

Current-voltage characteristics and the zero-resistance state in a two-dimensional electron gas

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We study the current-voltage characteristics (CVC) of two-dimensional electron gases with microwave-induced negative conductance in a magnetic field. We show that due to the Hall effect, strictly speaking, there is no distinction between N- and S-shaped CVC's. Instead, the observed CVC depends on the experimental setup with, e.g., an N-shaped CVC in a Corbino disk geometry corresponding to an S-shaped CVC in a Hall bar geometry in a strong magnetic field. We argue that instabilities of homogeneous states in regions of negative differential conductivity lead to the observation of zero resistance in Hall bars and zero conductance in Corbino disks, and we discuss the structure of current and electric-field domains.

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An interesting effect has been observed in recent papers:^{1,2} the resistance of a two-dimensional electron gas (2DEG) subjected to microwave irradiation drops to zero in some interval of an applied magnetic field B . This zero-resistance state (ZRS) is achieved at low magnetic fields where at the measurement temperatures, Shubnikov-de Haas oscillations are weak. Possible mechanisms for this phenomenon are discussed in a number of papers.³⁻⁹ It was shown in Refs. 4,7,8 that in the presence of irradiation and a magnetic field, the conductivity σ_{xx} may become negative in a weak electric field E_x . In addition, using a simple model, the authors of Ref. 7 have calculated the current-voltage characteristic $I(V)$ of an irradiated system with a density of states periodic in energy ε . They have demonstrated that not only regions with negative conductance G , but also regions with positive G , may have negative differential conductance $G_d = dI/dV$ on the current-voltage characteristic (CVC) curve. According to Ref. 7, the $I(V)$ curve is N shaped (or to be more exact, it can be regarded as a chain of the N-shaped CVC's), similar to the one shown in Fig. 1. This means that three voltages correspond to one current. While the authors of Ref. 7 do not show in detail how the ZRS arises in their approach, a phenomenological model for the ZRS was proposed in Ref. 5. Here, the authors assumed that the resistivity ρ_{xx} depends on the current density j in such a way that the dependence of electric field on current density, $E_x = \rho_{xx}(j)j_x$, is N shaped. This means that the inverse dependence $j_x(E_x)$ is S-shaped (three currents correspond to one voltage, cf. Fig. 3). It was shown that the states corresponding to the part of the CVC with negative differential conductance are unstable. The authors assumed that a stratification of the uniform current density occurs as a result of this instability and two current domains with opposite current directions arise in the Hall bar. With increasing total current I_x , the relative width of the two domains changes and the electric field E_x remains zero, that is, a ZRS is established in the system.

We note that the results presented in Refs. 4,7,8,5 are not completely new. Many years back, the absolute negative conductance was predicted in Refs. 10-12 (2DEG and 3DEG in a strong magnetic field under irradiation) and in Ref. 13 (a superlattice under irradiation). The instability of systems

with N- or S-shaped CVC in the absence of a magnetic field and the possible types of the electric-field or the current domains arising as a result of this instability were also studied three decades ago (see, for example, Ref. 14 and references therein). The instability of a system with S-shaped CVC in a magnetic field was studied in Ref. 15. As a result of the instability of a homogeneous state, in systems with N-shaped CVC, domains of constant electric field arise, whereas in systems with S-shaped CVC domains with constant current density appear. In the absence of a magnetic field, it was shown that a wide domain of constant electric field leads to a horizontal line on the CVC (zero differential conductance state), whereas a wide current domain in the sample leads to the appearance of a vertical line on the CVC (zero differential resistance state).^{14,16}

Currently, there does not appear to be a consensus as to what kind of CVC, N or S shaped, will lead to the ZRS in the presence of a magnetic field. In the present paper, we address this issue and analyze the form of the CVC as well as the

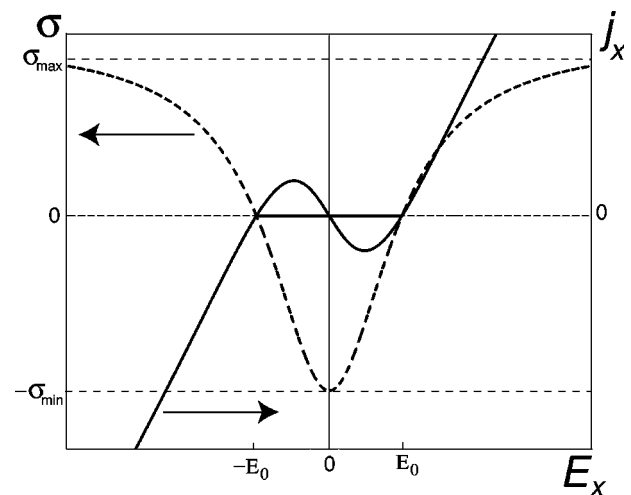


FIG. 1. Assumed N-shaped CVC (solid line) and the corresponding dependence of the conductivity σ on the electric field E_x (dashed line). Both the current component j_x and the conductivity σ are zero at $E_x = \pm E_0$. The solid horizontal line ($j_x = 0, |E_x| < E_0$) corresponds to a measurable part of the CVC in the Corbino disk measurements.

