

Critical fluctuation conductivity in layered superconductors in a strong electric field

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The paraconductivity, originating from critical superconducting order-parameter fluctuations in the vicinity of the critical temperature in a layered superconductor, is calculated in the frame of the self-consistent Hartree approximation for an arbitrarily strong electric field and zero magnetic field. The paraconductivity diverges less steep towards the critical temperature in the Hartree approximation than in the Gaussian one, and it shows a distinctly enhanced variation with the electric field. Our results indicate that high electric fields can be effectively used to suppress order-parameter fluctuations in high-temperature superconductors.

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I. INTRODUCTION

Due to their high critical temperature, small coherence length, and quasi-two-dimensional (quasi-2D) nature, the high-temperature superconductors (HTSC's) show a much more pronounced and therefore experimentally accessible effect of thermodynamic fluctuations in the critical region of the normal-superconducting transition. In general, an enhancement of the conductivity, denoted paraconductivity, is observed in HTSC's above T_c due to the presence of superconducting fluctuations. Outside the critical region, in the absence of a magnetic field and for small electric fields, the paraconductivity can be explained in terms of the Aslamazov-Larkin¹ theory of noninteracting, Gaussian fluctuations. The initial expressions for the paraconductivity have been extended for two-dimensional layered superconductors, a situation very much resembling the crystal structure in the cuprates, by Lawrence and Doniach.² However, it was shown that the fluctuation conductivity may be calculated in the linear-response approximation only for sufficiently weak fields, when they do not perturb the fluctuation spectrum.³ Reasonably high values of the electric field E can accelerate the fluctuating paired carriers to the depairing current and, thus, suppress the lifetime of the fluctuations, leading to a deviation from Ohm's law. In connection with the low-temperature superconductors, the nonlinearity has been studied theoretically for the isotropic case^{4,5} and also proved experimentally on thin aluminum films.⁶ The issue of the non-Ohmic fluctuation conductivity for a clean layered superconductor in an arbitrary electric field has been addressed by Varlamov and Reggiani,⁷ starting from a microscopic approach of Gor'kov⁸ for dirty isotropic superconductors. Essentially the same dependence on temperature and electric field has been recently⁹ derived, together with generalizations for the case of arbitrary dimension, based on the analytical derivation of the velocity distribution resulting from the Boltzmann equation for the fluctuating Cooper pairs.

The above-mentioned theories do not consider the interactions between fluctuations, so that fluctuations can be described by the Gaussian approximation. Thus, the quartic term in the Ginzburg-Landau free energy is neglected. This approximation is known to hold for temperature values not too close to the mean-field transition temperature, but it breaks down in the critical region, since the nonlinear char-

acter of the time-dependent Ginzburg-Landau (TDGL) equation cannot be neglected for high densities of fluctuation Cooper pairs. Several works¹⁰⁻¹⁸ have included the interaction between superconducting fluctuations in the critical transition region within different theoretical approaches. The simplest and most used one is the Hartree approximation which treats self-consistently the quartic term in the Ginzburg-Landau free-energy expansion. In this way expressions for the specific heat have been derived for bulk¹⁰ and layered¹¹ superconductors under a magnetic field, based on the functional integral approach. In the frame of the TDGL theory, Ullah and Dorsey¹³ computed the Nernst effect, thermopower, and longitudinal and Hall conductivities in the linear-response approximation for a layered superconductor in a magnetic field. Using the same relaxational dynamics of the TDGL approach, Dorsey¹⁴ provided expressions for the fluctuation conductivity in both the Ohmic and non-Ohmic regimes for isotropic superconductors of arbitrary dimensionality and in the absence of a magnetic field. More recently¹⁸ the effects of critical superconducting fluctuations on the scaling of the linear ac conductivity, $\sigma(\omega)$, of a bulk superconductor slightly above T_c in zero applied magnetic field have been investigated based on the dynamic renormalization-group method applied to the relaxational TDGL model of superconductivity, verifying explicitly the scaling hypothesis $\sigma(\omega, \xi)$ proposed originally by Fisher, Fisher, and Huse.¹² The essential features of the scaling and renormalization-group method have been also reviewed recently by Larkin and Varlamov.¹⁹ A more sophisticated approach based on the renormalization procedure and diagrammatic techniques has been developed for treating the non-Gaussian superconducting fluctuations by Ikeda, Ohmi, and Tsuneto¹⁵ and applied to the longitudinal conductivity, magnetization,¹⁶ and Hall conductivity¹⁷ in the linear-response approximation for a layered superconductor under magnetic field.

In this paper we shall address the problem of the non-Ohmic behavior of the non-Gaussian fluctuation conductivity for a layered superconductor, a topic that, to our present knowledge, has not yet been treated in the literature. While the nonlinear conductivity under arbitrarily strong electric field was derived for a layered system^{7,9} for Gaussian, non-interacting fluctuations, the effect of critical, strongly interacting fluctuations on the Ohmic and non-Ohmic conductiv-

ity was investigated¹⁴ only for isotropic systems of arbitrary dimensionality, but not for the layered Lawrence-Doniach model. The latter, however, would be required for comparison to experimental data on, e.g., $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$.

The paper is organized as follows. In Sec. II the TDGL equations are deduced for a layered superconducting system, with the explicit consideration of an arbitrarily strong electric field, oriented parallel to the layers. The fluctuation interaction term is also included in this model, within the self-consistent Hartree approximation. Section III presents the resulting equation solutions, obtained with the aid of the Green function technique (detailed in the Appendix). By considering in detail the necessary correction through the UV cutoff procedure, expressions for the fluctuation conductivity and self-consistent equation for the renormalized reduced temperature parameter are provided. Further, Sec. IV treats the limit cases of the linear response approximation, the no-cutoff limit, and also the isotropic 2D and 3D cases, recovering thus the results of previous theories. In Sec. V application of the model is illustrated by comparing the paraconductivity obtained in the present theory with that in the Gaussian fluctuation approximation for various applied electric fields. A comparison between the fluctuation suppression effects of the electric and magnetic fields is also illustrated. Finally, in Sec. VI, we summarize the main conclusions emerging from our analysis.

II. TDGL EQUATION FOR ARBITRARY ELECTRIC FIELD

For our purpose, we shall adopt the TDGL framework and treat the quartic term in the free-energy expansion within the simple self-consistent Hartree approximation. The starting point will be the Lawrence-Doniach expression of the Ginzburg-Landau (GL) free energy for a system of superconducting planes separated by a distance s , with a Josephson coupling between the planes, in the absence of a magnetic field,

$$\begin{aligned} \mathfrak{F} = \sum_n \int d^2x \left[a |\psi_n|^2 + \frac{\hbar^2}{2m} |\nabla \psi_n|^2 \right. \\ \left. + \frac{\hbar^2}{2m_c s^2} |\psi_n - \psi_{n+1}|^2 + \frac{b}{2} |\psi_n|^4 \right], \end{aligned} \quad (1)$$

where m and m_c are effective Cooper pair masses in the ab plane and along the c axis, respectively. The GL potential $a = a_0 \varepsilon$ is parametrized by $a_0 = \hbar^2 / 2m \xi_0^2$ and $\varepsilon = \ln(T/T_0) \approx (T - T_0)/T_0$, with T_0 being the mean-field transition temperature and ξ_0 the in-plane GL coherence length, extrapolated at $T=0$. The critical dynamics of the complex superconducting order parameter ψ_n in the n th plane will be described by the gauge-invariant relaxational time-dependent Ginzburg-Landau equation

