Effects of photoluminescence polarization in semiconductor quantum wells subjected to an in-plane magnetic field

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The strong effects of optical polarization anisotropy observed previously in quantum wells subjected to an in-plane magnetic field receive a complete description within the microscopic approach. The theory we develop involves two sources of optical polarization. The first source is due to correlations between electron and heavy-hole (HH) phases of ψ functions arising due to electron Zeeman spin splitting and joint manifestation of low-symmetry and Zeeman interactions of HH's in an in-plane magnetic field. In this case, four possible phase-controlled electron-HH transitions constitute the polarization effect, which can reach its maximal amount (± 1) at low temperatures when only one transition survives. The other polarization source stems from an admixture of excited light-hole states to HH's by low-symmetry interactions. The contribution of this mechanism to the total polarization is relatively small but can be independent of temperature and magnetic field. An analysis of the different mechanisms of HH splitting exhibits their strong polarization anisotropy. The joint action of these mechanisms can result in new peculiarities, which should be taken into account for an explanation of different experimental situations.

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I. INTRODUCTION

The linear optical polarization ρ of photoluminescence (PL) in quantum wells (QWs) is very sensitive to low-symmetry interactions V, which can be responsible for this polarization.[1–4] A typical situation corresponds to relatively weak V, which mixes the light-hole (LH) and heavy-hole (HH) states. By this virtue the polarization reaches a magnitude of about $\varepsilon = |V|/\Delta_{HL}$ (Δ_{HL} is the HH-LH energy splitting) without an external magnetic field.[2,3]

The strong polarization of luminescence from [001]-oriented quantum wells $Cd_{1-x}Mn_xTe/CdTe/Cd_{1-x}Mn_xTe$ and its π periodic anisotropy (i.e., dependence on sample rotation about the QW normal) has been observed in Refs. 1 and 4 under in-plane magnetic field $\vec{B}$. It has been assumed there that these properties are due to the C_{2v} symmetry potential of a hole in a QW. It was found that polarization and its anisotropy increase sharply with an increase of the in-plane magnetic field and reaches a few tens of a percent. This fact cannot be consistent with a small value of the ratio ε. Moreover, strong polarization effects as well as the significant contribution of the fourth harmonic (i.e., $\pi/2$-periodic component) of the aforementioned anisotropy for narrow QW's remains so far unexplained.[1] The phenomenological approach developed in Ref. 1 in terms of a representation of ρ bilinear in $\vec{B}$ cannot describe the strong effects when $\rho \approx 1$.

The complementary approach of Ref. 1 in terms of a pseudospin formalism requires the proper determination of pseudospin basic functions both for real electron spin operators and for the nonspin part of the interaction being responsible for optical transitions. Moreover, different kinds of HH interactions, determining the HH splitting under in-plane magnetic field, need further consideration. This means that for a correct description of the above experimental data it is necessary to have a microscopic theory in terms of actual electron and hole spins rather than pseudospins.

Here, we pay attention to the well-known fact that any interaction splitting the degenerate electron and HH levels imposes some phase correlations between the electron and hole wave functions. In addition to the above small contribution caused by LH-HH mixing, this correlation forms the polarization and its anisotropy associated with some pairs of distinguishable electron-hole optical transitions regardless of the spin level splitting value.

On the other hand, there are different possible interactions which are able to lift the HH degeneracy in a magnetic field.[5] Latter interactions impose their specific correlations between electron and hole ψ-function phases as well as the period and phase of optical polarization anisotropy (OPA), i.e., the polarization dependence on QW rotation about its normal. Thus, if one of the four possible electron-HH transitions prevails (for instance, due to low enough temperatures), one can expect the appearance of strong optical polarization.

In this paper, we provide a quantitative microscopic analysis of the optical polarization anisotropy caused by different low-symmetry interactions of a hole in QW's. First, we discuss the general expression for OPA in terms of electron and HH ψ-function phases. Then we show that the different interactions leading to HH splitting reveal various OPA dependences on in-plane magnetic field $\vec{B}$ rotation. This demonstrates the necessity to account for a joint contribution of the aforementioned terms to OPA resulting in qualitatively new peculiarities due to interference effects. Finally, in the framework of our theory, we explain quantitatively the most interesting experimental results that are accessible from the literature.

II. THEORETICAL BACKGROUND

A. Photoluminescence linear polarization

We are interested in the linear polarization of the PL spectrum that involves four optical transitions from two electron spin sublevels to two HH sublevels. To avoid the problems of these component spectral shifts in a magnetic field (see below) we assume that integral PL intensities I_α of polarization α can be extracted from experiment and associated with transition probabilities in terms of the thermal population of spin sublevels. We also assume that luminescence occurs due to a recombination of noninteracting electrons and holes. Actually, excitonic effects might have an influence on OPA because of the electron-hole exchange interaction $H_{ex} = \frac{2}{3}[\Delta_0 \vec{J}\vec{s} + \Delta_1 \Sigma_i J_i^3 s_i - \delta(J_x^3 s_y + J_y^3 s_x)]$ where Δ_0 is isotropic and Δ_1 and δ are anisotropic exchange constants.[5] This interaction leads to an energy level splitting of excitons consisting of electrons and HH's in zero magnetic field as well as to a spin state mixture.[6] So we can suspect some peculiarities due to exciton effects until the electron (hole) spin splitting in the effective fields does not exceed $|\Delta_1|$ and $|\delta|$. Since the manifestation of $H_{\rm ex}$ needs also significant differences in exciton levels populations, the excitonic effects are negligible in the case of strong enough magnetic fields or high enough temperatures. Hereafter we assume that the conditions for excitonic effects to be small are satisfied. The data for the magnitudes of exciton exchange interaction [being less than 1 μeV (Ref. 5)] give enough evidence to neglect $H_{\rm ex}$ in the very wide range of magnetic fields and temperatures.

Thus, according to definition,

$$\rho_\alpha = \frac{I_\alpha - I_{\alpha'}}{I_\alpha + I_{\alpha'}}, \tag{1}$$

where the plane of α' polarization is perpendicular to that of α polarization. Then, we introduce the reference frame associated with the main crystal axes so that $\overrightarrow{OZ}$ is parallel to the growth axis [001], while $\overrightarrow{OX} \| [100]$ and $\overrightarrow{OY} \| [010]$ lie in the QW plane.

