Electron-density stratification in two-dimensional structures tuned by an electric field

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We describe a kinetic instability mechanism which leads to electron-density stratification. The spatial period of the arising space-charge and field configuration is found to be inversely proportional to the electric field and can be tuned by applied voltage. The instability has no interpretation in the framework of a traditional hydrodynamic approach, since it arises from the modulation of the electron distribution function in both the coordinate and energy space. The phenomenon can be observed in two-dimensional nanostructures at relatively low electron densities.

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Recent progress in microelectronics has largely been associated with controlled fabrication of low-dimensional semiconductor systems, so there is much interest in the transport properties of semiconductor nanostructures of both classical and quantum nature. The classical transport phenomena involving density perturbations with characteristic scales larger than the elastic mean free path l are usually described in the framework of a hydrodynamic approach. The kinetic effects are believed to dominate on smaller scales. These effects are treated theoretically in terms of the Boltzmann equation. In this paper, we propose a purely classical instability mechanism, which involves relatively smooth (compared to l) density distributions but, surprisingly, requires a kinetic description. The instability leads to electron-density stratification (EDS) with a spatial period considerably exceeding l. In contrast to usual current and density instabilities in semiconductors,[1–3] the electron dynamics in this instability cannot be described on the basis of local hydrodynamic parameters, such as electronic density, drift velocity, and temperature. This implies that the phenomenon is of essentially kinetic nature. The effect can be observed in two-dimensional (2D) nanostructures with one type of free carriers (here we consider 2D quantum wells of n type) at relatively high applied voltages, when acoustic-phonon scattering is incapable to balance the heating by an electric field. At such voltages, the energy balance in the system is mostly controlled by the gain in the electric field and the loss by optical-phonon scattering. In this case, a homogeneous solution of the Boltzmann equation corresponds to usual Ohm's law. However, this solution is unstable, since the behavior of the system on the scales smaller than the optical-phonon emission length L_0 is essentially nonequilibrium. We will show that the bunching of the electron distribution function (DF) on these scales leads to EDS with a spatial period of the order of L_0.

Closely related striation phenomena are widely known from gas discharge physics. The striated discharge has been observed since the time of Faraday and is regarded as one of the most typical discharges.[4–8] In spite of this, there is still no consistent theory of striation. It has been realized in the past decades that the hydrodynamic description of striated discharges is valid only for very high electron densities when the collisions between electrons are frequent enough to provide the maxwellization of the electron DF.[9,10] At lower electron densities occurring in typical gas discharges, the phenomenon is of essentially kinetic nature. The DF perturbation in the striations varies in both space and along the energy axis,[11–17] and it is impossible to parametrize it in terms of the perturbations of electron density and temperature. The kinetic striation mechanism was first analyzed in Refs. 11–13. It was argued in Ref. 11 that the necessary conditions for kinetic stratification are the following: (a) the momentum relaxation is much faster than the energy relaxation; (b) the energy relaxation is mostly controlled by the energy gain in the external field F_0 and strong inelastic collisions with a large fixed energy transfer W_0; (c) there should exist a mechanism of a weak continuous energy loss. In a spatially modulated potential $U(z)=-F_0 z+\delta U(z)$, $\delta U(z)=\delta U(z+L)$, these conditions provide[11–13] a resonant DF response at $L=L_0/m$, where $L_0=W_0/F_0$, $m=1,2,\ldots$. This ''resonant'' behavior corresponds to the widely known empirical Novak rule.[6–8,18] The idea was put forward[11] that under the conditions (a)–(c), an instability develops leading to EDS with the same periods L_0/m. In Refs. 11–13, the electron kinetics was analyzed only in a fixed electric potential profile $U(z)$. However, a complete analysis of instability requires a self-consistent calculation of the potential perturbation $\delta U(z,t)$. Since the discharge field depends crucially on the ion motion and on the complex ion generation processes, even a linear instability problem for a gas discharge plasma is still lacking a self-consistent solution.