$$\Gamma_0^{-1} \left(\frac{\partial}{\partial t} + i \frac{e_0}{\hbar} \varphi \right) \psi_n = - \frac{\delta \mathfrak{F}}{\delta \psi_n^*} + \zeta_n(\mathbf{x}, t), \quad (2)$$

where the pair electric charge is $e_0 = 2e$ and the order-parameter relaxation rate Γ_0 , given by²⁰⁻²²

$$\Gamma_0^{-1} = \frac{\pi \hbar^3}{16m \xi_0^2 k_B T}, \quad (3)$$

is related to the lifetime of metastable Cooper pairs^{9,19} $\tau^{(\text{BCS})} \approx \pi \hbar / 16k_B(T - T_0)$ through the relation $\Gamma_0^{-1} = 2a \tau^{(\text{BCS})}$. As can be noticed, we define the relaxation rate Γ_0^{-1} as depending on the actual temperature T , while in many other works,^{3,19,23} it is defined as a function of the mean-field critical temperature T_0 . Of course the difference is negligible near the transition point, but one must recall that in the derivation of the GL equations from BCS theory,^{21,24,25} the temperature is usually approximated with the critical one, while the reduced temperature $\ln(T/T_0)$ is approximated with $(T - T_0)/T_0$. The same happens while the TDGL equation is obtained for temperatures near T_0 , as, for instance, in Refs. 23 and 26. In some more recent derivations of the TDGL equation,^{22,27} however, the original appearance of these parameters is preserved, so that the order-parameter relaxation rate is written as depending on the actual temperature. In this mathematically somewhat stricter sense, one could therefore write the lifetime of the metastable Cooper pairs as $\tau^{(\text{BCS})} = \pi \hbar / 16k_B T \ln(T/T_0)$.

The Langevin forces $\zeta_n(\mathbf{x}, t)$ introduced in Eq. (2) in order to model the thermodynamical fluctuations must satisfy the fluctuation-dissipation theorem and ensure that the system relaxes to the proper equilibrium distribution. This requirement is fulfilled if the Langevin forces $\zeta_n(\mathbf{x}, t)$ are correlated by the Gaussian white-noise law

$$\langle \zeta_n(\mathbf{x}, t) \zeta_{n'}^*(\mathbf{x}', t') \rangle = 2\Gamma_0^{-1} k_B T \delta(\mathbf{x} - \mathbf{x}') \delta(t - t') \frac{\delta_{nn'}}{s}, \quad (4)$$

where $\delta(\mathbf{x} - \mathbf{x}')$ is the two-dimensional delta function concerning the in-plane coordinates. Since we are interested in finding the conductivity for an arbitrary electric field, we cannot use the linear-response approximation, so we have to explicitly include the electric field in the model. In order to compute the in-plane fluctuation conductivity, we shall assume the field \mathbf{E} along the x axis (where x and y are the in-plane coordinates), generated by the scalar potential $\varphi = -Ex$. In the chosen gauge, the current density operator along the x direction in the n th plane will be given by

$$j_x^{(n)} = - \frac{ie_0 \hbar}{2m} [\psi_n^*(\mathbf{x}, t) \partial_x \psi_n(\mathbf{x}, t) - \psi_n(\mathbf{x}, t) \partial_x \psi_n^*(\mathbf{x}, t)], \quad (5)$$

so that after averaging with respect to the noise,

$$\langle j_x^{(n)} \rangle = - \frac{ie_0 \hbar}{2m} (\partial_x - \partial_{x'}) \langle \psi_n(\mathbf{x}, t) \psi_n^*(\mathbf{x}', t) \rangle |_{\mathbf{x}=\mathbf{x}'}. \quad (6)$$

The time-dependent GL equation (2) is written

$$\begin{aligned} & \Gamma_0^{-1} \frac{\partial \psi_n}{\partial t} - i \frac{e_0 \Gamma_0^{-1} E x}{\hbar} \psi_n - \frac{\hbar^2}{2m} \nabla^2 \psi_n + a \psi_n \\ & + \frac{\hbar^2}{2m_c s^2} (2\psi_n - \psi_{n+1} - \psi_{n-1}) + b |\psi_n|^2 \psi_n = \zeta_n(\mathbf{x}, t). \end{aligned} \quad (7)$$

As already mentioned, the quartic term in the thermodynamical potential will be treated in the Hartree approximation, in the same sense as applied also in previous works,^{13,20,28} namely, by replacing the cubic term $b |\psi_n|^2 \psi_n$ in Eq. (7) with $b \langle |\psi_n|^2 \rangle \psi_n$. In this way, the nonlinearity is decoupled, resulting in a linear problem with a modified (renormalized) GL potential $\tilde{a} = a + b \langle |\psi_n|^2 \rangle$, which implies a renormalized reduced temperature

$$\tilde{\varepsilon} = \varepsilon + \frac{b}{a_0} \langle |\psi_n|^2 \rangle. \quad (8)$$

The average $\langle |\psi_n|^2 \rangle$ is to be determined, in principle, self-consistently together with the parameter $\tilde{\varepsilon}$.

In order to simplify the following computations, we shall introduce the Fourier transform with respect to the two in-plane coordinates and the layer index, respectively, through the relations

$$\begin{aligned} \psi_n(\mathbf{x}, t) &= \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \int_{-\pi/s}^{\pi/s} \frac{dq}{2\pi} \psi_q(\mathbf{k}, t) e^{-i\mathbf{x} \cdot \mathbf{k}} e^{-iqns}, \\ \psi_q(\mathbf{k}, t) &= \int d^2 \mathbf{x} \sum_n s \psi_n(\mathbf{x}, t) e^{i\mathbf{x} \cdot \mathbf{k}} e^{iqns}, \end{aligned} \quad (9)$$

so that Eq. (7) becomes

$$\begin{aligned} & \left[\Gamma_0^{-1} \frac{\partial}{\partial t} - \frac{e_0 \Gamma_0^{-1} E}{\hbar} \frac{\partial}{\partial k_x} + \frac{\hbar^2 \mathbf{k}^2}{2m} + \tilde{a} \right. \\ & \left. + \frac{\hbar^2 \gamma^2}{ms^2} (1 - \cos qs) \right] \psi_q(\mathbf{k}, t) = \zeta_q(\mathbf{k}, t). \end{aligned} \quad (10)$$

We have introduced the anisotropy parameter $\gamma = \xi_{0c} / \xi_0 = \sqrt{m/m_c}$, with ξ_{0c} the out-of-plane GL coherence length extrapolated at $T=0$. The term $\zeta_q(\mathbf{k}, t)$ in Eq. (10) is the Fourier transform according to the rules (9) of the noise function $\zeta_n(\mathbf{x}, t)$. One can directly verify that the following correlation function holds:

$$\begin{aligned} \langle \zeta_q(\mathbf{k}, t) \zeta_q^*(\mathbf{k}', t') \rangle &= 2\Gamma_0^{-1} k_B T (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') \\ & \times \delta(q - q') \delta(t - t'). \end{aligned} \quad (11)$$

