Domain theory of self-induced transparency in a semiconductor superlattice

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A self-consistent description of the electrodynamics of a semiconductor superlattice interacting with electromagnetic field of arbitrary frequency and strength is presented. Central to the present formalism is a recasting of the proper boundary conditions for the Maxwell problem that enables a speedy and straightforward solution for the distribution of the internal field in the active superlattice medium. On application of the self-consistent approach we found a self-induced transparency region in the superlattice layer which takes the form of a domain. The propagation and evolution of such a domain, which can contain a coherent train of Bloch oscillations, dominate the electron dynamics of the superlattice. The important properties of the self-induced transparency domain are also discussed here.

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I. INTRODUCTION

In the past three decades much research was concerned with the theory of electron transport in a superlattice in a given field approximation. In particular, Ignatov and Romanov predicted the possibility of self-induced transparency in a superlattice in 1976.[1] Later, self-induced transparency was incorrectly associated with the macroscopic appearance (collapse of minibands[2,3]) of the electron dynamic localization.[4] Recently, the incorrectness of the connection between these two effects was proven in Ref. 5, where it was shown that self-induced transparency and dynamic localization are different phenomena with different physical origins, displaced in time from each other, and, in general they arise at different electric fields. In all the above-mentioned works the given field approximation was employed, along with a description of electron-scattering processes of various degrees of sophistication. The fact that within the given field approximation there is a possibility of missing important details pertinent to the inhomogeneous high-frequency field distribution in a superlattice was nevertheless ignored in these works.

The first effort to go beyond the given field approximation was made by Dodin, Zharov, and Ignatov,[6] in which the authors proposed to describe a volume electron current by means of an equivalent surface current, which led to a self-consistent equation system in real time, constituting a link between external field incident on the superlattice and the internal field in the superlattice. However, the latter approach is still limited since the internal field in the superlattice was considered homogeneous, and obviously this works when the superlattice layer width is much smaller than the skin layer width. The case of high-frequency irradiation of a semi-infinite superlattice when the incident frequency is much higher than the scattering frequency has been considered in Ref. 7, using an effective dielectric function. The nonlinear connection between the incident field and the internal field led to a reflection coefficient exhibiting hysteresis as a function of the incident field. Recently, Romanov and Romanova[8] summarized all the investigations related to self-induced, induced, and selective transparencies of superlattices of the last decade, with particular emphasis on the obvious contradictions between experimental observation of self-induced transparency[9] and theoretical results obtained by means of the given field approximation. A recent proposal[10] for a stripline terahertz Bloch oscillator[11] based on a superlattice (SL) with the weak barriers and SL layer width $L \approx \lambda/2$, which is necessary for a resonant condition, again highlighted the urgency for the inclusion of all transport effects in a superlattice, in particular, the self-induced transparency, within the inhomogeneous high-frequency field distribution in the superlattice as a function of given incident wave field.

In this work we present a self-consistent treatment of the electrodynamics, of the planar semiconductor superlattice, under the irradiation of an incident electromagnetic wave of arbitrary amplitude. We obtain the appropriate boundary conditions for the electromagnetic field, which allow us to solve the wave equation in the needed region (the active superlattice layer) only. We generalize the well-known material equations for the case of an inhomogeneous electric field. We apply this self-consistent approach to demonstrate the possibility of self-induced transparency in the case in which the incident wave frequency is much lower than electron-scattering frequency. Numerical solutions show a domain character of induced transparency with the possible efficient excitation of a coherent Bloch oscillation train.

