Kinetics and Boltzmann kinetic equation for fluctuation Cooper pairs

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The Boltzmann equation for excess Cooper pairs above T_c is derived in the framework of the time-dependent Ginzburg-Landau (TDGL) theory using Langevin's approach of the stochastic differential equation. The Newton dynamic equation for the momentum-dependent drift velocity is obtained and the effective drag force is determined by the energy-dependent lifetime of the metastable Cooper pairs. The Newton equation gives just the Drude mobility for the fixed momentum of Cooper pairs. It is shown that the comparison with the well-known result for Aslamazov-Larkin paraconductivity and BCS treatment of the excess Hall effect can give the final determination of all the coefficients of TDGL theory. As a result the intuitive arguments used for an interpretation of the experimental data for fluctuation kinetics are successively introduced. The presented simple picture of the degenerated Bose gas in τ approximation near the Bose-Einstein condensation temperature can be used for analysis of fluctuation conductivity for the cases of high frequency and external magnetic field for layered and bulk superconductors. The work of the Boltzmann equation is illustrated by frequency-dependent Aslamazov-Larkin conductivity in nanowires, in the two-dimensional case and in the case of strong electric field where the TDGL equation is solved directly. There are also derived explicit formulas for the current in the case of arbitrary time dependence of electric field up to the THz range, the distribution of fluctuation Cooper pairs for nonparabolic dispersion, the influence of the energy cut-off, and the self-consistent equation for the reduced temperature. The general theory is illustrated by formulas for fluctuation conductivity in nanowires and nanostructured superconductors.

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I. INTRODUCTION

For all high temperature superconductors the fluctuation phenomena can be observed and their investigation takes a significant part of the complete understanding of these materials; for a contemporary review on the fluctuation phenomena in superconductors see the review by Larkin and Varlamov.[1] The Ginzburg-Landau (GL) approach of the order parameter is an adequate tool to investigate the low-frequency behavior of fluctuations near T_c; for a review of the Gaussian GL fluctuations see Ref. 2. A lot of important papers on the fluctuation phenomena in superconductors and related topics have not been cited in these reviews, see for example Ref. 3. We wish to point out that the GL approach is the standard tool for the investigation of magnetic field penetration in superconductors[4] and even the non-Gaussian approach to critical fluctuations.[5]

Amidst all kinetic phenomena the fluctuation conductivity created by the metastable in the normal state Cooper pairs is probably best investigated. The Boltzmann equation is a standard tool for investigation of kinetic phenomena and the purpose of the present paper is to derive the Boltzmann equation for fluctuation Cooper pairs and to illustrate its work on the example of the fluctuation conductivity; a shortened version of the present research was presented in preliminary communications.[6] We rederive the frequency dependence of the Aslamazov-Larkin conductivity, fluctuation Hall effect at weak magnetic fields, and magnetoconductivity. We analyze the experimental data for indium oxide films and find significant deviation from the BCS weak coupling prediction. We are coming to the conclusion that a systematic investigation of the lifetime of fluctuation Cooper pairs will give important information for our understanding of the physics of superconductivity.

II. FROM THE TDGL EQUATION VIA BOLTZMANN EQUATION TO NEWTON EQUATION

Our starting point is the time-dependent Ginzburg Landau (TDGL) equation for the superconducting order parameter derived in the classical paper by Abrahams and Tsuneto[7]; see also Ref. 8 and references cited in the review by Larkin and Varlamov,[1]

$$\frac{(-i\hbar D_r)^2}{2m^*}\Psi + a\Psi + b|\Psi|^2\Psi = -\hbar\gamma(D_t\Psi - \zeta), \qquad (1)$$

where m^* and $|e^*|=2|e|$ are the mass and charge of the Cooper pairs, parameter γ describes the dissipation, and $\zeta(\mathbf{r},\mathbf{t})$ is the external noise in the TDGL equation. Here

$$-i\hbar D_r = -i\hbar\nabla - e^*\mathbf{A}/c,$$

$$i\hbar D_t = i\hbar\partial_t - e^*\varphi,$$

are the operators of kinetic momentum and energy, $\mathbf{A}$ is a vector potential, and φ is the potential.

Close to the critical temperature $a(T)\approx(T-T_c)a_0/T_c$, and $b\approx$const, where $a_0=\hbar^2/2m^*\xi^2(0)$, and $\xi(0)$ is coherence length.

