Negative intersubband absorption in biased tunnel-coupled wells

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We investigate the use of intersubband transitions between two weakly coupled shallow quantum wells to generate terahertz radiation. The wells are designed for independent contacting and biasing. The maximum gain (absorption) scales with the wavefunction overlap and difference between the well carrier densities. The absorption line shape is studied taking into account the interplay between inhomogeneous and homogeneous broadening mechanisms, including both large-scale variations in the coupled levels and also short-range scattering. The intersubband transitions are strongly renormalized due to Coulomb interactions (depolarization and exchange effects). Population inversion can be achieved through independent variation of the upper and lower well Fermi energy with applied bias. Numerical estimates of the negative absorption coefficient are obtained using realistic parameters, indicating the possible application of such a THz resonator to achieve stimulated emission.

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I. INTRODUCTION

Stimulated emission in the THz spectral region has been demonstrated in bulk *p*-type semiconductors during the past 20 years (see Ref. 1, and references therein). Different schemes for the realization of THz lasing based on intersubband transitions have also been considered during the last decade,[2,3] including the case of photon-assisted tunneling between neighboring QWs in resonant tunneling diode structures,[4] motivated by the improvement in device parameters and the successful application of cascade structures for mid-IR lasing.[5] Stimulated THz emission has recently been reported in tunnel-coupled cascade structures,[6] following previous investigations of the spontaneous THz emission.[7,8] The investigation of different mechanisms to achieve population inversion in structures with closely spaced levels is now timely. In this paper we study the conditions for the stimulated emission of THz radiation from independently contacted tunnel-coupled double quantum well (DQW) structures where population inversion can be realized by using the contacts to control the electron concentrations and quasi-Fermi levels in the upper (u-) and lower (l-) QW's [see Fig. 1(a)].

DQW structures with independent contacts have been investigated during the past decade[9,10] with the process of photon-assisted tunneling also considered recently.[11,12] We extend here previous studies of intersubband transitions in DQW's with a common Fermi level[13,14] to consider the case where there are nonequilibrium electron populations in the u- and l-QW's, described by quasi-Fermi levels ε_{Fu} and ε_{Fl}, and where an applied bias ensures $\varepsilon_{Fu}\neq\varepsilon_{Fl}$ [see Fig. 1(b), 1(c)]. By varying the bias, it is possible to achieve either absorption or stimulated emission regimes due to resonant transitions between the tunnel-coupled electronic states [see Fig. 1(b) and 1(c), respectively]. The modal gain can be maximized if the DQW structure is placed along the central axis of a THz resonator with ideal mirrors at $z=\pm L/2$, where L is the width of the resonator, with dielectric permittivity ϵ. A simplified formula for the gain coefficient g is given for the fundamental mode by (see Ref. 15 for details)

$$g=\frac{8\pi\Omega\,\mathrm{Re}\sigma_{\Delta\omega}}{c^2\sqrt{\epsilon(\Omega L/c)^2-\pi^2}},\quad \mathrm{Re}\sigma_{\Delta\omega}\simeq\frac{(ev_\perp)^2}{\hbar\Omega}\,\frac{\nu\delta n}{\Delta\omega^2+\nu^2}. \tag{1}$$

Here Ω is the resonant frequency, $\Delta\omega=\omega-\Omega$ is the detuning frequency, $v_\perp$ is the intersubband velocity, and δn is the population difference. Equation (1) assumes a Lorentzian line shape for the intersubband peak, with relaxation frequency ν. Numerical estimates for a GaAs-based weakly coupled QW system, with $v_\perp\simeq1.7\times10^6$ cm/s, give us $g_{\max}\sim5\ \mathrm{cm}^{-1}$ if $\delta n\simeq10^{11}\ \mathrm{cm}^{-2}$ and $\hbar\nu\sim0.2$ meV. Thus, a measurable gain is possible in the system under consider-

