Effects of fermion-boson interaction in neutral atomic systems

Nobuhiko Yokoshi and Susumu Kurihara
Department of Physics, Waseda University, Okubo, Shinjuku, Tokyo 169-8555, Japan
(Received 7 April 2003; published 4 August 2003)

We investigate the collective excitations of ^{3}He–^{4}He mixture films at zero temperature within the random phase approximation and linear response theory. In a low concentration regime of ^{3}He, a level repulsion between zero sound and third sound modes is derived, which opens the possibility to observe quantum mechanical coherence between a ^{3}He particle-hole pair and the condensate of ^{4}He. We also investigate the ^{3}He–^{4}He vertex corrections in the ladder approximation, and show that the third branch, the combined mode of fermionic particle-hole pair and the third sound quanta, provides a unique correction to the Landau f function. Some implications for a fermion-boson mixture of alkali atoms in a potential trap are discussed.

DOI: 10.1103/PhysRevB.68.064501 PACS number(s): 67.60.Fp, 67.57.Jj, 68.15.+e

A two-dimensional (2D) Fermi liquid is one of the most important subjects in condensed matter physics; it is in a deep connection with high-temperature superconductivity in cuprates and alkali-doped fullerenes, and with the quantum Hall effect. Free surfaces of superfluid ^{4}He bulk liquid or films provide ideal fields for 2D Fermi systems free from impurities and inhomogeneities in chemical potential due to the randomness of walls or substrates. In particular, phase separated ^{3}He–^{4}He mixture films have been the subject of theoretical and experimental interest for the last two decades.[1] At very low temperature, it is well known that ^{3}He atoms are bound to the surface of superfluid ^{4}He,[2] and behave as well-defined 2D Fermi liquids.[3,4] Therefore, they are regarded as good candidates for 2D fermion-boson systems. The nature of ^{3}He and ^{4}He films depends strongly on system parameters such as concentrations, temperature, and the van der Waals potential from substrates. This richness of the parameters has provided various suggestions such as Cooper pairing[5] and the dimerization[6–8] of ^{3}He, as well as the suppression of the superfluidity[9] and Casimir effect[10,11] of superfluid ^{4}He films.

In this 2D fermion-boson system, ^{3}He particles interact with one another through a thickness variation of ^{4}He films, i.e., the third sound driven by the van der Waals potential,[5] as well as direct interactions. A fascinating feature of this system is that characteristic energies can be easily tuned by varying the thickness of ^{3}He and ^{4}He films continuously. Therefore, it is expected that interference between excitations is enhanced, and the nonadiabatic effect becomes relevant when suitable parameters are chosen. In this paper, we report a theoretical study of the response to the spectrum of collective excitations when the thickness of the mixture films is varied at zero temperature. We will show that a level repulsion between zero sound and third sound modes takes place. Moreover, a third branch is derived, which we interpret as a combined mode of a fermionic particle-hole pair and a third sound phonon, by calculating the vertex function in some detail. We also calculate the contribution of this collective excitation to the Landau f function.

We assume that the system forms a phase-separated double layer, i.e., a normal ^{3}He liquid covering ^{4}He which consists of a superfluid layer and a nonsuperfluid "inert layer."[9,12] In this system, particles are sensitive to the modulation of the substrate van der Waals potential due to the thickness variation of ^{3}He and ^{4}He films, especially at low concentrations. We take account of this effect in interactions between particles and include other effects in hydrodynamic masses. An effective Hamiltonian which consists of third sound phonons and ^{3}He quasiparticles interacting with one another and with phonons was derived by one of the authors,[5] and has the following form:

$$H_{\rm eff}=\sum_{\mathbf{k},\sigma}\frac{\hbar^2k^2}{2m_3}c^\dagger_{\mathbf{k},\sigma}c_{\mathbf{k},\sigma}+v_3\sum_{\mathbf{q}}\rho_{\mathbf{q},\uparrow}\rho_{-\mathbf{q},\downarrow}+\sum_{\mathbf{q}}\hbar\omega_{\mathbf{q}}b^\dagger_{\mathbf{q}}b_{\mathbf{q}}+\sum_{\mathbf{q},\sigma}g_{\mathbf{q}}(b_{\mathbf{q}}+b^\dagger_{-\mathbf{q}})\rho_{-\mathbf{q},\sigma}, \tag{1}$$