The correlations of the white noise $\langle\zeta\rangle=0$,

$$\langle \zeta^*(\mathbf{r}_1,t_1)\zeta(\mathbf{r}_2,t_2)\rangle = \Gamma\,\delta(t_1-t_2)\,\delta(\mathbf{r}_1-\mathbf{r}_2), \tag{2}$$

are parametrized by fluctuation parameter Γ. The BCS theory gives

$$\gamma_{BCS} = \frac{\pi}{8}\frac{a_0}{T_c},$$

and that is why we parametrize $\gamma = \gamma_{BCS}\tau_{rel}$, by the dimensionless parameter $\tau_{rel} \simeq 1$, which describes the relative lifetime of fluctuation Cooper pairs.

The most simple is the case of free particles, which means $\mathbf{A}=\mathbf{0}$, $\varphi=0$, $b|\Psi|^2 \approx 0$. Introducing the Fourier transformation

$$\Psi(\mathbf{r},t) = \sum_p \frac{e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}}{\sqrt{\mathcal{V}}}\psi_p(t), \tag{3}$$

$$\zeta(\mathbf{r},t) = \sum_p \frac{e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}}{\sqrt{\mathcal{V}}}\zeta_p(t), \tag{4}$$

where

$$\sum_p \approx \mathcal{V}\int \frac{d^D p}{(2\pi\hbar)^D},$$

and

$$\langle \zeta_p^*(t_1)\zeta_q(t_2)\rangle = \Gamma\,\delta_{p,q}\delta(t_1-t_2),$$

we obtain the TDGL equation in momentum representation

$$(\varepsilon_p + a)\psi_p = -\hbar\gamma(d_t\psi_p - \zeta_p). \tag{5}$$

The solution of this reads

$$\psi_p(t) = e^{-t/2\tau_p}\left(\int_0^t e^{t'/2\tau_p}\zeta_p(t')dt' + \psi_p(0)\right), \tag{6}$$

where

$$\tau_p = \frac{\hbar\gamma}{2(\varepsilon_p + a(T))}, \quad \varepsilon_p = \frac{p^2}{2m^*} \tag{7}$$

are the momentum-dependent lifetime and kinetic energy of fluctuation Cooper pairs. The number of particles for every momentum can be found by noise averaging

$$n_p = \langle \psi_p^*(t)\psi_p(t)\rangle = n_p(0)e^{-t/\tau_p} + (1-e^{-t/\tau_p})\overline{n_p}, \tag{8}$$

where $n_p(0) = |\psi_p(0)|^2$ is the initial number. The time differentiation of this solution gives the well-known Boltzmann equation

$$\frac{d}{dt}n_p(t) = -\frac{1}{\tau_p}(n_p(t) - \overline{n_p}), \tag{9}$$

which can be considered in this physical situation as a consequence of the TDGL equation. The quantity

$$\overline{n_p} = n_p(t=\infty) = \Gamma\tau_p$$

gives the equilibrium number of particles. The fluctuation parameter Γ is related to dissipation parameter γ by the fluctuation-dissipation theorem, which here takes the form

$$\Gamma = \frac{2T}{\hbar\gamma} = \frac{\overline{n_p}}{\tau_p} = \frac{T}{a_0\tau_0}, \tag{10}$$

where

$$\overline{n_p} = \frac{T}{\varepsilon_p + a(T)}$$

is the Rayleigh-Jeans distribution.

Let us now analyze the influence of a weak electric field in the Boltzmann[9] equation

$$\partial_t n_p + e^*\mathbf{E}\cdot\partial_{\mathbf{p}} n_p = -\frac{1}{\tau_p}(n_p - \overline{n_p}). \tag{11}$$

For the solution we search in the form

$$n_p(t) = n(\mathbf{p},t) \approx \overline{n}[\mathbf{p} - m^*\mathbf{V}(\mathbf{p},t)], \tag{12}$$

and we obtain the Newton equation

$$m^* d_t \mathbf{V}_p(t) = e^*\mathbf{E} - \frac{m^*}{\tau_p}\mathbf{V}_p(t) \tag{13}$$

for the field of drift velocity in momentum space. The general formula for the current gives

$$\mathbf{j}_{\rm fl} = \sum_p e^*\frac{n_p}{\mathcal{V}}\mathbf{v}_p = \overleftrightarrow{\sigma}_{\rm fl}\cdot\mathbf{E}, \quad n_{\rm D} = \sum_p \frac{n_p}{\mathcal{V}}, \tag{14}$$

where n_D is the D-dimensional volume density of the fluctuation Cooper pairs. Substitution here of the shifted equilibrium distribution gives the well-known formula for the conductivity tensor[10]

$$\overleftrightarrow{\sigma}_{\rm fl} = e^{*2}\int \frac{d^D p}{(2\pi\hbar)^D}\frac{\mathbf{v}_p\otimes\mathbf{v}_p}{1/\tau_p - i\omega}\left(-\frac{\partial n_p}{\partial\varepsilon_p}\right), \tag{15}$$

where $\mathbf{v}_p = \partial_p\varepsilon_p = \mathbf{p}/m^*$ is the Cooper pairs' velocity. This is only a small fraction of the total conductivity

$$\sigma(T) = \sigma_{\rm N}(T) + \sigma_{\rm fl}(\epsilon), \quad \epsilon = \frac{T-T_c}{T_c}, \quad \sigma_{\rm fl} \ll \sigma_{\rm N}. \tag{16}$$

