Magnetic relaxation in partly penetrated critical states of type-II superconductors

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Magnetic relaxation in a thin flat type-II superconductor of arbitrary shape placed in an external magnetic field perpendicular to its plane is analyzed in the general case when flux penetration is not necessarily complete. We derive an expression for the magnetic relaxation rate that generalizes the well-known formula of Beasley *et al.* obtained for superconductors in the fully penetrated critical state. Our result allows one to extract the effective height of the activation barrier from experimental data even when the critical state has a complicated structure, in particular when the applied magnetic field is less than the field of full flux penetration into the sample.

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I. INTRODUCTION

There is a great body of experimental data on magnetic relaxation in type-II superconductors, see, e.g., Ref. 1 for a recent review. These data enable one to estimate the so-called effective depth of a flux-pinning well, U_0, using the expression[2] for the magnetic relaxation rate $R \equiv dM/d\ln t$:

$$R = -\frac{T}{U_0} M_c . \tag{1}$$

Here M is the magnetic moment of the sample, M_c is its value in the critical state, T is the temperature, and t is the time. This formula was first derived by Beasley, Labusch, and Webb,[2] for a superconducting cylinder in an external magnetic field applied along its axis. However, Eq. (1) is also valid for a plate in a field parallel to its plane[3] and for a thin strip or a thin disk in a transverse magnetic field.[4] The assumptions used in deriving formula (1) are the following: First, flux-line pinning should be isotropic and independent of the magnetic induction **B**; second, the ratio T/U_0 is small; and third, the fully penetrated critical state is reached in the sample. The magnetic relaxation rate of anisotropic superconductors was investigated in Refs. 5–8. When T/U_0 is not too small, deviations from the logarithmic decay, $\delta M \propto \ln t$, become observable[9] at large t. This so-called long-term magnetic relaxation was analyzed in Refs. 10–14. Here we shall go beyond the framework of the third assumption.

Formula (1) can be interpreted as coming from the logarithmic decay of the current density j:

$$j(t) = j_c\left(1 - \frac{T}{U_0}\ln t\right), \tag{2}$$

where j_c is the critical current density. Indeed, when j_c and U_0 are independent of the magnetic induction **B**, and the sample is in the fully penetrated critical state, one has $M(t) \propto j(t)$, and formula (1) follows from Eq. (2). It is clear from this consideration that in fact, the expression (1) is valid for any shape of the superconductor. But this formula fails when some partly penetrated critical state occurs in the superconductor. Such a state is realized if, e.g., the external magnetic field is less than the field of full flux penetration into the sample H_p or if in the initial critical state there is a flux front separating regions of the sample with $j = j_c$ and $j = -j_c$ (in the "virgin case" $H < H_p$ the front separates regions with $j = j_c$ and $j = 0$). During relaxation the front shifts, and this leads to an additional change of M as compared with the fully penetrated critical state, i.e., with Eq. (1). In this case the magnetic relaxation rate R generally depends on the shape of the front, and thus on the shape of the sample. Up to now, explicit formulas for R in a partly penetrated critical state have been available only for a plate[3] in a longitudinal field and for a disk[6] in axial field. In this paper we find an expression for R that generalizes Eq. (1) to partly penetrated critical states in flat superconductors of arbitrary shape.

II. MAGNETIC RELAXATION RATE

We define a flat superconductor here as a sample with a *constant* thickness d that is considerably less than the lateral extensions of the superconductor (but d should exceed the London penetration depth). In lateral directions the superconductor can have arbitrary shape. Note that platelet-shaped crystals of high-T_c superconductors satisfy these conditions. The external magnetic field H is assumed to be applied along the thickness (the z axis). Throughout the paper we also use the approximation $B = \mu_0 H$ (i.e., the magnetic field H is implied to exceed noticeably the lower critical field H_{c1}) and assume that j_c and U_0 are independent of **B**.

We begin our analysis with the situation when the external magnetic field is less than the field H_p, and the initial critical state originates from partial magnetic-flux penetration into the uniform state of the superconductor with $B = 0$. In this situation there is a flux front γ separating the inner region of the sample with $j = 0$ from its outer region where $j = j_c$ (Fig. 1). The critical state of this type was investigated in Ref. 15. As shown in that paper, the three-dimensional critical state problem in the flat superconductor can be split into a one-dimensional problem across the thickness of the sample and a two-dimensional problem of an infinitely thin superconductor of the same shape. It is essential here that in the considered case of constant sheet current $J_c = j_c d$ in the pen-