II. FORMULATION OF THE SUPERLATTICE ELECTRODYNAMICS

Consider the active media layer, for example, the planar superlattice, with width h_{SL}, which is located between two half spaces. The left half space $z \leqslant 0$ has the electric and magnetic permittivities ε_1, μ_1, and the right half space $z \geqslant h_{\mathrm{SL}}$ has the electric and magnetic permittivities ε_2, μ_2. The particular geometry of the system is sketched in Fig. 1. The superlattice layers lie in the (y,z) plane. The incident electromagnetic plane wave with polarization of electric field $\mathbf{E} = E_x \cdot \mathbf{x}_0$ parallel to the superlattice growth axis impinges from the left half space on the planar superlattice. Now we assume that the incident wavelength greatly exceeds the SL period d. This assumption gives $\partial/\partial x = 0$. Furthermore, since the superlattice is considered to be infinitely extended in the

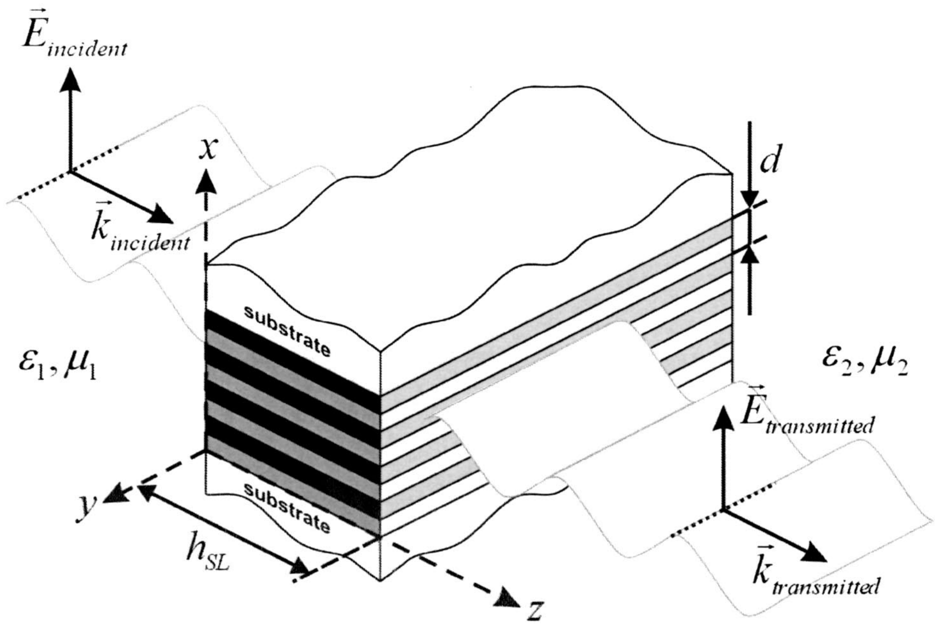

FIG. 1. Sketch of the geometry.

y direction, it follows then that $\partial/\partial y=0$. Our task is to investigate self-consistently the electrodynamic processes occurring in the superlattice due to strong nonlinear transport within the single-miniband approximation in the field of the incident wave.

Due to the proposed system symmetry it is clear that the electric field at any position z is a function of real time t and z and has only the E_x component. The wave equation can be written in the following form:

$$\frac{\varepsilon(z)\mu(z)}{c_0^2}\frac{\partial^2 E_x}{\partial t^2}-\frac{\partial^2 E_x}{\partial z^2}=-\mu(z)\mu_0\frac{\partial j_x}{\partial t}, \tag{1}$$

where $\mu_0=4\pi\times10^{-7}\ NA^{-2}$, $c_0=3\times10^8\ \mathrm{ms}^{-1}$. Expressions for permittivities are

$$\varepsilon(z)=\varepsilon_I \qquad (z<0),$$

$$\varepsilon(z)=\varepsilon_{\mathrm{SL}} \qquad (0<z<h_{\mathrm{SL}}),$$

$$\varepsilon(z)=\varepsilon_2 \qquad (z>h_{\mathrm{SL}}),$$

$$\mu(z)=\mu_1 \qquad (z<0),$$

$$\mu(z)=\mu_{\mathrm{SL}} \qquad (0<z<h_{\mathrm{SL}}),$$

$$\mu(z)=\mu_2 \qquad (z>h_{\mathrm{SL}}).$$

The conduction current is given by

$$j_x=0 \qquad (z<0),$$

$$j_x=en_eV \qquad (0<z<h_{\mathrm{SL}}),$$

$$j_x=0 \qquad (z>h_{\mathrm{SL}}),$$

where n_e is the electron concentration in the first miniband, $V=V(t,z)$ is an average electron velocity from the Boltzmann equation, and e is the electron charge. To proceed we need to specify the form of the boundary conditions. From the general integral Maxwell equations we have the following boundary conditions at $z=0$, $z=h_{\mathrm{SL}}$,