For thin superconducting films $D=2$ substituting

$$\frac{dp_x dp_y}{(2\pi\hbar)^2} = \frac{d(\pi p^2)}{(2\pi\hbar)^2} = \frac{m^*}{2\pi\hbar^2}d\varepsilon_p, \quad -\frac{\partial\overline{n}}{\partial\varepsilon} = \frac{T}{(\varepsilon+a)^2}, \tag{17}$$

we obtain the classical result by Aslamazov and Larkin[11]

$$\sigma_{\rm AL}(\epsilon) = \frac{e^2}{16\hbar}\tau_{\rm rel}\frac{T_c}{T-T_c} = \frac{e^2T}{\pi\hbar^2}\tau(\epsilon), \tag{18}$$

where

$$\tau(\epsilon)\equiv\tau(\mathbf{p}=\mathbf{0},\epsilon)=\frac{\pi\hbar}{16T_c}\frac{\tau_{\rm rel}}{\epsilon}=\frac{\tau_0}{\epsilon} \tag{19}$$

is the lifetime for Cooper pairs with zero momentum.

For the two-dimensional (2D) case conductivity is just the inverse resistance $\sigma^{(2D)}(T)=R_{\square}^{-1}(T)$. For conventional disordered superconductors normal conductivity can be approximated by residual conductivity far above T_c, for example, $T=3T_c$. In this approximation Aslamazov-Larkin conductivity can be rewritten in a convenient for experimental data processing form

$$\left(\frac{1}{R_{\square}(T)}-\frac{1}{R_{\square}(3T_c)}\right)^{-1}\approx\frac{16\hbar}{e^2\tau_{\rm rel}}\left(\frac{T}{T_c}-1\right). \tag{20}$$

Performing the linear regression fit of the data presented in Ref. 12 we have obtained that for indium oxide films $\tau_{\rm rel}=1.15$. This significant 15% deviation from the weak coupling BCS value is created by strong coupling effects. We conclude that analogous systematic investigations for thin films would be very helpful for our understanding of the dynamics of the order parameter in superconductors. Decreasing the lifetime and $\tau_{\rm rel}$ by depairing impurities or disorder for anisotropic gap superconductors definitely deserves a great attention.

III. FLUCTUATION CONDUCTIVITY IN DIFFERENT PHYSICAL CONDITION

A. High-frequency conductivity

For diagonal components of conductivity taking into account that $\mathrm{Tr}\hat{1}=D$, from the general formula Eq. (15), we obtain

$$\sigma_{\rm fl}=\frac{e^{*2}}{D}\int\frac{d^Dp}{(2\pi\hbar)^D}\frac{v_p^2}{1/\tau_p-i\omega}\left(-\frac{\partial n_p}{\partial\varepsilon_p}\right). \tag{21}$$

It is convenient to introduce a dimensionless frequency $z=\omega\tau(\epsilon)$. In order to derive the dimensionless complex conductivity $\varsigma(\omega)$ we need to solve the elementary integral

$$\varsigma(z)=\varsigma_1(z)+i\varsigma_2(z)=2\int_1^\infty\frac{x-1}{x^2(x+y)}dx \tag{22}$$

$$=\frac{2}{y}\left[\left(1+\frac{1}{y}\right)\ln(1+y)-1\right]$$

$$=\frac{2}{-iz}\left[\left(1+\frac{1}{-iz}\right)\ln(1-iz)-1\right] \tag{23}$$

where $x=p^2/2m^*a(\epsilon)+1$ is the kinetic energy of Cooper pairs taken into account from the ''chemical potential'' in $a(\epsilon)$ units, and $y=-iz=-i\omega\tau(\varepsilon)$ is the dimensionless Matsubara frequency $\zeta_{\rm M}=-i\omega$. The integral Eq. (22) is solved considering the Matsubara frequency y to be a real variable. Then we can make the analytical continuation to real frequencies substituting $y=-iz$ in the result Eq. (23). This method is very popular in the quantum field theory, but works effectively for classical problems as well. In such a way we obtain