In this paper we will demonstrate that the kinetic stratification can be, in principle, observed in low-dimensional semiconductor structures.[19] Moreover, it turns out that a relatively simple self-consistent analytical solution can be found for semiconductors. The main simplification follows from the fact that, in contrast to the gas discharge, a compensating positive charge is fixed and uniform. The stratification conditions (a)–(c) can be easily achieved in semiconductors. The momentum relaxation is usually fast compared to the energy relaxation. The requirements (b) and (c) are also usually satisfied, and the scattering by optical phonons with energy W_0 and the scattering by acoustic phonons work as strong inelastic and weak quasielastic energy relaxation mechanisms. We will show that this effect can be observed in 2D quantum wells of small thickness. The spatial periods of

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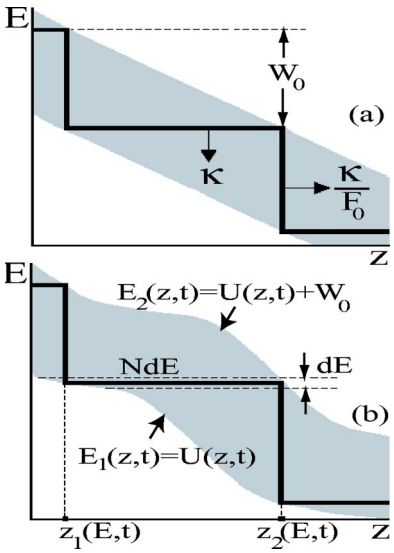

FIG. 1. (a) Motion of an electron in a uniform field, $U_0(z) = -F_0 z$. The diffusive ''staircase'' trajectories slowly drift in the z direction with velocity κ/F_0. (b) The motion in the modulated potential $U(z,t) = -F_0 z + \delta U(z,t)$.

arising EDS equal L_0/m and can be tuned by applied voltage. We assume that the lattice temperature T_0 (which is stabilized by a thermal bath) as well as the Fermi energy are small compared to W_0 (in what follows, we put $T_0 = 0$ for simplicity). The condition (b) requires that the electrons be ''hot'' and their energies be of the order of W_0, i.e., the electron gas is nondegenerate. We also assume that the electron concentration should be small and that the electron-electron collisions can be neglected.

Let us consider the motion of an electron in an external field F_0. The elastic scattering leads to a diffusion in the coordinate space. If the energy relaxation processes are ''turned off,'' the electron is infinitely heated by the field F_0, diffusing over the kinetic energy W. Evidently, this diffusion is strictly correlated with the diffusion in the coordinate space, since the full electron energy $E = W - F_0 z$ is conserved. In fact, an electron diffuses in the (z,W) space along the line $E = \text{const}$. The optical-phonon emission restricts the electron motion by a shell $0 < W < W_0$. Indeed, the kinetic energy increases in the process of diffusion with a constant total energy E. Reaching the point $W = W_0$, the electron loses the energy W_0 and starts a diffusive motion with a lower total energy $E - W_0$. The electron trajectory in the space (E,z) is shown in Fig. 1(a). The interaction with acoustic phonons plays the role of a friction force leading to a continuous loss of the electron energy with the rate κ due to spontaneous emission (since we have assumed that $T_0 = 0$). There are three time scales in our problem: the transport scattering time τ; the characteristic time of the electron heating by the electric field, $\tau_0 \sim L_0^2/D_0$ (here $L_0 = W_0/F_0$, $D_0 = W_0\tau/M$, and M is the electron effective mass); and the time W_0/κ characterizing the rate of energy loss due to the emission of acoustic phonons. We will assume that

$$\tau \ll \tau_0 \ll \frac{W_0}{\kappa}. \tag{1}$$

Inequality (1) implies that the acoustic-phonon scattering may be considered as a small perturbation. Due to the scattering, the ''staircase'' diffusive trajectories move slowly down (along the axis E) with velocity κ. This also means that the trajectories slowly drift with the velocity $s_0 = \kappa/F_0$ along the z axis [see Fig. 1(a)]. Since s_0 is inversely proportional to the applied field, one can say that the motion of trajectories demonstrates a negative differential mobility. It is well known that the negative differential mobility should lead to current instability,[1,2] but our case is more complicated than the Gunn instability, because one has to follow the motion of diffusive trajectories rather than of individual electrons. As for the latter, their average drift velocity obeys Ohm's law $v = F_0\tau/M$ and is much larger than s_0 [since inequalities (1) may be rewritten as $s_0 \ll v \ll \sqrt{W_0/M}$]. The fact that the instability can be observed in the Ohmic regime indicates that the effect is purely kinetic and cannot be described in terms of hydrodynamic parameters.