The electron (or HH) spin splitting $\omega = \omega_e$ (or $\omega = \omega_h$) is assumed to be described by the following matrix Hamiltonian in a certain basis $|n\rangle$, $n = 1, 2$,

$$||H_{n,n'}|| = \frac{\omega}{2}\begin{pmatrix} 0 & e^{-i\theta} \\ e^{i\theta} & 0 \end{pmatrix}, \tag{2}$$

where $\omega = 2|H_{1,2}|$ and $\sin\theta = -2\,{\rm Im} H_{1,2}/\omega$. The eigenvalues and eigenfunctions of the Hamiltonian (2) are

$$E_\pm = \pm\frac{1}{2}\omega, \quad \psi^\pm = \frac{1}{\sqrt{2}}(\pm e^{-i\theta/2}|1\rangle + e^{i\theta/2}|2\rangle). \tag{3}$$

In the case of an electron subjected to an in-plane magnetic field $\vec{B} = B\{\cos\varphi, \sin\varphi, 0\}$, $\psi^\pm \equiv \psi_c^\pm$, the Hamiltonian $H = \vec{G}_e \vec{s}$ takes the form (2) in a representation of $|1\rangle = S\uparrow$ and $|2\rangle = S\downarrow$, where S is a periodic part of the conductivity-band Bloch function and $\uparrow$ and $\downarrow$ are the eigenstates of spin projection s_z. Here $\theta = \varphi$ is an angle between $\vec{B}$ and $\overrightarrow{OX}$, and the effective magnetic field $\vec{G}_e = \omega_e \vec{B}/B$ (in energy units) is a sum of the external magnetic field and so-called exchange field emerging from the exchange interaction of electron with magnetic ions.

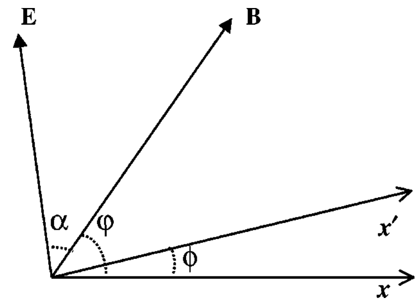

FIG. 1. The relative positions of the crystal axis x, the axis x' of the C_{2v} interaction (19), direction of the in-plane magnetic field $\vec{B}$, and the line E of the intersection of the plane of linear polarization with the (001) plane and the angles between these lines.

In the case of HH's ($\psi^\pm \equiv \psi_v^\pm$) the basis $|1\rangle = L_+\uparrow$, $|2\rangle = -L_-\downarrow$ corresponds to a $\pm 3/2$ projection of HH angular momentum on the z direction, $L_\pm = (1/\sqrt{2})(X \pm iY)$, where X and Y are the periodic parts of the valence-band Bloch functions. The dependence $\theta = \theta(\varphi)$ has to be found for each specific form of the HH Hamiltonian (see below).

The operator of the interband optical transition with the polarization plane rotated relative to the magnetic field $\vec{B} = B\{\cos\varphi, \sin\varphi, 0\}$ by angle α about the $\overrightarrow{OZ}$ axis (see Fig. 1) takes the form

$$\hat{V}_\alpha = p_- e^{i(\varphi+\alpha)} + p_+ e^{-i(\varphi+\alpha)}, \tag{4}$$

where $p_\pm = \frac{1}{2}(e_x \pm i e_y)$ and e_x and e_y are transformed as x and y.

Using the definitions (3) and (4), one can easily find the matrix element $M_{k,j}^\alpha = \langle \psi_c^k | \hat{V}_\alpha | \psi_v^j \rangle$ of the electrodipole optical transition between electron states ψ_c^k, $k = \pm 1$, and HH states ψ_v^j, $j = \pm 1$, as well as the corresponding probability $W_{k,j}^\alpha = |M_{k,j}^\alpha|^2$

$$W_{k,j}^\alpha \propto \begin{cases} \sin^2(3\varphi/2 + \alpha - \theta/2), & k = j, \\ \cos^2(3\varphi/2 + \alpha - \theta/2), & k \neq j, \end{cases} \tag{5}$$

where an unimportant dimensional coefficient has been dropped. Similarly, one can find the optical transition probability $W_{k,j}^{\alpha'}$ for the perpendicular polarization plane, which formally means the substitution $\alpha \to \alpha' = \alpha + \pi/2$ in Eq. (5). Since the contribution of each of optical transition $k \to j$ to the total PL intensity I_α is proportional to spin sublevel populations of electrons $P_e^k \propto e^{-k\omega_e/2T_e}/(e^{\omega_e/2T_e} + e^{-\omega_e/2T_e})$ and HH $P_h^j \propto e^{-j\omega_h/2T_h}/(e^{\omega_h/2T_h} + e^{-\omega_h/2T_h})$ (T_e and T_h are the electron and HH spin temperatures in energy units that can differ from lattice temperature T), the general Equation (1) assumes the following form:

$$\rho_{\alpha}^{(0)}=\frac{\sum_{k,j}P_e^kP_h^j(W_{k,j}^{\alpha}-W_{k,j}^{\alpha'})}{\sum_{k,j}P_e^kP_h^j(W_{k,j}^{\alpha}+W_{k,j}^{\alpha'})}. \tag{6}$$

Substitution of Eq. (5) and expressions for P_e^k and P_h^j into Eq. (6) leads after some algebra to the following simple result:

$$\rho_{\alpha}^{(0)}=-P_{eh}\cos(3\varphi+2\alpha-\theta), \tag{7}$$

$$P_{eh}=\tanh(\omega_e/2T_e)\tanh(\omega_h/2T_h). \tag{8}$$

Note that Eq. (7) does not describe all possible polarization effects in QW's. A closer look at the derivation of Eq. (7) shows that low-symmetry perturbations V of HH basic wave functions $|1\rangle$ and $|2\rangle$ should also be taken into account along with HH splitting in spite of the small value $\varepsilon=|\langle m|V|m\pm\Delta m\rangle|/\Delta_{HL}$, where $|m\rangle$ and $|m\mp\Delta m\rangle$ are nonperturbed HH ($|m|=3/2$) and LH ($|m\mp\Delta m|=1/2$) basic functions. Doing so needs a distinction between the case $\Delta m=1$, leading to HH splitting in the third order, and the case $\Delta m=2$, leading to the formation of an effective g factor in the first order. Thus, the perturbation V gives rise to corrections $\delta\rho_{\alpha}\sim\varepsilon^{3-\Delta m}$ to the total polarization that can be now written as a sum