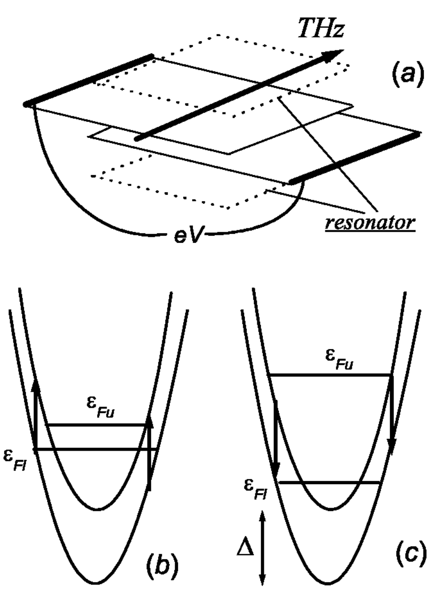

FIG. 1. Schematic figure showing an independently contacted double QW structure placed in a THz resonator (a). The dispersion relations of tunnel-coupled the wells with different doping levels (ε_{Fu} and ε_{Fl} are the Fermi levels in u- and l-QW's) for the cases of absorption (b) and stimulated emission (c).

ation, justifying a more careful examination of this scheme of stimulated emission, as performed below. The integrated absorption/emission is determined by the wavefunction overlap between the two QW's, while the broadening of the resonant peak is influenced by short-range scattering and long-range inhomogeneities (see Ref. 12 for a discussion of the influence of the homogeneous and inhomogeneous contributions). The transition energy is calculated to be strongly renormalized due to the Coulomb interaction.[14] Below we take into account both the high-frequency self-consistent electric field (depolarization) and exchange contributions. We also examine how the efficiency of the interaction with THz radiation can be enhanced if a multi-DQW structure can be placed into a THz waveguide.

The analysis below is divided into two sections. The basic equations describing the resonant response of the structure under consideration are presented in Sec. II, including the Coulomb interaction and large-scale inhomogeneities. The numerical results for intersubband absorption and the discussion of the conditions to achieve stimulated THz emission in a resonator are given in Sec. III, with our conclusions presented in Sec. IV.

II. SELF-CONSISTENT RESPONSE

The response of the electrons in a DQW to a transverse electric field $E_\perp \exp(-i\omega t)$ is described by the linearized quantum kinetic equation[16]

$$(-i\omega+i\mathbf{v}\cdot\nabla_x)\widehat{\delta f}_{\mathbf{px}}+\frac{i}{\hbar}[\widetilde{h},\widehat{\delta f}]_{\mathbf{px}}+\frac{i}{\hbar}[\widetilde{\delta h},\hat{\rho}]_{\mathbf{px}}=I_c(\widehat{\delta f}|\mathbf{px}), \tag{2}$$

which determines the modification of the density matrix $\widehat{\delta f}_{\mathbf{px}}\exp(-i\omega t)$ in the Wigner representation. In Eq. (2) $\hat{\rho}$ is the equilibrium density matrix, while $\mathbf{v}=\mathbf{p}/m$ is the in-plane velocity of electrons with effective mass m. The collison integral, $I_c(\widehat{\delta f}|\mathbf{px})$ is replaced below by $-\nu\widehat{\delta f}_{\mathbf{px}}$, where ν is the relaxation frequency due to homogeneous broadening, and $[\ \ldots,\ldots\ ]_{\mathbf{px}}$ is the commutator in the Wigner representation. Equation (2) uses the effective Hamiltonian:

$$\widetilde{h}=\hat{h}+\sum_{\mathbf{Q}}{}'v_Q[e^{-i\mathbf{Q}\cdot\mathbf{r}}sp(e^{i\mathbf{Q}\cdot\mathbf{r}}\hat{\rho})-e^{-i\mathbf{Q}\cdot\mathbf{r}}\hat{\rho}e^{i\mathbf{Q}\cdot\mathbf{r}}], \tag{3}$$

which includes the Coulomb interaction to second order, with the perturbation operator given by

$$\widetilde{\delta h}=\frac{ie}{\omega}E_\perp\hat{\mathrm{v}}_\perp+\sum_{\mathbf{Q}}{}'v_Q[e^{-i\mathbf{Q}\cdot\mathbf{r}}sp(e^{i\mathbf{Q}\cdot\mathbf{r}}\widehat{\delta\rho})-e^{-i\mathbf{Q}\cdot\mathbf{r}}\widehat{\delta\rho}e^{i\mathbf{Q}\cdot\mathbf{r}}]. \tag{4}$$