$$E_x(t,z=0-0)=E_x(t,z=0+0),$$

$$E_x(t,z=h_{\mathrm{SL}}-0)=E_x(t,z=h_{\mathrm{SL}}+0),$$

$$H_y(t,z=0-0)=H_y(t,z=0+0),$$

$$H_y(t,z=h_{\mathrm{SL}}-0)=H_y(t,z=h_{\mathrm{SL}}+0). \tag{2}$$

The last two boundary equations can be transformed to a more convenient form. In the case when $\partial\mu_1/\partial t=\partial\mu_{\mathrm{SL}}/\partial t=\partial\mu_2/\partial t=0$ we obtain boundary conditions in the following form:

$$E_x(t,z=0-0)=E_x(t,z=0+0),$$

$$E_x(t,z=h_{\mathrm{SL}}-0)=E_x(t,z=h_{\mathrm{SL}}+0),$$

$$\frac{1}{\mu_1}\frac{\partial E_x}{\partial z}(t,z=0-0)=\frac{1}{\mu_{\mathrm{SL}}}\frac{\partial E_x}{\partial z}(t,z=0+0),$$

$$\frac{1}{\mu_{\mathrm{SL}}}\frac{\partial E_x}{\partial z}(t,z=h_{\mathrm{SL}}-0)=\frac{1}{\mu_2}\frac{\partial E_x}{\partial z}(t,z=h_{\mathrm{SL}}+0). \tag{3}$$

The boundary conditions (3) are more convenient than Eq. (2) because they contain only electric field that exists in our system. However, these boundary conditions (3) are not completely satisfactory yet. They do not allow a solution of the wave Eq. (1) within the active media layer only $(0<z<h_{\mathrm{SL}})$ and do not allow us to concern ourselves with the field outside the active media layer. This is a very important issue related to the boundary Eq. (3). If we want to code our system in real time with some computational technique or even to arrive at an analytical solution, it is much more advantageous to solve the wave equation only within the finite

space (superlattice). To achieve this goal we can use the fact that in free space the solution of wave Eq. (1) represents two independent running waves: incident and reflected waves in half space $z<0$, and a transmitted wave in half space $z>h_{\mathrm{SL}}$:

$$E_x(t,z)=E_{\mathrm{inc}}(t,z)+E_{\mathrm{reflect}},(t,z), \qquad z<0,$$

$$\frac{\sqrt{\varepsilon_1\mu_1}}{c_0}\frac{\partial E_{\mathrm{inc}}}{\partial t}+\frac{\partial E_{\mathrm{inc}}}{\partial z}=0,$$

$$\frac{\sqrt{\varepsilon_1\mu_1}}{c_0}\frac{\partial E_{\mathrm{reflect}}}{\partial t}-\frac{\partial E_{\mathrm{reflect}}}{\partial z}=0,$$

$$E_x(t,z)=E_{\mathrm{transm}}(t,z), \qquad z>h_{\mathrm{SL}},$$

$$\frac{\sqrt{\varepsilon_2\mu_2}}{c_0}\frac{\partial E_{\mathrm{transm}}}{\partial t}+\frac{\partial E_{\mathrm{transm}}}{\partial z}=0. \tag{4}$$

