# Resonant photon-assisted tunneling between independently contacted quantum wells

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The line shape and magnitude of the low-temperature, photon-assisted tunneling current is examined in a nonideal double-quantum-well structure with independent contacts. We include inhomogeneous broadening effects due to processes such as heterointerface roughness or nonuniform doping and scattering by the short-range potential. We show that the interplay between the inhomogeneous and homogeneous broadening mechanisms can modify the line shape of the resonant tunnel current between a predominantly Gaussian and a Lorentzian peak shape. Numerical estimates of the linewidths and comparison between amplitudes of the dark and photon-assisted contributions to the tunneling current are presented. We conclude that the integrated total photon-assisted tunneling current is maximized relative to the dark current when the inhomogeneous broadening is minimized, while the peak current value depends both on the homogeneous and inhomogeneous scattering processes.

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## I. INTRODUCTION

Photon-assisted tunneling of electrons has been studied in superlattices,<sup>1</sup> in double-barrier structures,<sup>2</sup> and in tunnelcoupled low-dimensional heterostructures: double quantum wells (DQW's),<sup>3</sup> wires,<sup>4</sup> and dots.<sup>5</sup> Recently, photon-assisted tunneling processes have also been considered between independently contacted quantum wells<sup>6</sup> (see Fig. 1). This last system is related to the 2D-2D tunneling transistor demonstrated in Ref. 7, so that this case may be considered as a phototunneling transistor. The results of Ref. 6 were derived for the high-temperature case where the shape of the tunneling current peak is dominated by homogeneous broadening processes. Here, we consider the low-temperature case when nonscreened large-scale variations of the coupled levels<sup>8,9</sup> lead to an inhomogeneous broadening of the tunneling current peak. The resonant tunneling current is expected to increase substantially in the low-temperature range, making this case of interest for tunable detection of THz radiation.

In the present paper, we examine the line shape of the photon-induced resonances in the tunneling current in nonideal DQW's with independent contacts to each QW. The scheme of photoinduced tunneling is shown in Fig. 1(a). The inhomogeneous broadening of the resonant tunneling current induced by the large-scale variations of levels, which cannot be screened completely (see Refs. 8 and 10), is taken into account in addition to the usual homogeneous broadening due to short-range scattering. The interplay between these two mechanisms of broadening leads to a change in the line shape of the photoinduced tunneling current from a Lorentzian to a predominantly Gaussian shape. In this case, the tail of the dark current is determined by the homogeneous broadening while the amplitude of the photon-assisted peak is determined by both mechanisms of broadening. Numerical estimates of the linewidth and comparison of the dark and photon-assisted current are performed for the cases of inhomogeneous broadening due to large-scale random variations in the widths of the upper (u) and lower (l) QW's and due to nonuniform doping [see the band diagram in Fig. 1(b)].

The paper is organized in the following way. In Sec. II we

introduce and evaluate the general formula for the photonassisted tunneling current, based on a weak interwell tunneling approach. Numerical estimates of the line shape of the resonant peak are presented in Sec. III, using a Green's function approach in the Wigner representation. The discussion of the assumptions used and the conclusions are given in the last section. We present in two appendixes a brief evaluation of the Wigner representation for the single-particle Green's function, and we discuss the expression for the inhomogeneous broadening due to heterointerface roughness and to nonuniform doping.

#### **II. PHOTON-ASSISTED TUNNELING**

We describe the interwell tunnel coupling using a oneelectron matrix Hamiltonian written in a basis of u and lorbitals as follows (see Ref. 8 for details):

$$\begin{vmatrix} \Delta_t / 2 + \hat{p}^2 / 2m + U_{u\mathbf{x}} & T \\ T & -\Delta_t / 2 + \hat{p}^2 / 2m + U_{l\mathbf{x}} \end{vmatrix} = \hat{h}_t + T \hat{\sigma}_x,$$
(1)