As far as the elastic collisions are dominant [see Eq. (1)], the DF is almost isotropic,[20] $f(z,W,\varphi,t) \approx f_i(z,W,t) + f_a(z,W,t)\cos(\varphi)$. Here f_i is the isotropic part of the DF, $f_a\cos(\varphi)$ is the small anisotropic correction, and φ is the angle between the electron velocity and applied field. Denote $J(z,W,t) = \sqrt{W/2M} f_a$. The equations for f_i and J can be written as[1,2,20]

$$J = -D(W)\left(\frac{\partial f_i}{\partial z} + F\frac{\partial f_i}{\partial W}\right), \tag{2}$$

$$\frac{\partial f_i}{\partial t} + \frac{\partial J}{\partial z} + F\frac{\partial J}{\partial W} = \kappa\frac{\partial f_i}{\partial W}, \tag{3}$$

where $D(W) = W\tau/M$ is an energy-dependent diffusion coefficient. For simplicity, we assume that τ and κ are energy independent (see below). The boundary conditions (BC) for Eqs. (2) and (3) read as $f_i|_{W=W_0} = 0$, $(FJ - \kappa f_i)|_{W=0} = FJ|_{W=W_0}$. Here $F(z,t) = -\partial U(z,t)/\partial z$, and $U(z,t)$ is potential energy, which includes both the self-consistent potential created by the electrons and the external potential $U_0(z) = -F_0 z$. The condition $f_i|_{W=W_0} = 0$ corresponds to the limit of a very strong interaction with optical phonons (''black wall'' condition). The second boundary condition is related to the conservation of the number of particles involved in inelastic collisions.[21] Equations (2) and (3) at $\kappa = 0$ have a homogeneous stationary solution $J = J_0 = n_0 v/W_0$, $f_i = f_{i0} = (n_0/W_0)\ln(W_0/W)$, where n_0 is the stationary electron concentration (we assume the following normalization $\int_0^{W_0} f_{i0} dW = n_0$). Since $\int_0^{W_0} J_0 dW = n_0 v$, the stationary solution corresponds to the Ohmic regime. According to Ref. 11, we rewrite Eqs. (2) and (3) in the variables (E,z,t),

$$J = -D(E-U)\frac{\partial f_i}{\partial z}, \tag{4}$$

$$\frac{\partial f_i}{\partial t} + \frac{\partial J}{\partial z} = \left(\kappa - \frac{\partial U}{\partial t}\right)\frac{\partial f_i}{\partial E}, \tag{5}$$

where $E=W+U(z,t)$ is the full energy of a particle. The motion of a particle in the space of new variables is restricted by the curves $E=E_1(z,t)=U(z,t)$, $E=E_2(z,t)=W_0+U(z,t)$ [See Fig. 1(b)]. BC can be rewritten as $f_i|_{E=E_2(z,t)}=0$, $(FJ-\kappa f_i)|_{E=E_1(z,t)}=FJ|_{E=E_2(z,t)}$. Since the total energy of an electron changes slowly (with the characteristic time W_0/κ), it will be useful to introduce the electron-density distribution along the E axis:

$$N(E,t)=\int_{z_1(E,t)}^{z_2(E,t)} dz f_i, \tag{6}$$

where $z_1(E,t)$, $z_2(E,t)$ are the inverse functions of $E_1(z,t)$, $E_2(z,t)$, respectively, and the quantity NdE represents the number of electrons along the ''staircase'' trajectories restricted by E and $E+dE$ [see Fig. 1(b)]. The stationary value of N is given by $N_0=n_0/F_0$. Introducing the notation $I_E(t)=J(z_2,E,t)$ (the stationary I_E being equal to J_0), we find from Eq. (5)

$$J(z_1,E,t)=I_E(t)+\int_{z_1}^{z_2} dz\left[\frac{\partial f_i}{\partial t}-\left(\kappa-\frac{\partial U}{\partial t}\right)\frac{\partial f_i}{\partial E}\right].$$