The last step is the substitution of the spatial partial derivatives of the electric fields of running waves by temporal partial derivatives in Eqs. (3) from (4). And finally we obtain the wave equation, with *universal boundary conditions*, which need to be solved only within the space interval of interest ($0<z<h_{\mathrm{SL}}$):

$$\frac{\varepsilon_{\mathrm{SL}}\mu_{\mathrm{SL}}}{c_0^2}\frac{\partial^2 E_x}{\partial t^2}-\frac{\partial^2 E_x}{\partial z^2}=-\mu_{\mathrm{SL}}\mu_0\frac{\partial j_x}{\partial t},$$

$$\sqrt{\frac{\varepsilon_1}{\mu_1}}\left[\frac{\partial E_x}{\partial t}(t,0)-2\frac{dE_{\mathrm{inc}}}{dt}(t)\right]=\frac{c_0}{\mu_{\mathrm{SL}}}\frac{\partial E_x}{\partial z}(t,0),$$

$$-\sqrt{\frac{\varepsilon_2}{\mu_2}}\frac{\partial E_x}{\partial t}(t,h_{\mathrm{SL}})=\frac{c_0}{\mu_{\mathrm{SL}}}\frac{\partial E_x}{\partial z}(t,h_{\mathbf{SL}}), \tag{5}$$

where $E_{\mathrm{inc}}(t)\equiv E_{\mathrm{inc}}\,(t,z=0)$ is the incident field which is actually impinging on the left side of the superlattice. This is a completely self-consistent physical approach to investigate the real time interaction between the incident field and an active medium.

III. MATERIAL EQUATIONS WITH INHOMOGENEOUS ELECTRIC FIELD IN THE MEDIA

To investigate the interaction between the quantum superlattice and the incident field with the self-consistent equation system (5) we need the material equations in the medium (SL). This specifies the relation between the conduction current in the superlattice and the electric field. We treat this within the Boltzmann equation for one-miniband electron transport with the assumption that

$$eE(z,t)d<\Delta,\Delta_g,$$

$$\hbar\omega_{\mathrm{inc}}<\Delta,\Delta_g,$$

where Δ (Δ_g) is the width of the first allowed (forbidden) miniband, and d is the period of the superlattice. The Boltzmann equation in our case can be written as

$$\frac{\partial f}{\partial t}+v_z\frac{\partial f}{\partial z}+eE(z,t)\frac{\partial f}{\partial p}=St[f],$$

where $f(t,z,p)$ is the electron distribution function, and $\varepsilon(p)=\Delta/2[1-\cos(pd/\hbar)]$ is the energy spectrum of electrons in the miniband taken in the tight-binding approximation, $v=\partial\varepsilon/\partial p$, $v_z=p_z/m_z$, m_z is an effective mass in z direction. A collision integral is taken in the effective relaxation-time approximation,

$$St[f]=-\frac{\{f(t,z,p)-f_0(p)\}}{\tau_\Sigma}-\frac{\{f(t,z,p)-f(t,z,-p)\}}{2\tau_{ee}},$$

where τ_Σ is the electron energy relaxation time, τ_{ee} describes the electron-electron scattering, $f_0(p)=d/2\{\pi\hbar I_0(\Delta/2k_BT)\}^{-1}\exp\{(\Delta/2k_BT)\cos(pd/\hbar)\}$ is the equilibrium distribution function, k_B is the Boltzmann's constant, and T is the superlattice temperature. The average electron velocity and energy are defined by the following expressions:

$$V(t,z)=\int v(p)f(t,z,p)dp,$$

$$\Sigma(t,z)=\int \varepsilon(p)f(t,z,p)dp,$$

where integrals are taken within the first Brillion zone $-(\pi\hbar/d)\leqslant p\leqslant(\pi\hbar/d)$. The expressions for the full time derivatives of the average velocity and energy can be directly obtained as

$$\begin{aligned}\frac{dV}{dt}(t,z)&=\int\frac{d}{dt}[v(p)f(t,z,p)]dp=\int\frac{d}{dt}[v(p)]f(t,z,p)dp\\&\quad+\int v(p)\frac{d}{dt}[f(t,z,p)]dp\\&=eE(t,z)\int\frac{\partial^2\varepsilon(p)}{\partial p^2}f(t,z,p)dp+\int v(p)St[f]dp\\&=\frac{eE(t,z)}{m_{\mathrm{eff}}(\Sigma)}-\frac{V(t,z)}{\tau_V},\end{aligned} \tag{6a}$$