where *u* and *l* are used as labels for the upper and lower wells, respectively. Here *T* is the tunnel matrix element,  $\hat{\sigma}_j$  is the *j*th component of the Pauli matrix, *m* is the electron effective mass, and the potential energies  $U_{jx}$  describe both the short-range potentials responsible for the scattering and the nonscreened large-scale variations of the energy levels. The interlevel splitting under an electric field  $E_{\perp} \cos \omega t$ (pointing perpendicular to the QW plane) is given by  $\Delta_t$  $= \Delta + eE_{\perp}Z \cos \omega t$ , where *Z* is the distance between the centers of the wave functions in the *u* and *l* QW's, and  $\Delta$  is the level splitting without tunneling. The tunneling current density  $J_{\perp}(t)$  is written through the density matrix  $\hat{\rho}_t$  according to

$$J_{\perp}(t) = \frac{2|e|T}{\hbar L^2} \operatorname{Sp}\langle\langle \hat{\sigma}_{y} \hat{\rho}_{t} \rangle\rangle, \qquad (2)$$



FIG. 1. (a) Schematic view of an independently contacted DQW structure under a transverse ac field and (b) band diagram of the photon-assisted tunneling process. One- and two-photon transitions are shown on the right and left, respectively;  $\varepsilon_{Fu}$  and  $\varepsilon_{Fl}$  are the Fermi levels in the upper (*u*) and lower (*l*) QW's.

where  $L^2$  is the normalization area and we have used the interwell current operator  $|e|(T/\hbar)\hat{\sigma}_y$ ; see Ref. 11. Here  $\langle\langle\cdots\rangle\rangle$  means the average over the short-range and large-scale potential variations and Sp... includes the average over the in-plane electronic states in both QW's and a summation over the matrix variable.

It is convenient to separate  $\hat{\rho}_t$  into the diagonal and nondiagonal parts  $[\hat{\rho}_t]$  and  $\{\hat{\rho}_t\}$ , respectively, and to transform the equation for the density matrix into the system<sup>12</sup>

$$\frac{\partial [\hat{\rho}_{t}]}{\partial t} + \frac{i}{\hbar} [\hat{h}_{t}, [\hat{\rho}_{t}]]_{-} + \frac{iT}{\hbar} [\hat{\sigma}_{x}, \{\hat{\rho}_{t}\}]_{-} = 0,$$

$$\frac{\partial \{\hat{\rho}_{t}\}}{\partial t} + \frac{i}{\hbar} [\hat{h}_{t}, \{\hat{\rho}_{t}\}]_{-} + \frac{iT}{\hbar} [\hat{\sigma}_{x}, [\hat{\rho}_{t}]]_{-} = 0, \qquad (3)$$

where  $[\cdots]_{-}$  stands for the commutator of the two operators. Since the tunneling current (2) is written through the nondiagonal component of the density matrix, we can express  $\{\hat{\rho}_i\}$  through the diagonal component  $[\hat{\rho}_i]$  as follows:

$$\{\hat{\rho}_t\} = \frac{iT}{\hbar} \int_{-\infty}^t dt' e^{\delta t'} \hat{S}(t,t') [\hat{\sigma}_x, [\hat{\rho}_t]]_- \hat{S}(t,t')^+.$$
(4)

Here  $\delta \rightarrow +0$  and  $\hat{S}(t,t')$  is the evolution operator which is determined by the equation  $i\hbar \partial \hat{S}(t,t')/\partial t = \hat{h}_t \hat{S}(t,t')$  with the initial condition  $\hat{S}(t,t) = 1$ . After substitution of Eq. (4) into the equation for  $[\hat{\rho}_t]$ , we obtain a closed system of equations for the diagonal components of the density matrix,  $\hat{\rho}_{jt}$ , with j = u, l. We neglect any heating of the electrons by the photon-assisted tunneling and use below the equilibrium distributions in the *u* and *l* QW's,  $\hat{\rho}_j^{(0)}$ . Thus, the diagonal part of the density matrix is written as  $\hat{\rho}_u^{(0)}\hat{P}_u + \hat{\rho}_l^{(0)}\hat{P}_l$ , where  $\hat{P}_u = (1 + \hat{\sigma}_z)/2$  and  $\hat{P}_l = (1 - \hat{\sigma}_z)/2$  are the projection operators.