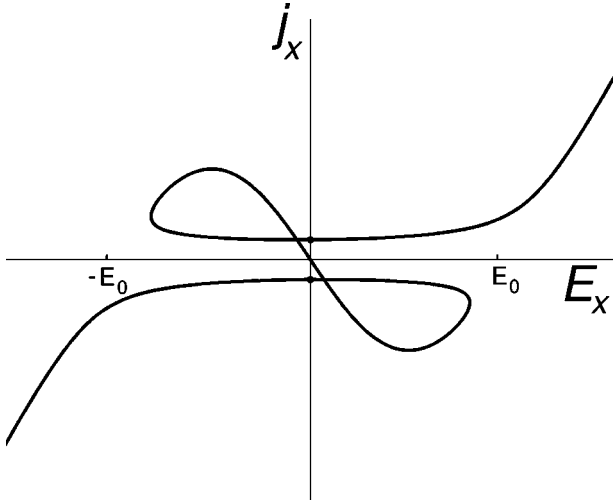


FIG. 2. The CVC in the case of a weak magnetic field. Here, $\sigma_{min} = \sigma_{max} = 10\sigma_H$. The curve intersects the line $E_x = 0$ at the points $j_x = 0, \pm\sigma_H E_0$.

type of domains in different experimental setups. We consider a 2DEG in a magnetic field B and assume that the conductivity $\sigma_{xx} \equiv \sigma$ depends on the electric field E , and the Hall conductivity $\sigma_{xy} \equiv \sigma_H$ is independent of the electric field E . We are not going to analyze the origin of this type of nonlinear behavior and just note that this kind of dependence is discussed in Ref. 7. In a two-dimensional system, the electric field E has two components $E_{x,y}$ so that the conductivity σ depends on $E = (E_x^2 + E_y^2)^{1/2}$. If one of two components vanishes identically (for example, $E_y = 0$), as it happens in measurements on the Corbino disk, then $j_x = \sigma(E_x)E_x$. We assume that the $\sigma(E_x)$ dependence corresponds to an N-shaped CVC (see Fig. 1). We will show that the form of the CVC in experiments on a Hall bar depends strongly on the magnetic field, becoming S-shaped in the limit of strong Hall effect: $\sigma_H \gg |\sigma|$. This regime $\omega_c \tau_i \gg 1$, where ω_c is the cyclotron frequency and τ_i is the momentum relaxation time, is relevant to the experiments under consideration. We note in passing that in the experiments, due to the predominant forward-scattering nature of the disorder, the single particle lifetime τ_s is much shorter than τ_i .¹ In the following, we will discuss the form of the CVC and the structure of nonhomogeneous states (electric-field and current domains) in different experimental setups.

We consider a 2DEG in a magnetic field, assuming first that the current density j and the electric field E are homogeneous in the sample. The current density $j_{x,y}$ depends on the electric field $E_{x,y}$ via the components σ and σ_H of the conductivity tensor

$$j_{x,y} = \sigma(E)E_{x,y} \pm \sigma_H E_{y,x}. \quad (1)$$

If the sample has the shape of a Corbino disk, the electric field E_y in the azimuthal direction is zero, and the CVC shown in Fig. 1 is N shaped. If we consider a sample in the shape of a Hall bar, then, in general, both components $E_{x,y}$ are finite. However, the current component j_y in the Hall bar is zero and, hence, we can express the component E_y through