$$\begin{aligned}\frac{d\Sigma}{dt}(t,z)&=\int\frac{d}{dt}[\varepsilon(p)f(t,z,p)]dp\\&=\int\frac{d}{dt}[\varepsilon(p)]f(t,z,p)dp\\&\quad+\int\varepsilon(p)\frac{d}{dt}[f(t,z,p)]dp\\&=eE(z,t)\int v(p)f(t,z,p)dp+\int\varepsilon(p)St[f]dp\\&=eV(t,z)E(t,z)-\frac{(\Sigma-\Sigma_{\mathrm{thermal}})}{\tau_\Sigma},\end{aligned} \tag{6b}$$

where $\Sigma_{\mathrm{thermal}}=\Delta/2(1-\mu_{\mathrm{thermal}})$ is the average thermal electron energy in the absence of an electric field, $\mu_{\mathrm{thermal}}=[I_1(\Delta/2k_BT)/I_0(\Delta/2k_BT)]$, $I_{0,1}(x)$ are modified Bessel functions $(1/\tau_V)=(1/\tau_\Sigma)+(1/\tau_{ee})$, and $m_{\text{eff}}(\Sigma)=m_0/(1-2\Sigma/\Delta)$ is the energy-dependent effective electron mass, which is determined by the energy spectrum in the miniband and physically related to the Bragg reflection. $m_0=(2\hbar^2/\Delta d^2)$ is the effective electron mass at the miniband bottom.

The material Eqs. (6a) and (6b) must be rewritten in the partial derivative form. Because of the symmetry, the operator of the full time derivative d/dt, which is, in general, $(d/dt)=(\partial/\partial t)+\mathbf{V}(\mathbf{r},t)(\partial/\partial\mathbf{r})$, equals the partial time derivative $(d/dt)=(\partial/\partial t)$. The expression for the partial time derivative of the electron current, which is a part of the wave Eq. (1), generally contains two terms,

$$\frac{\partial j_x}{\partial t}=en_e\frac{\partial V}{\partial t}+eV\frac{\partial n_e}{\partial t},$$

but $\partial n_e/\partial t$ decays very quickly in time according to the Maxwell relaxation-time process $(\partial n_e/\partial t)\approx-[(n_e-N_D)/\tau_{\text{Maxwell}}]$ and $\tau_{\text{Maxwell}}=(\varepsilon_{SL}\varepsilon_0/\sigma)\sim 2.7\times10^{-15}$ s at doping electron concentration in superlattice $N_D=10^{17}\ \text{cm}^{-3}$ and effective scattering frequency $f_{\text{scatt}}=1/(2\pi\tau_\Sigma)=1.0$ THz. Thus, the second term can be neglected and $n_e=N_D$ for the physical processes with characteristic time $\tau\gg\tau_{\text{Maxwell}}$.

IV. SOLUTIONS OF THE SELF-CONSISTENT SYSTEM OF THE PARTIAL DIFFERENTIAL EQUATIONS

Now we can formulate the self-consistent electrodynamic system, to be solved within the planar superlattice, which describes the interaction between the incident field and the superlattice:

$$\frac{\varepsilon_{\text{SL}}\mu_{\text{SL}}}{c_0^2}\frac{\partial^2 E}{\partial t^2}-\frac{\partial^2 E}{\partial z^2}=-\mu_{\text{SL}}\mu_0 e n_e\frac{\partial V}{\partial t},$$

$$\frac{\partial V}{\partial t}(t,z)=\frac{eE}{m_{\text{eff}}(\Sigma)}-\frac{V}{\tau_V},$$

$$\frac{\partial \Sigma}{\partial t}(t,z)=eVE-\frac{(\Sigma-\Sigma_{\text{thermal}})}{\tau_\Sigma},$$

$$\sqrt{\frac{\varepsilon_1}{\mu_1}}\left[\frac{\partial E}{\partial t}(t,0)-2\frac{dE_{\text{inc}}}{dt}(t)\right]=\frac{c_0}{\mu_{\text{SL}}}\frac{\partial E}{\partial z}(t,0),$$

$$-\sqrt{\frac{\varepsilon_2}{\mu_2}}\frac{\partial E}{\partial t}(t,h_{\text{SL}})=\frac{c_0}{\mu_{\text{SL}}}\frac{\partial E}{\partial z}(t,h_{\text{SL}}). \tag{7}$$