Here v_Q is the Coulomb matrix element, $\hbar\mathbf{Q}$ is the momentum transfer, $\mathbf{r}$ the position vector, and the symbol $sp\cdots$ indicates an averaging over electron states. The 2×2 operators $\hat{h}$ and $\hat{\mathrm{v}}_\perp$ in Eqs. (2),(3) describe the single-particle Hamiltonian and the transverse velocity operator (see explicit expressions in Refs. 12,17). The interwell contribution to the current density induced by the field $E_\perp\exp(-i\omega t)$ is given by $J_{\omega\mathbf{x}}=(2e/S)\Sigma_{\mathbf{p}}\mathrm{tr}\hat{\mathrm{v}}_\perp\widehat{\delta f}_{\mathbf{px}}$, where S is the normalization area and $\mathrm{tr}\cdots$ stands for the trace over the upper (u-) and lower (l-) QW states.

Below we calculate $J_{\omega\mathbf{x}}$ for weakly coupled DQW's with an accuracy of the order of T^2, where T is the tunneling matrix element. Since $\hat{\mathrm{v}}_\perp$ is proportional to T, we write

$$J_{\omega\mathbf{x}}=i\frac{2ev_\perp}{S}\sum_{\mathbf{p}}[\delta f_{ul}(\mathbf{px})-\delta f_{lu}(\mathbf{px})], \tag{5}$$

where $v_\perp=ZT/\hbar$ and Z is the interlevel spatial separation. The nondiagonal components of the density matrix in Eq. (4), $\delta f_{ul}(\mathbf{px})$ and $\delta f_{lu}(\mathbf{px})$, are considered with an accuracy of the order of T. For such a case, we use in Eq. (3) the diagonal Hamiltonian $\hat{h}$ determined through the matrix elements $\varepsilon_{jp}+w_{j\mathbf{x}}, j=u,l$. Here $\varepsilon_{jp}=\varepsilon_j+p^2/2m$ describes the carrier dispersion in the jth QW with the random potentials $w_{j\mathbf{x}}$ and including the self-consistent contributions from $\widetilde{h}$, which are proportional to $sp(e^{i\mathbf{Q}\cdot\mathbf{r}}\hat{\rho})$. Using a basis of self-consistent wave functions we can transform Eq. (2) into the system

$$\begin{aligned}&[\omega+i\nu-\omega_{ul}(p\mathbf{x})+i\mathbf{v}\cdot\nabla_x]\delta f_{ul}(\mathbf{px})\\&\qquad-\delta h_{ul}(\mathbf{px})(f_{lp\mathbf{x}}-f_{up\mathbf{x}})/\hbar=0,\end{aligned} \tag{6}$$

$$\begin{aligned}&[\omega+i\nu+\omega_{ul}(p\mathbf{x})+i\mathbf{v}\cdot\nabla_x]\delta f_{lu}(\mathbf{px})\\&\qquad-\delta h_{lu}(\mathbf{px})(f_{up\mathbf{x}}-f_{lp\mathbf{x}})/\hbar=0.\end{aligned}$$

Here $f_{j\mathbf{x}p}=\theta(\varepsilon_{jF}-\varepsilon_{jp}-w_{j\mathbf{x}})$ is the zero-temperature Fermi distribution in the jth QW, with Fermi energy ε_{jF} and with the interlevel energy separation $\hbar\omega_{ul}(p\mathbf{x})$ given by

$$\begin{aligned}\hbar\omega_{ul}(p\mathbf{x})=&\ \varepsilon_{ul}+w_{u\mathbf{x}}-w_{l\mathbf{x}}\\&-\sum_{j=u,l}\int\frac{d\mathbf{p}_1}{2\pi m}f_{jp_1\mathbf{x}}\left[M_{ujju}\left(\frac{|\mathbf{p}-\mathbf{p}_1|}{\hbar}\right)\right.\\&\left.-M_{ljjl}\left(\frac{|\mathbf{p}-\mathbf{p}_1|}{\hbar}\right)\right].\end{aligned} \tag{7}$$