$$\rho_{\alpha}=\rho_{\alpha}^{(0)}+\delta\rho_{\alpha}. \tag{9}$$

The explicit form of $\delta\rho_{\alpha}$ depends on specific form of the interaction leading to HH-LH mixing. The comparison of contributions of two terms to Eq. (9) permits us to conclude that electron-HH spin correlations ($\rho_{\alpha}^{(0)}$ term) dominate in OPA at sufficiently low temperatures. However, the $\delta\rho_{\alpha}$ term can dominate at high temperatures or zero (small) magnetic field. In the subsequent discussion we concentrate on two important cases: polarization ρ_0 along a magnetic field direction with $\alpha=0°$ and polarization ρ_{45} in the plane rotated relatively $\vec{B}$ by $\alpha=45°$.

B. Spectral dependence of linear polarization

In this subsection we discuss the effects of spectral shifts of electron-HH optical transitions caused by spin splitting (3). A simplest situation corresponds to the electron-HH optical line splitting into four plainly distinguishable components with polarizations $\rho_{\alpha k,j}=(W_{k,j}^{\alpha}-W_{k,j}^{\alpha'})/(W_{k,j}^{\alpha}+W_{k,j}^{\alpha'})=-kj\cos(3\varphi+2\alpha-\theta)$ and intensities $I_{kj}\propto P_e^kP_h^j$. If these components overlap with each other (i.e., the splitting of spectral components is smaller than their linewidth σ_{kj}), the polarization depends on the spectral position at the contour of a composite line of optical transitions. Thus it is convenient to determine the spectral-dependent polarization

$$\rho_{\alpha}(\omega)=\frac{I_{\alpha}(\omega)-I_{\alpha'}(\omega)}{I_{\alpha}(\omega)+I_{\alpha'}(\omega)}. \tag{10}$$

The intensity $I_{\alpha}(\omega)$ of the optical transition at frequency ω depends on the line shape of each electron-HH transition $f_{k,j}(\omega)=f(\omega-(\omega_0+k\omega_e/2-j\omega_h/2))$, where the possible dependence of ω_0 on magnetic field describes the effect of the line center-of-mass shift in a magnetic field $\vec{B}$. We assume also that line shapes $f_{k,j}(\omega)$ with linewidth $\sigma_{kj}=\sigma$ are the same for all transitions $|k\rangle\rightarrow|j\rangle$. So Eq. (10) takes the form

$$\rho_{\alpha}^{(0)}(\omega)=\frac{\sum_{k,j}f_{k,j}(\omega)(W_{k,j}^{\alpha}-W_{k,j}^{\alpha'})P_e^kP_h^j}{\sum_{k,j}f_{k,j}(\omega)(W_{k,j}^{\alpha}+W_{k,j}^{\alpha'})P_e^kP_h^j}. \tag{11}$$

In the case of small magnetic field shifts ω_e and $\omega_h\ll\sigma$, the line shape function can be expanded into a power series

$$f_{k,j}(\omega)\approx f(\Delta\omega)-\frac{1}{2}f'(\Delta\omega)(k\omega_e-j\omega_h)+\frac{1}{8}f''(\Delta\omega)(k\omega_e-j\omega_h)^2, \tag{12}$$

where $f'(\omega)$ and $f''(\omega)$ are the first and second derivatives of $f(\omega)$ respectively, and $\Delta\omega=\omega-\omega_0$. Substitution of this expansion into Eq. (11) with respect to Eq. (5) results in a net effect similar to that of Eq. (7) where P_{eh} should be replaced by

$$P_T(\omega)=\tanh\left(\frac{\omega_e}{2T_e}\right)\tanh\left(\frac{\omega_h}{2T_h}\right)--\frac{\sigma f'(\Delta\omega)}{2f(\Delta\omega)}\left[\frac{\omega_h}{\sigma}\tanh\left(\frac{\omega_e}{2T_e}\right)-\frac{\omega_e}{\sigma}\tanh\left(\frac{\omega_h}{2T_h}\right)\right]-\frac{\sigma^2f''(\Delta\omega)}{4f(\Delta\omega)}\frac{\omega_e\omega_h}{\sigma^2}. \tag{13}$$

The latter equation displays a sharp polarization dependence on the detuning $\Delta\omega$. Moreover, this dependence is determined by the specific line shape.In the case of a Gaussian or Lorentzian shape,

$$P_T^{\mathrm{G}}(\omega)\simeq P_{eh}+\left(\frac{1}{2}-\frac{\Delta\omega^2}{\sigma^2}\right)\frac{\omega_e\omega_h}{\sigma^2}$$

or

$$P_T^{\mathrm{L}}(\omega)\simeq P_{eh}+\frac{\sigma^2-3\Delta\omega^2}{2(\sigma^2+\Delta\omega^2)^2}\frac{\omega_e\omega_h}{\sigma^2},$$

respectively.

Finally, we have to consider an intermediate case $\omega_h\ll\sigma\ll\omega_e$. This is because the inequality $\omega_h\ll\omega_e$ usually takes place in a wide range of magnetic fields. Two components $k=\pm$ of different electron spin states have intensities $I_k\propto P_e^k$ and opposite signs of polarization. Their OPA is described in a form similar to that of Eq. (7) but with

$$P_{T,k}(\omega)=k\left[\tanh\left(\frac{\omega_h}{2T_h}\right)-\frac{\sigma f'(\Delta\omega)}{2f(\Delta\omega)}\frac{\omega_h}{\sigma}\right]. \tag{14}$$

instead of P_{eh}.

The difference between the Gaussian and Lorentzian shapes of spectral lines becomes evident if we consider their asymptotics $P^{\mathrm{G}}_{T,k}(\omega)\simeq k[\tanh(\omega_h/2T_h)+\omega_h\Delta\omega/\sigma^2]$ and $P^{\mathrm{L}}_{T,k}(\omega)\simeq k[\tanh(\omega_h/2T_h)+\omega_h\Delta\omega/(\Delta\omega^2+\sigma^2)]$.

