condensate function is not small at energies  $|\epsilon| \leq \epsilon_{sg}$ . The exchange field shifts this energy interval by the large value  $J$  so that at low energies the condensate function (both singlet and triplet) is small if  $\gamma_b$  is not too small. Note also that the amplitude of the TC is smaller than the amplitude of the SC in a  $NSN$  structure since it contains a large parameter  $\kappa_{\pm} \sim \sqrt{J}$  in the denominator [see Eqs. (28) and (29)].

For completeness we finally note that the change of the local DOS in the ballistic case ( $J\tau \gg 1$ ) was considered in Ref. 31 and in the pure ballistic case ( $\tau \rightarrow \infty$ ) in Ref. 32. It turns out that the results in these two cases differ greatly from those obtained in the present paper for a diffusive system ( $J\tau \ll 1$ ).

### III. JOSEPHSON CURRENT IN A $F/S/F/S/F$ STRUCTURE

In this section we calculate the Josephson current between the  $S$  layers of a  $FSFSF$  structure. We assume again that the thickness of the  $F$  layers  $d_F$  is much larger than  $\xi_J$  [Eq. (21)]. In this case the Josephson coupling between the  $S$  layers is due to the long-range part of the TC. Therefore the supercurrent in the transverse direction is unusual, since it is caused by the triplet component of the condensate that is odd in frequency and even in momentum.

At the same time, the in-plane superconductivity is caused mainly by the ordinary singlet component. Therefore the macroscopic superconductivity due to the Josephson coupling between the layers is an interesting combination of the singlet superconductivity within the layers and the odd triplet superconductivity in the transversal direction.

We will see that the unusual character of the superconductivity in the transversal direction leads to peculiarities of the Josephson effect. For example, if the bias current flows through the terminal superconducting layer  $S_0$  and  $S_A$  (see Fig. 7), the supercurrent is zero because of the different symmetry of the condensate in  $S_0$  and  $S_A$ . In order to observe the Josephson effect in this structure the bias current has to pass through the layers  $S_A$  and  $S_B$ , as shown in Fig. 7. The

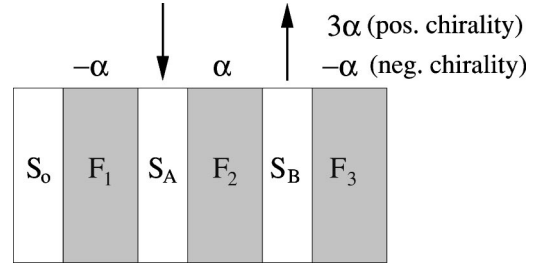


FIG. 7. The multilayered structure considered. The arrows show the bias current. In the case of positive (negative) chirality the magnetization vector  $\mathbf{M}$  of the layer  $F_3$  makes an angle  $3\alpha$  ( $-\alpha$ ) with the  $z$  axis, i.e., in the case of positive chirality the vector  $\mathbf{M}$  rotates in one direction if we go over from one  $F$  layer to another, whereas it oscillates in space in the case of negative chirality.

supercurrent between  $S_A$  and  $S_B$  is nonzero because each superconductor has its “own” TC and the phase difference  $\varphi$  is finite.

The Josephson current  $I_S$  is given by the expression

$$I_S = (L_y L_z) \sigma_F \text{Tr}(\hat{\tau}_3 \hat{\sigma}_0) \sum_{\omega} \check{f} \partial_x \check{f}. \quad (35)$$

This current was calculated for the case of small angles  $\alpha$  in Ref. 17. Here  $L_y L_z$  is the area of the interface and  $\sigma_F$  is the conductivity of the  $F$  layer. The simplest way to calculate  $I_S$  is to assume a weak coupling between the  $S$  layers, which corresponds to the case when the condition  $d_F > \xi_T$  holds. In this case the long-range part of the TC is given by the sum of two terms each of those is induced by the layers  $S_A$  and  $S_B$  in Fig. 7:

$$\check{f}(x) = \check{f}_A(x) + \check{S} \check{U} \check{f}_B(x - d_S - d_F) \check{U}^+ \check{S}^+, \quad (36)$$

where

$$\check{f}_A(x) = e^{-\kappa_{\omega}(x-d_S)} (b_1 i \hat{\tau}_1 \hat{\sigma}_1 + b_0 i \hat{\tau}_2 \hat{\sigma}_0) \quad (37)$$