where the correlation between superfluid and inert layers and the surface tension of the films is neglected for simplicity. These approximation may be allowed, except the region where the structure normal to substrate becomes remarkable.[13] Here, $\rho_{\mathbf{q},\sigma}=\Sigma_{\mathbf{k}}c^\dagger_{\mathbf{k},\sigma}c_{\mathbf{k}+\mathbf{q},\sigma}$ is the Fourier transformation of the density operator and $\sigma=\uparrow,\downarrow$ and N_4 are spin indices and the number of ^{4}He atoms respectively. The spectrum of the third sound phonon and coupling energies are respectively given by

$$(\hbar\omega_{\mathbf{q}})^2=(\hbar c_{\rm B}q)^2+\left(\frac{\hbar^2q^2}{2m_4}\right)^2, \tag{2}$$

$$v_3=\frac{3u_3}{2n_3^2(d+h_3+h_4)^4S}, \tag{3}$$

$$g_{\mathbf{q}}=\frac{1}{\sqrt{N_4}}\frac{3u_3h_4}{n_3(d+h_3+h_4)^4}\left(\frac{\hbar^2q^2}{2m_4\hbar\omega_{\mathbf{q}}}\right)^{1/2}, \tag{4}$$

where the subscripts 3 and 4 denote ^{3}He and ^{4}He; n_i, and h_i are the average bulk densities and average thickness of the films, and m_i and u_i are the hydrodynamic masses and characteristic van der Waals energies. S and d are the surface area and thickness of the inert layer. In the equations above, $c_{\rm B}$ is the velocity of third sound phonon which strongly depends on concentrations through h_3 and h_4, and can be obtained by[5,14]

$$c_{\mathrm{B}}=(1-\Delta)\frac{3u_4h_4}{m_4n_4(d+h_4)^4},$$
$$\Delta=\frac{u_3}{u_4}\left[1-\left(\frac{d+h_4}{d+h_3+h_4}\right)^4\right]. \tag{5}$$

It is suggested that the third sound velocity [Eq. (5)] needs some corrections when the submonolayer regime of ^{3}He is considered.[15] We neglect these corrections here, because they do not play a crucial role in this paper.

A similar model has been studied in detail to investigate the equation of state and correlation energy of ^{3}He,[16,17] whereas we will focus on the low energy collective behaviors of the mixture below. To investigate the spectrum of collective excitations, we shall consider a 2×2 susceptibility matrix, which is in connection with area density fluctuations (thickness variation) in linear response theory, in which interactions are treated within the random phase approximation (RPA).[18,19] The unperturbed susceptibility and interaction matrix of the mixtures has the following forms

$$\hat{\chi}^{(0)}=\begin{pmatrix}\chi_3 & 0\\ 0 & \chi_4\end{pmatrix},\quad \hat{V}=\begin{pmatrix}v_3 & g_{\mathbf{q}}\\ g_{\mathbf{q}} & 0\end{pmatrix}. \tag{6}$$

Here, χ_3 and χ_4 are the susceptibilities of 2D pure fermion and boson systems, and at zero temperature they are given as

$$\chi_3(\mathbf{q},\omega)=\frac{S}{(2\pi)^2}\int \mathrm{d}^2\mathbf{k}\frac{n(\epsilon_{\mathbf{k}-\mathbf{q}/2})-n(\epsilon_{\mathbf{k}+\mathbf{q}/2})}{\epsilon_{\mathbf{k}-\mathbf{q}/2}-\epsilon_{\mathbf{k}+\mathbf{q}/2}+\hbar\omega+i0^+}$$
$$=-\frac{N(0)}{2}\left(1+\frac{\frac{i\omega}{v_{\mathrm{F}}q}}{\sqrt{1-\left(\frac{\omega}{v_{\mathrm{F}}q}\right)^2}}\right) \tag{7}$$

