Influence of uniaxial anisotropy on a quantum *XY* spin-glass model with ferromagnetic coupling

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In the replica symmetric approximation and the static limit in Matsubara "imaginary time," we have investigated a quantum *XY* spin-glass system with the ferromagnetic coupling and the uniaxial anisotropy numerically. We found that, for $S=1$, under the uniaxial anisotropy, the spin-glass phase breaks into two phases: a longitudinal spin-glass phase, and a spin-glass phase; the mixed phase of the spin glass and the ferromagnet breaks into two phases: a mixed phase of the longitudinal spin glass and the longitudinal ferromagnet, a mixed phase of the spin glass and the ferromagnet; the peak in the curve of the specific heat versus temperature is split into two peaks: the peak of uniaxial anisotropy and the peak of the ferromagnetic coupling. The system will be dominated by the random exchange interaction if the probability of the random exchange interaction taking negative value is greater than 15.87%. In the absence of the uniaxial anisotropy, there is a mean-interaction translational invariance in the spin-glass phase and the paramagnetic phase. In the presence of the uniaxial anisotropy, there is a mean-interaction translational invariance in the spin-glass phase, the longitudinal spin-glass phase and the paramagnetic phase. In these phases, the entropy, the specific heat and the susceptibility do not depend on the mean-interaction.

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I. INTRODUCTION

The properties of the spin glass have usually been interpreted in terms of the Sherrington-Kirkpartrick (SK) infinite-range model treated in various extensions and approximations. Quantum spin glasses were studied for the first time by Sommers[1] and by Bray *et al.*[2] independently who treated an isotropic quantum Heisenberg spin-glass model. Though the infinite-range SK model is rather unrealistic, the quantum problems are usually treated in the "static" limit, in which the noncommutativity of spin is neglected partially, the essential properties,[1–6] even the detailed properties[7,8] of the thermodynamic and magnetic quantities of the spin glass systems are revealed. Recently, a large number of experimental and theoretical studies on the spin glasses have been obtained.[9–24] These works focus their attentions on the aging,[25,26] the memory,[27,28] the nonequilibrium dynamics,[29,30] and Griffiths singularities,[31] even on the nonlinear response.[32,33]

As is known, various anisotropies play important roles in the spin glass systems. A strong anisotropy of the magnetic susceptibility have been found in hexagonal metallic spin-glass systems experimentally.[34–36] These systems behave in an Ising-type, *XY*-like or Heisenberg-type manner, depending on the sign and the magnitude of the single-spin uniaxial anisotropy energy

$$h_i = -DS_{yi}^2. \tag{1}$$

The long-range SK spin glass model with local uniaxial anisotropy has been studied by Bray[2] and by Cragg and Sherrington.[37] Its quantum version has been discussed by Usadel *et al.*[38,39] They showed that the phase diagram of these systems is function of the impurity concentration and the strength of the anisotropy.

In Refs. 37 and 38, the authors discussed the phase diagram and the magnetic susceptibility of the random SK model with the uniaxial anisotropy, but they neglect the thermodynamic quantities. Motivated by this problem, we revisit this model, and focus our attention on the thermodynamic and magnetic quantities, and discuss the differences between the *XY* spin glass with and without the uniaxial anisotropy. In order to map the mean interaction-temperature phase diagram, we let the Gaussian probability distribution of the random exchange interaction be asymmetric.

II. MODEL HAMILTONIAN AND METHOD

The Hamilton operator for the quantum *XY* spin glass with ferromagnetic couplings and uniaxial anisotropy is given by

$$H = -\sum_{i<j} J_{ij}(S_{ix}S_{jx}+S_{iy}S_{jy}) - D\sum_i {S_{iy}}^2, \tag{2}$$

where the sums extend over all distinct pairs of (i,j). For a given pair (i,j), the exchange interaction J_{ij} is a random parameter with Gaussian probability distribution,

$$P(J_{ij}) = \left(\frac{N}{2\pi J^2}\right)^{1/2} \exp\left(-\frac{N(J_{ij}-J_0)^2}{2J^2}\right), \tag{3}$$

where J_0 and J are defined as the mean and the variance of the exchange interaction respectively. The spin operators S_i obey the standard spin commutation relations.