By substituting Eq. (4) with the equilibrium density matrix into Eq. (2) and performing the summation over the discrete variable j we transform the expression for the tunneling current density into

$$J_{\perp}(t) = \left(\frac{T}{\hbar}\right)^{2} \int_{-\infty}^{t} dt' e^{\delta t'} \frac{2e}{L^{2}} \operatorname{sp}\langle\langle \hat{S}_{u}(t,t')(\hat{\rho}_{u}^{(0)} - \hat{\rho}_{l}^{(0)}) \times \hat{S}_{l}(t,t')^{+} + \hat{S}_{l}(t,t')(\hat{\rho}_{u}^{(0)} - \hat{\rho}_{l}^{(0)}) \hat{S}_{u}(t,t')^{+}\rangle\rangle,$$
(5)

where sp... means the average over the in-plane electronic states in the u, l QW's and  $\hat{S}_j(t,t')$  is the evolution operator for the *j*th QW. Separating out the level splitting factor, we write  $\hat{S}_i(t,t')$  in the form

$$\hat{S}_{j}(t,t') = \exp\left(\mp \frac{i}{2\hbar} \int_{t'}^{t} d\tau \Delta_{\tau}\right) e^{-i\hat{h}_{j}(t-t')/\hbar}.$$
(6)

Here  $\hat{h}_j = \hat{p}^2/2m + U_{j\mathbf{x}}$ , and - and + correspond to the u and l QW's, respectively. The current density averaged over one period,  $J_T = (\omega/2\pi) \int_{-\pi/\omega}^{\pi/\omega} dt J_{\perp}(t)$ , is obtained as

$$J_{T} = \frac{2|e|}{L^{2}} \left(\frac{T}{\hbar}\right)^{2} \sum_{k=0}^{\infty} \left[J_{k} \left(\frac{eE_{\perp}Z}{\hbar\omega}\right)\right]^{2} \int_{-\infty}^{0} d\tau e^{\delta\tau + i(\Delta/\hbar + k\omega)\tau} \\ \times \left\langle \left\langle \operatorname{sp}e^{i\hat{h}_{u}\tau/\hbar}(\hat{\rho}_{u}^{(0)} - \hat{\rho}_{l}^{(0)})e^{-i\hat{h}_{l}\tau/\hbar}\right\rangle \right\rangle + \text{c.c.}$$
(7)

after expansion of the field-dependent factor in Eq. (6) (with  $J_k(x)$  the *k*th-order Bessel function] and after integration overtime.

Using the eigenstates of the *j*th QW,  $\hat{h}_j |j\lambda\rangle = \varepsilon_{j\lambda} |j\lambda\rangle$ , and performing the integration over  $\tau$  in Eq. (7) we present  $J_T$  as a sum of *k*-photon contributions

$$J_T = \sum_{k=-\infty}^{\infty} \left[ J_k \left( \frac{eE_{\perp}Z}{\hbar \omega} \right) \right]^2 J(\Delta + k\hbar \omega), \qquad (8)$$

where the current density  $J(\Delta)$  is given by

$$J(\Delta) = \frac{\pi |e|}{\hbar L^2} (2T)^2 \left\langle \left\langle \sum_{\lambda,\lambda'} |(l\lambda|u\lambda')|^2 \delta(\varepsilon_{u\lambda'} - \varepsilon_{l\lambda} + \Delta) \right. \right. \\ \left. \times (f_{l\lambda}^{(0)} - f_{u\lambda'}^{(0)}) \right\rangle \right\rangle.$$
(9)

This expression coincides with the tunneling current in the absence of the oscillating electric field [see Eq. (8) in Ref. 8].