and

$$\chi_4(\mathbf{q},\omega)=\frac{2\omega_{\mathbf{q}}/\hbar}{\omega^2-\omega_{\mathbf{q}}^2+i0^+}, \tag{8}$$

where $\epsilon_{\mathbf{k}}=\hbar^2\mathbf{k}^2/2m_3-\epsilon_{\mathrm{F}}$ with ϵ_{F} being the Fermi energy and $n(\epsilon_{\mathbf{k}})$, v_{F}, and $N(0)=m_3S/\pi\hbar^2$ are the Fermi-Dirac distribution, the Fermi velocity, and the density of states at the Fermi surface of ^{3}He, respectively. In the equations above, terms of order q^4 are neglected since we are interested in low energy excitations.

Within the RPA treatment, the susceptibility matrix of mixture films can be obtained as a solution of $\hat{\chi}=\hat{\chi}^{(0)}+\hat{\chi}^{(0)}\hat{V}\hat{\chi}$,[19] which is

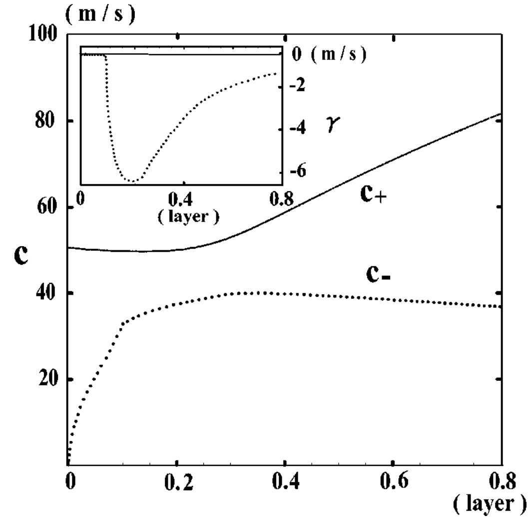

FIG. 1. Real and imaginary parts (inset) of the spectrum are plotted with $\omega/q=c+i\gamma$ as functions of the concentration of the ^{3}He. Here we fix the thickness of ^{4}He films to $d=25.3$ μmol/m^2 and $h_4=25.8$ μmol/m^2, and the graphite sheet is modeled as a substrate.

$$\hat{\chi}_{\mathrm{RPA}}=\frac{1}{1-v_3\chi_3-2g_{\mathbf{q}}^2\chi_3\chi_4}\times\begin{pmatrix}\chi_3 & g_{\mathbf{q}}\chi_3\chi_4\\ g_{\mathbf{q}}\chi_3\chi_4 & \chi_4(1-v_3\chi_3)\end{pmatrix}, \tag{9}$$

where the factor 2 in the last term of the denominator comes as a result of summing over the spin indices. If $g_{\mathbf{q}}=0$, it can easily be seen that this susceptibility matrix becomes diagonal, and that its elements are usually the RPA susceptibility, $\chi_3/(1-v_3\chi_3)$, and χ_4. Therefore, two poles of this matrix correspond to the spectrum of two eigenmodes; one is the zero sound branch of ^{3}He due to the nonlinearity of the van der Waals potential, and the other is third the sound branch of ^{4}He. For the third sound phonon, this RPA treatment is equivalent to solving Dyson's equation in the Migdal approximation,[20] where the effective self-energy of the third sound is $2g_{\mathbf{q}}^2\chi_3/(1-v_3\chi_3)$. At zero temperature and long-wave length limits, the dispersion relation and damping rate are calculated numerically, and the results are shown in Fig. 1. Here, we employ the parameters u_i and d, modeling the double layer system on the flat surface of graphite.[3] For the films, we fix the thickness of ^{4}He films, and alter the ^{3}He concentration from a small fraction of the monolayer to one layer (a monolayer corresponds to the density 10.6 μmol/m^2 for ^{3}He and 12.9 μmol/m^2 for ^{3}He[8]). One can see that these two branches exhibit a level repulsion characteristic of reactively coupled oscillators. Hence, the two branch are well hybridized and show a clear level splitting around the region $h_3\approx0.3$ layer (~3.2 μmol/m^2), where c_{B} crosses the bare zero sound velocity $c_{\mathrm{F}}=[(1+\tilde{v}/2)/(\sqrt{1+\tilde{v}})]v_{\mathrm{F}}$ with $\tilde{v}=N(0)v_3$. As the concentration of ^{3}He particles increases further, one can see from the dynamical structure function of ^{4}He, $\mathrm{S}^{(44)}(\mathbf{q},\omega)$, defined as