The equation system (7) has been numerically analyzed. The goal of the numerical investigation was to investigate the possibility of observing the well-known effect of self-induced transparency, which is believed to happen only in case of high-frequency (THz) superlattice pumping, when the incident wave frequency exceeds the scattering frequency.[12] More interestingly, and perhaps from a more practical point of view, we studied the case in which incident wave frequency is much lower than the characteristic scattering frequency in the superlattice ($f_{\text{scatt}}\simeq 1.0$ THz) at room temperature.

The dynamics of self-induced transparency in a superlattice depends on the interrelations among three spatial scales. The first two are the superlattice width h_{SL}, which can remain arbitrary in our consideration, and the width of skin layer h_{skin}. The relation between h_{skin} and h_{SL} plays an important role because of the effective metallic behavior of the superlattice with respect to the incident-field distortion when the average electron is near the bottom of the miniband. The skin layer width can be estimated as $h_{\text{skin}}=1/|\text{Im}\,\tilde{k}_{\text{SL}}|$, where $\tilde{k}_{\text{SL}}=\omega_{\text{inc}}\sqrt{\tilde{\varepsilon}_{\text{SL}}(\omega_{\text{inc}})}/c_0$, $\tilde{\varepsilon}_{\text{SL}}(\omega_{\text{inc}})=\varepsilon_{\text{SL}}-[\omega_{pe}^2/\omega_{\text{inc}}(\omega_{\text{inc}}-i\tau_V^{-1})]$, $\omega_{\text{inc}}=2\pi f_{\text{inc}}$, f_{inc} is the frequency of the incident wave, $\omega_{pe}=\sqrt{\mu_{\text{thermal}}(e^2 n_e/\varepsilon_0 m_0)}$ is the effective electron plasma frequency in the metallic (not transparent) state, and $\varepsilon_0=8.85\times10^{-12}\ \text{Fm}^{-1}$.

The self-consistent approach allows to take into account an appearance of inhomogeneous internal field in the superlattice under irradiation, when the superlattice width is the order of skin layer width. Therefore, the case of a wide superlattice is of special interest for the self-consistent approach. Numerical investigations in this case show that inhomogeneous electric field in the SL leads to *self-induced transparency, which always appears with a domain character*. This means that induced transparency occurs only within finite superlattice space (which can be considered as the domain size), near the irradiated SL surface, i.e., inside the skin layer, and propagates with a velocity which is approximately the phase velocity of the electromagnetic wave in the superlattice $v_{\text{domain}}\simeq v_{\text{phase}}=(c_0/\sqrt{\varepsilon_{\text{SL}}\mu_{\text{SL}}})$, leaving behind a region of transparent medium. The local transition time for the transition from metallic to transparent state is close to the electron-scattering time, therefore the third space size affecting transparency behavior is the effective domain size $h_{\text{domain}}=v_{\text{domain}}/f_{\text{scatt}}$.