Equation (10) may be solved using the Green function method, as done also by Tucker and Halperin²⁹ and Dorsey¹⁴ in order to solve the relaxational TDGL equation with a Langevin noise for isotropic superconductors. Our derivation differs, however, by the fact that it is applied for a layered superconductor and, also, through the differently chosen potential gauge. We denote thus by $R_q(\mathbf{k}, t; k'_x, t')$ the Green function for the Eq. (10), which satisfies

$$\begin{aligned} & \left[\Gamma_0^{-1} \frac{\partial}{\partial t} - \frac{e_0 \Gamma_0^{-1} E}{\hbar} \frac{\partial}{\partial k_x} + \frac{\hbar^2 k_x^2}{2m} + a_1 \right] R_q(\mathbf{k}, t; k'_x, t') \\ & = \delta(k_x - k'_x) \delta(t - t'), \end{aligned} \quad (12)$$

where we have introduced the notation

$$a_1 \equiv \tilde{a} + \frac{\hbar^2 k_y^2}{2m} + \frac{\hbar^2 \gamma^2}{ms^2} (1 - \cos qs). \quad (13)$$

Thus, the solution of Eq. (10) will then be written

$$\psi_q(\mathbf{k}, t) = \int dt' \int dk'_x R_q(\mathbf{k}, t; k'_x, t') \zeta_q(k'_x, k_y, t'). \quad (14)$$

The Green function $R_q(\mathbf{k}, t; k'_x, t')$ is computed in the Appendix and given by Eq. (A7), so that the relation (14) for the Fourier-transformed order parameter will be written, accordingly,

$$\begin{aligned} \psi_q(\mathbf{k}, t) &= \Gamma_0 \exp \left\{ \frac{\hbar \Gamma_0}{e_0 E} \left[\frac{\hbar^2 k_x^3}{6m} + a_1 k_x \right] \right\} \int dt' \theta(t - t') \\ & \times \exp \left\{ - \frac{\hbar \Gamma_0}{e_0 E} \left[\frac{\hbar^2 \left(k_x + \frac{e_0 E}{\hbar} [t - t'] \right)^3}{6m} \right. \right. \\ & \left. \left. + a_1 \left(k_x + \frac{e_0 E}{\hbar} [t - t'] \right) \right] \right\} \\ & \times \zeta_q \left(k_x + \frac{e_0 E}{\hbar} [t - t'], k_y, t' \right). \end{aligned} \quad (15)$$

One can notice that the solution (15) fulfills causality, due to the retarded character of the Green function $R_q(\mathbf{k}, t; k'_x, t')$. It can be also simplified further,

$$\begin{aligned} \psi_q(\mathbf{k}, t) &= \Gamma_0 \int_0^\infty d\tau \exp \left\{ -\Gamma_0 \left[a_1 \tau + \frac{\tau \hbar^2}{2m} \left(k_x + \frac{e_0 E \tau}{2\hbar} \right)^2 \right. \right. \\ & \left. \left. + \frac{e_0^2 E^2}{24m} \tau^3 \right] \right\} \zeta_q \left(k_x + \frac{e_0 E \tau}{\hbar}, k_y, t - \tau \right). \end{aligned} \quad (16)$$

Now, following Eq. (6), the current density can be written as

$$\begin{aligned} \langle j_x^{(n)} \rangle &= - \frac{e_0 \hbar}{m} 2\Gamma_0 k_B T \int \frac{dk_x}{2\pi} \int \frac{dk_y}{2\pi} \int_{-\pi/s}^{\pi/s} \frac{dq}{2\pi} k_x \int_0^\infty d\tau \\ & \times \exp \left\{ -\Gamma_0 \left[2 \left(\tilde{a} + \frac{\hbar^2 k_y^2}{2m} + \frac{\hbar^2 \gamma^2}{ms^2} [1 - \cos qs] \right) \tau \right. \right. \\ & \left. \left. + \frac{\tau \hbar^2}{m} \left(k_x + \frac{e_0 E \tau}{2\hbar} \right)^2 + \frac{e_0^2 E^2}{12m} \tau^3 \right] \right\}, \end{aligned} \quad (17)$$

while the averaged density of fluctuating Cooper pairs is written, analogously,

$$\begin{aligned} \langle |\psi_n|^2 \rangle &= 2\Gamma_0 k_B T \int \frac{dk_x}{2\pi} \int \frac{dk_y}{2\pi} \int_{-\pi/s}^{\pi/s} \frac{dq}{2\pi} \int_0^\infty d\tau \\ &\times \exp \left\{ -\Gamma_0 \left[2 \left(\tilde{a} + \frac{\hbar^2 k_y^2}{2m} + \frac{\hbar^2 \gamma^2}{ms^2} [1 - \cos qs] \right) \tau \right. \right. \\ &\left. \left. + \frac{\tau \hbar^2}{m} \left(k_x + \frac{e_0 E \tau}{2\hbar} \right)^2 + \frac{e_0^2 E^2}{12m} \tau^3 \right] \right\}, \quad (18) \end{aligned}$$

where we have taken into account expression (11) for the Fourier-transformed noise correlation function and, also, replaced the notation a_1 with its value (13).

Before proceeding further and solve the integrals over momentum variables in Eqs. (17) and (18), we must recall the inherent ultraviolet (UV) divergence of Ginzburg-Landau theory, which is not valid on length scales less than the zero-temperature coherence length ξ_0 . The short-wavelength fluctuations break down the ‘‘slow variation condition’’ for the superconducting order parameter, a central hypothesis of the GL approach.³⁰ This difficulty can be solved by applying an UV cutoff to the fluctuation spectrum, a procedure introduced from the beginning in GL theory.³¹

III. SOLUTIONS WITH CONSIDERATION OF THE UV CUTOFF

The classical^{4,28,32} procedure is to suppress the short-wavelength fluctuating modes through the *momentum cutoff* condition

$$\mathbf{k}^2 < c \xi_0^{-2}, \quad (19)$$

where the dimensionless cutoff factor c is close to unity. Also a *total energy cutoff* was suggested,^{33–35} which eliminates the most energetic fluctuations and not only those with short wavelengths,

$$\mathbf{k}^2 + \xi^{-2}(\varepsilon) < c \xi_0^{-2}. \quad (20)$$

Heuristically, the adequacy of the total-energy cutoff can be justified on the grounds of the Gaussian GL approach by taking into account that the probability of each fluctuating mode is controlled by its total energy $\hbar^2 \mathbf{k}^2 / 2m + a_0 \varepsilon$, and not only by its momentum.^{9,19} Very recently³⁶ it was suggested that the physical meaning of the ‘‘total-energy’’ cutoff follows from the uncertainty principle, which imposes a limit to the confinement of the superconducting wave function. It must be, however, mentioned that in the critical fluctuation region, the two cutoff prescriptions almost coincide quantitatively, due to the low reduced-temperature value ε with respect to the factor c .

For simplicity, in the following we shall apply the cutoff procedure in its classical form (19) on the \mathbf{k} -plane momentum integrals in Eqs. (17) and (18). If necessary, one can get easily also the result that would correspond to the ‘‘total-energy’’ cutoff (20) by simply replacing $c \rightarrow c - \tilde{\varepsilon}$. Although for the Lawrence-Doniach model the z -axis momentum q

also contributes to the fluctuation mode energy with the term $(\hbar^2 \gamma^2 / ms^2)(1 - \cos qs)$, the inclusion of a momentum cutoff in this direction is not necessary, since the z -axis spectrum is already modulated through $-\pi/s \leq q \leq \pi/s$. One can thus perform the integral over q momentum in Eqs. (17) and (18) and get

$$\begin{aligned} &\int_{-\pi/s}^{\pi/s} \frac{dq}{2\pi} \exp \left[-2\Gamma_0 \tau \frac{\hbar^2 \gamma^2}{ms^2} (1 - \cos qs) \right] \\ &= \frac{1}{s} \exp \left(-\frac{2\Gamma_0 \hbar^2 \gamma^2 \tau}{ms^2} \right) I_0 \left(\frac{2\Gamma_0 \hbar^2 \gamma^2 \tau}{ms^2} \right), \quad (21) \end{aligned}$$

where we used the identity $I_0(x) = (1/2\pi) \int_{-\pi}^{\pi} e^{x \cos \theta} d\theta$ for the modified Bessel function $I_0(x)$.