To carry out the average over the random bonds, we use the replica method and the imaginary time functional technique. In the replica symmetry approximation, the static free energy function per spin is given by

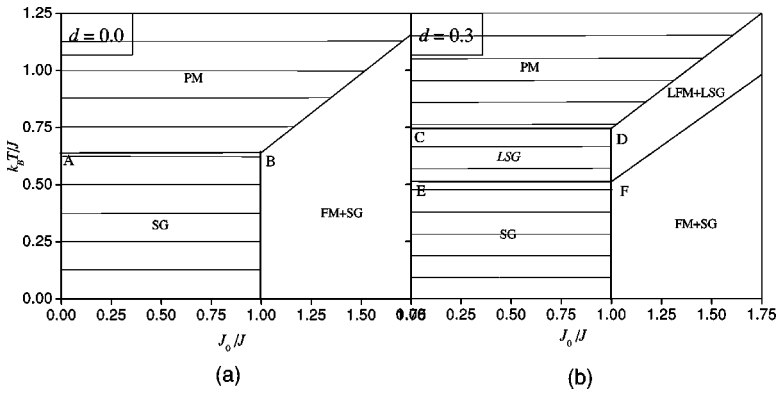

FIG. 1. The phase diagram for $S=1$, (a) for uniaxial anisotropy $u=0.0$, (b) for $u=0.3$.

$$f/J=\frac{J\beta}{4}(R_T^2+R_L^2-Q_T^2-Q_L^2)+\frac{1}{2}j_0(M_T^2+M_L^2)-\frac{1}{J\beta}\int D\mathbf{x}\ln L(\mathbf{x}), \tag{4}$$

where $j_0=J_0/J$ and $\beta=1/k_BT$, T is the temperature, k_B is Boltzmann constant. For $S=1$, the function $L(x)$ is defined as

$$L(\mathbf{x})=\int D\mathbf{x}'\sum_{n=0}^{3}\exp(J\beta\lambda_n), \tag{5}$$

with

$$\lambda_n=\frac{2}{3}d+\frac{2}{3}\sqrt{d^2+3A^2+3B^2}\times\cos\left\{\frac{2n\pi}{3}+\frac{1}{3}\arccos\left[-\frac{2d^3+9A^2d-18B^2d}{2(d^3+3A^2+3B^2)^{3/2}}\right]\right\},$$

$$d=D/J,\quad A=a_1x+a_2x'+j_0M_T,\quad B=a_3y+a_4y'+j_0M_L,$$

$$a_1=\sqrt{Q_T},\quad a_2=\sqrt{R_T-Q_T},\quad a_3=\sqrt{Q_L},\quad a_4=\sqrt{R_L-Q_L}. \tag{6}$$

The R_T, R_L, Q_T, Q_L and M_T, M_L represent the transverse and longitudinal components of the self-interactions, the spin-glass order parameter and the magnetization respectively, and are determined by

$$R_T=\frac{1}{(J\beta a_2)^2}\int D\mathbf{x}\frac{1}{L(\mathbf{x})}\int D\mathbf{x}'(x'^2-1)\sum_{n=1}^{3}\exp(J\beta\lambda_n), \tag{7}$$

$$R_L=\frac{1}{(J\beta a_4)^2}\int D\mathbf{x}\frac{1}{L(\mathbf{x})}\int D\mathbf{x}'(y'^2-1)\sum_{n=1}^{3}\exp(J\beta\lambda_n), \tag{8}$$

$$Q_T=\frac{1}{(J\beta a_2)^2}\int D\mathbf{x}\left(\frac{1}{L(\mathbf{x})}\int D\mathbf{x}'x'\sum_{n=1}^{3}\exp(J\beta\lambda_n)\right)^2, \tag{9}$$

$$Q_L=\frac{1}{(J\beta a_4)^2}\int D\mathbf{x}\left(\frac{1}{L(\mathbf{x})}\int D\mathbf{x}'y'\sum_{n=1}^{3}\exp(J\beta\lambda_n)\right)^2, \tag{10}$$

$$M_T=\frac{1}{J\beta a_1}\int D\mathbf{x}x\ln L(\mathbf{x}), \tag{11}$$

$$M_L=\frac{1}{J\beta a_3}\int D\mathbf{x}y\ln L(\mathbf{x}). \tag{12}$$

These functions are obtained from f in Eq. (4) at the saddle point with respect to the spin self-interactions, the spin-glass order parameters and the magnetization.