We say that the active medium is transparent if the electron current equals zero. Therefore it is desirable to analyze the dynamics of transition processes inside the SL from the metallic state to the transparent state in the space-time dependence of electron current or velocity $V(z,t)$ as it is shown in Fig. 2, where we present a development of transparency in a GaAs/GaAlAs ($\varepsilon_{\text{SL}}=13$, $\mu_{\text{SL}}=1$) planar superlattice with the following parameters: $d=5$ nm, $\Delta=0.1$ eV, with relatively low electron concentration $n_e=10^{16}\ \text{cm}^{-3}$ [Fig. 2(a)] and the highly doped case $n_e=10^{17}\ \text{cm}^{-3}$ [Fig. 2(b)] at room temperature in vacuum $\varepsilon_1=\varepsilon_2=\mu_1=\mu_2=1$. The superlattice width $h_{\text{SL}}=158\ \mu$m is chosen so as to exceed the skin layer in both cases: $h_{\text{skin}}^A=118$, $h_{\text{skin}}^B=37\ \mu$m. Also we suppose that electron-electron scattering can be neglected, $f_V=f_\Sigma\equiv f_{\text{scatt}}=1.0$ THz. In the figures the electron velocity is taken in units of $V_0\equiv(\Delta d\mu_{\text{thermal}}/2\hbar)=2.9\times10^7$ cm/s, the position inside the superlattice in units of $Z_0\equiv v_{\text{phase}}/f_{\text{scatt}}=83\ \mu$m, and time in units of the scattering period $T_0\equiv 1/f_{\text{scatt}}=10^{-12}$ s. We choose the incident field in a form, that is very close to the harmonic form with frequency f_{inc}

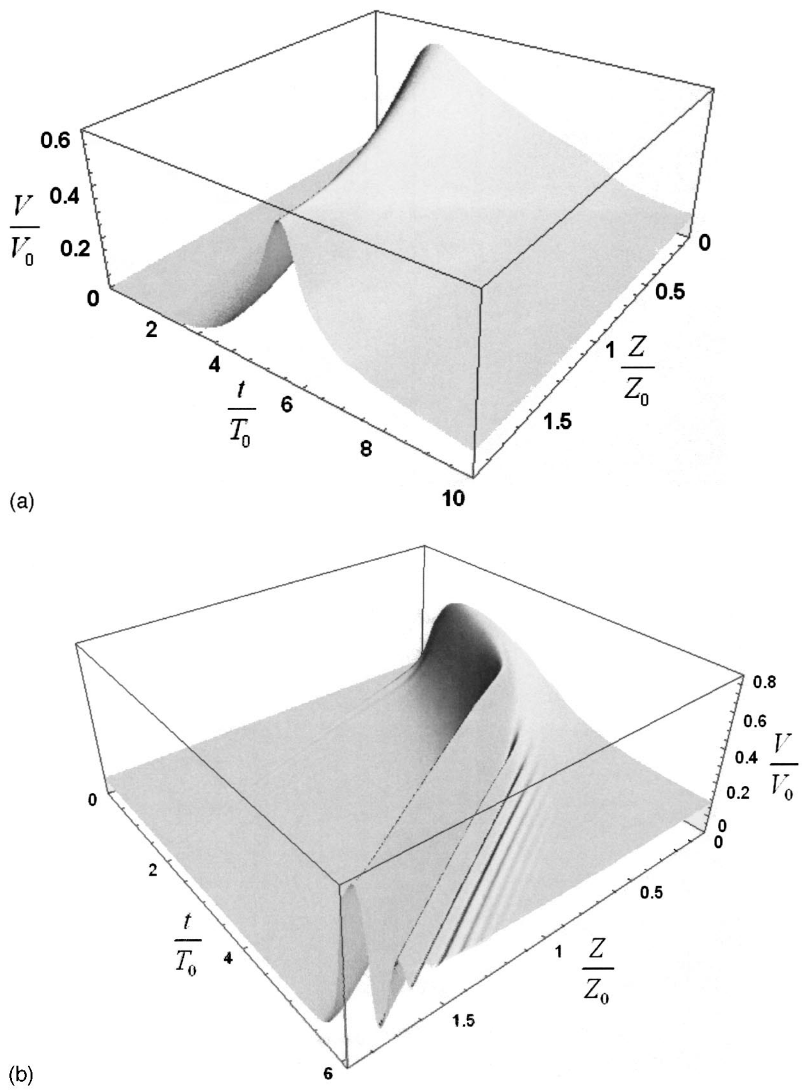

FIG. 2. (a) Self-induced transparency domain without Bloch oscillations. (b) Self-induced transparency domain with Bloch oscillations.