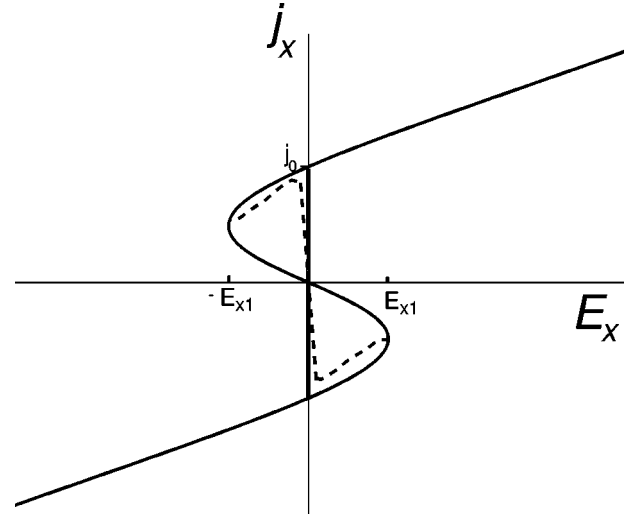


FIG. 3. The S-shaped CVC (solid line) for the case of large magnetic field. Here, $\sigma_{min} = \sigma_{max} = 0.1\sigma_H$. The curve intersects the line $E_x = 0$ at the points $j_x = 0, \pm\sigma_H E_0$. The solid vertical line ($E_x = 0, |j_x| < j_0$) corresponds to the stable measurable state on a Hall bar. For a discussion of the dashed line, see text.

$E_x : E_y = E_x \sigma_H / \sigma(E)$. Inserting it into the expression for the current density j_x , we get the dependence of j_x on E_x ,

$$j_x = \frac{\sigma^2(E) + \sigma_H^2}{\sigma(E)} E_x. \quad (2)$$

For the component E_x of the electric field, we get

$$|E_x| = \frac{|\sigma(E)|}{\sqrt{\sigma^2(E) + \sigma_H^2}} E. \quad (3)$$

Equations (2) and (3) determine the form of the CVC for the Hall bar. We plot the resulting CVC in Figs. 2 and 3 for the case of low and high magnetic field, respectively. We see that in a weak field, $\sigma_{min,max} \gg \sigma_H$, the CVC for almost all currents follows the form of the $I(V)$ characteristic shown in Fig. 1. However at low currents, $j_x \approx \sigma_H E_0$, the shape of the CVC changes drastically. This kind of the CVC cannot be assigned either to the N- or S-shaped types of the CVC. Associated with the loops is a change of the Hall angle from small values at large negative j_x to a value close to π near $j_x = E_x = 0$ and back to small values at large j_x .

With increasing magnetic field, the form of the CVC changes, and finally at $\sigma_H \gtrsim \sigma_{min,max}$, is transformed into an S-shaped $I(V)$ characteristic (Fig. 3). The $I(V)$ curve crosses the y axis at the currents $j_x = 0, \pm j_0$, where $j_0 = \sigma_H E_0$. The characteristic field E_{x1} at which $dj_x(E_x)/dE_x = \infty$ is determined by $E_x = E_{x1}(E_1)$, where E_1 satisfies the equation $E_1 d\sigma(E_1)/dE_1 = -\sigma(E_1)$ and is of the order of $E_{x1} = a E_0 \sigma_{min}/\sigma_H$ [$a = 2/3^{3/2}$ for a parabolic dependence of $\sigma(E)$ on E at small E]. The corresponding value of the current is $j_x(E_{x1}) = j_0/\sqrt{3}$ in the same approximation. In the limit of high magnetic field, $\omega_c \tau_i \gg 1$, the Hall angle does not change appreciably and oscillates around $\pi/2$.

In order to check the applicability of this model to the experiments,^{1,2} it would be interesting to measure the CVC

in two configurations. If the CVC is measured on a Corbino disk, the $I(V)$ curve should have the form shown in Fig. 1, that is, the N-shaped form. However, due to instability of a homogeneous state, a horizontal line ($j_x=0$) should be observed in the interval $-E_0 < E_x < E_0$ (zero-conductance state). In principle, one can also observe the part with negative absolute and differential conductance if short pulses of the voltage are applied to the sample. This method of measuring the CVC of a homogeneous sample was used in studies of the Gunn effect a long time ago (see Ref. 14 and references therein). The duration of the voltage pulses should be shorter than a characteristic time for the buildup of the electric-field domains, but longer than the period of the ac field applied to the sample. In dc measurements, the domains of the electric field $E_x(x)$ should appear as it happens, for example, in the Gunn effect. In the system under consideration, the electric field has different direction in each domain. A variation of the total voltage leads to a change of the relative widths of the domains; the current remains negligible (zero-conductance state). Due to the Hall effect, the electric-field domains also carry current densities. Since the strength of the electric field is given by the critical field E_0 , where the Hall angle is $\pi/2$, the associated currents always flow perpendicular to the direction of the electric field. Thus, in a magnetic field, current and electric-field domains exist simultaneously.