We shall therefore apply the cutoff by performing the momentum integral from Eq. (17) for $\mathbf{k}^2 < c \xi_0^{-2}$,

$$\begin{aligned} &\int_{\mathbf{k}^2 < c \xi_0^{-2}} \frac{d^2 \mathbf{k}}{(2\pi)^2} k_x \exp \left\{ -2\Gamma_0 \tau \frac{\hbar^2}{2m} \left[k_y^2 + \left(k_x + \frac{e_0 E \tau}{2\hbar} \right)^2 \right] \right\} \\ &= e^{-\Gamma_0 e_0^2 E^2 \tau^3 / 4m} \frac{1}{(2\pi)^2} \int_0^{\sqrt{c} \xi_0^{-1}} dk k^2 e^{-\tau \Gamma_0 \hbar^2 k^2 / m} \\ &\times \int_{-\pi}^{\pi} d\varphi \cos \varphi e^{-\Gamma_0 \tau^2 \hbar e_0 E k \cos \varphi / m} \\ &= -e^{-\Gamma_0 e_0^2 E^2 \tau^3 / 4m} \frac{1}{2\pi} \left(\frac{m a_0}{\hbar^2} \right)^{3/2} \\ &\times \int_0^c dw \sqrt{2w} e^{-2\tau \Gamma_0 a_0 w} I_1 \left(\frac{\Gamma_0 \tau^2 e_0 E}{m} \sqrt{2m a_0 w} \right), \quad (22) \end{aligned}$$

where we introduced the new dimensionless variable $w = \hbar^2 k^2 / 2m a_0$ and used the first-order modified Bessel function $I_1(x) = (1/2\pi) \int_{-\pi}^{\pi} d\varphi \cos \varphi e^{x \cos \varphi}$. The current density (17) will be written eventually, after considering Eqs. (21) and (22) and introducing the new integration variable $u = 2a_0 \Gamma_0 \tau$,

$$\begin{aligned} j(\tilde{\varepsilon}, E) &= \frac{ek_B T}{\pi \hbar s \xi_0} \int_0^\infty du I_0 \left(\frac{2\gamma^2 \xi_0^2}{s^2} u \right) e^{-(\tilde{\varepsilon} + 2\gamma^2 \xi_0^2 / s^2) u} \\ &\times e^{-4(\pi e \xi_0 E / 16 \sqrt{3} k_B T)^2 u^3} \\ &\times \int_0^c dw \sqrt{w} e^{-uw} I_1 \left(\frac{\pi e \xi_0}{8 k_B T} E u^2 \sqrt{w} \right), \quad (23) \end{aligned}$$

where we have also expressed the parameters Γ_0 and a_0 with the aid of the in-plane coherence length ξ_0 .

In order to infer the fluctuation conductivity $\sigma = j/E$, we shall use the fact that the function $I_1(x)$ consists only of odd argument powers and satisfies, namely, the identity $I_1(x) = (x/2)[I_0(x) - I_2(x)]$, so that we can finally write the paraconductivity under the form

$$\begin{aligned} \sigma(\tilde{\varepsilon}, E) &= \frac{e^2}{16\hbar s} \int_0^\infty du I_0\left(\frac{ru}{2}\right) u^2 e^{-(\tilde{\varepsilon}+r/2)u-4(E/E_0)^2 u^3} \\ &\quad \times \int_0^c dw w e^{-uw} \left[I_0\left(2\sqrt{3}\frac{E}{E_0}u^2\sqrt{w}\right) \right. \\ &\quad \left. - I_2\left(2\sqrt{3}\frac{E}{E_0}u^2\sqrt{w}\right) \right], \end{aligned} \quad (24)$$

where we have introduced the notation

$$r = \frac{4\mathcal{V}^2\xi_0^2}{s^2} = \left(\frac{2\xi_0 c}{s}\right)^2 \quad \text{and} \quad E_0 = \frac{16\sqrt{3}k_B T}{\pi e \xi_0} \quad (25)$$

for the anisotropy parameter r and the characteristic electric field E_0 , respectively. Relations (25) can be expressed also as depending on microscopical parameters, like the Fermi velocity v_F and the electronic interlayer hopping energy J . By identifying the in-plane GL coherence length ξ_0 from the microscopic derivation of the GL equation in the two-dimensional case, one has in the clean limit¹⁹ $\xi_0 = [7\zeta(3)/32]^{1/2} \hbar v_F / \pi k_B T_0$, which implies for $T \approx T_0$

$$E_0 = 64 \sqrt{\frac{6}{7\zeta(3)}} \frac{k_B^2 T_0^2}{e \hbar v_F} \quad \text{and} \quad r = \frac{7\zeta(3)}{8\pi^2} \frac{J^2}{k_B^2 T_0^2}, \quad (26)$$

where we have also used in the same approximations (clean limit and $T \approx T_0$) the expression for the anisotropy parameter r from Ref. 37 as a function of the interlayer electron hopping energy J .

In an analogous manner as presented above for the current density, one can apply the cutoff procedure to the momentum integral in Eq. (18) and obtain

$$\begin{aligned} &\int_{\mathbf{k}^2 < c\xi_0^{-2}} \frac{d^2\mathbf{k}}{(2\pi)^2} \exp\left\{-2\Gamma_0\tau \frac{\hbar^2}{2m} \left[k_y^2 + \left(k_x + \frac{e_0 E \tau}{2\hbar}\right)^2\right]\right\} \\ &= e^{-\Gamma_0 e_0^2 E^2 \tau^3 / 4m} \frac{m a_0}{2\pi \hbar^2} \int_0^c dw e^{-2\tau \Gamma_0 a_0 w} \\ &\quad \times I_0\left(\frac{\Gamma_0 \tau^2 e_0 E}{m} \sqrt{2m a_0 w}\right), \end{aligned} \quad (27)$$

so that the averaged density of Cooper pairs, Eq. (18), becomes

$$\begin{aligned} \langle |\psi_n(\mathbf{x}, t)|^2 \rangle &= \frac{m k_B T}{2\pi \hbar^2 s} \int_0^\infty du I_0\left(\frac{ru}{2}\right) e^{-(\tilde{\varepsilon}+r/2)u-4(E/E_0)^2 u^3} \\ &\quad \times \int_0^c dw e^{-uw} I_0\left(2\sqrt{3}\frac{E}{E_0}u^2\sqrt{w}\right), \end{aligned} \quad (28)$$

where we passed from τ to the variable u and used the notation (25). One can easily prove that without the UV cutoff

(i.e., for $c \rightarrow \infty$), the expression would be divergent. Replacing $c \rightarrow c - \tilde{\varepsilon}$ would in turn correspond to an energy cutoff under the form (20). The self-consistent equation (8) for the parameter $\tilde{\varepsilon}$ will therefore be written

$$\begin{aligned} \tilde{\varepsilon} &= \ln \frac{T}{T_0} + g T \int_0^\infty du I_0\left(\frac{ru}{2}\right) e^{-(\tilde{\varepsilon}+r/2)u-4(E/E_0)^2 u^3} \\ &\quad \times \int_0^c dw e^{-uw} I_0\left(2\sqrt{3}\frac{E}{E_0}u^2\sqrt{w}\right), \end{aligned} \quad (29)$$

where the factor

$$g = \frac{2\mu_0 \kappa^2 e^2 \xi_0^2 k_B}{\pi \hbar^2 s} \quad (30)$$

has been computed by taking into account the expression of the quartic term coefficient,³⁰ $b = \mu_0 \kappa^2 e_0^2 \hbar^2 / 2m^2$, with κ being the Ginzburg-Landau parameter $\kappa = \lambda_0 / \xi_0$.