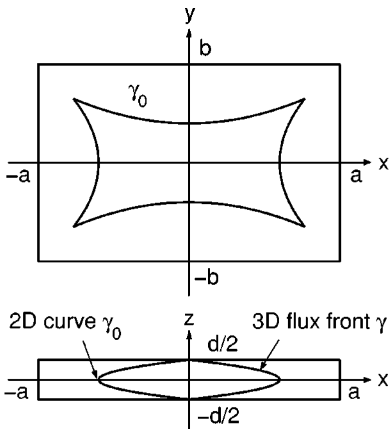

FIG. 1. The front of the magnetic flux penetrating into a thin rectangular superconductor plate with pinning when the applied perpendicular magnetic field is increased from zero. The top figure shows the two-dimensional (2D) curve γ_0 that forms the equator of the three-dimensional flux front γ shown in the lower plot. Inside this 3D front the magnetic induction **B** and the current density are exactly zero, and inside the curve γ_0 the perpendicular component of B_z is practically zero when $d \ll a,b$ for a thin plate with dimensions $2a\times 2b\times d$.

etrated region (i.e., outside the curve γ_0 in Fig. 1), the solution of the two-dimensional problem is completely determined by the ratio H/J_c.[15–17] Thus, the magnetic moment in the critical state, M_c, can be written in the form

$$M_c = M_c^{\rm sat} F(H/J_c),$$

$$M_c^{\rm sat} = C J_c, \tag{3}$$

where $M_c^{\rm sat}$ is the saturation value of M_c, reached in the fully penetrated critical state; C is some geometrical factor which depends on the shape and lateral dimensions of the flat superconductor, and the function $F(x)$ is determined by the shape of the sample.

It was shown by Beasley, Labusch, and Webb[2] that in the case of a cylinder, the equations for the magnetic relaxation allow separation of the temporal and spatial variables to lowest order in the small parameter $T/U_0 \ll 1$. Recently, this result was extended to superconductors of other shapes.[4,6,13,18,19] It is important that this separation works also in the partly penetrated critical state.[6,19] Taking into account these results, we find that expression (2) remains true even for partial penetration.

Replacing J_c in Eq. (3) by $j(t)d$ with $j(t)$ from formula (2), differentiating the obtained expression with respect to $\ln t$, and using the identity

$$\frac{dF}{dJ_c} = -\frac{H}{J_c}\frac{dF}{dH},$$

we arrive at the expression,

$$R \equiv \frac{dM}{d\ln t} = -\frac{T}{U_0}\left(M_c - H\frac{dM_c}{dH}\right) \tag{4}$$

that generalizes Eq. (1) to the case of partly penetrated critical states. Note that formula (4) is also valid when the magnetic field lies in the plane of the superconductor.

Within our approximation $j_c=$ const, the magnetic moment M_c becomes constant and is equal to its saturation value $M_c^{\rm sat}$ at $H \geq H_p$. Then, the second term in Eq. (4) vanishes, and this formula goes over into Eq. (1). It is the second term in Eq. (4) that takes into account the above-mentioned shift of the flux front during relaxation. In obtaining this term we have used the fact that in the critical state a shift of the front also occurs when the external magnetic field changes. This has enabled us to express the shift caused by relaxation in terms of the shift caused by a change of H. Besides this, the second term takes into account the relaxation of the flux-line directions[19,20] when the critical state contains so-called rotating flux lines.[15]

III. EXAMPLES

Consider flat superconductors of the following shapes: a disk of radius b_0, a strip of width $2b_0$, and an elliptic-shaped platelet with the axes $2b_0$ and $2a_0$ ($a_0 > b_0$). The exact solutions of the critical state problem are known for the disk[21] and for the strip,[22–24] while for elliptic-shaped platelets an approximate analytical solution was recently obtained[25] that gives the magnetic moment M_c with very high accuracy. Using these solutions and formula (4), we present here explicit formulas for the magnetic relaxation rate R in these three cases.

A. Disk

In the disk the two-dimensional front γ_0, Fig. 1, is a circle of radius b that is the following function of H:[21]

$$\tilde{b} \equiv \frac{b}{b_0} = \frac{1}{\cosh(H/H_d)} \tag{5}$$

with $H_d = J_c/2$. As to the magnetic moment in the critical state, M_c, one has[21]

$$M_c^{\rm disk} = M_c^{\rm sat}\frac{2}{\pi}\left(\arccos\tilde{b} + \tilde{b}\tanh\frac{H}{H_d}\right) \tag{6}$$

with $M_c^{\rm sat} = -\pi J_c b_0^3/3$ and field of full penetration $H_p = H_d \ln(4b_0/d)$.[26] From Eqs. (4)–(6) we find

$$R^{\rm disk} = -\frac{T}{U_0} M_c^{\rm sat}\frac{2}{\pi}\left(\arccos\tilde{b} + \tilde{b}\tanh\frac{H}{H_d} - 2\tilde{b}^3\frac{H}{H_d}\right). \tag{7}$$

This expression exactly coincides with Eq. (43) of Ref. 6 where $R^{\rm disk}$ was obtained in a different way.