$$\varsigma_1(z)=\frac{2}{z^2}\left[z\arctan(z)-\frac{1}{2}\ln(1+z^2)\right]=\frac{2}{\pi}\mathcal{P}\int_0^\infty\frac{x\varsigma_2(x)}{x^2-z^2}dx, \tag{24}$$

$$\varsigma_2(z)=\frac{2}{z^2}\left[\arctan(z)-z+\frac{z}{2}\ln(1+z^2)\right]$$

$$=-\frac{2z}{\pi}\mathcal{P}\int_0^\infty\frac{\varsigma_1(x)}{x^2-z^2}dx. \tag{25}$$

Then the frequency-dependent conductivity reads

$$\sigma_{\rm 2D}(\epsilon,\omega)=\sigma_{\rm AL}(\epsilon)\varsigma(\omega\tau(\epsilon)). \tag{26}$$

The integral Eq. (21) can be solved for arbitrary dimension

$$\sigma_{\rm D}(\epsilon,\omega)=\sigma_{\rm D}(\epsilon)\varsigma_{\rm D}(z), \tag{27}$$

cf. the paper by Dorsey[13]

$$\sigma_{\rm D}(\epsilon)=4\frac{\Gamma(2-D/2)}{(4\pi)^{D/2}}\frac{e^2}{\hbar}[\xi(\epsilon)]^{2-D}\frac{T\tau(\epsilon)}{\hbar}, \tag{28}$$

where $\xi(\epsilon)\equiv\xi(0)/\sqrt{\epsilon}$ is the temperature-dependent coherence length. The conductivity in this case has the form

$$\varsigma_{\rm 1,D}(z)=\frac{8}{D(D-2)z^2}\left[1-(1+z^2)^{D/4}\cos\left(\frac{D}{2}\arctan z\right)\right],$$

$$\varsigma_{\rm 2,D}(z)=\frac{8}{D(D-2)z^2}\left[-\frac{D}{2}z+(1+z^2)^{D/4}\sin\left(\frac{D}{2}\arctan z\right)\right]. \tag{29}$$

B. Hall effect

The fluctuation Hall conductivity also can be derived in the framework of the Boltzmann kinetic equation. We have to take into account a small imaginary part α of the γ parameter in the TDGL equation, i.e., $\gamma\to\gamma+i\alpha$, and $\alpha\ll\gamma$. The solution[6] of the kinetic equation gives

$$\sigma_{xy}(\epsilon)=\frac{Z}{3}\omega_c\tau(\epsilon)\sigma_{\rm AL}(\epsilon)\propto\tau^2(\epsilon), \tag{30}$$

where $\omega_c=e^*B/m^*c$ is the ''cyclotron'' frequency and

$$Z=-\,\mathrm{Im}\frac{1}{\gamma+i\alpha}\approx\frac{\alpha}{\gamma^2}\ll1. \tag{31}$$

This result agrees with microscopic calculations.[1] Due to the small value of the parameter α, the fluctuation Hall effect is difficult to observe. With fitting of α and m^* from the experimental data finishes the complete determination of parameters of TDGL theory.

C. Magnetoconductivity

It is interesting to mention that the classical formula for the conductivity Eq. (15) correctly works even for strong magnetic fields. We only have to substitute the momentum integration with summation on discrete Landau levels, taking into account the density of Landau magnetic subbands

$$\epsilon_p \to \epsilon_n = \hbar\omega_c\left(n+\frac{1}{2}\right) = a_0(2n+1)h, \tag{32}$$

where

$$h = \frac{\hbar\omega_c}{2a_0} = \frac{B_z}{B_{c2}(0)} \tag{33}$$

is the dimensionless magnetic field and

$$B_{c2}(0) = -T_c\frac{d}{dT}B_{c2}(T)|_{T_c} \tag{34}$$

is the linear extrapolation. In the numerator of Eq. (15) we have to substitute the classical velocity with the oscillator matrix elements of the momentum. Analogously for the energy-dependent lifetime we have to average on neighboring levels. Due to the triviality of the oscillator problem these substitutions can be performed in only one way, and Aslamazov-Larkin conductivity Eq. (18) is substituted by the magnetoconductivity of Abrahams, Prange, and Stephen[14] (APS)

$$\sigma_{\mathrm{AL}}(\epsilon) = \frac{e^2T\tau_0}{\pi\hbar}\frac{1}{\epsilon} \to \sigma_{\mathrm{APS}}(\epsilon,h) = \frac{e^2T\tau_0}{\pi\hbar}f(\epsilon,h), \tag{35}$$

i.e., $1/\epsilon$ has to be substituted by APS function

$$f(\epsilon,h) = \frac{2}{h^2}\left[\epsilon F\left(\frac{1}{2}+\frac{\epsilon}{2h}\right) - \epsilon F\left(1+\frac{\epsilon}{2h}\right) + h\right]. \tag{36}$$

This two-dimensional result can be easily generalized for layered and bulk superconductors using the layering operator introduced in Ref. 2.