It is well known that an in-plane magnetic field contributes to the linear polarization of the optical spectra of absorption, reflectivity, etc. In these cases the effect of the correlations of wave function phases can become apparent also due to HH and electron spin splitting. Unlike the case of PL, here the optical transitions occur between electronic states of completely populated valence band and empty conductivity band. Therefore, Eqs. (13) and (14) describe this situation in the limit T_e, $T_h\to\infty$. However, the polarization of reflectivity spectra should be still described in terms of a standard equation for the reflection coefficient with transition probabilities (5). In subsequent calculations, we primarily focus on PL polarization [Eqs. (7) and (8)] since it has been thoroughly studied experimentally in the literature.

III. HH INTERACTIONS

Here we consider the HH interactions sequentially according to their contribution to lowering the symmetry of the QW potential. Also, we assume that the above interactions are small perturbations as compared to Δ_{HL}. In the general case (i.e., without the latter assumption) the consideration of OPA can be performed only numerically. Such a consideration, however, is beyond the scope of the present paper. On the other hand, such numerical calculations have been fulfilled in Ref. 7 in the context of a description of experiments of Ref. 4. Namely, in Ref. 7, the important case of an arbitrary relation between the effective magnetic field acting on the hole G_h and HH-LH splitting Δ_{HL} has been considered, while other low-symmetry interactions have still been assumed to be small.

A. Zeeman interaction

The Zeeman interaction is isotropic in terms of the hole effective angular momentum $J=3/2$:

$$V_Z=\vec{G}_h\vec{J}=G_h(J_X\cos\varphi+J_Y\sin\varphi). \tag{15}$$

Here $\vec{G}_h$ is an effective in-plane magnetic field acting on the hole in energy units. This field can include the effects of carrier-ion (hole-ion) exchange interaction in the case of diluted magnetic semiconductor (DMS) quantum structure.[8,9] In the case of [001]-oriented QW's V_Z does not split the HH states in first and second orders in perturbation. Third order can be represented by some effective Hamiltonian with Pauli matrices $\vec{\sigma}$ in the base $|1\rangle$ and $|2\rangle$, calculated in a second order of perturbation theory according to the Löwdin procedure (see Ref. 10):

$$V_Z^{(3)}=\frac{3}{4}\Delta_{HL}h^3(\sigma_x\cos 3\varphi+\sigma_y\sin 3\varphi),\quad h=G_h/\Delta_{HL}. \tag{16}$$

Equation (16) gives *isotropic* HH splitting $\omega_Z=\frac{3}{2}\Delta_{HL}h^3$ and ψ-function phase $\theta=3\varphi$. According to Eq. (7) this corresponds to isotropic (i.e., independent of angle φ) polarization $\rho_\alpha^{(0)}=-P_{eh}\cos 2\alpha$. The polarization is maximal along or across the magnetic field direction ($\alpha=0$ or 90°) and is absent for $\alpha=45°$, which can be also expected from symmetry considerations. If the magnetic field is weak enough, one can find $P_{eh}\simeq G_h^3G_e/T_eT_h\Delta_{HL}^2\propto B^4$. In this case the contribution from the LH admixture can be more important. The calculation of the LH contribution to HH polarization stemming from the LH admixture to the basic functions $|1\rangle$ and $|2\rangle$ gives rise to the corrections $\delta\rho_0^{(2)}=-h^2$ and $\delta\rho_{45}^{(2)}=0$.

B. Non-Zeeman interaction with a magnetic field

The symmetry of the Luttinger Hamiltonian admits the existence of a non-Zeeman interaction of holes with a magnetic field in the form

$$V_q=q_1G_h(J_x^3\cos\varphi+J_y^3\sin\varphi), \tag{17}$$

where q_1 is a relatively small parameter reflecting the interaction between valence and Γ_{15} conductivity bands.[11] If we derive Eq. (17) in terms of the approach of Ref. 12, the external magnetic field B substitutes G_h on the right-hand side of Eq. (17) so that the Luttinger parameter q appears in the form $q_1=qB/G_h$. If we supplement the consideration of Ref. 12 by mechanisms sensitive to the effects of exchange field influence, the latter equality can be modified. Nevertheless, we preserve the notation V_q in the form (17) to unify the form of expressions for interactions of different type.

The Hamiltonian (17) has nonzero matrix elements between HH states $|3/2\rangle$ and $|-3/2\rangle$, which defines the effective HH Hamiltonian in first order in perturbation (17):

$$V_q^{(1)}=\frac{3}{4}\Delta_{HL}q_1h(\sigma_x\cos\varphi-\sigma_y\sin\varphi). \tag{18}$$

Comparison of Eq. (18) with Eq. (2) gives *isotropic* HH splitting $\omega_q=\frac{3}{2}q_1G_h$ and ψ-function phase $\theta=-\varphi$. Therefore interaction (17) can be responsible for the fourth harmonic of OPA (7), $\rho_\alpha^{(0)}=-P_{eh}\cos(4\varphi+2\alpha)$, which coincides with a cubic anisotropy of the Luttinger Hamiltonian. Note that the in-plane g tensor $g^{\perp}_{\mu\nu}$ [which can be defined in terms of Eq. (18) for HH pseudospin[11] $\tilde{s}_x=\sigma_x/2$ and $\tilde{s}_y=-\sigma_y/2$] is isotropic—i.e., $g^{\perp}_{xx}=g^{\perp}_{yy}$, $g^{\perp}_{xy}=g^{\perp}_{yx}=0$.

C. Potentials of C_{2v} symmetry

Most OPA experiments performed up to now have found some π-periodic component of OPA. It was assumed that such kind of anisotropy is due to the potentials of C_{2v} symmetry in a hole Hamiltonian.[1,2,4,5] We consider two reasons for the appearance of the C_{2v} hole potential in a QW. First is a C_{2v} component (so-called interface C_{2v} potential) of the heterojunction potential inherent to [001]-oriented structures composed of zinc-blende semiconductors.[13,14] In QW structures with common anion (cation), the contributions of two interface potentials compensate each other. However, this compensation is not complete in the case of nonidentical barriers or interface profile.[2] Such a kind of interaction can be written in terms of a hole angular momentum in the form[13] $V_{if}=t_{if}\{J_x,J_y\}$, where t_{if} is the interaction constant ($|t_{if}|\ll\Delta_{HL}$), $\{J_x,J_y\}=\frac{1}{2}(J_xJ_y+J_yJ_x)$. Additionally to V_{if}, there is also a C_{2v} potential $V_d=d\varepsilon_{x_dy_d}\{J_{x_d},J_{y_d}\}$ caused by in-plane strains.[5] Here d is a deformation potential; $\varepsilon_{x_dy_d}$ is the strain with x_d and y_d principal axes forming some angle with [100] and [010] directions. Actually, we do not need to consider V_{if} and V_d separately since their sum is also the C_{2v} potential V_t, which takes the canonical form in terms of total amplitude T_t and axes x' and y' forming angle ϕ with [100] and [010] directions:

$$V_t=T_t\{J_{x'},J_{y'}\}. \tag{19}$$

As an illustration, Fig. 1 shows a position of the coordinate axes defining the angles φ, ϕ, and α.