$$\hat{S}(\mathbf{q},\omega)=-\frac{1}{\pi}\mathrm{Im}\hat{\chi}(\mathbf{q},\omega)=\begin{pmatrix}S^{(33)} & S^{(34)}\\ S^{(43)} & S^{(44)}\end{pmatrix}, \tag{10}$$

that the third sound spectrum evolves continuously from the upper branch $(+)$ to the lower one $(-)$, and moves into the particle-hole continuum of ^{3}He (see Fig. 2). This situation results in the strong damping of the bosonic third sound by resonantly absorbing the ^{3}He particle-hole pair. This behavior of the spectrum has not been verified.[15,21] To our knowledge, there is no experimental result carefully investigating the region where the velocities of two components are close to each other.

As for the damping rate, the beginning of the damping of the hybridized mode can be seen in the discontinuous concentration dependence of the sound velocity in the lower branch. Unlike conventional electron-phonon systems in metals, the ratio of the sound velocities of the two components is of order unity; therefore the effect of this damping is not small and has a possibility to be observed (see the inset of Fig. 1). Indeed, the imaginary part of the spectrum can be easily estimated to have a behavior $\sim -[\lambda/(2+\tilde{v}) \times[c_{\mathrm{B}}/\sqrt{v_{\mathrm{F}}^2-c_{\mathrm{B}}^2}](c_{\mathrm{B}}q)$ at a high concentration regime of ^{3}He, i.e., $c_{\mathrm{B}}<v_{\mathrm{F}}$. Here, $\lambda=-N(0)g_{\mathbf{q}}^2\chi_4(\mathbf{q},0)>0$ is a dimensionless phonon exchange coupling constant, which is determined by the Fermi energy and the van der Waals potential.[5] Around the region $c_{\mathrm{B}}\approx v_{\mathrm{F}}$, it has a strong peak which amounts to about 15% of the spectrum. In the 3D case, a similar level repulsion and a damping of the bosonic mode are discussed by Yip, with dilute boson-fermion mixture gases of alkali atoms, such as ^{39}K–^{40}K and ^{6}Li–^{7}Li, in mind.[18] In his system, fermions are spin polarized because of magnetic trapping and fermion-fermion interaction is not included, hence zero sound, due to phonon exchange interaction only, is discussed.

Having established the existence of the level repulsion and the damping within the RPA treatment, we will discuss the effect of the vertex function for the ^{3}He–^{4}He exchange interaction in more detail. Unlike conventional metals, the "Debye energy" is comparable to or greater than ϵ_{F} in our system. This situation opens the possibility of a scenario in which nonadiabatic effects are relevant, and new qualitative phenomena arise from vertex corrections. Here, we treat the vertex function in ladder approximation,[22] i.e., as a solution of the Bethe-Salpeter equation

$$\Gamma(\mathbf{q},i\omega_m)=g_{\mathbf{q}}-k_{\mathrm{B}}T\sum_{ik_n}\sum_{\mathbf{k}} g_{\mathbf{k}-\mathbf{p}}^2\chi_4(\mathbf{p}-\mathbf{k},ip_l-ik_n) \times G(\mathbf{k},ik_n)G(\mathbf{k}-\mathbf{q},ik_n-i\omega_m)\Gamma(\mathbf{q},i\omega_m), \tag{11}$$