D. Strong electric fields

Using the optical gauge

$$\varphi = 0, \quad \mathbf{A} = -t\mathbf{E},$$

the TDGL equation Eq. (1) reads

$$d_u\psi_q(u) = -\frac{1}{2}[(q+fu)^2+\epsilon]\psi_q(u) + \overline{\zeta}_q(u), \tag{37}$$

where we are introducing dimensionless variables for the $u = t/\tau_0$ time, $q = p\xi(0)/\hbar$ momentum, $f = e^*E\tau_0\xi(0)/\hbar$ electric field, and $\overline{\zeta}_q(u) = \tau_o\zeta_p(t)$ noise. We have a linear ordinary differential equation which can be solved for arbitrary $f(u)$, i.e., for arbitrary time dependence of the electric field. For constant electric field the TDGL equation has the solution

$$\psi_q(u) = \left\{\int_0^u \exp\left[\frac{1}{2}\int_0^{u_1}[(q+fu_2)^2+\epsilon]du_2\right]\overline{\zeta_q(u_1)}du_1 + \psi_q(0)\right\}\exp\left[-\frac{1}{2}\int_0^u[(q+fu_3)^2+\epsilon]du_3\right]. \tag{38}$$

In order to obtain the static $[t \gg \tau(\epsilon)]$ momentum distribution we have to perform the noise averaging

$$n_k = \lim_{u\to\infty}\langle|\psi_{q+fu}(u)|^2\rangle = \frac{T}{a_0}\int_0^\infty \exp\left[-(k^2+\epsilon)v + fkv^2 - \frac{1}{3}f^2v^3\right]dv, \tag{39}$$

where $u_1 = u - v$ and

$$k = q + fu = (p - e^*A)\frac{\xi(0)}{\hbar}$$

is the dimensionless kinetic momentum. This distribution can be directly derived from Boltzmann equation Eq. (11) for fluctuation Cooper pairs.[15] In Ref. 15 it was demonstrated that substitution of the momentum distribution Eq. (39) in the formula for the current density Eq. (14) gives the result which agrees with the formula by Dorsey[13]; cf. also the paper by Gor'kov[16]

$$j(E_x) = \frac{e^2\tau_{\mathrm{rel}}E_x}{16\hbar[2\pi^{1/2}\xi(0)]^{D-2}}\int_0^\infty \frac{\exp(-\epsilon u - gu^3)}{u^{(D-2)/2}}du, \tag{40}$$

where

$$g \equiv \frac{f^2}{12}, \quad f = \frac{e^*E_x\xi(0)\tau_0}{\hbar} = \frac{\pi}{8}\frac{eE_x\xi(0)}{T_c}\tau_{\mathrm{rel}}.$$

Differentiating the upper expression we obtain differential conductivity

$$\sigma_{\mathrm{diff}} = \frac{dj(E_x)}{dE_x} = \frac{e^2\tau_{\mathrm{rel}}}{16\hbar[2\sqrt{\pi}\xi(0)]^{D-2}} \times \int_0^\infty \frac{1-2gu^3}{u^{(D-2)/2}}\exp(-\epsilon u - gu^3)du. \tag{41}$$

Applying a voltage $U(t) = U_{\mathrm{DC}} + U_{\mathrm{AC}}\cos\omega t$ to the nanowire, the differential conductivity can be easily determined measuring the AC component for the current if $U_{\mathrm{AC}} \ll U_{\mathrm{DC}}$. Cooling the sample the differential conductivity will decrease, then at some temperature it will be annulated and what will happen at further cooling is an interesting experimental question.