The potential (19) does not lift the $\pm 3/2$ HH state degeneracy but results in their mixing with $\mp 1/2$ LH states in the first order of perturbation theory. This generates some temperature– and magnetic-field-independent polarization[2] $\delta\rho_\alpha^{(1)}$ with respect to the polarization plane α: $\delta\rho_\alpha^{(1)}=-t\sin 2(\varphi-\phi+\alpha)$, where $t=T_t/\Delta_{HL}$.[15]

In the presence of a magnetic field, the potential V_t generates an effective in-plane g factor for HH's.[5] This effect can be taken into account in lowest order as the interference of V_Z, Eq. (15), and V_t, Eq. (19). In terms of Pauli matrices, the HH splitting is described by the effective Hamiltonian

$$V_{ht}^{(2)}=-\frac{3}{2}\Delta_{HL}ht[\sigma_x\sin(\varphi+2\phi)-\sigma_y\cos(\varphi+2\phi)], \tag{20}$$

which defines the phase $\theta=\varphi+2\phi+\pi/2$ and *isotropic* HH splitting $\omega_{ht}=3\Delta_{HL}ht$. In spite of this fact, representation of Eq. (20) in the form of a Zeeman interaction for pseudospin $\tilde{s}$ yields an anisotropic g tensor. In the $x'y'$ reference frame one can find $g^\perp_{x'x'}=g^\perp_{y'y'}=0$, and $g^\perp_{x'y'}=g^\perp_{y'x'}$.[16] So, the effect of C_{2v} OPA (7) is described by $\rho_\alpha^{(0)}=-P_{eh}\sin 2(\varphi-\phi+\alpha)$. If a magnetic field is sufficiently weak, one can find $P_{eh}\simeq 3G_hG_e/4T_eT_h\propto B^2$.[1]

Here we once more emphasize that the nature of the polarization $\rho_\alpha^{(0)}$ calculated with nonperturbed basic functions $|\pm 3/2\rangle$ is different from that of $\delta\rho_\alpha^{(1)}$ calculated with LH admixture in first order. Thus, one can easily imagine a situation when $\rho_\alpha^{(0)}\gg\delta\rho_\alpha^{(1)}$ despite the fact that $V_{ht}^{(2)}$ generates $\rho_\alpha^{(0)}$ in second perturbation order in V_Z and V_t, while $\delta\rho_\alpha^{(1)}$ arises in first order in V_t.

D. Random potential of HH localization

The PL in QW's is known to stem from localized excitonic (hole) states, which are formed by a random potential (interface roughness, defects, etc.) In the general case, the profile of this potential (and therefore the hole density) is not symmetric in the QW plane. In-plane asymmetry of localized hole ψ function leads to mixing of HH Ψ_H and LH Ψ_L states, which results in the appearance of a finite HH g factor.[11] The corresponding Hamiltonian, describing the splitting of localized HH's can be represented as some C_{2v} potential (see Ref. 17 for details)

$$V_{h\kappa}^{(2)}=\Delta_{HL}h\kappa[\sigma_x\cos(\varphi+2\phi_\kappa)+\sigma_y\sin(\varphi+2\phi_\kappa)]. \tag{21}$$

Here ϕ_κ determines the axes ξ and η for canonical representation (19) of the $V_{h\kappa}^{(2)}$ potential by means of an angle between x' and ξ, $\kappa=\overline{\gamma}\hbar^2(K_\xi^2-K_\eta^2)/m_0\Delta_{HL}$ and $K_\nu^2=\langle\Psi_H|\Psi_L\rangle\langle\Psi_L|-\partial^2/\partial\nu^2|\Psi_H\rangle$, $\nu=\xi,\ \eta$; $\gamma_2<\overline{\gamma}<\gamma_3$, and γ_2 and γ_3 are Luttinger parameters. In the case of an axially symmetric ψ function, the equality $K_\xi^2=K_\eta^2$ gives zero $V_{h\kappa}^{(2)}$ potential. In the general case, $K_\xi^2\neq K_\eta^2$, the potentials (20) and (21) can be combined into a single quantity $V_{ht\kappa}^{(2)}$ that takes the form (20) with effective amplitude t_κ and angle ϕ_r instead of t and ϕ:

$$t_\kappa=\sqrt{(t+\kappa\cos 2\phi_\kappa)^2+(\kappa\sin 2\phi_\kappa)^2}, \tag{22}$$

$$\phi_r=\frac{1}{2}\arcsin\frac{\kappa\sin 2\phi_\kappa}{t_\kappa}+\phi. \tag{23}$$

It should be emphasized that Eq. (21) has to do with a single hole localized on some fluctuation of a random QW potential. The observable PL polarization is the result of the addition of a great number of localized state contributions with random amplitudes and principal axis directions. Thus, a complete description of PL polarization has to include an averaging over the parameters κ and ϕ_κ with some distribution functions. If there are no preferential directions for HH localization in the QW plane, the potential $V_{h\kappa}^{(2)}$ cannot lead to OPA. Moreover, numerical analysis shows that the random potential (21) can greatly suppress the magnitude of polarization and therefore amplitude of OPA as soon as it exceeds other regular (nonrandom) interactions. In a subsequent discussion we omit the contribution of $V_{h\kappa}^{(2)}$ into OPA for simplicity. Nevertheless, consideration of the random potential influence may be in need of a description of realistic experimental situations. The magnetic polaron effect[18] should also be considered as an intensification factor for the parameter κ.