where $G(\mathbf{k},ik_m)$ is a bare single-particle propagator of ^{3}He quasiparticles, with ik_n and $i\omega_m$ being fermionic and bosonic Matsubara frequencies. We assume that the vertex function depends only on the energy-momentum transfer. Here, the external fermion frequency ip_l is set to zero after an analytic continuation. This approximation may be justified when low energy particle-hole excitations give the dominant contribution. As for the external wave vector $\mathbf{p}$, we introduce q_{c} as a cutoff of the wave number transfer $|\mathbf{p}-\mathbf{k}|$,[23] which is of the order of the inverse of the coherence length of ^{4}He films. At zero temperature and the long wavelength limit, this equation is solved analytically and the vertex function has the following forms:

$$\Gamma(\mathbf{q},\omega)=\frac{g_{\mathbf{q}}}{1+\dfrac{\lambda}{2}\left(C(\mathbf{q},\omega)+\dfrac{\dfrac{i\omega}{v_{\mathrm{F}}q}}{\sqrt{1-\left(\dfrac{\omega}{v_{\mathrm{F}}q}\right)^2}}\right)}, \tag{12}$$

$$C(\mathbf{q},\omega)=\frac{1}{2}\left(\frac{1}{\sqrt{1-\left(\dfrac{v_{\mathrm{F}}q}{c_{\mathrm{B}}q_{\mathrm{c}}-\omega}\right)^2}}+\frac{1}{\sqrt{1-\left(\dfrac{v_{\mathrm{F}}q}{c_{\mathrm{B}}q_{\mathrm{c}}+\omega}\right)^2}}\right). \tag{13}$$

Here, we neglect the polarization contribution of the second term on the right hand side of Eq. (11), which has the same dynamical ($\omega\to 0$, $\mathbf{q}=0$) and static ($\omega=0$, $\mathbf{q}\to 0$) limits, because it gives only quantitative difference in the denominator of the vertex function. Obviously, the Migdal approximation breaks down qualitatively in the region $\omega\approx v_{\mathrm{F}}q$ and the vertex function diverges to $\pm\infty$, as $\omega\to\omega_{\mathrm{C}}$ from above and below, respectively. Here, ω_{C} is obtained as

$$\omega_{\mathrm{C}}\simeq\frac{1+\lambda/2}{\sqrt{1+\lambda}}\left(v_{\mathrm{F}}q-\frac{\lambda(1+\lambda^2/2)}{8(1+\lambda/2)^2}\frac{(v_{\mathrm{F}}q)^3}{(c_{\mathrm{B}}q_{\mathrm{c}})^2}\right). \tag{14}$$

This means that the dynamical phonon-mediated ^{3}He–^{3}He interaction becomes strongly enhanced and even changes its sign. Its behavior resembles the Feshbach resonance in alkali atom gases,[24] but this frequency dependent "potential" cannot be used in a Hamiltonian formalism such as the scattering length of alkali atoms, since the strong frequency dependence of interaction imposes one to take the strong retardation effect into account.[22] We expect this breakdown of perturbation theory not to lead to a violation of Fermi liquid theory. Indeed, the pole provides only a subdominant correction to the imaginary part of single-particle self-energy that behaves as $\mathrm{Im}\Sigma^p(k_F,\omega)\sim\lambda^3\epsilon_F(\hbar\omega/\epsilon_F)^2$ for $\omega\to 0^+$ (subscript p denotes the pole contribution). Moreover, nonzero quasiparticle residue Z at the Fermi surface can be derived with use of the Kramers-Kronig relation. Thus the quasiparticle is well defined, even with the singularity in the vertex function.

We shall investigate the effect of this phonon exchange vertex function to the spectrum of excitations. The singularity of $\Gamma(\mathbf{q},\omega)$ in the long wavelength limit implies the existence of a low-lying excitation which obeys Bose-Einstein statistics. We interpret the pole as the excitation energy of an additional collective mode. Physically, this third branch corresponds to a combined mode of the ^{3}He particle-hole pair and the third sound phonon. It may be considered as the pole