In the above, the abbreviation denotes

$$\int D\mathbf{x}A(\mathbf{x})\equiv\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dxdy\exp\left(-\frac{x^2+y^2}{2}\right)A(\mathbf{x}). \tag{13}$$

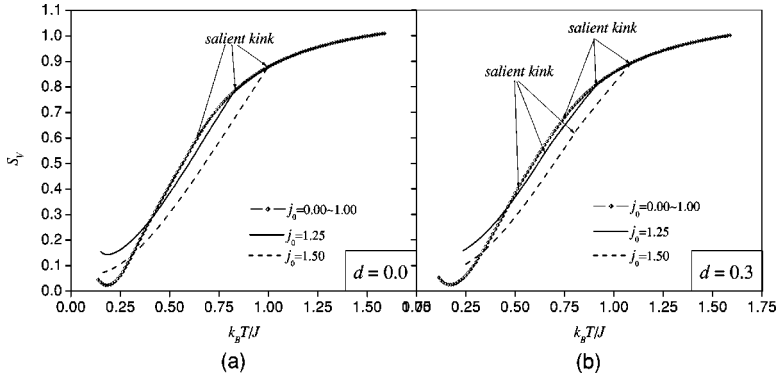

FIG. 2. The curves of the entropy depending on the temperature for $S=1$, (a) for uniaxial anisotropy $u=0.0$, (b) for $u=0.3$.

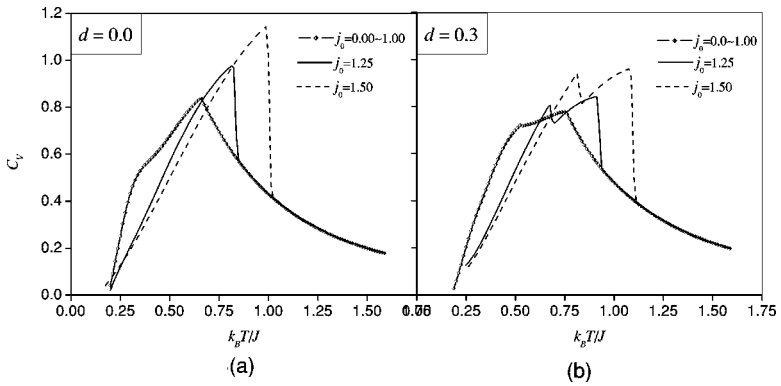

FIG. 3. The functions of the specific heat as the temperature for $S=1$, (a) for uniaxial anisotropy $u=0.0$, (b) for $u=0.3$.

According to Eq. (4), it is straightforward to get the entropy

$$S/k_B=\frac{(J\beta)^2}{4}(R_T^2+R_L^2-Q_T^2-Q_L^2)+\int Dx \ln L(x)$$

$$-J\beta\int Dx\frac{1}{L(x)}\sum_{n=1}^{3}\lambda_n \exp(J\beta\lambda_n). \quad (14)$$

In the above calculations, we have used the saddle condition of the free energy with respect to the spin self-interactions, the spin-glass order parameters, and the magnetization.

By using Eq. (14), the specific heat can be calculated from

$$C_V=-\beta\frac{dS}{d\beta}. \quad (15)$$

We do not give the final expression of the specific heat because of its complication.

III. NUMERICAL RESULTS AND ANALYSIS

Equations (7)–(12) can be solved self-consistently when d and j_0 are given. Therefore, the entropy, the specific heat and the susceptibility are determined. The phase diagram is obtained too. Now we wish to report our numerical results in this section.