#### **III. LINE SHAPE OF THE TUNNELING CURRENT**

Following the considerations in Ref. 8, we convert Eq. (9) through the retarded (*R*) and advanced (*A*) Green's functions in the *j*th QW's,  $G_{j\varepsilon}^{R,A}(\mathbf{x},\mathbf{x}')$ , as follows:

$$J(\Delta) = \frac{|e|T^2}{\pi \hbar L^2} \int d\varepsilon [\theta(\varepsilon_{Fl} - \varepsilon) - \theta(\varepsilon_{Fu} - \varepsilon + \Delta)]$$
$$\times \sum_{a,b=R,A} (-1)^r \int d\mathbf{x} \int d\mathbf{x}'$$
$$\times \langle G^a_{l\varepsilon}(\mathbf{x}, \mathbf{x}') G^b_{u\varepsilon - \Delta}(\mathbf{x}', \mathbf{x}) \rangle, \qquad (10)$$

where r=1 for a=b and r=0 for  $a\neq b$ . Here we have averaged over the short-range potentials in the *u* and *l* QW's with  $\langle \cdots \rangle$  denoting an averaging over the large-scale potential. It is convenient to introduce  $G_{j\varepsilon}^{R,A}$  in the Wigner representation and to transform the last factor in Eq. (10) into the form

$$\sum_{a,b=R,A} (-1)^r \int d\mathbf{x} \int \frac{d\mathbf{p}}{(2\pi\hbar)^2} \langle G^a_{l\varepsilon}(\mathbf{p},\mathbf{x}) G^b_{u\varepsilon-\Delta}(\mathbf{p},\mathbf{x}) \rangle.$$
(11)

The Wigner representation for the Green's functions is outlined in Appendix A. The result for the retarded function is

$$G_{j\varepsilon}^{R}(\mathbf{p},\mathbf{x}) = \frac{i}{\hbar} \int_{-\infty}^{0} d\tau \exp\left[\frac{i}{\hbar} (\varepsilon_{p} + w_{j\mathbf{x}} - \varepsilon - i\gamma_{j})\tau\right],$$
(12)

where  $\varepsilon_p = p^2/2m$  is the kinetic energy. The advanced function is given by  $G_{j\varepsilon}^A(\mathbf{p}, \mathbf{x}) = G_{j\varepsilon}^R(\mathbf{p}, \mathbf{x})^*$ .

Substituting Eq. (12) into Eq. (11) and carrying out the averaging over the large-scale potential variations we obtain

$$\sum_{a,b=R,A} (-1)^r \langle G_{l\varepsilon}^a(\mathbf{p},\mathbf{x}) G_{u\varepsilon-\Delta}^b(\mathbf{p},\mathbf{x}) \rangle$$

$$= \frac{1}{\hbar^2} \int_{-\infty}^0 d\tau_1 \int_{-\infty}^0 d\tau_2 e^{(\gamma_l \tau_1 + \gamma_u \tau_2)/\hbar} \\ \times \{ e^{i(\varepsilon_p - \varepsilon)(\tau_1 - \tau_2)/\hbar} e^{-i\Delta\tau_2/\hbar} K_-(\tau_1, \tau_2) \\ - e^{i(\varepsilon_p - \varepsilon)(\tau_1 + \tau_2)/\hbar} e^{i\Delta\tau_2/\hbar} K_+(\tau_1, \tau_2) \} + \text{c.c.}, \quad (13)$$

where the damping factors due to the large-scale potential variations are given by  $K_{\pm}(\tau_1, \tau_2) = \langle \exp[i(w_{lx}\tau_1 \pm w_{ux}\tau_2)/\hbar] \rangle$ . The averaged result (13) does not depend on **x**, so that we can set  $\int d\mathbf{x}/L^2 = 1$  in Eq. (10). Performing the integration over momenta after substituting Eq. (13) into Eq. (10) we transform  $J(\Delta)$  to obtain