The Coulomb kernel is calculated using the electron envelope functions, φ_{az}, in the u- and l-QW's (Ref. 14)

$$M_{abcd}(q)=\frac{1}{a_Bq}\int dz\varphi_{az}\varphi_{bz}\int dz'\varphi_{cz'}\varphi_{dz'}e^{-q|z-z'|}, \tag{8}$$

where a_B is the effective Bohr radius, q is the in-plane component of the wave vector $\mathbf{Q}=(\mathbf{q},q_\perp)$ and we have integrated over the component of wave vector, $q_\perp$, perpendicular to the QW plane. The perturbation operator (4) is then evaluated for Eq. (6) using the kernel of Eq. (8):

$$\delta h_{jj'}(\mathbf{px}) \pm \frac{eE_\perp \mathrm{v}_\perp}{\omega} = \int \frac{d\mathbf{p}_1}{2\pi m} \sum_{ab} \delta f_{ab}(\mathbf{p}_1\mathbf{x}) \left[2M_{jj'ba}(0) - M_{jabj'}\left(\frac{|\mathbf{p}-\mathbf{p}_1|}{\hbar} \right) \right] \simeq - \int \frac{d\mathbf{p}_1}{2\pi m} \delta f_{jj'}(\mathbf{p}_1\mathbf{x}) M_{jjj'j'} \times \left(\frac{|\mathbf{p}-\mathbf{p}_1|}{\hbar} \right), \tag{9}$$

where we have neglected the overlap of the u and l orbitals on the right hand side and the signs "$+$" or "$-$" correspond to the cases $j=l, j'=u$, or $j=u, j'=l$. This expression differs essentially from the single QW case, so that the character of the Coulomb renormalization is then different from in Ref. 16.

It is convenient to introduce new functions $\delta f^{(\pm)}_{\mathbf{px}} = \delta f_{ul}(\mathbf{px}) \pm \delta f_{lu}(\mathbf{px})$, so that the induced current density takes the form $J_{\omega\mathbf{x}} = i(2e\mathrm{v}_\perp/L^2)\Sigma_{\mathbf{p}} \delta f^{(-)}_{\mathbf{px}}$ and the pair of equations in Eq. (6) are transformed to:

$$[\omega + i\nu + i\mathbf{v}\cdot\nabla_x]\delta f^{(+)}_{\mathbf{px}} - \omega_{ul}(p\mathbf{x})\delta f^{(-)}_{\mathbf{px}} - \theta_{p\mathbf{x}} \int \frac{d\mathbf{p}_1}{2\pi\hbar m} M_{uull}\left(\frac{|\mathbf{p}-\mathbf{p}_1|}{\hbar} \right) \delta f^{(-)}_{\mathbf{p}_1\mathbf{x}} = -\frac{2eE_\perp \mathrm{v}_\perp}{\hbar\omega}\theta_{p\mathbf{x}},$$

$$[\omega + i\nu + i\mathbf{v}\nabla_x]\delta f^{(-)}_{\mathbf{px}} - \omega_{ul}(p\mathbf{x})\delta f^{(+)}_{\mathbf{px}} - \theta_{p\mathbf{x}} \int \frac{d\mathbf{p}_1}{2\pi\hbar m} M_{uull}\left(\frac{|\mathbf{p}-\mathbf{p}_1|}{\hbar} \right) \delta f^{(+)}_{\mathbf{p}_1\mathbf{x}} = 0. \tag{10}$$

Here $\theta_{p\mathbf{x}} \equiv f_{up\mathbf{x}} - f_{lp\mathbf{x}}$ is the population factor and we have used the symmetry property $M_{uull} = M_{lluu}$. Below we approximate the Coulomb kernels as $M_{uull}(q) \simeq \exp(-qZ)/a_B q, M_{uuuu}(q) = M_{llll}(q) \simeq 1/a_B q$ and introduce the functions $\varphi^{(\pm)}_{p\mathbf{x}}$ according to $\delta f^{(\pm)}_{\mathbf{px}} = -(2eE_\perp v_\perp/\hbar\omega)\theta_{p\mathbf{x}}\varphi^{(\pm)}_{p\mathbf{x}}$. Equation (10) can then be rewritten as

$$(\omega + i\nu)\varphi^{(+)}_{p\mathbf{x}} - \omega_{ul}(\mathbf{x})\varphi^{(-)}_{p\mathbf{x}} + \int \frac{d\mathbf{p}_1}{2\pi m} \frac{\theta_{p_1\mathbf{x}}}{a_B|\mathbf{p}-\mathbf{p}_1|} \times [\varphi^{(-)}_{p\mathbf{x}} - e^{-|\mathbf{p}-\mathbf{p}_1|Z/\hbar}\varphi^{(-)}_{p_1\mathbf{x}}] = 1, \tag{11}$$