The above equation (24) for the paraconductivity, together with the self-consistent equation (29), both valid for an arbitrary strong electric field and with the explicit inclusion of the UV cutoff, are the main results of this paper. A short comment would be useful regarding the application of the cutoff procedure to the \mathbf{k} integrals in Eqs. (27) and (22). It could seem more appealing to apply the cutoff on the translated wave-vector magnitude, such as $k_y^2 + [k_x + (e_0 E \tau / 2\hbar)]^2 < c\xi_0^{-2}$, as was approximately done, although in a different gauge, by Kajimura and Mikoshiba,³⁸ while studying the paraconductivity in an arbitrary electric field in the two-dimensional case. This would certainly simplify the calculations and consequently the factors that depend on the cutoff parameter c , but would not really correspond to the actual meaning of the UV cutoff, which is to assure the slow variation condition for the order parameter by eliminating the rapidly oscillating modes in the Fourier expansion (9). It can be seen, for instance, that when the dummy variable τ grows towards ∞ , the component k_x would have to approach $-\infty$ in order to preserve the cutoff condition in translated form. It can be, moreover, verified that the ‘‘translated’’ cutoff would not give the correct result in the $E=0$ limit; namely, the third term in Eq. (31) below would be missing.

IV. LIMIT CASES

A. Linear-response limit

It is worth comparing to previous results what Eqs. (24) and (29) become in the zero-field limit $E=0$. Taking into account that $I_0(0)=1$ and $I_2(0)=0$ and using the integral form for the modified Bessel function $I_0(ru/2)$, one obtains for the linear-response conductivity

$$\begin{aligned}
\sigma(\tilde{\varepsilon})|_{E=0} &= \frac{e^2}{16\hbar s} \int_0^\infty du (1 - e^{-cu} - cue^{-cu}) \\
&\quad \times I_0\left(\frac{r}{2}u\right) \exp\left(-\tilde{\varepsilon}u - \frac{r}{2}u\right) \\
&= \frac{e^2}{16\hbar s} \left[\frac{1}{\sqrt{\tilde{\varepsilon}(\tilde{\varepsilon}+r)}} - \frac{1}{\sqrt{(\tilde{\varepsilon}+c)(\tilde{\varepsilon}+c+r)}} \right. \\
&\quad \left. - \frac{c(c+\tilde{\varepsilon}+r/2)}{[(c+\tilde{\varepsilon}+r)(c+\tilde{\varepsilon})]^{3/2}} \right], \quad (31)
\end{aligned}$$

which is the Lawrence-Doniach² formula for the fluctuation conductivity of a layered superconductor, with the inclusion of the UV cutoff. A formally identical expression was also inferred by Carballeira *et al.*³⁵ for Gaussian fluctuations (i.e., with $\tilde{\varepsilon} = \varepsilon$) in order to fit the HTSC paraconductivity in the high-reduced-temperature region.

The integral in Eq. (29) can also be easily performed in the limit $E=0$, and it yields the relation

$$\begin{aligned}
\tilde{\varepsilon} - \ln \frac{T}{T_0} &= \frac{2\mu_0\kappa^2 e^2 \xi_0^2 k_B T}{\pi\hbar^2 s} \int_0^c \frac{dx}{\sqrt{(x+\tilde{\varepsilon})(x+\tilde{\varepsilon}+r)}} \\
&= \frac{4\mu_0\kappa^2 e^2 \xi_0^2 k_B T}{\pi\hbar^2 s} \ln \frac{\sqrt{\tilde{\varepsilon}+c} + \sqrt{\tilde{\varepsilon}+c+r}}{\sqrt{\tilde{\varepsilon}} + \sqrt{\tilde{\varepsilon}+r}}, \quad (32)
\end{aligned}$$

which matches the formula found by Mishonov and Penev,²⁸ with the only difference that in Ref. 28 the temperature T is approximated with T_0 in the logarithm prefactor. It can be easily verified that Eq. (32) is also coincident with an analogous equation found previously by Ullah and Dorsey¹³ (UD) for the renormalized reduced temperature $\tilde{\varepsilon}$ if one considers the zero-magnetic-field limit in the UD formula. In this limit, the sum over Landau levels transforms, with the aid of the Euler-MacLaurin summation formula, to the integral from Eq. (32) if one takes also into account that the cutoff used by UD corresponds with $c=2$.

We have thus shown that from our Eqs. (24) and (29) one can infer in the limit $E \rightarrow 0$ the already known results for conductivity and renormalized reduced temperature in the Ohmic approximation.

B. No-cutoff limit

The cutoff procedure is crucial for calculating the averaged fluctuating Cooper pairs density, since Eq. (18) yields a divergent τ integral when one performs the \mathbf{k} -momentum integrals on the entire \mathbf{k} plane.

The result (24) for the paraconductivity remains, however, finite even if one removes the cutoff (i.e., for $c \rightarrow \infty$), although it will give then a larger paraconductivity than in the cutoff case, especially for higher reduced temperatures ($\varepsilon \geq 0.1$). Turning back to Eq. (17) with the cutoff removed,

one is allowed to translate the integral variable k_x so that $k_x + (e_0 E \tau / 2\hbar) \rightarrow k_x$, and after performing the Poisson \mathbf{k} integrals, one obtains eventually for the fluctuation in-plane conductivity in the presence of an arbitrary electric field E , without considering the UV cutoff, the simpler relation

$$\sigma(\tilde{\varepsilon}, E) = \frac{e^2}{16\hbar s} \int_0^\infty du I_0\left(\frac{r}{2}u\right) e^{-\tilde{\varepsilon}u - (r/2)u - (E/E_0)^2 u^3}. \quad (33)$$

Expression (33) is not new. In its form, it is essentially similar to the ones found by Varlamov and Reggiani⁷ and Mishonov *et al.*⁹ for the case of Gaussian fluctuations if one neglects the difference which consists in the presence of the renormalized parameter $\tilde{\varepsilon}$ in Eq. (33) instead of the reduced temperature $\varepsilon = \ln(T/T_0)$. It must be stated, however, that Ref. 7, based on a microscopical approach,⁸ defines an out-of-plane coherence length larger by a factor of $\sqrt{2}$ than the commonly used one. Nevertheless, its result corresponds to our Eq. (33) when expressed through Eqs. (26) in the microscopical parameters v_F and J (the latter is denoted in Ref. 7 by w). Mishonov *et al.*⁹ signalize differences in one of their intermediary results with respect to Refs. 8 and 7, but they find the same dependence of the non-Ohmic conductivity on the reduced temperature ε and the electric field E by solving the Boltzmann equation for the velocity distribution of the fluctuating Cooper pairs. Their formula differs mathematically from Eq. (33) only by the presence of two extra factors before the integral: namely, the ratio between a and b coherence lengths (since Ref. 9 takes into account also a general in-plane anisotropy) and the ratio T/T_0 , which in fact comes artificially only if one writes the relaxation rate Γ_0 with T_0 instead of T [see the comment after Eq. (3)].