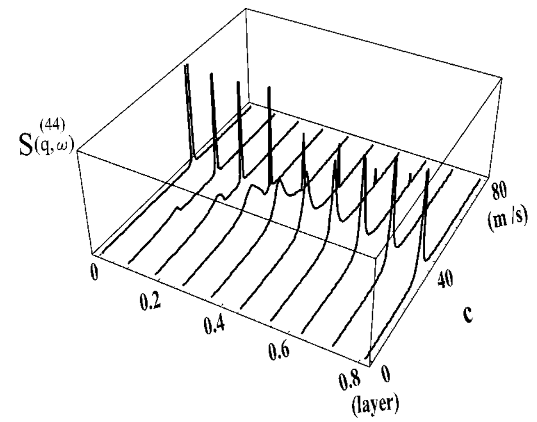

FIG. 2. The Dynamical structure function $S^{(44)}(q,\omega)$ for a fixed wave number q is plotted as a function of the concentration of ^{3}He and the velocity of excitation. The third sound evolves to the lower branch, and the spectral weight $S^{(44)}(q,\omega)$ is broadened as concentration of the ^{3}He quasiparticle increase. Parameters u_i, d, and h_4 are the same as in Fig. 1.

of $D_C(\mathbf{q},\omega)$, which is Fourier transformation of $-\Sigma_\sigma\langle T_t[\phi_{\mathbf{q}}(t)\rho_{-\mathbf{q},\sigma}(\mathbf{0})+\rho_{\mathbf{q},\sigma}(t)\phi_{-\mathbf{q}}(0)]\rangle$.[20] Here, T_t is Wick's usual time ordering operator, $\phi_{\mathbf{q}}(t)$ is the field operator of the third sound phonon, and $\langle\cdots\rangle$ denotes the average for the ground state of the mixture. It should be noted that this ''susceptibility'' contains multiphonon processes, thus the relation to area density fluctuations is no longer linear. The third spectrum obtained by the numerical calculation, lies above particle-hole continuum and between the upper and lower branches. Furthermore, one can see from the dynamical structure function $S^C(\mathbf{q},\omega)=-1/\pi\mathrm{Im}D_C$ that the combined mode branch has a considerable spectral weight except for the region where the zero sound and third sound are well hybridized (see Fig. 3). The behavior of this spectral weight causes the resonant nature of $S^C(\mathbf{q},\omega)$ at $\omega\approx c_B q$. Indeed, the combined mode is completely suppressed as $\omega_C\to c_B q$. Physically, this behavior causes the mode-mode repulsion.[18]

We also investigate the correction of the combined mode to the Landau Fermi liquid parameter. At zero temperature, it is obtained as $f_{\sigma,\sigma'}(\theta)=\delta\Sigma_\sigma(\mathbf{k},\epsilon_{\mathbf{k}})/\delta n_{\sigma'}(\epsilon_{\mathbf{k}'})$ with $\mathbf{k}\cdot\mathbf{k}'=k_F^2\cos\theta$.[25] Since the combined mode does not contribute to $f_{\uparrow\downarrow}(\theta)$, we can obtain the Landau f function as follows[25]:

$$f_{\uparrow\uparrow}(\theta)=f_{\uparrow\downarrow}(\theta)-\mathrm{P.}\int\frac{ds}{\pi}\frac{\lambda}{s}\frac{\mathrm{Im}\Gamma(\mathbf{k}-\mathbf{k}',s)}{g_{\mathbf{k}-\mathbf{k}'}}. \tag{15}$$

Here, P. denotes the principal value of the integral. The pole contribution, i.e., the collective mode contribution, comes from the delta function part of ImΓ, and is obtained as

$$N(0)[f^p_{\uparrow\uparrow}(\theta)-f^p_{\uparrow\downarrow}(\theta)]\approx-\frac{\lambda^3}{16}\left[1+\frac{\lambda}{2}\frac{(v_F k_F)^2}{(c_B q_c)^2}\sin^2\left(\frac{\theta}{2}\right)\right], \tag{16}$$