If the CVC is measured on a Hall bar, it has the form shown in Fig. 3 with a vertical line in the interval $-j_0 < j_x < j_0$ (zero-resistance state). The part of the $I(V)$ curve with negative conductance can also be measured if one applies short current pulses. The pulsed measurements would allow one to determine a characteristic time required for the buildup of the current filaments. It is worth noting that a switching of the system into a nonhomogeneous state has much in common with a first-order phase transition (see Ref. 14). Therefore, even in dc measurements it is possible, in principle, to measure “superheated” and “supercooled” states which correspond to parts of the CVC with negative absolute, but positive differential conductance. As in the case of a first-order phase transition, the result of a particular measurement will depend on factors that influence the nucleation of a new phase (homogeneity of the sample, boundary conditions, etc). If the current filament is created in the middle of the sample, the CVC has the form shown in Fig. 3 by the dashed line.¹⁴ This means that the observed CVC may have a hysteresis: with decreasing current, it goes along the upper part of the CVC, passes through the intersection point j_0 , and reaches a point $j(E_{x1})$. When j_x decreases further, a jump to a curve shown by the dashed line occurs and the CVC follows this line to the region of negative currents. In this configuration, as it is noted in Ref. 5, the current domains (current filaments) with opposite current directions are elongated in the x direction. The stratification of the current density, as we mentioned above, is natural in systems with S-shaped $I(V)$ characteristic. In the presence of a strong magnetic field, the electrical-field domains arise simultaneously in the transverse direction (the Hall field domains).

In order to find the form of the electric-field and current domains, one needs to derive the full set of equations gov-

erning the space and time dependence of all quantities of interest (electric field, current density, etc.). At present there is no such theory. Here, we suggest a simple phenomenological model that allows one to map this problem to the problem considered earlier. We assume that the Coulomb screening length is less than the thickness of the 2DEG. In this case, the relation between the concentration of electrons n and the electric field \mathbf{E} is given by the Poisson equation

$$\nabla \cdot \mathbf{E} = \frac{4\pi e}{\epsilon} (n - n_0), \quad (4)$$

where n_0 is the electron density in the uniform case. Consider the case of a Corbino disk. Then $E_y = 0$, and the component j_x is given by

$$j_x = \sigma(E_x)E_x - eD\partial_x n + \frac{\epsilon}{4\pi} \partial_t E_x. \quad (5)$$

Here, the second term is the diffusion current and the third term is the displacement current. This simplified model cannot be applied directly to the real experimental situation because it implies a local approximation for the current density, that is, for example, the screening length $l_{\text{scr}} = \sqrt{D/(4\pi\sigma_{\text{max}}/\epsilon)}$ is assumed to be longer than the mean free path (obviously this is not the case in real samples). However, this model in our opinion captures the main constituent of the system qualitatively. One can eliminate the electron density from Eqs. (4) and (5) and obtain for a stationary or steadily moving domain of the electric field

$$\frac{4\pi}{\epsilon} j_x = \frac{4\pi}{\epsilon} \sigma_0(E_x)E_x - D\partial_{\xi\xi}^2 E_x - \partial_{\xi} E_x \left(s - \frac{\sigma_0(E_x)E_x}{en_0} \right). \quad (6)$$

Here, $\sigma_0(E_x) = \sigma(E_x)n_0/n$ and $\xi = x - st$. We assumed that $E_x(x, t) = E_x(\xi)$. One can see that Eq. (5) agrees almost completely with an equation describing the Gunn effect (see Ref. 14). The only difference is that the $\sigma_0(E_x)$ dependence is different in both cases; in particular, the symmetry point of the N-shape part of the CVC in this case is located at the origin of the coordinates (E_x, j_x) . This means that the topology of solutions in the $(E_x, \partial_{\xi} E_x)$ plane remains unchanged and, therefore, many conclusions about CVC and forms of the electric-field domains in the Gunn effect are valid for our system (however, domains in our system are almost motionless). For instance, wide stationary domains with the electric field equal to $\pm E_0$ are described by the phase-space trajectory (the separatrix)

$$\frac{1}{r} \left[\mathcal{E}_{\zeta} - \frac{1}{r} \ln(1 + r\mathcal{E}_{\zeta}) \right] = U(1) - U(\zeta), \quad (7)$$

where $\zeta = \xi/l_{\text{scr}}$, $r = E_0/(el_{\text{scr}}n_0)$, $\mathcal{E} = E_x/E_0$ is the normalized electric field, $U(\mathcal{E}) = \int_0^{\mathcal{E}} d\mathcal{E}_1 j(\mathcal{E}_1)/j_m$, $j_m = \sigma_{\text{max}}E_0$, and $j(\mathcal{E}_1) = \sigma(\mathcal{E}_1)\mathcal{E}_1$ is the CVC. The screening length l_{scr} in our model determines the width of the domain wall. We will not

study here in detail the form of domains arising in the system. Note only that this analysis can be carried out similarly to that presented in Refs. 14 and 16.