IV. INTERFERENCE EFFECTS

The foregoing analysis has shown a few important polarization properties attributed to different kinds of HH interactions in QW's. We have found that HH splitting is isotropic for each HH Hamiltonian (16), (18), and (20). However, they reveal different OPA's, which points to a possible interplay between these contributions. Let us consider a joint manifestation of interactions that can split HH states:

$$V=V_Z^{(3)}+V_q^{(1)}+V_{ht}^{(2)}. \tag{24}$$

First, we should find the total module $\omega_h/2$ and phase θ for the matrix element:

$$V_{1,2}=\frac{1}{2}(\omega_Ze^{-i3\varphi}+\omega_qe^{i\varphi}+\omega_{ht}e^{-i(\varphi+2\phi+\pi/2)}). \tag{25}$$

After some algebra, the HH splitting can be rewritten in terms of two expressions Q and R:

$$\omega_h = \frac{3}{2}\Delta_{HL} h\sqrt{Q^2+R^2}, \tag{26}$$

$$Q = 2t\cos 2(\varphi-\phi) - q_1 \sin 4\varphi, \tag{27}$$

$$R = h^2 + 2t\sin 2(\varphi-\phi) + q_1\cos 4\varphi. \tag{28}$$

In a similar manner, we calculate the polarization (7) where θ has to be found from definition (25). Thus, after some transformations, we obtain ρ_α for two values of α:

$$\rho_\alpha = \begin{cases} -P_{eh}\dfrac{R}{\sqrt{Q^2+R^2}} - h^2 - t\sin 2(\varphi-\phi), & \alpha = 0°, \\ -P_{eh}\dfrac{Q}{\sqrt{Q^2+R^2}} - t\cos 2(\varphi-\phi), & \alpha = 45°. \end{cases} \tag{29}$$

For completeness, here we also take into account the corrections for LH-HH mixing. Equation (29) with Eqs. (8) and (26) are the final results of our calculations that cover the most practical important cases.

One can see that HH splitting (26) reveals magnetic anisotropy with finite magnitudes of t and q_1 despite the isotropic character of HH splitting of each of terms (24) taken separately. Moreover, the effective HH transversal g factor $\tilde{g}_\perp = \omega_h / G_h$ can be turned to zero for some direction and amplitude of a magnetic field. To demonstrate this, we note that the equation $\tilde{g}_\perp = \frac{3}{2}\sqrt{Q^2+R^2} = 0$ can be identically rewritten in the form of a set of two equations $Q=0$ and $R=0$ in terms of variables φ (the angle) and h (the amplitude). The simplest solution of this system can be obtained in the case of $\phi=0$, namely, $\varphi = \pm\pi/4$ and $h = h_c \equiv \sqrt{\pm 2t + q_1}$. The latter expression separates also the magnetic field magnitude for the case where the Zeeman interaction is dominant ($h \gg h_c$) (which fixes the relatively small amplitude of the polarization anisotropy) with the case $h \ll h_c$, where the polarization oscillations due to the contributions, proportional to the product of Zeeman and non-Zeeman (and/or C_{2v}) interactions, would not be suppressed by the isotropic Zeeman interaction.

V. DISCUSSION: COMPARISON WITH EXPERIMENT

As is obvious from Eq. (29), the anisotropy and possible random degeneration of HH splitting do not influence OPA for high temperatures $T_h \gg \omega_h$ (or $P_{eh} \propto \omega_h$). In this case we may expect the additive contributions of different aforementioned OPA mechanisms: $\rho_0^{(0)} \simeq -\frac{3}{2}R(G_h/T_h)\tanh(\omega_e/2T_e)$, $\rho_{45}^{(0)} \simeq -\frac{3}{2}Q(G_h/T_h)\tanh(\omega_e/2T_e)$; i.e., the amplitude of second harmonic of OPA is proportional to t while that of the fourth harmonic is proportional to q_1.

If HH splitting is not small (as compared to temperature T_h) in some range of angles φ, Eq. (29) shows a qualitatively different character of OPA. Namely, higher harmonics with large amplitudes can appear. This can be regarded as a manifestation of higher powers in an expansion of $\tanh(\omega_h/2T_h)$. Figure 2 reports some calculated curves of OPA and corresponding effective g-factor anisotropy demonstrating a new net effect without the influence of the cubic anisotropy of V_q.

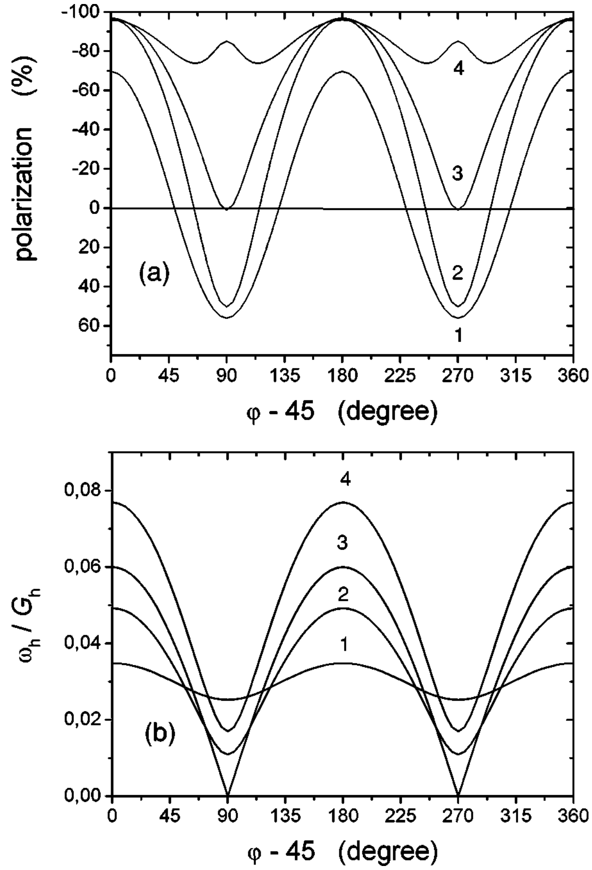

FIG. 2. The OPA (a) and transversal effective HH g factor $\tilde{g}_\perp = \omega_h/G_h$ (b) calculated for $q_1=0$, $t=0.01$, $\Delta_{HL}=125$ meV, $T_e = T_h = 2$ K, and few magnitudes of magnetic field strength $h = -c\sqrt{2t}$: $c=0.4$ (curve 1), $c=0.8$ (curve 2), $c=1$ (curve 3), and $c=1.25$ (curve 4).