is the long-range part of the TC induced by the layer  $S_A$ . The coefficient  $b_1$  is given by Eq. (29) and  $b_0 = -(\tan \alpha) b_1$ . If the  $S_{A,B}/F$  interfaces are identical as well as the superconductors  $S_A$  and  $S_B$ , the function  $\check{f}_B$  is equal to  $\check{f}_A$  if one replaces the exponential function  $\exp[-\kappa_{\omega}(x-d_S)]$  by  $\exp[\kappa_{\omega}(x-d_S-d_F)]$ . The phase of the  $S_A$  layer is set to be zero and the phase of  $S_B$  is  $\varphi$ . This phase has been taken into account by the gauge transformation performed with the help of the matrix  $\check{S} = \hat{\tau}_0 \cos(\varphi/2) + i \hat{\tau}_3 \sin(\varphi/2)$ . The magnetizations  $\mathbf{M}$  of the layers  $F_1$  and  $F_2$  make an angle  $\mp \alpha$  with the  $z$  axis, respectively. For the direction of  $\mathbf{M}$  in  $F_3$  we consider two cases: (a) the direction of magnetization is  $-\alpha$  (negative chirality) or (b)  $2\alpha$  (positive chirality). In the latter case the matrix  $\check{U}$  in Eq. (36) is given by

$$\check{U} = \hat{\tau}_0 \hat{\sigma}_3 \cos \alpha + i \hat{\tau}_3 \hat{\sigma}_2 \sin \alpha. \quad (38)$$

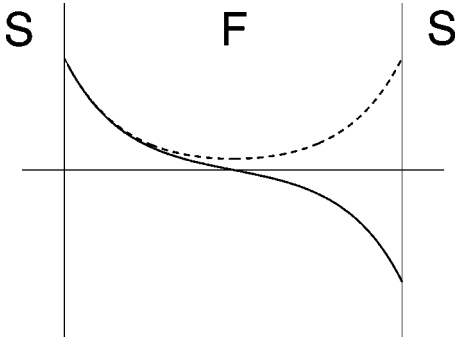


FIG. 8. The spatial dependence of the amplitude of the TC  $f_1(x)$  in the case of positive (solid line) and negative (dashed line) chirality.

In the case of negative chirality,  $\tilde{U}$  is the unit matrix and one has to change the sign of  $\alpha$  in the expression for the function  $\tilde{f}_B$  [Eq. (36)]. In Fig. 8 we show schematically the spatial dependence of  $f_1(x)$ .

Substituting Eq. (36) into Eq. (35) one obtains after simple transformations  $I_S = I_c \sin \varphi$ , where

$$eR_F I_c = \pm 2\pi T \sum_{\omega} \kappa_{\omega} d_F b_1^2(\alpha) (1 + \tan^2 \alpha) e^{-d_F \kappa_{\omega}}, \quad (39)$$

where  $b_1(\alpha)$  is given in Eq. (29) and the  $+$  ( $-$ ) sign corresponds to positive (negative) chirality. In the case of negative chirality the critical current is negative ( $\pi$  contact). It is important to emphasize that the nature of the  $\pi$  contact differs from that predicted in Refs. 4 and observed in Ref. 7. In our case the negative Josephson coupling is due to the TC and can be realized in S/F structures with negative chirality. This gives a unique opportunity to switch experimentally between the 0 and  $\pi$  contacts by changing the angles of the mutual magnetization of the layers. It is worth mentioning that another effect concerning the chirality of the  $\mathbf{M}$  vector was studied by the authors in Ref. 33. It was shown that the resistance of a multidomain ferromagnetic wire depends on the chirality of the  $\mathbf{M}$  variation in space.

In Fig. 9 we plot the dependence of  $I_c$  on the angle  $\alpha$ . If the orientation of  $\mathbf{M}$  is parallel ( $\alpha=0$ ) or antiparallel ( $\alpha$

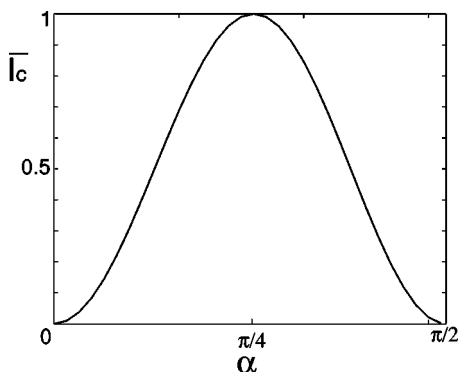


FIG. 9. Dependence of the critical current (normalized with respect to the maximum value) on the angle  $\alpha$ . We have chosen the same values as in Fig. 4.