In summary, we have studied the form of the current-voltage characteristic $I_x(V_x)$ in the presence of nonlinearity and magnetic field. The Hall effect removes the distinction between N- and S-shaped CVC's characteristic for zero-field systems. Instead, the shape of the CVC depends on boundary conditions imposed by the experimental setup. We showed that an N-shaped CVC in the Corbino disk geometry corresponds for sufficiently strong magnetic fields to an S-shaped

CVC in the Hall bar geometry. Homogeneous states with negative differential conductivity are unstable to the formation of domains. Again, due to the Hall effect these domains are characterized by both a constant electric field and a constant current density. In these domains, the Hall angle is 90° . As a result, we expect the measurements of zero conductance in Corbino disks and zero resistance in Hall bars to be manifestations of the same physical phenomenon.

After the preparation of this manuscript we became aware of Ref. 17 that reports the observation of an apparently zero-conductance state in a Corbino disk geometry.

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- ¹R.G. Mani, J.H. Smet, K. von Klitzing, V. Narayanmurti, W.B. Johnson, and V. Umansky, *Nature (London)* **420**, 646 (2002); cond-mat/0303034 (unpublished).
- ²M.A. Zudov, R.R. Du, L.N. Pfeiffer, and K.W. West, *Phys. Rev. Lett.* **90**, 046807 (2003); M.A. Zudov, R.R. Du, J.A. Simmons, and J.R. Reno, *Phys. Rev. B* **64**, R201311 (2001).
- ³J.C. Phillips, cond-mat/0212416 (unpublished); cond-mat/0303181 (unpublished); cond-mat/0303184 (unpublished).
- ⁴A.C. Durst, S. Sachdev, N. Read, and S.M. Girvin, cond-mat/0301569 (unpublished).
- ⁵A.V. Andreev, I.L. Aleiner, and A.J. Millis, cond-mat/0302063 (unpublished).
- ⁶P.W. Anderson and W.F. Brinkman, cond-mat/0302129 (unpublished).
- ⁷J. Shi and X.C. Xie, cond-mat/0302393 (unpublished); cond-mat/0303141 (unpublished).
- ⁸A.A. Koulakov and M.E. Raikh, cond-mat/0302465 (unpublished).
- ⁹S.A. Mikhailov, cond-mat/0303130 (unpublished).
- ¹⁰V.I. Ryzhii, *Fiz. Tverd. Tela* **11**, 2577 (1969) [*Sov. Phys. Solid State* **11**, 2078 (1970)]; V.I. Ryzhii, R.A. Suris, and B.S. Shchamkhalova, *Fiz. Tekh. Poluprovodn.* **20**, 2078 (1986) [*Sov. Phys. Semicond.* **20**, 1299 (1987)].
- ¹¹V.F. Elesin and E.A. Manykin, *Fiz. Tverd. Tela* **8**, 3621 (1966) [*Sov. Phys. Solid State* **8**, 2891 (1967)]; V.F. Elesin, *Zh. Eksp. Teor. Fiz.* **55**, 792 (1965) [*Sov. Phys. JETP* **28**, 410 (1969)].
- ¹²V.V. Vyurkov and P.V. Domnin, *Fiz. Tekh. Poluprovodn.* **13**, 1951 (1979) [*Sov. Phys. Semicond.* **13**, 1137 (1979)].
- ¹³V.V. Pavlovich and E.M. Epshtein, *Fiz. Tekh. Poluprovodn.* **10**, 2001 (1976) [*Sov. Phys. Semicond.* **10**, 1196 (1976)].
- ¹⁴A.F. Volkov and Sh.M. Kogan, *Usp. Fiz. Nauk* **96**, 633 (1968) [*Sov. Phys. Usp.* **11**, 881 (1969)].
- ¹⁵Sh.M. Kogan, *Fiz. Tverd. Tela* **10**, 1536 (1968) [*Sov. Phys. Solid State* **10**, 1213 (1969)].
- ¹⁶A.F. Volkov and Sh.M. Kogan, *Zh. Eksp. Teor. Fiz.* **52**, 1647 (1967) [*Sov. Phys. JETP* **25**, 1095 (1967)].
- ¹⁷C.L. Yang, M.A. Zudov, T.A. Knuutila, R.R. Du, L.N. Pfeiffer, and K.W. West, cond-mat/0303472 (unpublished).