Very interesting experimental data had been obtained in Ref. 1, where second and visible fourth harmonics of the OPA were detected in PL of a 20-Å CdTe QW with semimagnetic $Cd_{1-x}Mn_xTe$ barriers. The exchange interaction with magnetic ions in the barriers and interfaces enlarges the HH splitting, which makes it possible to reach a significant magnitude of HH and electron spin polarization at liquid helium temperatures.[8] A quantitative analysis on the basis of Eqs. (29) and (26) shows that the parameters $h=-0.056$, $\omega_e = 0.014\Delta_{HL}$, $T_e = T_h = \Delta_{HL}/720$, $t=0.001$, and $q_1 = -0.0006$ describe nicely the OPA experimental data of Ref. 1 (see Fig. 3) assuming that $\Delta_{HL}=125$ meV and electron and hole spin temperatures are equal to the lattice temperature $T=2$ K. These calculations explain also the effect of great OPA amplification from 0.1% (which could not be detected in Ref. 1 because of experimental errors) to 15% under the magnetic field action. On the other hand, we cannot assign the above magnitudes to microscopic parameters of the structure under consideration because some parameters (such

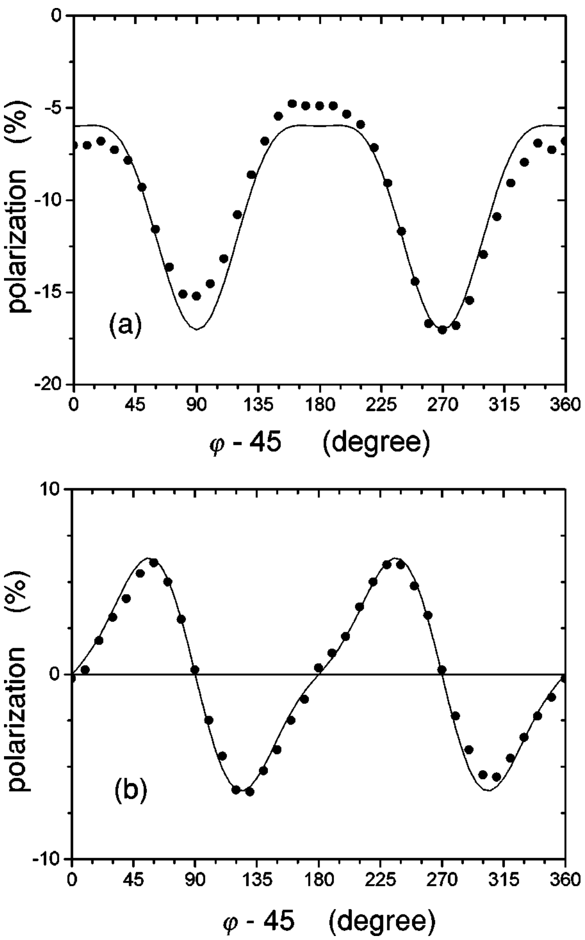

FIG. 3. Comparison of the OPA calculated in terms of Eq. (29) (solid lines) with experimental data (points) of Kusrayev *et al.* [Ref. 1, Fig. 3(b) therein] recorded parallel ($\alpha = 0°$) to the magnetic field polarization plane (a) and plane with $\alpha = 45°$ rotated relative to $\alpha = 0°$ by 45° (b). The fitting parameters see in the text.

as temperature, HH-LH splitting, effective electron and hole fields) are missing in Ref. 1. The example of the experimental data fitting shown in Fig. 3 gives a demonstration of the proposed theory adaptability for a nontrivial experimental situation. Besides that, the calculations show high sensitivity of the OPA [Eq. (29)] to independent variation of different parameters. For instance, the small change of parameter t form 0.0009 to 0.0011 leads to a significant modification of the OPA curves. Nevertheless, fairly good agreement with the experiments[1] can be achieved for other t values if the unknown parameters in Ref. 1 (like the exchange-enhanced magnetic filed and HH-LH splitting) take other values.

A successful description of the experiment raises the question about the relationship of the above obtained parameters with those for QW's with other widths. There are a few mechanisms of the QW width L_w influence on OPA. (i) The carrier exchange interaction with magnetic ions along with an external magnetic field B contributes to the effective fields G_e and G_h. Note that in the case of nonmagnetic QW's with semimagnetic barriers, this exchange interaction is proportional to the overlap of the envelopes of the carrier wave functions with barriers. Thus, one can expect a significant decrease of this exchange contribution to the effective fields in a very wide nonmagnetic QW. On the other hand, the values of the exchange fields can have an additional component with a weaker dependence on the QW width related to the so-called interface smoothing.[8,19] However, one can expect a reduction of the magnetic field influence on the OPA with QW broadening, especially for contributions of $V_Z^{(3)}$, Eq. (16), and $V_{ht}^{(2)}$, Eq. (20), which are proportional to $G_h^3 G_e$ and $G_h G_e$, respectively. A similar reduction of the $V_q^{(1)}$ [Eq. (18)] contribution may not be so important (being proportional to BG_e) if this term of the Luttinger Hamiltonian is described in the model of Ref. 12. In such a case this mechanism contributes to increasing the role of the fourth harmonic with respect to the second harmonic of OPA with a broadening of the QW. (ii) The HH-LH splitting depends primarily on L_w. In the case of infinitely high QW barriers, one can expect $\Delta_{HL} \propto 1/L_w^2$. This increases the contribution of terms $V_Z^{(3)}$, $V_{ht}^{(2)}$, and $V_{h\kappa}^{(2)}$ to OPA with QW broadening but does not influence the term $V_q^{(1)}$. This mechanism contributes to decreasing the role of the fourth harmonic with respect to the second harmonic of OPA with a broadening of the QW. (iii) Narrow QW's are most favorable for carrier localization on the fluctuations of the random potential as well as for polaron formation. This increases the role of the random potential (21) that can suppress OPA, as has been mentioned in Sec. III.