IV. CURRENT FUNCTIONAL: SELF-CONSISTENT APPROXIMATION AND ENERGY CUT-OFF

The self-consistent approximation for the reduced critical temperature[2,13] in the one-dimensional (1D) case reads

$$\epsilon_{\rm ren}=\ln\frac{T}{T_0}+\frac{b}{a_0}n_{1D}=\ln\frac{T}{T_0}+\epsilon_{1G}\mathcal{N}_1(\epsilon_{\rm ren},f), \qquad (42)$$

where n_{1D} is the bulk density of the fluctuation Cooper pairs when we have 1D fluctuations in a wire with cross section $S\ll\xi^2(\epsilon)$ and

$$\epsilon_{1G}\equiv\frac{\mu_0\lambda^2(0)\xi(0)e^2T_c}{\sqrt{\pi}S\hbar^2}=\frac{k_B}{8\sqrt{\pi}\Delta C\xi(0)S}, \qquad (43)$$

where $\lambda(\epsilon)=\lambda(0)/\sqrt{-\epsilon}$ is the temperature-dependent penetration depth and ΔC is the jump of the specific heat at T_c per unit volume. For numerical calculations the function

$$\mathcal{N}_1(\epsilon,f))\equiv\int_0^\infty\exp(-\epsilon v-gv^3)\frac{dv}{\sqrt{v}} \qquad (44)$$

has to be programmed as

$$\mathcal{N}_1(\epsilon,f)=2\int_0^\infty\exp(-gz^6-\epsilon z^2)dz, \quad z^2=v. \qquad (45)$$

Analogously for the thin superconducting film with thickness $d_f\ll\xi(\epsilon)$ the equation for reduced temperature at zero electric field takes the form[2]

$$\epsilon_{\rm ren}=\ln\frac{T}{T_0}+\frac{b}{a_0}n_{2D}=\ln\frac{T}{T_0}+\epsilon_{2G}\mathcal{N}_2(\epsilon_{\rm ren}), \qquad (46)$$

where n_{2D} is the volume density of the fluctuation Cooper pairs having 2D fluctuations,

$$\epsilon_{2G}\equiv\frac{k_B}{4\pi\Delta C\xi^2(0)d_f}=2\pi\mu_0\frac{T_c}{d_f}\left(\frac{\lambda(0)}{\Phi_0}\right)^2 \qquad (47)$$

is the 2D Ginzburg number, and

$$\mathcal{N}_2(\epsilon)\equiv\ln\left(\frac{c+\epsilon}{\epsilon}\right). \qquad (48)$$

As simplest possible application of these results we have to mention nanostructured superconductors, e.g., nanowires,[17] similar to those used for long time investigation of phase slip centers[18] of the superconducting phase. We are pointing out that paraconductivity is a property of the normal phase.

For general (nonparabolic) dispersion we can derive from the TDGL equation the formula for the distribution of fluctuation Cooper pairs

$$n_k(u)=\frac{T}{a_0}\int_0^u\exp\left\{-\int_{u_1}^u\left[\frac{\varepsilon(k(u_2))}{a_0}+\epsilon\right]du_2\right\}du_1 + \overline{n}_k(0)\exp\left\{-\int_{u_1}^u\left[\frac{\varepsilon(k(u_2))}{a_0}+\epsilon\right]du_2\right\}, \qquad (49)$$

where the dimensionless kinetic momentum and the vector potential are

$$k(u)=q+\overline{A}(u), \quad \overline{A}(u)=-\frac{e^*\xi(0)}{\hbar}A(t).$$

In the case of parabolic dispersion and arbitrary time dependence of the electric field we can write Eq. (14) for the current functional in the form

$$j[A]=\frac{\sqrt{\pi}\hbar e}{m^*\xi(0)}\left[\frac{T}{a_0}\int_0^u F_A[u_1]du_1+\overline{n}_k(0)F_A[0]\right], \qquad (50)$$

where for brevity we introduce the functionals

$$F_A[u_1]\equiv\left(\frac{\overline{A}(u)}{\sqrt{u-u_1}}-\frac{B_A[u_1]}{(u-u_1)^{3/2}}\right) \qquad (51)$$

$$\times\exp\left[\frac{(B_A[u_1])^2}{u-u_1}-G(u_1)\right] \qquad (52)$$

and

$$B_A[u_1]\equiv\int_{u_1}^u\overline{A}(u_2)du_2,$$

$$G_A[u_1]\equiv-\epsilon(u-u_1)+\int_{u_1}^u(\overline{A}(u_2))^2du_2.$$

Local GL theory with an energy cut-off for the kinetic energy $\varepsilon_p<a_0c$ [or for the kinetic energy taken into account from the chemical potential $\varepsilon_p+a(T)<a_0c$] is very often used for describing the fluctuation phenomena[2]

$$\varepsilon(k)=\begin{cases}a_0k^2, & |k|<\Lambda\\ a_0c, & |k|>\Lambda\end{cases} \qquad (53)$$