$$(\omega + i\nu)\varphi^{(-)}_{p\mathbf{x}} - \omega_{ul}(\mathbf{x})\varphi^{(+)}_{p\mathbf{x}} + \int \frac{d\mathbf{p}_1}{2\pi m} \frac{\theta_{p_1\mathbf{x}}}{a_B|\mathbf{p}-\mathbf{p}_1|} \times [\varphi^{(+)}_{p\mathbf{x}} - e^{-|\mathbf{p}-\mathbf{p}_1|Z/\hbar}\varphi^{(+)}_{p_1\mathbf{x}}] = 0,$$

where $\omega_{ul}(\mathbf{x}) \equiv \Delta/\hbar + \delta w_{\mathbf{x}}$ and $\delta w_{\mathbf{x}} \equiv w_{u\mathbf{x}} - w_{l\mathbf{x}}$ is the random contribution to the interlevel energy Δ (Fig. 1). In Eqs. (11) we have also neglected non-local effects, omitting the spatial derivatives $\mathbf{v}\cdot\nabla_x$ and $\varphi^{(\pm)}$ are only dependent on $|\mathbf{p}|$ because the Coulomb contributions in Eqs. (11) do not depend on the $\mathbf{p}$-plane angle.

Introducing the local conductivity $\sigma_{\omega\mathbf{x}}$ by the standard relation $J_{\omega\mathbf{x}} = \sigma_{\omega\mathbf{x}} E_\perp$ we express the response (5) through $\varphi^{(-)}_{p\mathbf{x}}$ according to

$$\sigma_{\omega\mathbf{x}} = \frac{2(ev_\perp)^2}{i\hbar\omega} n^{(-)}_{\mathbf{x}\omega}, \quad n^{(\pm)}_{\mathbf{x}\omega} = 2\int \frac{d\mathbf{p}}{(2\pi\hbar)^2} \theta_{p\mathbf{x}}\varphi^{(\pm)}_{p\mathbf{x}}. \tag{12}$$

The conductivity appears to be parametrically dependent on $\omega_{ul}(\mathbf{x})$ in the local approximation, when $n^{(\pm)}_{\mathbf{x}\omega} = n^{(\pm)}_{\omega}[\omega_{ul}(\mathbf{x})]$. The averaging over the random contribution to the interlevel energy $\delta w_{\mathbf{x}}$ is performed according to

$$\langle n^{(-)}_{\omega\mathbf{x}} \rangle = \frac{\hbar}{2\sqrt{\pi}\Gamma} \int d\omega' n^{(-)}_{\omega}(\omega') \exp\left[-\left(\frac{\hbar\omega' - \Delta}{2\Gamma} \right)^2 \right], \tag{13}$$

where $\langle \cdots \rangle$ denotes the averaging procedure[18] and $\Gamma = \sqrt{\langle \delta w^2_{\mathbf{x}} \rangle /2}$ is the inhomogeneous broadening energy. Thus, the averaged conductivity (12), σ_ω, is given by $\sigma_\omega = 2(ev_\perp)^2 \langle n^{(-)}_\omega \rangle / i\hbar\omega$. The relative absorption of a DQW structure, introduced as the ratio of the energy flux absorbed compared to that transmitted through the structure,[19] is given by $\xi_\omega = (4\pi/c\sqrt{\epsilon})\mathrm{Re}\sigma_\omega$.

III. RESULTS

Here we consider the average response of the tunnel-coupled DQW's and their potential to generate stimulated THz emission. Before discussion of the numerical results based on Eqs. (11)–(13), we consider simple analytical formulas for the high-concentration limit $\varepsilon_{u,lF} \gg |\varepsilon_u - \varepsilon_l|$.

A. High-concentration case

For the high-concentration case, the system of integral equations (11) is transformed below into the balance equations for $n^{(\pm)}_{\mathbf{x}\omega}$. We can neglect the momentum dependence of $\varphi^{(\pm)}_{p\mathbf{x}}$ in the narrow region close to $\varepsilon_{u,lF}$ and we obtain the system of balance equations

$$\begin{vmatrix} \omega + i\nu & -\Omega_{\mathbf{x}} \\ -\Omega_{\mathbf{x}} & \omega + i\nu \end{vmatrix} \begin{vmatrix} n^{(+)}_{\mathbf{x}\omega} \\ n^{(-)}_{\mathbf{x}\omega} \end{vmatrix} = \begin{vmatrix} \Delta n_{\mathbf{x}} \\ 0 \end{vmatrix}, \tag{14}$$