Taking into account that $\partial z_1/\partial E=-1/F(z_1,t)$, $\partial z_1/\partial t=(\partial U/\partial t)/F(z_1,t)$, and with BC, we get the continuitylike equation that governs the electron motion along the total energy axis:

$$\frac{\partial N}{\partial t}-\frac{\partial}{\partial E}\left(N\left[\kappa-\left\langle\frac{\partial U}{\partial t}\right\rangle\right]\right)=I_{E+W_0}(t)-I_E(t). \tag{7}$$

Here the angular brackets mean the averaging over z,

$$\left\langle\frac{\partial U}{\partial t}\right\rangle=\frac{1}{N}\int_{z_1}^{z_2} dz\frac{\partial U}{\partial t}f_i(z,E).$$

Next, we consider the deviations from the stationary solution in the linear approximation. A small periodic potential molulation along the coordinate, $U-U_0=\delta U_q\exp(-i\omega t+iqz)$, induces the energy dependence of the quantities I_E, N in the form $I_E-J_0=\delta I_q\exp(-i\omega t-iqE/F_0)$, $N-N_0=\delta N_q\exp(-i\omega t-iqE/F_0)$. We will demonstrate that for $q\approx q_m=\pm 2\pi m/L_0$ (where $m=1,2,\ldots$), the imaginary part of ω is positive, which implies that the stationary solution is unstable. For $q=q_m$, the solution is a periodic function of energy and $I_{E+W_0}(t)=I_E(t)$. Then the linearization of Eq. (7) yields

$$\omega_m=\frac{\kappa}{F_0+\Delta F_m}q_m, \tag{8}$$

where $\Delta F_m=-iq_mN_0\langle\delta U_m\rangle/\delta N_m$. We see that the physics of the problem is governed by the only parameter $\langle\delta U_m\rangle/\delta N_m$ (the subscript m implies that all quantities are taken at $q=q_m$). The instability $[\mathrm{Im}(\omega_m)>0]$ occurs at $\mathrm{Re}\langle\delta U_m\rangle/\delta N_m>0$. In order to find this parameter, one should go beyond the averaged kinetic equation (7) and solve Eqs. (4) and (5) together with the Poisson equation. As long as Eq. (8) is proportional to a small parameter κ, one can simplify the solution of Eqs. (4) and (5) assuming $\kappa=0$ and neglecting $\partial f_i/\partial t$ and $\partial U/\partial t$, since Eq. (8) provides $\omega\sim\kappa$. Then Eq. (5) reduces to $\partial J/\partial z=0$, which implies $J(z,E,t)=I_E(t)$. As a result, Eq. (4) yields

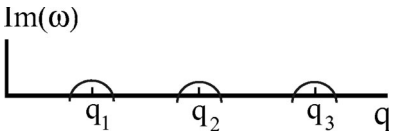

FIG. 2. The instability increment as a function of q. Instability regions correspond to $q\approx q_m=\pm 2\pi m/L_0$.

$$f_i(z,E,t)=I_E(t)\int_z^{z_2}\frac{dz'}{D[E-U(z',t)]}. \tag{9}$$

The small variation of the distribution function δf_i can be found by linearizing this equation with respect to δI_m, δU_m, the functions z_1 and z_2 being also linearized. The Poisson equation gives us the proportionality between δU_m and the Fourier transform δn_m of the variation of the electron concentration $\delta n(z,t)=\delta\int_{E_1}^{E_2}dE f_i$. Here the variation includes the variation of δf_i as well as that of the integration limits $E_1(z,t)$ and $E_2(z,t)$. In this paper, we restrict ourselves to the case of a 2D semiconductor quantum well, assuming that the dielectric constant ϵ is the same inside and outside the quantum well. For such a structure, $\delta U_m=(2\pi e^2/\epsilon|q_m|)\delta n_m$. Using now the linearized equations (6) and (9), one can find the relation between δN_m and δU_m. To calculate the parameter ΔF_m, one should also average the variation of the potential $\delta U_m\exp(iq_mz)$ with the stationary distribution function $f_{i0}(E,z)$, since we solve the problem in a linear approximation. After cumbersome but rather straightforward calculations, we eventually get