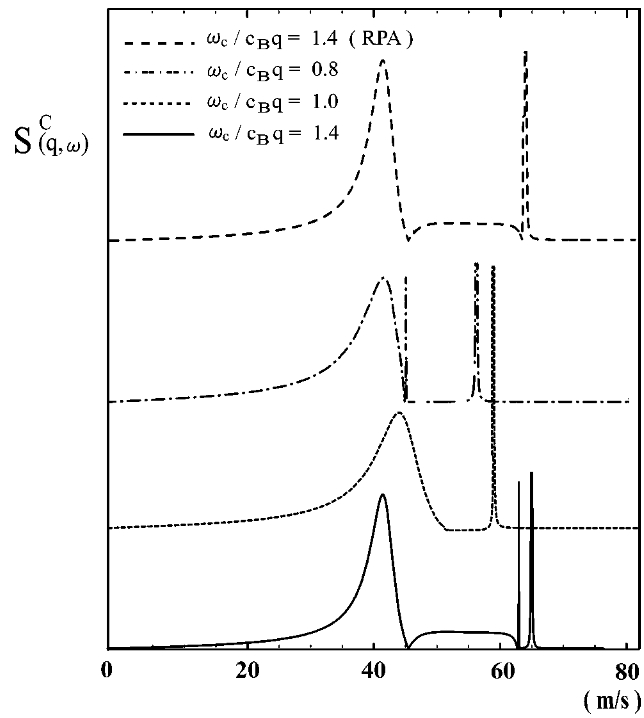

FIG. 3. The dynamical structure function of the combined mode, $S^C(q,\omega)$, is plotted for three parameter regimes as a function of ω/q. One can see that there exists a sharp spectral weight between the third sound and zero sound, which does not occur within the RPA treatment. At $\omega_C/c_B q=1.4$, the spectral weight of the third branch is found to be about 30% of all. At $\omega_C/c_B q=1.4$, the spectral weight of the third branch is found to be about 30% of all. At the region $\omega_C\approx c_B q$, the combined mode is strongly suppressed by the mode-mode repulsion effect.

In addition, the contribution from the particle-hole continuum of the vertex function can be obtained from the real part of Γ with use of Kramers-Kronig relation, and has the following form:

$$N(0)(f^{ph}_{\uparrow\uparrow}(\theta)-f^{ph}_{\uparrow\downarrow}(\theta))\approx\frac{-\lambda}{1+\frac{\lambda}{2}C\left[2k_F\sin\left(\frac{\theta}{2}\right),0\right]}. \tag{17}$$

These contributions are not singular. Additionally, the pole gives rise to an additional factor λ^2 to the correction for the Landau f function. Therefore, the third branch contribution may become important in the strong coupling region.

As for the observability, this third branch may be observed under a strong van der Waals potential, since it is inherently related to the interaction. Therefore, an experiment with low-thickness superfluid ^{4}He films and a substrate which has a strong van der Waals potential, such as gold, is desired. A recent experiment made it possible to evaluate the Landau Fermi liquid parameter as a function of the ^{3}He concentration[4]; therefore the third branch contribution [Eq. (16)] has the possibility to be observed at the low-concentration regime of ^{3}He. Further, it should be noted that there is no restriction in system components in our method; therefore, these discussions can be easily applied to quasi-2D alkali atom gases in which phase separation does not occur. Regarding the existence of the third branch, an alkali gas system may be better suited for experimental verification because fermion-boson interaction can be almost freely changed as well as other system parameters. In such a case, zero sound with higher spin channel may be taken into account.[26]

In conclusion, we have investigated the effects of a phonon exchange process on the collective excitations of ^{3}He and ^{4}He at absolute zero. Two remarkable features have been found. One is the level repulsion between the zero sound and third sound due to ^{3}He–^{4}He interaction. This repulsion becomes remarkable in the region where the sound velocities of the two components are close to each other. This situation results in a hybridization of two eigenmodes and a finite level splitting in the spectrum. Furthermore, the damping of the third sound, resonant decay into the particle-hole pair excitation, is also shown to be significant when the third sound velocity is smaller than v_F. Also, Migdal's theorem breaks down when the nonadiabatic effect of ^{3}He–^{4}He interaction is considered. A third branch comes in; this may be interpreted as a combined mode of a particle-hole pair and third sound quanta. The concentration dependence of the dynamical structure factor and the correction to the Landau Fermi liquid parameter of this collective excitation are also discussed.