$=\pi/2$ ) the amplitude of the triplet component is zero, and therefore there is no coupling between the neighboring S layers, i.e.,  $I_c=0$ . For any other angle between the magnetizations the amplitude of the TC is finite. This leads to a nonzero critical current. At  $\alpha=\pi/4$  (perpendicular orientation of  $\mathbf{M}$ )  $I_c$  reaches its maximum value.

The weak coupling assumption ( $d_F > \xi_T$ ) leads to an exponential decay of  $I_c$  with increasing  $d_F$  [Eq. (39)]. In the case  $d_F \leq \xi_T$ , Eq. (39) is not valid. One can easily obtain  $I_c$  for the case of an arbitrary  $d_F$  and small  $\alpha$ . It turns out that in this case Eq. (39) remains valid if the exponential factor  $\exp(-\kappa_{\omega} d_F)$  is replaced by  $\cosh^{-2}(\kappa_{\omega} d_F/2)$  and in the expression for  $b_1$  [Eq. (29)]  $\Theta_F$  is replaced by  $\Theta_F/2$ .

In order to estimate the value of the critical current  $I_c$ , we use Eq. (39). If  $d_F$  exceeds the length  $\xi_T$  (for example,  $d_F/\xi_T=2$ ) only the term with  $n=0$  (i.e.,  $\omega=\pi T_c^*$ ) is important in the sum. In this case one obtains

$$\frac{eR_F I_c}{T_c^*} = \frac{4}{\pi} \left( \frac{\Delta}{T_c^*} \right)^2 e^{-\kappa_T d_F C}, \quad (40)$$

where the factor  $C$  can be easily expressed in terms of  $M_{\pm}$ ,  $T_{\pm}$ , etc. Thus,  $C$  depends on many parameters such as  $\gamma$ ,  $\gamma_b$ ,  $\kappa_J$ , etc. We estimate  $C$  for values of these parameters similar to those which were used in Ref. 27:  $\gamma_b=0.5$ ,  $\gamma=0.1$ ,  $d_S \kappa_S=0.4$ ,  $d_F \kappa_{\omega}=1.5$ ,  $\kappa_{\omega}/\kappa_S=3$ . We get  $C=10^{-2}-10^{-3}$  for  $\kappa_J d_S=5-10$ . The expression (40) for  $I_c$  also contains the parameters  $(\Delta/T_c^*)^2$  and  $\exp(-d_F \kappa_T)$  which are also small. We note, however, that if  $d_F \leq \xi_T$ , the exponential function is replaced by a numerical factor of the order of 1. The factor  $(\Delta/T_c^*)^2$  is also of the order 1 if the temperature is not close to  $T_c^*$ . Taking  $\sigma_F^{-1}=60 \mu\Omega \text{ cm}$  (cf. Ref. 27) and  $d_F \sim \xi_T \sim 200 \text{ nm}$  we obtain  $I_c \sim 10^4-10^5 \text{ A cm}^{-2}$ ; that is, the critical current is a measurable quantity (see experimental works<sup>7</sup>) and the detection of the TC is possible.

#### IV. EFFECT OF SPIN-ORBIT INTERACTION

So far the only interaction we have considered in the ferromagnet is the exchange field  $J$  acting on the conducting electrons. However, in reality spin-orbit interactions that appear due to interactions of electron spins with spin-orbital impurities may become important. Following again the notation of Ref. 6 we write an additional term in the Hamiltonian that describes the spin-orbit part as<sup>34,35</sup>

$$H_{so} = \frac{U_{so}}{2p_{Fn,s,p,n',s',p'}} \sum c_{nsp}^{\dagger} (\mathbf{p} \times \mathbf{p}') (\check{S}_{ss'}^{nn'}) c_{n's'p'}, \quad (41)$$

where  $\check{S}=(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\tau}_3 \hat{\sigma}_3)$  and  $\mathbf{p}$  and  $\mathbf{p}'$  are the momenta before and after scattering at the impurities. Although in general the characteristic energy of the spin-orbit interaction is much smaller than the exchange energy, it can be comparable to the superconducting gap  $\Delta$  and therefore this effect should be taken into account when describing the supercurrent.