B. Strip

In the strip the two-dimensional front γ_0 comprises two straight lines separated by a distance $2b$. This b is still described by Eq. (5) but with $H_d = J_c/\pi$, while M_c per unit length of the strip is given by[22–24]

$$M_c^{\rm strip}=M_c^{\rm sat}\sqrt{1-\tilde{b}^2}=M_c^{\rm sat}\tanh\frac{H}{H_d} \quad (8)$$

with $M_c^{\rm sat}=-J_c b_0^2$ and penetration field $H_p=H_d[1+\ln(2b_0/d)]$.[26] Formulas (4) and (8) then give

$$R^{\rm strip}=-\frac{T}{U_0}M_c^{\rm sat}\left(\tanh\frac{H}{H_d}-\frac{(H/H_d)}{\cosh^2(H/H_d)}\right). \quad (9)$$

C. Elliptic-shaped platelet

In elliptic-shaped platelets the two-dimensional front γ_0 approximately is an ellipse with the shorter axis $2b$ implicitly determined by the following relation:[25]

$$\frac{H}{H_d}=\int_{\tilde{b}}^{1}\frac{E(k)db'}{b'\sqrt{1-b'^2}}, \quad (10)$$

where $\tilde{b}=b/b_0$, $H_d=J_c/\pi$, $E(k)$ is the complete elliptic integral of the second kind with

$$k^2=1-\left(\frac{b'e}{\cos(e\arccos b')}\right)^2$$

and $e=b_0/a_0$. The magnetic moment M_c is now

$$M_c^{\rm ell}=-\frac{2}{3}J_c a_0 b_0^2\left[\frac{\cos(\arcsin\tilde{b}+e\arccos\tilde{b})}{1-e}+\frac{\cos(\arcsin\tilde{b}-e\arccos\tilde{b})}{1+e}\right]. \quad (11)$$

When $\tilde{b}\to 0$, it follows from this formula that

$$M_c^{\rm sat}=-\frac{4}{3}J_c a_0 b_0^2\frac{\cos(e\pi/2)}{1-e^2}. \quad (12)$$

Using expressions (10) and (11), we find:

$$H\frac{dM_c^{\rm ell}}{dH}=-\frac{2}{3}J_c a_0 b_0^2\frac{H}{H_d}\frac{\tilde{b}}{E(k)}[\sin(\arcsin\tilde{b}+e\arccos\tilde{b})+\sin(\arcsin\tilde{b}-e\arccos\tilde{b})] \quad (13)$$

with $k^2=1-[be/\cos(e\arccos\tilde{b})]^2$. Insertion of Eqs. (11) and (13) into Eq. (4) yields the magnetic relaxation rate $R^{\rm ell}$ of elliptic-shaped platelets.

D. Generalizations

The H dependences of the magnetic relaxation rates for the disk, for the strip, and for the elliptic-shaped platelet are presented in Fig. 2. With high accuracy of about 10^{-2}, all these dependences can be approximated by the following formula:

$$R=-\frac{T}{U_0}M_c^{\rm sat}\left(\tanh\frac{H}{H_1}-\frac{(H/H_1)}{\cosh^2(H/H_1)}\right), \quad (14)$$