C. Isotropic limit

Results analogous to Eqs. (24) and (29) for the isotropic 2D case can be easily derived by simply taking the limit $\xi_{0c} = 0$ (or $r \rightarrow 0$) and identifying the interlayer distance s with the film thickness d . We will have thus

$$\begin{aligned}
\sigma^{(2D)}(\tilde{\varepsilon}, E) &= \frac{e^2}{16\hbar d} \int_0^\infty du u^2 e^{-\tilde{\varepsilon}u - 4(E/E_0)^2 u^3} \int_0^c dw w \\
&\quad \times e^{-uw} \left[I_0\left(2\sqrt{3} \frac{E}{E_0} u^2 \sqrt{w}\right) - I_2\left(2\sqrt{3} \frac{E}{E_0} u^2 \sqrt{w}\right) \right], \quad (34)
\end{aligned}$$

$$\begin{aligned}
\tilde{\varepsilon} - \ln \frac{T}{T_0} &= \frac{2\mu_0\kappa^2 e^2 \xi_0^2 k_B T}{\pi\hbar^2 d} \int_0^\infty du e^{-\tilde{\varepsilon}u - 4(E/E_0)^2 u^3} \\
&\quad \times \int_0^c dw e^{-uw} I_0\left(2\sqrt{3} \frac{E}{E_0} u^2 \sqrt{w}\right). \quad (35)
\end{aligned}$$

Equations (34) and (35) differ from the analogous results of Ref. 38 because there only an approximate form of the cutoff procedure was applied, as we have already mentioned at the end of Sec. III.

If one neglects the cutoff ($c \rightarrow \infty$), Eq. (35) becomes divergent, while Eq. (34) takes the already known^{5,9,14} form

$$\sigma_{\text{no cut}}^{(2D)}(\tilde{\varepsilon}, E) = \frac{e^2}{16\hbar d} \int_0^\infty du e^{-\tilde{\varepsilon}u - (E/E_0)^2 u^3}, \quad (36)$$

with the specification that in Eq. (36) the renormalized parameter $\tilde{\varepsilon}$ is present, instead of the reduced temperature ε .

If, on the contrary, one preserves the cutoff but takes the linear response limit $E \rightarrow 0$, Eqs. (34) and (35) become

$$\sigma^{(2D)}(\tilde{\varepsilon})|_{E=0} = \frac{e^2}{16\hbar d} \left[\frac{1}{\tilde{\varepsilon}} - \frac{1}{\tilde{\varepsilon} + c} - \frac{c}{(c + \tilde{\varepsilon})^2} \right], \quad (37)$$

$$\tilde{\varepsilon} - \ln \frac{T}{T_0} = \frac{2\mu_0 \kappa^2 e^2 \xi_0^2 k_B T}{\pi \hbar^2 d} \ln \frac{\tilde{\varepsilon} + c}{\tilde{\varepsilon}}. \quad (38)$$

A formally identical expression to Eq. (37) is also to be found in Ref. 35 for Gaussian fluctuations (i.e., with $\tilde{\varepsilon} = \varepsilon$), while Eq. (38) is implicitly contained in the results of Ref. 28.

Results for the isotropic 3D case cannot be obtained by just imposing the 3D condition $s \rightarrow 0$ (or $r \rightarrow \infty$) to Eqs. (24) and (29), because these equations were calculated by assuming that a cutoff in the z direction is not necessary for layered superconductors. However, in the 3D case, a cutoff for the k_z momentum is as necessary as for the k_x and k_y components. The calculations can be performed according to the same scheme as for the layered case, and the results are presented here for completeness:

$$\begin{aligned} \sigma^{(3D)}(\tilde{\varepsilon}, E) &= \frac{e^2}{8\pi\hbar\xi_0} \int_0^\infty du u^2 e^{-\tilde{\varepsilon}u - 4(E/E_0)^2 u^3} \\ &\times \int_0^c dw w^{3/2} e^{-uw} \left[\frac{\cosh\left(2\sqrt{3}\frac{E}{E_0}u^2\sqrt{w}\right)}{\left(2\sqrt{3}\frac{E}{E_0}u^2\sqrt{w}\right)^2} \right. \\ &\left. - \frac{\sinh\left(2\sqrt{3}\frac{E}{E_0}u^2\sqrt{w}\right)}{\left(2\sqrt{3}\frac{E}{E_0}u^2\sqrt{w}\right)^3} \right], \quad (39) \end{aligned}$$

$$\begin{aligned} \tilde{\varepsilon} - \ln \frac{T}{T_0} &= \frac{2\mu_0 \kappa^2 e^2 \xi_0^2 k_B T}{\pi^2 \hbar^2} \int_0^\infty du e^{-\tilde{\varepsilon}u - 4(E/E_0)^2 u^3} \\ &\times \int_0^c dw w^{1/2} e^{-uw} \frac{\sinh\left(2\sqrt{3}\frac{E}{E_0}u^2\sqrt{w}\right)}{2\sqrt{3}\frac{E}{E_0}u^2\sqrt{w}}. \quad (40) \end{aligned}$$

Similarly to the 2D case, if one neglects the cutoff ($c \rightarrow \infty$), the term on the right-hand side of Eq. (40) becomes divergent, while Eq. (39) takes the expression

$$\sigma_{\text{no cut}}^{(3D)}(\tilde{\varepsilon}, E) = \frac{e^2}{32\sqrt{\pi}\hbar\xi_0} \int_0^\infty du \frac{1}{\sqrt{u}} e^{-\tilde{\varepsilon}u - (E/E_0)^2 u^3}, \quad (41)$$

already known^{5,9,14} for Gaussian fluctuations (i.e., with $\tilde{\varepsilon} = \varepsilon$).

In the linear response limit ($E \rightarrow 0$) but with the cutoff preserved, Eqs. (39) and (40) become

$$\begin{aligned} \sigma^{(3D)}(\tilde{\varepsilon})|_{E=0} &= \frac{e^2}{48\pi\hbar\xi_0} \left[\frac{3 \arctan(\sqrt{c/\tilde{\varepsilon}})}{\sqrt{\tilde{\varepsilon}}} - \frac{3\tilde{\varepsilon}\sqrt{c}}{(\tilde{\varepsilon} + c)^2} - \frac{5c^{3/2}}{(\tilde{\varepsilon} + c)^2} \right], \quad (42) \\ \tilde{\varepsilon} - \ln \frac{T}{T_0} &= \frac{4\mu_0 \kappa^2 e^2 \xi_0^2 k_B T}{\pi^2 \hbar^2} \left[\sqrt{c} - \sqrt{\tilde{\varepsilon}} \arctan\left(\sqrt{\frac{c}{\tilde{\varepsilon}}}\right) \right]. \quad (43) \end{aligned}$$

Equation (42) matches thus formally the expression found³⁵ for Gaussian fluctuations ($\tilde{\varepsilon} = \varepsilon$).

V. RESULTS OF THE MODEL

The renormalization procedure required for our present results consists thus in using the reduced-temperature parameter $\tilde{\varepsilon}$, renormalized by solving Eq. (29), in the conductivity expression (24). This procedure causes the critical temperature to shift towards lower temperatures. In analogy with the Gaussian fluctuation case, we shall adopt as definition for the critical temperature $T_c(E)$ the vanishing of the reduced temperature, $\tilde{\varepsilon} = 0$. Thus, in the absence of an electric field we can use Eq. (32) taken at $T = T_c(0)$ and $\tilde{\varepsilon} = 0$, so that one gets

$$T_0 = T_c(0) \left(\sqrt{\frac{c}{r}} + \sqrt{1 + \frac{c}{r}} \right)^{2gT_c(0)}. \quad (44)$$

In practice, one knows the actual critical temperature $T_c(0) \equiv T_{c0}$ measured at very low electrical field, so that Eq. (44) allows us to estimate the bare mean-field characteristic temperature T_0 . Then, having the parameter T_0 , one can use Eqs. (29) for any temperature T and field E in order to find the actual renormalized $\tilde{\varepsilon}(T, E)$ and, further, the conductivity $\sigma(T, E)$.