where $\Delta n_{\mathbf{x}}$ is the difference in concentration between the u- and l-QW's. The renormalized frequency $\Omega_{\mathbf{x}}$ is given by

$$\Omega_{\mathbf{x}} = \omega_{ul}(\mathbf{x}) - \int \frac{d\mathbf{p}_1}{2\pi m} \frac{\theta_{p_1\mathbf{x}}}{a_B|\mathbf{p}-\mathbf{p}_1|} (1 - e^{-|\mathbf{p}-\mathbf{p}_1|Z/\hbar})\Big|_{p=p_F} = \omega_{ul}(\mathbf{x}) - \frac{\Delta n_{\mathbf{x}}}{a_B p_F \rho_{2D}} \mathcal{A}(Zp_F/\hbar),$$

$$\mathcal{A}(y) = \int_0^\pi \frac{d\varphi}{\pi} \frac{1 - e^{-y\sqrt{2(1-\cos\varphi)}}}{\sqrt{2(1-\cos\varphi)}}, \tag{15}$$

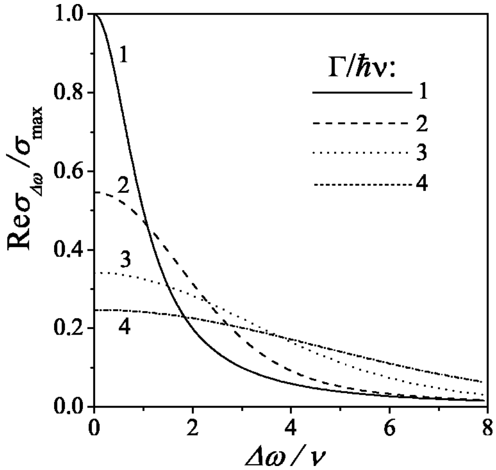

FIG. 2. Transformation of the line shape of $\mathrm{Re}\sigma_{\Delta\omega}$ upon an increase in the contribution of the inhomogeneous broadening given by the ratio $\Gamma/\hbar\nu$.

where ρ_{2D} is the 2D density of states, and we have also used $\int d\mathbf{p}\theta_{p\mathbf{x}}/(2\pi\hbar)^2=\Delta n_{\mathbf{x}}/2$. The function $\mathcal{A}(y)$ can be approximated as $0.6y^{0.4}$ with an accuracy of about 5%.

The conductivity (12) averaged over the 2D plane is then given by

$$\sigma_\omega=i\frac{2(ev_\perp)^2}{\hbar\omega}\left\langle\frac{\Delta n_{\mathbf{x}}\Omega_{\mathbf{x}}}{\Omega_{\mathbf{x}}^2-(\omega+i\nu)^2}\right\rangle \simeq-\frac{(ev_\perp)^2\delta n}{\hbar\Omega}\int_{-\infty}^{0}dte^{i\Delta\omega t+\nu t}e^{-(\Gamma t/\hbar)^2}, \tag{16}$$

where the denominator has been expressed by means of an integral over time as $(E-i\gamma)^{-1}=(i/\hbar)\int_{-\infty}^{0}dt\exp[(iE+\gamma)t/\hbar]$ for the near-resonant region. The averaged redistribution of concentration between the u- and l-QW's δn is given by $\delta n=\langle\Delta n_{\mathbf{x}}\rangle=\rho_{2D}(\varepsilon_{Fu}-\varepsilon_{Fl})$. The averaged resonant frequency Ω in (16) is given by

$$\hbar\Omega=\Delta-\frac{\delta n}{\rho_{2D}}\frac{\mathcal{A}(Zp_F/\hbar)}{a_Bp_F}. \tag{17}$$