In the Born approximation the self-energy is given by



$$\check{\Sigma}_{so} = n |U_{so}|^2 \langle G \rangle_{so},$$

where

$$\langle G \rangle_{so} = \nu \int d\xi_p \int \frac{d\Omega}{4\pi} (\mathbf{n} \times \mathbf{n}') \check{S} G \check{S} (\mathbf{n} \times \mathbf{n}'). \quad (42)$$

Here  $\mathbf{n}$  is a unit vector parallel to the momentum. Including this term in the quasiclassical equations is straightforward and the resulting Usadel equation takes the form<sup>34</sup>

$$\begin{aligned} & -iD \partial_r (\check{g} \partial_r \check{g}) + i(\hat{\tau}_3 \partial_r \check{g} + \partial_r \check{g} \hat{\tau}_3) + [\check{\Delta}, \check{g}] \\ & + J[\check{n}, \check{g}] + \frac{i}{\tau_{so}} [\check{S} \hat{\tau}_3 \check{g} \hat{\tau}_3 \check{S}, \check{g}] = 0, \end{aligned} \quad (43)$$

where

$$\frac{1}{\tau_{so}} = \frac{1}{3} \nu n \pi \int \frac{d\Omega}{4\pi} |U_{so}|^2 \sin^2 \theta \quad (44)$$

is the spin-orbit scattering time.

As before, one can linearize Eq. (43) in the  $F$  layer and obtain equations for the condensate function  $\check{f}$  similar to Eqs. (15-16) but now including the spin-orbit interaction term. The solution again has the form

$$\check{f}(x) = i\hat{\tau}_2 \otimes [f_0(x)\hat{\sigma}_0 + f_3(x)\hat{\sigma}_3] + i\hat{\tau}_1 \otimes f_1(x)\hat{\sigma}_1. \quad (45)$$

The functions  $f_i(x)$  are given by  $f_i(x) = \sum_j b_j \exp[\kappa_j x]$ , where the new eigenvalues  $\kappa_j$  are

$$\kappa_{\pm}^2 = \pm \frac{2i}{D_F} \sqrt{J^2 - \left(\frac{4}{\tau_{so}}\right)^2} + \frac{4}{\tau_{so} D_F}, \quad (46)$$

$$\kappa_0^2 = \kappa_{\omega}^2 + 2 \left( \frac{4}{\tau_{so} D_F} \right). \quad (47)$$

We see from these equations that the singlet and triplet components are affected by the spin-orbit interaction, making the decay of the condensate in the ferromagnet faster. In the limiting case  $4/\tau_{so} > J, T_c$  both components penetrate over the same distance  $\xi_{so} = \sqrt{\tau_{so} D_F}$  and therefore the long-range effect is suppressed. In this case the characteristic oscillations of the singlet component are destroyed.<sup>35</sup> In the more interesting case  $4/\tau_{so} \sim T_c < J$ , the singlet component is not affected and penetrates over distances of the order  $\xi_J$ . At the same time, the triplet component is more sensitive to the spin-orbit interaction and the penetration length equals  $\min(\xi_{so}, \xi_T) > \xi_J$ .

The spin-orbit interaction is relevant in systems with large- $Z$  elements. The characteristic spin-orbit energy  $1/\tau_{so}$  also depends on the scattering concentration and density of states [cf. Eq. (44)]. Experimental data concerning this energy are still unclear and controversial, mainly due to the difficulty to separate the contribution of the spin-orbit from other scattering types. From numerical band structure calculations one can estimate the parameter  $J\tau_{so}$ . For example, for a typical magnetic transition metal, like Fe, in the dirty limit  $J\tau_{so} \sim 10^2$ , while for dirty Gd  $J\tau_{so} \sim 10$  (see Ref. 36 and references therein). Thus, according to our model,

material-like transition metals are better candidates in order to observe the predicted effects. Thus, provided the spin-orbit interaction is not very strong, the penetration of the triplet condensate over the long distances discussed in the previous sections is still possible, although the penetration length is reduced.

## V. CONCLUSION

We studied odd  $s$ -wave triplet superconductivity that may arise in  $S/F$  multilayered structures with a noncollinear orientation of magnetizations. It was assumed that the orientation of the magnetization is not affected by the superconductivity (e.g., the energy of the magnetic anisotropy is much larger than the superconducting energy). The analysis was carried out in the dirty limit ( $J\tau \ll 1$ ) when the Usadel equation is applicable.