In order to illustrate the main features of the Hartree approximation for the critical fluctuation model, we shall take as example a common material, like optimally doped $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$. Typical values for the characteristic parameters are then $s = 1.17$ nm for the interlayer distance, $\xi_0 = 1.2$ nm and $\xi_{0c} = 0.14$ nm for the zero-temperature in-plane and out-of-plane coherence lengths, respectively, κ

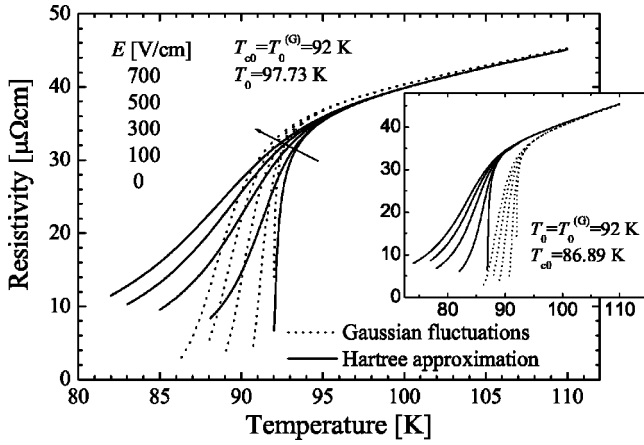


FIG. 1. Resistivity in the Gaussian theory (dotted curves) and in the Hartree approximation for the interacting fluctuations (solid curves) for different values of the applied electric field. The following parameters were used: interlayer distance $s = 1.17$ nm; zero-temperature in-plane and out-of-plane coherence lengths $\xi_0 = 1.2$ nm and $\xi_{0c} = 0.14$ nm, respectively; Ginzburg-Landau parameter $\kappa = 70$; and zero-field critical temperature $T_{c0} = 92$ K. The UV cutoff parameter $c = 1$ was used. The arrow indicates the increasing electric field direction. The inset illustrates the critical temperature shift introduced by the Hartree model if the mean-field transition temperature T_0 were kept identical with the one in the Gaussian approach, $T_0^{(G)}$.

$= 70$ for the Ginzburg-Landau parameter, and $T_{c0} = 92$ K for the critical temperature under very small electric field. We also choose for convenience a linear temperature extrapolation for the normal-state resistivity which vanishes at $T = 0$ and has a typical value $\rho_N = 84 \mu\Omega \text{ cm}$ at $T = 200$ K.

In Fig. 1 the results of the Hartree approximation for the critical fluctuations are compared to the ones obtained from the Gaussian fluctuation theory. The zero-field critical temperature T_{c0} in the Hartree model was considered identical to the mean-field critical temperature $T_0^{(G)}$ in the Gaussian approximation in order to have the zero-field transition at the same temperature in both theories. This identification causes the mean-field transition temperature T_0 in the Hartree model to shift upwards with respect to T_{c0} . For our chosen parameters this shift was found to be

$$T_0 - T_{c0} = 5.733 \text{ K}, \quad (45)$$

while taking a cutoff parameter $c = 1$. The difference between the two temperatures depends on choice of the cutoff parameter; namely, it increases with the c value and becomes divergent for no cutoff ($T_0/T_{c0} \rightarrow \infty$ for $c \rightarrow \infty$). It can be noticed in Fig. 1 that the curves obtained in the Hartree approximation are less steep than those in the Gaussian one and show a significantly broadened transition region in the presence of strong applied electric fields. In addition, we find that the paraconductivity in the renormalized model is more sensitive to the electric field, showing a more pronounced suppression of the fluctuations at high fields in the lower part of

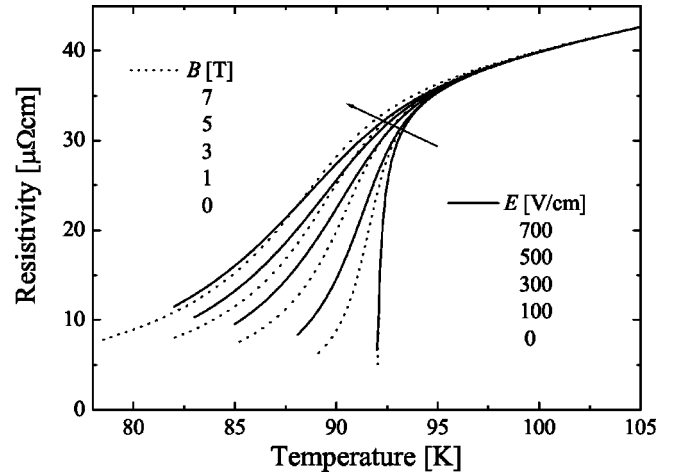


FIG. 2. Comparison between the transition broadening effect of the electric (solid lines) and magnetic (dotted lines) field, respectively, according to the renormalized Hartree approximation. The same parameters as in Fig. 1 were used. The magnetic field effect was calculated in the linear-response (zero-electric-field) limit according to the UD model (Ref. 13). The UV cutoff parameter $c = 1$ in our model corresponds to limiting the sum on the Landau levels at the index $1/2h$ in the UD model ($h = 2\pi\xi_0^2 B/\Phi_0$ being the reduced magnetic field). The arrow indicates the increasing electric and magnetic field direction.

the transition. However, above the zero-field transition temperature, the paraconductivity in the Hartree model is always higher than the one in the Gaussian approximation, due essentially to the critical temperature redefinition from T_0 to T_{c0} . If one preserved instead the same mean-field transition temperature $T_0 = T_0^{(G)}$ as in the Gaussian approximation, one could then visualize the critical temperature shift introduced by the Hartree approach, as shown in the inset of Fig. 1. Equation (44) would give then $T_{c0} = 86.894$ K, and the paraconductivity would be always lower than the one in the Gaussian approximation.

For illustration, we also give in Fig. 2 a comparison between the results of our model, applicable for layered superconductors in arbitrary electric fields in the absence of a magnetic field, and the ones of the complementary model of Ullah and Dorsey,¹³ which treats in the same Hartree approximation the case of an arbitrary magnetic field in the linear-response (zero-electric-field) limit. As one can easily notice, the same well-known “fan shape” transition broadening, encountered when a magnetic field is applied, can be also predicted for the presence of a sufficiently strong electric field. We argue thus that high electric fields can be used to suppress order-parameter fluctuations in HTSC’s as effectively as a magnetic field.

This comparison between the fluctuation suppression effects of the electric and magnetic fields could give also a rough estimation on how broad the validity domain of our model could be. It is known, for instance, that the renormalized fluctuation model¹⁵ for layered superconductors can successfully fit the resistivity curves of YBCO and BSCCO single crystals in magnetic fields up to about 10 T in a tem-

perature range that covers approximately the two superior thirds of the transition region. This extends from a few K in lower fields up to more than 10 K in higher fields, measured down from the resistivity onset point in zero field.¹⁵ The lowest third of the transition, where, experimentally, the resistivity slope becomes steeper, is instead affected by flux pinning effects and does not fit into GL theory. We can therefore assume, based on the similarities illustrated in Fig. 2, that also in high electric fields the renormalized fluctuation model based on the TDGL approach may have its validity in a temperature range at least as broad as in the case when a magnetic field is applied. Since under high electric fields (and, consequently, high current densities) the pinning of the self-field flux lines is overcome by the high Lorentz force, we can expect that the validity of the presented model might extend for even lower temperatures.