In contrast to the case of a DQW structure with a common Fermi level, when $\varepsilon_{Fu}-\varepsilon_{Fl}=\Delta$, the renormalized resonant frequency Ω is now dependent on δn. Note also that $\sigma_{\Delta\omega}=0$ if $\delta n=0$, as is clear from Eq. (16) and Fig. 1. For the near-resonant spectral region, where $|\Delta\omega|\sim\nu,\Gamma/\hbar\ll\Omega$, we can neglect $\mathrm{Im}\sigma_{\Delta\omega}$ and the line shape of $\mathrm{Re}\sigma_{\Delta\omega}$ depends only on the relative contribution of the inhomogeneous and homogeneous broadening mechanisms, given by the ratio $\Gamma/\hbar\nu$. Figure 2 shows how the line shape of $\mathrm{Re}\sigma_{\Delta\omega}$ changes on increasing the ratio $\Gamma/\hbar\nu$, transforming from a Lorenzian towards a Gaussian line shape. The asymptotic formulas for the limiting cases where either collisions or nonuniform broadening mechanisms dominate are given by

$$\mathrm{Re}\sigma_{\Delta\omega}=-\sigma_{\max}\begin{cases}[(\Delta\omega/\nu)^2+1]^{-1}, & \hbar\nu\gg\Gamma,\\ (\sqrt{\pi}\hbar\nu/2\Gamma)\exp[-(\hbar\Delta\omega/\Gamma)^2/2], & \hbar\nu\ll\Gamma.\end{cases} \tag{18}$$

Here and in Fig. 2 we have introduced the peak conductivity $\sigma_{\max}=(ev_\perp)^2\delta n/(\hbar\Omega\nu)$.

B. Negative absorption and gain

Now, we turn to the general case based on the numerical solution of Eqs. (11)–(13). We consider the relative absorption peak in a DQW structure with two wells of width 120 Å, separated by a barrier of 125 Å. This is close to the sample design of Ref. 10, but we assume $Al_{0.12}Ga_{0.88}As$ barriers, for which $v_\perp\simeq1.7\times10^6$ cm/s. We assume $\Gamma\simeq\hbar\nu\simeq0.2$ meV (see the numerical estimates in Ref. 12) while the level splitting between the u- and l-QW's is chosen as $\Delta=10$ meV. The distribution of concentration over the u- and l-QW's is described by n_u+n_l and δn. The relative absorption ξ_ω of the assumed structure is shown in Fig. 3 as a function of $\hbar\omega$ for different n_u+n_l and $\delta n=\pm10^{11}$ cm^{-2} or ±5

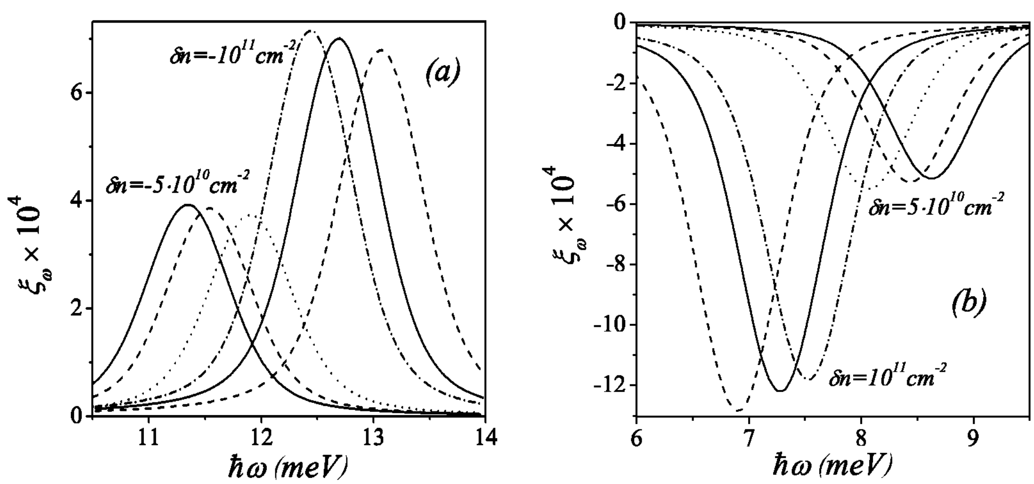

FIG. 3. The relative absorption ξ_ω for the absorption (a) or stimulated emission (b) regimes under total concentrations 2, 4, 6, and 8×10^{11} cm^{-2} (dotted, dashed, solid, and dash-dotted curves, respectively) and $\delta n=\pm10^{11}$ cm^{-2} or $\pm5\times10^{10}$ cm^{-2}.