It was shown that for all values of  $\alpha$  the condensate function consists of singlet (SC) and triplet (TC) components. Even in the case of a homogenous magnetization ( $\alpha=0$ ), in addition to the SC, the TC with the zero projection onto the  $z$  axis arises. In this case, both the SC and the TC decay in the  $F$  layers over a short distance given by  $\xi_J = \sqrt{D_F/J}$ . If the magnetization vectors  $\mathbf{M}$  are not collinear  $\alpha \neq 0, \pi/2$ , all projections of the TC appear, in particular, those with non-zero projection on the  $z$  axis. In this case, the TC penetrates the  $F$  layer over a long distance  $\xi_T = \sqrt{D_F/2\pi T}$ . In the presence of the spin-orbit interaction this penetration length is given by  $\min(\xi_{so}, \xi_T)$ , where  $\xi_{so} = \sqrt{\tau_{so} D_F}$ . Generally, this length may be much larger than  $\xi_J$ .

Thus, if the condition  $d_F \gg \xi_J$  is fulfilled the Josephson coupling between neighboring  $S$  layers is only due to the TC. Therefore in this case a new type of superconductivity may arise in the multilayered structures with noncollinear magnetizations. The supercurrent within each  $S$  layer is caused by the SC, whereas the supercurrent across the layers is caused by the triplet condensate, which is odd in the frequency  $\omega$  and even in the momentum.

The TC in our case is completely different from the triplet condensate found in  $\text{Sr}_2\text{RuO}_4$ .<sup>14</sup> In the latter case one has a  $p$  wave, even in  $\omega$ , triplet superconductivity, which is suppressed by impurity scattering. In contrast, the TC we have considered is not affected by nonmagnetic impurities. The reason for the existence of the long-range TC is the fact that if  $\alpha \neq 0$ , the SC and the TC are coupled and, in addition to  $\kappa_{\pm} = \xi_J^{-1}(1 \pm i)$ , the eigenvalue  $\kappa_T = \xi_T^{-1}$  appears. The latter corresponds to the long-range penetration of the TC in the ferromagnet.

The triplet superconductivity in  $S/F$  structures possesses an interesting property: the Josephson current depends on the chirality of the magnetization  $\mathbf{M}$ : If the  $\mathbf{M}$  vector rotates in only one direction (the positive chirality), the critical current  $I_c$  is positive. If the direction of the  $\mathbf{M}$  vector oscillates in space (the negative chirality) then  $I_c < 0$ . In the latter case spontaneously circulating currents must arise in the structure. This result can be explained as follows: if the chirality is positive the averaged  $\mathbf{M}$  vector  $\langle \mathbf{M} \rangle$  is zero and the  $S/F$  structure behaves as a superconductor with anisotropic properties (the singlet superconductivity along the layers and the

triplet superconductivity across them). In the case of negative chirality the average in space yields a nonzero magnetization  $\langle \mathbf{M} \rangle \neq \mathbf{0}$ . In such a superconductor with a built-in magnetic moment the circulating currents arise as they arise in superconductors of the second type in the mixed state.

Note also that in a single Josephson *FSFSF* junction a nonzero magnetic field exists also inside the junction, and this causes Meissner currents. However, the experiment of Ref. 7 on *SFS* junctions shows that the observed Fraunhofer pattern corresponds to  $\langle \mathbf{M} \rangle = 0$  in the *F* layer. This behavior according to the authors of Ref. 7 may be attributed to a multidomain structure.

It would be interesting to carry out experiments on *S/F* structures with noncollinear magnetization in order to observe this new type of superconductivity. As follows from a semiquantitative analysis, the best conditions to observe the Josephson critical current caused by the TC are high interface transparency (small  $\gamma_b$ ) and low temperatures. These

conditions are a bit beyond our quantitative study. Nevertheless, all qualitative features predicted here (angle dependence, etc.) should remain as a general case when one has to deal with the nonlinear Usadel equation.

Another type of experiment that may detect the triplet condensate is measuring the local density of states. As we have shown in Sec. II, the long-range TC may be detected by measuring the local DOS of the *F* layers.

*Note added.* Recently, a paper<sup>37</sup> appeared in which a detailed study of the critical temperature in a *FSF* structure with noncollinear magnetizations in the *F* layers has been presented.

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