$$J(\Delta) = |e| \left(\frac{T}{\hbar}\right)^2 \rho_{2D} \int_{\varepsilon_{Fu}+\Delta}^{\varepsilon_{Fl}} d\varepsilon \int_{-\infty}^0 dt e^{(\gamma_l + \gamma_u)t/\hbar} \\ \times \left\{ e^{-i\Delta t/\hbar} K_-(t,t) + i \int_{-2t}^{2t} d\tau e^{(\gamma_l - \gamma_u)\tau/2\hbar} \\ \times \left[ \frac{\mathcal{P}}{\tau} e^{-i\varepsilon\tau/\hbar - i\Delta(t-\tau/2)/\hbar} K_-\left(t + \frac{\tau}{2}, t - \frac{\tau}{2}\right) \right] \\ - \frac{\mathcal{P}}{t} e^{-i2\varepsilon t/\hbar + i\Delta(t-\tau/2)/\hbar} K_+\left(t + \frac{\tau}{2}, t - \frac{\tau}{2}\right) \right] + \text{c.c.},$$

$$(14)$$

where  $\mathcal{P}$  means the principal value. Under the condition  $\varepsilon_{Fj} \gg \gamma_j$  and for the inhomogeneous broadening energy introduced below, we find that the integral over  $\tau$  in Eq. (14) is negligible and  $J(\Delta)$  can be written as



FIG. 2. Resonant one-photon contributions to the tunneling current given by Eq. (16) for the cases of absorption (a) and stimulated emission (b) processes. The three curves show the calcualted response. The ratios of the inhomogeneous and homogeneous broadening factors are  $\Gamma/\gamma=0.3$ , 1, and 3, respectively.

$$J(\Delta) \simeq 2|e| \left(\frac{T}{\hbar}\right)^2 \delta n(\Delta) \operatorname{Re} \int_{-\infty}^0 dt e^{(\gamma - i\Delta)t/\hbar} e^{-(\Gamma t/\hbar)^2/2}.$$
(15)

where  $\delta n(\Delta) \equiv \rho_{2D} \int_{\varepsilon_{Fu}+\Delta}^{\varepsilon_{Fl}} d\varepsilon = \rho_{2D} [\varepsilon_{Fl} - (\varepsilon_{Fu}+\Delta) \theta(\varepsilon_{Fu}+\Delta)]$  is the filling factor, and the homogeneous and inhomogeneous broadening energies are introduced as  $\gamma \equiv \gamma_l + \gamma_u$  and  $\Gamma = \sqrt{\langle (w_{lx} - w_{ux})^2 \rangle}$ , respectively.

We restrict our treatment to the weak excitation case  $|e|E|Z \ll \hbar \omega$  when the dark (photonless, k=0) and onephoton  $(k = \pm 1)$  contributions are only essential in Eq. (8) and  $J_T \simeq J(\Delta) + (eE_\perp Z/2\hbar\omega)^2 [J(\Delta + \hbar\omega) + J(\Delta - \hbar\omega)].$ We consider below the vicinity of the photon-assisted peak of the tunneling current, under the condition  $|\Delta \pm \hbar \omega|$  $\ll \hbar \omega$ . Thus, the peaks which appear due to the absorption and stimulated emission processes (with k=1 and k=-1, respectively) are described by  $J_T^{(+)} \simeq J(\hbar \omega) + (eE_{\perp}Z/$  $\times 2\hbar\omega)^2 J(\Delta + \hbar\omega)$  and  $J_T^{(-)} \simeq J(-\hbar\omega) + (eE_\perp Z/2\hbar\omega)^2 J(\Delta + \hbar\omega)$  $-\hbar\omega$ ). It is important to note that under the condition  $\hbar\omega$  $\gg \Gamma^2 / \gamma$  the tail of the dark contribution is determined by the homogeneous broadening, so that  $J(\pm \hbar \omega)$  $\simeq 2|e|(T/\hbar\omega)^2 \delta n_+ \gamma/\hbar$  with the characteristic concentrations  $\delta n_{\pm} = \delta n (\Delta \simeq \pm \hbar \omega)$ . Introducing the detuning energies  $\Delta \pm \hbar \omega$ , we describe the photon-assisted peaks  $J_T^{(\pm)}$  as follows:

$$J_{T}^{(\pm)} \approx 2|e| \left(\frac{T}{\hbar}\right)^{2} \left[ \delta n_{\pm} \frac{\gamma/\hbar}{\omega^{2}} - \delta n_{F} \left(\frac{eE_{\perp}Z}{2\hbar\omega}\right)^{2} \\ \times \int_{-\infty}^{0} dt e^{\gamma t/\hbar - (\Gamma t/\hbar)^{2}/2} \cos(\Delta/\hbar \pm \omega) t \right].$$
(16)