We can see that mechanisms (i) and (ii) predict opposite dependences of $V_Z^{(3)}$ and $V_{ht}^{(2)}$ contributions on QW widening. These contributions decrease with decreasing of G_e and G_h and increase with decreasing of Δ_{HL}. At the same time, the term $V_q^{(1)}$ does not depend on Δ_{HL} and has a peculiar dependence on G_h. Thus the combination of the above dependences can result in either a suppression or enhancement of the fourth harmonic of the OPA (described by the $V_q^{(1)}$) as compared to the second harmonic (described by the $V_{ht}^{(2)}$) with QW broadening. Note that experiment in Ref. 1 shows the significant role of the OPA fourth harmonic only in the narrowest QW.

The aforementioned analysis shows that the situation with a dependence of the OPA on L_w may be quite complex. The most adequate approach to this problem seems to consist in independent determination of Δ_{HL} as well as G_e and G_h from magneto-optical measurements in Faraday geometry of experiment. Then, the microscopic constants t, ϕ, and q_1 in Hamiltonian (24) should be found from a comparison of general expressions (29) with experimentally observed OPA. The data of Ref. 1 do not allow one to accomplish this scenario due to the lack of values of Δ_{HL}, G_e, and G_h.

VI. CONCLUSION

We have developed a microscopic theory of OPA in QW's subjected to an in-plane magnetic field. Two types of optical polarization contributions should be distinguished. The first is due to an admixture of LH to HH states. This effect is small in accordance with the predictions of perturbation theory. A HH splitting in a magnetic field determines the other type of polarization mechanism owing to phase correlations of electron and hole ψ functions. This effect can lead to almost 100% polarization for suitably distinguishable four spectral lines of electron-HH optical transitions in spite of the relatively small interactions ($\ll\Delta_{HL}$) responsible for HH splitting. Besides that, we have considered the spectral properties of OPA. These polarization peculiarities turn out to be sensitive to the details of the PL (absorption, reflectivity, etc.) line shape in the case of a relatively small Zeeman splitting.

Our theory considers the Zeeman interaction, non-Zeeman HH splitting, and C_{2v} potentials to be the sources of different OPA. Their joint manifestation reveals peculiar OPA behavior due to interference effects. A random potential localizing HH's should be considered separately as a depolarization factor of the PL.

We predict and describe some effects: namely, (i) the anisotropy of HH splitting (or g factor) due to the interference of different HH potentials, (ii) the manifestation of fourth and higher harmonics in OPA caused by only the C_{2v} potential (Fig. 2), (iii) polarization suppression under the conditions of the crossing (anticrossing) of HH levels, and (iv) the depolarization effect due to a random potential influence. Our theory gives a full qualitative description for many important experimental details of OPA found earlier.

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[1] Yu.G. Kusrayev, A.V. Koudinov, I.G. Aksyanov, B.P. Zakharchenya, T. Wojtowicz, G. Karczewski, and J. Kossut, Phys. Rev. Lett. **82**, 3176 (1999).

[2] A. Kudelski, A. Golnik, J.A. Gaj, F.V. Kyrychenko, G. Karczewski, T. Wojtowicz, Yu.G. Semenov, O. Krebs, and P. Voisin, Phys. Rev. B **64**, 045312 (2001).

[3] S.M. Ryabchenko, Yu.G. Semenov, A.V. Komarov, T. Wojtowicz, G. Cywiński, and J. Kossut, Physica E (Amsterdam) **13**, 24 (2002).

[4] S.M. Ryabchenko, O.V. Terletskij, Yu.G. Semenov, and F.V. Kyrychenko, in *Optical Properties of 2D Systems with Interacting Electrons*, edited by W. Ossau and R. Suris (Kluwer Academic, Dordrecht, to be published 2003).

[5] G.E. Pikus and F.G. Pikus, Solid State Commun. **89**, 319 (1994).

[6] G.E. Pikus and F.G. Pikus, J. Lumin. **54**, 279 (1993).

[7] F.V. Kyrychenko, S.M. Ryabchenko, and Y.G. Semenov (unpublished).

[8] *Diluted Magnetic Semiconductors*, Vol. 25 of *Semiconductors and Semimetals*. edited by J.K. Furdyna and J. Kossut (Academic, Boston, 1988).

[9] J. Kossut, W. Dobrovolski, in *Handbook of Magnetic Materials*, edited by K.H.J. Bushow (Elsevier, Amsterdam, 1993).

[10] G.L. Bir and G.E. Pikus, *Symmetry and Strain Induced Effects in Semiconductors* (Wiley, New York, 1974).

[11] X. Marie, T. Amand, P. Le Jeune, M. Paillard, P. Renucci, L.E. Golub, V.D. Dymnikov, and E.L. Ivchenko, Phys. Rev. B **60**, 5811 (1999).

[12] C. Hensel and K. Suzuki, Phys. Rev. Lett. **22**, 838 (1969).

[13] E.L. Ivchenko, A.A. Toropov, and P. Voisin, Fiz. Tverd. Tela **40**, 1925 (1998) [Phys. Solid State **40**, 1748 (1998)].

[14] D.R. Yakovlev, A.V. Platonov, E.L. Ivchenko, V.P. Kochereshko, C. Sas, W. Ossau, L. Hansen, A. Waag, G. Landwehr, and L.W. Molenkamp, Phys. Rev. Lett. **88**, 257401 (2002).

[15] In the case of $B=0$ we assume that infinitesimal $\vec{B}$ determines only a plane that forms an angle φ with the $\overrightarrow{OX}$ direction.

[16] In Ref. 1, the principal axes x'' and y'' of the C_{2v} interaction rotated relative to x' and y' by 45° about the z axis are used: $g^{\perp}_{x''x''}=-g^{\perp}_{y''y''}$, $g^{\perp}_{x''y''}=g^{\perp}_{y''x''}=0$.

[17] Yu.G. Semenov, K.N. Borysenko, and K.W. Kim, Phys. Rev. B **66**, 113302 (2002).

[18] I.A. Merkulov, D.R. Yakovlev, K.V. Kavokin, G. Makh, B. Kuhn-Heinrich, W. Ossau, A. Waag, and G. Landwehr, Fiz. Tverd. Tela **39**, 2079 (1997) [Phys. Solid State **39**, 1859 (1997)].

[19] J. Siviniant, F.V. Kyrychenko, Y.G. Semenov, D. Coquillat, D. Scalbert, and J.P. Lascaray, Phys. Rev. B **59**, 10 276 (1999).