Figure 1 shows the mean interaction-temperature phase diagrams for $d=0.0$ and $d=0.3$. In the absence of the uniaxial anisotropy, as shown in Fig. 1(a), the phase diagram has three regions: the region of the spin-glass (SG) phase where $Q_T\neq 0$, $Q_L\neq 0$, $M_T=0$, $M_L=0$, the region of the paramagnetic (PM) phase where $Q_T=0$, $Q_L=0$, $M_T=0$, $M_L=0$ and the region of the mixed (FM+SG) phase of the ferromagnet and the spin-glass where $Q_T\neq 0$, $Q_L\neq 0$, $M_T\neq 0$, $M_L\neq 0$. The data of the boundary of the PM and SG phases is lie on a good straight line AB. But in the presence of the uniaxial anisotropy, as shown in Fig. 1(b), the SG phase breaks into a longitudinal SG phase where $Q_T=0$, $Q_L\neq 0$, $M_T=0$, $M_L=0$ and a SG phase where $Q_T\neq 0$, $Q_L\neq 0$, $M_T=0$, $M_L=0$, the FM+SG phase breaks into a longitudinal FM+SG phase where $Q_T=0$, $Q_L\neq 0$, $M_T=0$, $M_L\neq 0$ and a FM+SG phase where $Q_T\neq 0$, $Q_L\neq 0$, $M_T\neq 0$, $M_L\neq 0$. The data of the boundary of the PM and LSG phases lie on a straight line CD while the data of the boundary of the LSG and SG phases lie on a straight line EF.

Figure 2 exhibits the dependence of the entropy on temperature for different j_0. The entropy is positive in the temperature region considered, not as other models such as Heisenberg model in which the entropy usually goes negative at lower temperatures. The reason may be that the model considered here is an *XY* model. In Fig. 2(a), the diamond line corresponds to $j_0=0.00$, $j_0=0.25$, $j_0=0.50$, $j_0=0.75$, and $j_0=1.00$. The other two correspond to $j_0=1.25$ and 1.50, respectively. There is a salient kink in each of these three curves. The salient kink in diamond line corresponds to the transition from SG to PM. The salient kinks in the other two lines correspond to the transition from FM+SG to PM. In Fig. 2(b), the diamond line corresponds to $j_0=0.00$, $j_0=0.25$, $j_0=0.50$, $j_0=0.75$, and $j_0=1.00$. The others correspond to $j_0=1.25$ and 1.50, respectively. There are two salient kinks in each of these three curves [the left salient kink

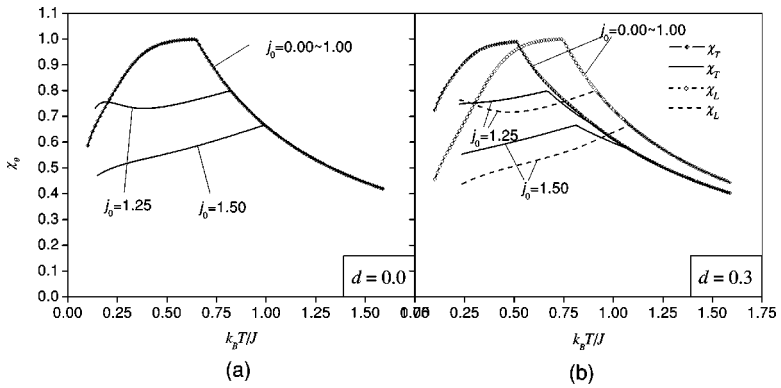

FIG. 4. The dependence of the susceptibility on the temperature for $S=1$, (a) for uniaxial anisotropy $u=0.0$, (b) for $u=0.3$.

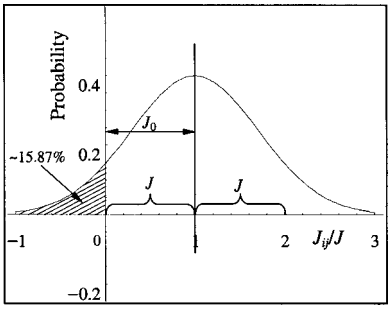

FIG. 5. A Gaussian distribution where $J_0 = J$.

can be noticed in Fig. 2(b) if hold the paper in a gracing angle]. The salient kinks in diamond line correspond to the transition (the left) from SG to LSG and to the transition (the right) from LSG to PM, respectively. The salient kinks in the other two lines correspond to the transition from SG+FM to LSG+LFM and to the transition from LSG+LFM to PM, respectively.