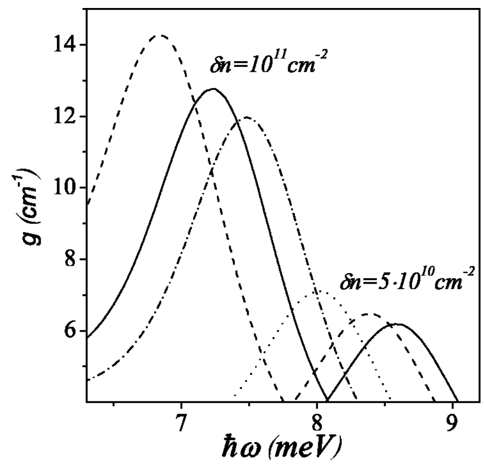

FIG. 4. The gain, g, versus $\hbar\omega$ for different concentrations $n_u + n_l$ (the same curves as in Fig. 3) and δn.

$\times 10^{10}$ cm^{-2}. One can see not only the transformation between the absorption [if $\delta n < 0$, Fig. 3(a)] and the stimulated emission [if $\delta n > 0$, Fig. 3(b)] regimes but also a visible shift of the absorption/emission peaks due to the Coulomb renormalization effect. Note, that the shift of the absorption peak increases with $|\delta n|$ and decreases with $n_u + n_l$.

By comparison with the recently demonstrated[6] cavity losses in a tunnel-coupled cascade structure, it isunlikely that a single pair of QW's will provide sufficient gain to achieve lasing. We therefore estimate here the modal gain that could be achieved in a multi-DQW structure. The practical implementation of such a structure remains to be addressed, regarding how to extend the techniques in Ref. 10 to enable many DQW's in series, or else to design a DQW cascadelike structure which would achieve an equivalent population inversion between each pair of wells. It is nevertheless of value to estimate the gain that might be achieved in such a structure. The interwell current in a *N*-layer structure is given by $J_\omega \simeq N\sigma_{\Delta\omega}E_\perp$, where the conductivity in Eq. (1) is replaced by $N\sigma_{\Delta\omega}$. Figure 4 shows the calculated gain in a six-layer DQW structure versus $\hbar\omega$ for the values of $n_u + n_l$ and δn considered in Fig. 3 and for $L \simeq 25$ μm. The maximal gain value is proportional to $\delta n T^2$ and to the inverse broadening energy, so that g can be enhanced in structures with more strongly coupled QW's (reduced barrier width and/or Al composition in the barrier.) Since the calculated gain appears comparable to the experimental value in Ref. 6 it could be possible to realize the stimulated emission regime in structures which are of order a millimetre in length, so long as additional losses are not significant.

IV. SUMMARY AND CONCLUSIONS

We have considerd here the interaction of a biased independently contacted DQW structure with a THz radiation field, and estimated the conditions for the stimulated emission regime to be obtained. Both the Coulomb renormalization of the intersubband transition energy and the homogeneous and inhomogeneoeus broadening mechanisms are taken into account. The enhancement of the emission due to the THz waveguide effect is also considered for the case of a multi-DQW structure placed in the center of a resonator.

Let us briefly discuss the assumptions used in our calculations. The DQW's are described with an accuracy of the order T^2 for the case of independently-contacted DQW's with a thick barrier. The high-frequency response was considered self-consistently, with an accuracy of the order of e^2 terms, which has been successfully used both for a single QW with large-scale nonuniformities[15] and for a DQW structure (see Ref. 14); these results show that the approach used is valid for DQW structures with a broadening energy $\geqslant 0.1$ meV). The balance equations (14) are valid for the high-concentration limit. The appearance of long-range potential fluctuations in the DQW's and the corresponding assumptions are discussed in Ref. 12 while the validity of the local approximation was considered in Ref. 16. We assume that the cavity losses for the stimulated mode do not depend on the electron concentration in the DQW's because there is no free carrier absorption. The introduction of different quasi-Fermi levels in the independently contacted *u*- and *l*-QW's supposes that the in-plane currents balance the interwell tunneling current (see Ref. 20 for an examination of size-dependent phenomena in DQW structures). All these simplifications do not change either the character of the THz response or the numerical estimates for the stimulated emission regime conditions.

To conclude, we have examined the conditions for realization of the stimulated emission regime using a multi-DQW structure with independent contacts, placed at the center of a THz wave guide. In spite of the technologically complicated structure considered, the results obtained clearly demonstrate the possibility to use independently contacted DQW's as a coherent source of THz radiation.

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