In the discussions above, we have focused on the low energy collective behavior and we have not considered the two-body scattering problem. It is expected that the interaction between ^{3}He and ^{4}He induces the effective ^{3}He–^{3}He attraction and leads to spin-singlet and neutral superfluid formation.[5–8] In future work we will consider the possibility that nonadiabatic phonon exchange may either drive the system into a superfluid transition or polaron formation.

The authors would like to thank M. Nishida, S. Tsuchiya, B. H. Valtan, and A. Morales for useful discussions.

[1] R. B. Hallock, in *Progress in Low Temperature Physics*, edited by W. P. Halperin (Elsevier-North-Holland, Amsterdam, 1995), Vol. XIV, p. 321, and references cited therein.

[2] F. M. Gasparini, B. Bhattacharyya, and M. J. DiPirro, Phys. Rev. B **29**, 4921 (1982).

[3] M. Dann, J. Nyéki, B. P. Cowan, and J. Saunders, Phys. Rev. Lett. **82**, 4030 (1999).

[4] P.-C. Ho and R. B. Hallock, Phys. Rev. Lett. **87**, 135301 (2001).

[5] S. Kurihara, J. Phys. Soc. Jpn. **51**, 3844 (1982); **52**, 1311 (1983).

[6] K. Miyake, Prog. Theor. Phys. **69**, 1794 (1983).

[7] E. Bashkin, N. Pavloff, and J. Treiner, J. Low Temp. Phys. **99**, 659 (1995).

[8] E. Krotscheck, J. Paaso, M. Saarela, and K. Schörkhuber, Phys. Rev. Lett. **85**, 2344 (2000); Phys. Rev. B **64**, 054504 (2001).

[9] G. A. Csáthy and M. H. W. Chan, Phys. Rev. Lett. **87**, 045301 (2001); G. A. Csáthy, E. Kim, and M. H. W. Chan, *ibid.* **88**, 045301 (2002).

[10] M. Krech and S. Dietrich, Phys. Rev. A **46**, 1886 (1992).

[11] R. Garcia and M.H.W. Chan, Phys. Rev. A **83**, 1187 (1999); **88**, 086101 (2002).

[12] F. M. Ellis, R. B. Hallock, M. D. Miller, and R. A. Guyer, Phys. Rev. Lett. **46**, 1461 (1981).

[13] E. Krotscheck, Phys. Rev. B **32**, 5713 (1985).

[14] R. A. Guyer and M. D. Miller, Phys. Rev. Lett. **47**, 349 (1981).

[15] R. H. Anderson, M. D. Miller, and R. B. Hallock, Phys. Rev. B **59**, 3345 (1999).

[16] R. H. Anderson and M. D. Miller, Phys. Rev. B **40**, 2109 (1989); **48**, 10 426 (1993).

[17] R. H. Anderson and M. D. Miller, Phys. Rev. B **66**, 014526 (2002).

[18] S.-K. Yip, Phys. Rev. A **64**, 023609 (2001).

[19] E. Kroscheck and M. Saarela, Phys. Rep. **232**, 1 (1993).

[20] See, e.g., A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Dover, New York, 1963).

[21] J. M. Valles, Jr., R. M. Heinrichs, and R. B. Hallock, Phys. Rev. Lett. **56**, 1704 (1986).

[22] J. R. Schrieffer, *Theory of Superconductivity* (Benjamin, New York, 1964).

[23] M. Botti, E. Cappelluti, C. Grimaldi, and L. Pietronero, Phys. Rev. B **66**, 054532 (2002).

[24] E. Tiesinga, B. J. Verhaar, and H. T. C. Stoof, Phys. Rev. A **47**, 4114 (1993).

[25] J. R. Engelbrecht, M. Randeria, and L. Zhang, Phys. Rev. B **45**, 10 135 (1992).

[26] S.-K. Yip and T.-L. Ho, Phys. Rev. A **59**, 4653 (2002).