where

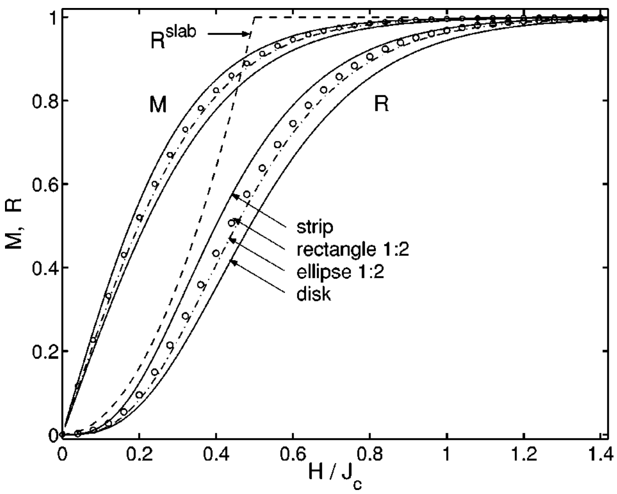

FIG. 2. The H dependences of the magnetic moments M and relaxation rates $R=dM/d\ln t$, Eq. (4), calculated for the thin disk and strip (solid lines), an elliptic-shaped platelet with $b_0/a_0=0.5$ (dash-dotted line), and for a thin rectangle with side ratio 1:2 (circles). The dashed line shows $R^{\rm slab}$, Eq. (16). The magnetic field H is measured in units of J_c, M in units $M_c^{\rm sat}$, and R in units $-(T/U_0)M_c^{\rm sat}$.

$$H_1=J_c\frac{E(k)}{\pi}\frac{\cos(e\pi/2)}{1-e^2} \quad (15)$$

and $k^2=1-e^2$, $e=b_0/a_0$. This approximation is good since the magnetic moments of the disk, strip, and of elliptic-shaped platelets in increasing H can be well fitted by the dependence[25] $M_c=M_c^{\rm sat}\tanh(H/H_1)$. In Fig. 2 we also show the magnetic relaxation rate of a rectangular platelet $R^{\rm rec}$ obtained from formula (4) using the magnetic moment of thin rectangles computed as in Ref. 16. The $R^{\rm rec}$ can be also fitted by Eq. (14) if one takes an appropriate scale H_1. Note that if the magnetic field lies in the plane of the samples, one has $M^{\rm slab}=M_c^{\rm sat}H(2H_p-H)/H_p^2$ (per unit area of the superconductor and for $0\leqslant H\leqslant H_p$) and[3]

$$R^{\rm slab}=-\frac{T}{U_0}M_c^{\rm sat}\frac{H^2}{H_p^2}, \quad (16)$$

where $H_p=J_c/2$ and $M_c^{\rm sat}=-J_c d/4$.

It follows from the formulas of this section that $R^{\rm disk}$, $R^{\rm strip}$ and $R^{\rm ell}$ (and $R^{\rm rec}$) are all proportional to H^3 when $H\ll J_c$. On the other hand, if the magnetic field lies in the plane of the samples, the magnetic relaxation rate R is proportional to H^2. This difference is due to the different current distributions in the critical states for transverse and longitudinal geometries.[17] Note that the dependence $R\propto H^3$ was observed for YBaCuO crystals in Ref. 3, but in that paper this finding was explained by a dependence of the critical current density j_c on B. Our results show that the proportionality of R to H^3 can be due to the geometry of the experiments.

We have analyzed above the relaxation rate in the partly penetrated critical state which results from the increase of the external magnetic field from zero to $H<H_p$. We now consider the situation when the superconductor was initially in a

spatially uniform state with $B_{\rm ini}=\mu_0 H_{\rm ini}\neq 0$, and then the external field increases or decreases, but $|H-H_{\rm ini}|<H_p$. For example, this situation may occur when remanent magnetization is investigated. In this case the function F in Eq. (3) depends on $|H-H_{\rm ini}|/J_c$, and we obtain the following generalization of Eq. (4):

$$R=-\frac{T}{U_0}\left(M_c-(H-H_{\rm ini})\frac{dM_c}{dH}\right). \qquad (17)$$

One more type of partially penetrated critical state occurs when the external field initially increased up to $H_{\rm ini}$ so that the superconductor was in the fully penetrated critical state, and then the external field decreases down to H with $H_{\rm ini}-H<H_p$. Note that if one measures a magnetization loop of the superconductor, this situation necessarily takes place during the transition from the ascending branch to the descending branch of the loop. In this case a flux front in the flat superconductor appears, which separates regions with $j=\pm j_c$. Since in such a critical state the function F in Eq. (3) still depends on $(H_{\rm ini}-H)/J_c$, one finds that formula (17) remains valid in this case. Interestingly, it follows from Eq. (17) that at some $M_c\neq 0$ the magnetic relaxation rate vanishes in this critical state.

Finally, we hypothesize the following: When j_c and U_0 are constant, formulas (3) seem to hold true not only for flat superconductors but also for samples of an *arbitrary shape* if one substitutes j_cL for J_c. Here L is some scale of length that can depend on the direction of the magnetic field. Then, we again arrive at expressions (4) and (17). In other words, it is quite possible that whatever shape the superconductor has, these expressions describe the magnetic relaxation rates in both fully and partly penetrated critical states and thus generalize formula (1).

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