In a few previous papers,^{39,40} the non-Ohmic fluctuation conductivity in high electric fields was reported to be experimentally proved by confronting the measured paraconductivity to the scaling laws predicted by Gaussian fluctuation models.^{4,7} However, the broadening of the temperature dependence of the resistive transition with respect to the increasing electric field and the breaking of the mean-field (Gaussian) theory in the immediate vicinity of T_c , signalized by Ref. 39, indicate that a renormalized (non-Gaussian) fluctuation model, such as the one presented in this paper, might be more appropriate. From the experimental viewpoint, applying electric fields of a few hundreds V/cm on cuprates may, however, be not an easy task, since the dissipated power density would attain levels of the order of GW cm^{-3} . On the one hand, high electric fields are necessary in order to put into evidence the non-Ohmic fluctuation conductivity, while, on the other hand, they produce high dissipation and can increase the sample temperature at values where nonlinearity is no longer discernable. In this connection, using short current pulses at high current densities (a few MA cm^{-2}) seems to be a better alternative to the dc and ac measurements.

VI. CONCLUSIONS

In summary, we have treated in this paper the critical fluctuation conductivity for a layered superconductor in zero magnetic field in the frame of the self-consistent Hartree approximation for an arbitrary electric field magnitude. The main results of our work are the formulas (24) for the fluctuation conductivity and (29) for the renormalized reduced-temperature parameter. In both equations the UV cutoff of the Ginzburg-Landau model was explicitly considered. In the linear-response limit ($E \rightarrow 0$), the corresponding expressions, Eqs. (31) and (32), reduce to the previous results of existing theories. Qualitatively, the temperature characteristics at different electrical fields in the Hartree approximation turn out to be less steep than those in the Gaussian one; they show a more pronounced suppression of the fluctuations at high fields in the lower part of the transition and a higher paraconductivity above the zero-field transition temperature than the Gaussian fluctuation model. All these features are quantitatively important for commonly used HTSC's, so that ex-

perimental investigations could be able to discern easily between the applicability of this model in competition with the Gaussian fluctuation approximation.

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APPENDIX: GREEN FUNCTION FOR THE TDGL EQUATION

Equation (12) can be solved easier for the Fourier transform of the Green function with respect to time,

$$R_q(\mathbf{k}, \omega; k'_x, t') = \int dt R_q(\mathbf{k}, t; k'_x, t') e^{i\omega(t-t')}, \quad (\text{A1})$$

which satisfies the equation

$$\left[-i\omega\Gamma_0^{-1} - \frac{e_0\Gamma_0^{-1}E}{\hbar} \frac{\partial}{\partial k_x} + \frac{\hbar^2 k_x^2}{2m} + a_1 \right] R_q(\mathbf{k}, \omega; k'_x, t') = \delta(k_x - k'_x). \quad (\text{A2})$$

One finds for the differential equation (A2) the solution

$$R_q(\mathbf{k}, \omega; k'_x, t') = A_q(\mathbf{k}, \omega; k'_x, t') \times \exp\left\{ \frac{\hbar\Gamma_0}{e_0E} \left[\frac{\hbar^2 k_x^3}{6m} + (a_1 - i\omega\Gamma_0^{-1})k_x \right] \right\}, \quad (\text{A3})$$

where the derivative of the coefficient $A_q(\mathbf{k}, \omega; k'_x, t')$ must satisfy

$$\begin{aligned} \frac{\partial A_q}{\partial k_x}(\mathbf{k}, \omega; k'_x, t') &= -\frac{\hbar\Gamma_0}{e_0E} \exp\left\{ -\frac{\hbar\Gamma_0}{e_0E} \left[\frac{\hbar^2 k_x^3}{6m} + (a_1 - i\omega\Gamma_0^{-1})k_x \right] \right\} \\ &\quad \times \delta(k_x - k'_x) \\ &= -\frac{\hbar\Gamma_0}{e_0E} \exp\left\{ -\frac{\hbar\Gamma_0}{e_0E} \left[\frac{\hbar^2 k_x'^3}{6m} + (a_1 - i\omega\Gamma_0^{-1})k_x' \right] \right\} \\ &\quad \times \delta(k_x - k'_x). \end{aligned} \quad (\text{A4})$$

The solution for the coefficient $A_q(\mathbf{k}, \omega; k'_x, t')$, which remains nondivergent when $k'_x \rightarrow -\infty$, is

$$A_q(\mathbf{k}, \omega; k'_x) = \frac{\hbar \Gamma_0}{e_0 E} \theta(k'_x - k_x) \times \exp\left\{-\frac{\hbar \Gamma_0}{e_0 E} \left[\frac{\hbar k_x'^3}{6m} + (a_1 - i\omega \Gamma_0^{-1}) k'_x \right]\right\}, \quad (\text{A5})$$

where $\theta(k'_x - k_x)$ is the Heaviside step function, so that the Fourier transform of the Green function will be

$$R_q(\mathbf{k}, \omega; k'_x, t') = \frac{\hbar \Gamma_0}{e_0 E} \theta(k'_x - k_x) \exp\left\{\frac{\hbar \Gamma_0}{e_0 E} \left[\frac{\hbar^2 (k_x^3 - k_x'^3)}{6m} + (a_1 - i\omega \Gamma_0^{-1})(k_x - k'_x) \right]\right\}. \quad (\text{A6})$$

Now we can apply the inverse Fourier transform to regain the Green function depending on time t and obtain

$$\begin{aligned} R_q(\mathbf{k}, t; k'_x, t') &= \int \frac{d\omega}{2\pi} R_q(\mathbf{k}, \omega; k'_x, t') e^{-i\omega(t-t')} \\ &= \frac{\hbar \Gamma_0}{e_0 E} \theta(k'_x - k_x) \\ &\quad \times \exp\left\{\frac{\hbar \Gamma_0}{e_0 E} \left[\frac{\hbar^2 (k_x^3 - k_x'^3)}{6m} + a_1 (k_x - k'_x) \right]\right\} \\ &\quad \times \int \frac{d\omega}{2\pi} \exp\left\{-i\omega \left[t - t' + \frac{\hbar}{e_0 E} (k_x - k'_x) \right]\right\} \\ &= \frac{\hbar \Gamma_0}{e_0 E} \theta(k'_x - k_x) \\ &\quad \times \exp\left\{\frac{\hbar \Gamma_0}{e_0 E} \left[\frac{\hbar^2 (k_x^3 - k_x'^3)}{6m} + a_1 (k_x - k'_x) \right]\right\} \\ &\quad \times \delta\left(t - t' + \frac{\hbar}{e_0 E} [k_x - k'_x]\right). \quad (\text{A7}) \end{aligned}$$

The form (A5) for the coefficient $A_q(\mathbf{k}, \omega; k'_x)$ and, namely, the presence of the Heaviside function $\theta(k'_x - k_x)$ assures that the Green function $R_q(\mathbf{k}, \omega; k'_x, t')$ in Eq. (A6) does not diverge and provides also the retarded character in Eq. (A7), i.e., $R_q(\mathbf{k}, t; k'_x, t') = 0$ for $t < t'$.

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