where the dimensionless constant $\Lambda\equiv\sqrt{c}\simeq1$. For $YBa_2Cu_3O_{7-\delta}$ the recent investigations[19] of high-frequency fluctuation conductivity determined $\Lambda\approx0.5$. Then the functional which participates in the formula for the current Eq. (50) takes the form

$$F_A[u_1] = -\frac{\sinh(2\Lambda B_A[u_1])}{\sqrt{u-u_1}}\exp\left[-(u-u_1)\left(c+\frac{(B_A[u_1])^2}{(u-u_1)^2}\right)\right] + \left(\overline{A}(u)-\frac{B_A[u_1]}{(u-u_1)}\right)\exp\left(\frac{(B_A[u_1])^2}{u-u_1}-G_A[u_1]\right) \times\int_{-\Lambda-A(u)}^{\Lambda-A(u)}\exp\left[-(u-u_1)\left(q+\frac{B_A[u_1]}{u-u_1}\right)^2dq\right]. \qquad (54)$$

These rather complicated formulas are necessary for investigation of paraconductivity in the THz range. In the next section we will give an illustration for the important one-dimensional case.

V. FLUCTUATION CONDUCTIVITY IN NANOWIRES

The recent development of the technology of the performance of nanowires made it possible and even indispensable for the investigation of fluctuation conductivity. In this section we will analyze in detail the general results in 1D case.

The integrants in the momentum distribution Eq. (39) is actually age distribution for fluctuation Cooper pairs

$$\mathcal{F}(v;k,\epsilon,f)=\exp\left[-(k^2+\epsilon)v+fkv^2-\frac{1}{3}f^2v^3\right], \quad (55)$$

the variable v is the age in units $\tau(\epsilon)$. Time integration returns us to the momentum distribution, which using the dimensional variables

$$k_\epsilon\equiv\frac{k}{|\epsilon|^{1/2}}, \quad f_\epsilon\equiv\frac{f}{|\epsilon|^{3/2}} \quad (56)$$

reads

$$\overline{n}(k;\epsilon,f)=\frac{n_T}{|\epsilon|}\mathcal{F}_\pm(k_\epsilon,f_\epsilon), \quad (57)$$

where

$$\mathcal{F}_\pm(k_\epsilon,f_\epsilon)\equiv\int_0^\infty\exp\left[-(k_\epsilon^2\pm1)x+f_\epsilon k_\epsilon x^2-\frac{1}{3}f_\epsilon^2x^3\right]dx \quad (58)$$

and $x=|\epsilon|u$.

For the case $\epsilon=0$, $T=T_c$ or when $f\to\infty$, i.e.,

$$f_\epsilon=e^*E_x\xi(\epsilon)\tau(\epsilon)/\hbar\gg1, \quad \xi(\epsilon)=\xi(0)/|\epsilon|^{1/2} \quad (59)$$

we obtain

$$n(k,f)=\frac{n_T}{f^{2/3}}\mathcal{F}_0(k_f), \quad k_f=\frac{k}{f^{1/3}}, \quad (60)$$

where in

$$\mathcal{F}_0(k_f)\equiv\int_0^\infty\exp\left[-k_f^2y+k_fy^2-\frac{1}{3}y^3\right]dy \quad (61)$$

we use the transformation $y=f^{2/3}v$.

For the 1D case using Eq. (40), we express the current

$$j(E_x,\epsilon)=\frac{\sqrt{\pi}e^2}{8\hbar}\tau_{\rm rel}\xi(0)E_x\mathcal{J}(\epsilon,f) \quad (62)$$

where

$$\mathcal{J}(\epsilon,f)=\int_0^\infty\exp(-\epsilon v-gv^3)\sqrt{v}\,dv, \quad g\equiv\frac{f^2}{12}. \quad (63)$$

The fluctuational current for Cooper pairs above and under the critical temperature is

$$j=\frac{\pi e^2\tau_{\rm rel}\xi(0)E_x}{16\hbar|\epsilon|^{3/2}}\mathrm{s}_\pm(f_\epsilon), \quad (64)$$

where

$$\mathrm{s}_\pm(f_\epsilon)=\frac{2}{\sqrt{\pi}}\int_0^\infty\exp(\mp v_\epsilon-g_\epsilon v_\epsilon^3)\sqrt{v_\epsilon}dv_\epsilon \quad (65)$$

is the dimensionless function, which depends on the strength of the electric field. For convenience we use $g_\epsilon=f_\epsilon^2/12$ and $v_\epsilon=v|\epsilon|$. We wish to point out the normalization and the asymptotics

$$\mathrm{s}_+(0)=1, \quad \mathrm{s}_\pm(f_\epsilon\to\infty)\sim\frac{4}{\sqrt{3}f_\epsilon}. \quad (66)$$