Figure 3 illustrates the dependence of the specific heat on temperature for different j_0. For $d=0.0$, as shown in Fig. 3(a), in every line, there is a peak corresponding to the salient kink in Fig. 2(a), it is caused by j_0. But when the uniaxial anisotropy is present, see Fig. 3(b), it splits into two peaks. These two peaks are caused by d (the left) and j_0 (the right) respectively. It is worthy to be noticed that the right peaks seem to look like that in the curves for the ordered system[40] and that of the $Co_2(OH)PO_4$ sample.[36] This may be caused by the fact that the system is dominated by the ordered exchange interaction.

The influence of uniaxial anisotropy on the system can also be seen in the susceptibility, Fig. 4 shows this effect. In Fig. 4(a), the curves of the transverse and longitudinal susceptibilities are coincided. But in the presence of the uniaxial anisotropy, as shown in Fig. 4(b), they are separated by the uniaxial anisotropy. It can be seen that the curves approach to the same asymptote in the high temperature region. This feature agrees with Refs. 34 and 35. But there is a cusp in the longitudinal susceptibility. This may be caused by the fact that we have set $d=0.3$, which is smaller than that in Ref. 37.

From the above discussions, one can see that the entropy, the specific heat and the susceptibility do not depend on j_0 for $0<j_0\leqslant 1$. According to the definition of $j_0=J_0/J$, the condition $0<j_0\leqslant 1$ determines a Gaussian distribution of the random exchange interaction, in which the mean J_0 of the Gaussian distribution is smaller than the variance J of it. This means that the probability for $J_{ij}<0$ must be greater than 15.87%, see Fig. 5. The fact that the entropy, the specific heat and the susceptibility do not depend on j_0 for $0<j_0 \leqslant 1$ demonstrates that, for $d=0.0$, there is a j_0 translational invariance in the SG phase and the PM phase; for $d=0.3$, there is a j_0 translational invariance in the SG phase, the LSG phase and the PM phase. The invariance in these phases can be represented by clusters of straight lines as shown in Fig. 1. Along these lines, the entropy, the specific heat, and the susceptibility will be the same. And then, the system is dominated by the disorder interaction.

As we know, different values of j_0 give different spin glass samples. The difference between different samples, if there is any, cannot be observed in a single sample. So this j_0 translational invariance is never noticed by previous studies. The fact mentioned above tells us that the quenched samples will show no difference if their probability of the random exchange interaction taking negative value is greater than 15.87%. That is to say, once the probability is beyond 15.87%, the sample will give the same spin glass behavior. This property makes the preparation of spin glass samples easier than expected.

IV. CONCLUSIONS

In summary, we have investigated the quantum *XY* spin-glass system with the ferromagnetic coupling and the uniaxial anisotropy numerically. We found that, under the uniaxial anisotropy, the SG phase breaks into two phases: LSG phase and SG phase; the SG+FM phase breaks into two phases: LSG+LFM phase and SG+FM phase. The peak in the curve of the specific heat versus temperature is split into two peaks: the peak of uniaxial anisotropy and the peak of the ferromagnetic coupling. The system will be dominated by the disorder exchange interaction J_{ij} if the probabilities for $J_{ij}<0$ is greater than 15.87%. For $d=0.0$, there is a j_0 translational invariance in the SG phase and the PM phase; for $d=0.3$, there is a j_0 translational invariance in the SG phase, the LSG phase and the PM phase. The entropy, the specific heat and the susceptibility do not depend on the mean interaction in these phases.

In the above, we have set $S=1$, but these results are not completely held for half-integer spins. For $S=1/2$, the uniaxial anisotropy cannot make the phase break. But the independence of j_0 is still held. For $S=3/2$, the results mentioned above are all not held. As for short-ranged model, it remains an open problem.

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