$=30$ GHz (at this frequency it is still possible to focus radiation with a metal wave guide) and amplitude $E_{\mathrm{inc}}^{\mathrm{max}}=157$ kV/cm case (A), $E_{\mathrm{inc}}^{\mathrm{max}}=392$ kV/cm case (B) slightly modulated to satisfy null initial conditions $E_{\mathrm{inc}}(t=0)=0$, $(dE_{\mathrm{inc}}/dt)(t=0)=0$. We should note that none of the results depends on the form of the modulation, which confirms universal transparency behavior and good convergence of the numerical code employed. From both pictures we clearly see time-space formation of the skin effect on the initial phase transforming to the propagating transparency domain. The propagation velocity of the transparency in case A almost equals phase velocity, in contrast with case B where $v_{\mathrm{domain}} \simeq 0.5 v_{\mathrm{phase}}$, which means that the effective domain size in case B, $h_{\mathrm{domain}} \simeq 41.6\ \mu\mathrm{m}=0.5 Z_0$, is half of that in case A, 83 $\mu\mathrm{m}=Z_0$. The current does not disappear completely after transition to the transparent state, which is in accordance with the result of the given field approximation in Ref. 5.

To excite a domain of induced transparency the incident flux must exceed some critical value, which depends on the superlattice parameters. For every value of SL width that exceeds the skin layer width the threshold amplitude of incident field is the same, which implies that induced transparency is a surface effect. For case A, $E_{\mathrm{thresh}}^{A}=94.2$ kV/cm: for case B, $E_{\mathrm{thresh}}^{B}=337.5$ kV/cm at $f_{\mathrm{inc}}=30$ GHz. Such a high amplitude of the electric field will be the main obstacle for experimental observation of self-induced transparency in a wide-miniband ($\Delta \simeq 0.1$ eV) planar superlattice. We should note that the question of validity of the hydrodynamic description of electron transport does not appear here. In both cases the internal field in the SL at the transition to the transparent state is five times smaller than the maximum allowed field for this description, $E_{\mathrm{max}}=(\Delta/ed)=200$ kV/cm. The threshold amplitude of the incident field decreases when the SL width becomes less than that of the skin layer. For example, for case B SL parameters and width are $h_{\mathrm{SL}}=h_{\mathrm{skin}}^{\mathrm{B}}/4.5=8.3\ \mu\mathrm{m}$, $E_{\mathrm{thresh}}=78.5$ kV/cm.

The main difference between transparency dynamics in cases A and B is clearly seen. In case B the transparency dynamics occurs with effective excitation of Bloch oscillation train with frequency $f_{\mathrm{Bloch}}=8.0 f_{\mathrm{scatt}}=8$ THz. Similar results have been recently reported in Ref. 13, where an equivalent screen current approximation in a lateral superlattice has been used and it was argued that the transition to the transparent state always occurs with Bloch oscillation train excitation if the superlattice is in a bistable state, and the incident flux with frequency $f_{\mathrm{inc}} \approx f_{\mathrm{scatt}}$ exceeds some critical value, which depends on surface concentration $n_s = n_e h_{\mathrm{SL}}$. Here we generalize this effect in the case of a planar superlattice and point out that the threshold incident flux for the onset of transparency without Bloch oscillation is less than the threshold incident flux for transparency with Bloch oscillation train excitation. For example, in Fig. 2(a) the transparency occurs without excitation of Bloch oscillation, but if we increase the incident-field amplitude up to $E_{\mathrm{inc}} = 160$ kV/cm, the transparency occurs with excitation Bloch oscillation similar to that in Fig. 2(b). Furthermore, due to the domain behavior of self-induced transparency the excitation of Bloch oscillations with maximum efficiency at the given superlattice parameters can occur only if the width of the SL exceeds the domain size at these parameters.

V. CONCLUSION

We have studied numerically the dynamics of self-induced transparency in a planar superlattice within a self-consistent electrodynamic formalism, which needs to be solved only within the superlattice. We demonstrated the possibility of self-induced transparency in the case in which the incident wave frequency is much lower than the electron-scattering frequency in the superlattice. We found domain-like character of the self-induced transparency, which can contain a coherent train of Bloch oscillations. The important properties of the domain are also discussed here.

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