Using the strong field asymptotics $f_\epsilon\gg1$, we obtain the fluctuational current at T_c

$$j(f_\epsilon\to\infty)=\frac{2}{\sqrt{3}}\frac{eT_c}{\hbar}. \quad (67)$$

We wish to point out that $j(f_\epsilon\to\infty)/T_c$ is universal and contains only fundamental physical constants. All material constants like $\xi(0)$, $\lambda(0)$ or the cross section of the nanowire S are cancelled. For experimental data processing it is necessary to perform linear regression of the IV curve

$$j_{\rm tot}=\frac{U}{R_N(T_c)}+\mathrm{sign}(U)\times24.22\ \text{nA}\ \ T_c[\mathrm{K}], \quad (68)$$

where the second term is the universal fluctuational current at strong electric fields around T_c. In order to avoid the thermoelectric effect coming from different materials forming the contacts to the nanowire one can analyze the current harmonics predicted by Eq. (68). In such a way the investigation of fluctuation current in nanowires can be used as a high accuracy test for applicability of the TDGL equation for nanostructured superconductors.

Using Eq. (14) and Eq. (39) we derive the 1D dimensional density

$$n_{1D}=\frac{n_T}{2\sqrt{\pi}\xi(0)S}\mathcal{N}_1(\epsilon,f), \quad (69)$$

where $\mathcal{N}_1(\epsilon,f)$ is previously defined in Eq. (44), when we considered without a derivation the self-consistent equation for the reduced temperature. The analysis of temperature and electric field dependence is reduced to two functions of one variable f_ϵ above and below the T_c

$$n_{1D}=\frac{n_T}{2\xi(0)S\sqrt{|\epsilon|}}N_\pm(f_\epsilon), \quad (70)$$

where

$$\mathcal{N}_\pm(f_\epsilon)=\frac{1}{\sqrt{\pi}}\int_0^\infty e^{\mp v_\epsilon-g_\epsilon v_\epsilon^3}\frac{dv_\epsilon}{\sqrt{v_\epsilon}}, \quad N_+(f_\epsilon=0)=1. \quad (71)$$

Analogously at T_c, where $\epsilon=0$ and $f_\epsilon\gg1$ we obtain

$$n_{1D}(f)=\frac{\Gamma(1/6)n_T}{2^{2/3}3^{5/6}\sqrt{\pi}\xi(0)S}\frac{1}{f^{1/3}}. \quad (72)$$

Choosing a typical set of parameters for the Sn nanowire: $\xi(0)=1000$Å, $\lambda(0)=1000$Å, $D=500$ Å, $S=\pi D^2/4$ according Eq. (43) we obtain for the 1D Ginzburg number $\epsilon_{1G}=4.3\times10^{-5}$. This parameter is essential for the numerical solution of the equation for renormalized reduced temperature Eq. (42).

VI. DISCUSSION AND CONCLUSION

Solving in parallel the TDGL equation and the Boltzmann equation we obtained coinciding results: not only for the linear case of Aslamazov-Larkin conductivity, but for the cases of strong electric fields, arbitrary time dependence of the electric field, nonparabolic momentum dependence of energy of Cooper pairs, energy cut-off, self-consistent equation for the renormalized reduced temperature, frequency dependence of the fluctuation conductivity etc. The number of fluctuation Cooper pairs which participates in the Boltzmann equation and the formulas for the current is actually the diagonal element of the order parameter correlator[19] $n_k(t)=C[p_{\rm kin}=p-e^*A(t);t,t]$. One can also easily check that the entropy of fluctuation Cooper pairs η is increasing with the time $d\eta/dt\geqslant0$; the capital η in the η-theorem by Boltzmann is often spelled as Latin *H*. Our self-consistent formula for the fluctuation conductivity of a superconducting nanowire can be directly used for the experimental data processing. In such a way we conclude that the Boltzmann equation for fluctuation Cooper pairs reproduces the known results of the fluctuation theory in the normal phase and it is a adequate tool to predict new phenomena related to metastable Cooper pairs, like negative differential conductivity in the fluctuation regime predicted in Ref. 15 and strong electric field effect in nanostructured superconductors where the heating effects are reduced. Our universal result Eq. (68) for a fluctuational current in a nanowire under strong electric field shows that the Boltzmann equation for the fluctuation Cooper pairs will become an indispensable tool for understanding the electronic processes in nanostructured superconductors.

Note added in proof. Citation of some very recent papers and graphical material will be given in the cond-mat version of the present work.[20]

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