$$\omega_m=s_0q_m+i\frac{\kappa}{W_0}\frac{\lambda_m|\alpha_m|^2}{1+\lambda_m\alpha_m}, \tag{10}$$

where $\alpha_m=\int_0^{L_0}dy[1-\exp(iq_my)]/y$, $\lambda_m=e^2n_0/m\epsilon F_0$. It is easy to check that $\mathrm{Im}(\omega_m)>0$ for any m. Thus, the stationary solution is unstable for $q=q_m$. For a low field, $\lambda_m\gtrsim 1$, the increment is field independent, $\mathrm{Im}(\omega_m)\sim\kappa/W_0$. One can show that for $q\approx q_m$, the spectrum reads as $\omega(q)=\omega_m+(q-q_m)v-iD^*(q-q_m)^2/4$, where

$$D^*=D_0\left(1+\frac{\lambda_m\alpha_m^*}{1+\lambda_m\alpha_m}\frac{2}{iq_mL_0}\right)$$

(we have neglected the small corrections of the order of κ to v and D^*). This implies that the instability exists only in the vicinity of q_m (see Fig. 2), leading to EDS with periods L_0/m.

Next, we discuss the possibility of observing the effect in a 2D n-type quantum well with a relatively large amount of short-range impurities, which determine the electron-transport time τ. In this case τ is energy independent, as was assumed above. The instability increment is proportional to the rate of the energy loss, κ, which can be calculated in the full analogy with the 3D case.[2] For an infinitely deep rectangular quantum well of width a, the calculations yield

$\kappa = C_0^2 \pi^2 M / \rho a^3 \hbar$. Here C_0 is a deformation potential constant, ρ is the crystal density. This result justifies our assumption that κ does not depend on W. Also, we see that κ rapidly increases with decreasing a. The law $\kappa \sim a^{-3}$ can be understood from simple estimates. The momentum transfer from an electron to a phonon in the direction normal to the quantum well is of the order of $\hbar/a$. The emission of the phonon leads to the energy loss $\sim \hbar S/a$, where S is the sound velocity. The rate of the energy loss via the emission of longitudinal phonons may be neglected due to a small factor $k_{||}a$, where $k_{||}$ is the in-plane wave vector of the 2D electron. We find that κ is proportional to the integral over dq_z of the product of the energy loss $\hbar S/a$ and the squared matrix element $V_q^2 \sim q \sim 1/a$ The upper limit of the integral is of the same order, $1/a$, yielding $\kappa \sim a^{-3}$. This means that the instability is more likely to be observed in thin 2D structures. On the other hand, the instability is suppressed by electron-electron collisions, which leads to the DF maxwellization. Thus, the instability condition is given by $\mathrm{Im}(\omega_m) > 1/\tau_{ee}$, where τ_{ee} is the characteristic time of electron-electron scattering. A crude estimation of τ_{ee} for hot electrons with the characteristic energy W_0 gives $\tau_{ee}^{-1} \sim e^4 n_0 / \epsilon^2 \hbar W_0$. Having in mind Eqs. (1) and (10), one can see that for low electron densities $e^4 n_0 / \epsilon^2 \hbar < \kappa$, there is a certain field range, in which the instability can be observed. Simple estimations for GaAs and GaN show that for thin quantum wells, $a \approx 30$ Å, the electron concentration is restricted by a small but quite reasonable value of $\sim 10^{10}$ cm^{-2}.

The development of the instability must result in tunable current oscillations in the terahertz range, which may lead to important practical applications (see Ref. 22 for a review). One can roughly estimate the oscillation frequency as $\nu \sim dw(q)/dqL_0 \sim \tau_0^{-1} \sim F_0^2$. We see that ν can be tuned by applied voltage. It follows from Eq. (1) that the oscillations in the terahertz range are possible at $\tau \lesssim 10^{-12}$ s.

In conclusion, we have presented a self-consistent linear theory of kinetic stratification. We have shown that the spatial periods of the strata equal $W_0/F_0 m$ and can be tuned by applied voltage.

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