Here we use  $\delta n_F = \rho_{2D}(\varepsilon_{Fu} - \varepsilon_{Fl})$  with  $\varepsilon_{Fu} - \varepsilon_{Fl} \gg |\Delta \pm \hbar \omega| \sim \Gamma, \gamma$ —i.e., conditions appropriate to a nonsymmetrically doped DQW structure.

The spectral dependence of the photoinduced current is shown in Fig. 2 for a structure with parameters close to those used in Ref. 7. We consider a GaAs/Al<sub>0.3</sub>Ga<sub>0.7</sub>As-based DQW structure, where the two QW's, each of width 120 Å and with carrier concentrations  $4 \times 10^{11}$  and  $2 \times 10^{11}$  cm<sup>-2</sup>, respectively, are separated by a 125-Å barrier. The homogeneous broadening energy  $\gamma$  is chosen to be 0.16 meV,



FIG. 3. Linear intensity dependence of the one-photon resonant currents  $J_{max}$  vs coupled pump power density under the conditions  $\Delta - \hbar \omega = 0$  (solid lines) and  $\Delta + \hbar \omega = 0$  (dashed lines) for the same ratios of  $\Gamma / \gamma$  as in Fig. 2.

equivalent to a mobility around  $10^5 \text{ cm}^2/\text{V}$  s. We have plotted the current density due to absorption [Fig. 2(a)] or stimulated emission [Fig. 2(b)] processes under resonant excitation with  $\hbar \omega = 10$  meV and with the coupling field in the structure,  $E_{\perp} \simeq 45$  V/cm, equivalent to a coupled pump power of  $10^{\circ}$  W cm<sup>-2</sup>. Note that the dark current is bigger for case (a) while the relative contribution of the photonassisted current is bigger for case (b). The photon-assisted contributions to  $J_T^{(\pm)}$  are suppressed at a fixed frequency as the relative contribution of inhomogeneous broadening increases, although the integrated photon-assisted current would remain unchanged for a "broadband" pump. Explicit expressions for  $\Gamma$  due to heterointerface roughness and nonuniform doping are presented in Appendix B: from Eq. (B3) we obtain  $\Gamma_r \simeq 0.39$  meV for the one-monolayer variations of the heterointerfaces,  $\overline{\delta d} \approx 2.5$  Å, while Eq. (B5) gives  $\Gamma_d$  $\simeq 0.32$  meV for the case of a 2% variation in the donor concentrations placed outside the *u* and *l* QW's. Since  $J_T^{(\pm)}$  $\propto T^2$ , both the photon-assisted and dark currents can be increased by about two orders of magnitude in a GaAs/Al<sub>0.2</sub>Ga<sub>0.8</sub>As/GaAs-based DQW structure with the same dimensions.

The one-photon approximation used in Eq. (16) is valid up to a coupled pumping level of the order of 100 kW/cm<sup>2</sup> for 10 meV photons; at this level, the arguments of the Bessel functions in Eq. (8) become comparable with unity and consideration of multiphoton transitions becomes essential. Figure 3 illustrates the linear dependence of the peak currents  $J_{max}$ , at  $\Delta \mp \hbar \omega = 0$ , versus the THz pump power coupled into the structure for the above presented parameters. One can see that the photon-assisted and dark currents are comparable for coupled pump levels of the order of 100 W/cm<sup>2</sup>.

The efficiency of a detector based on the photon-assisted tunneling process in DQW's with independent contacts will be directly proportional to the efficiency with which the electric field of an incident terahertz beam couples to the interwell transition. This coupling efficiency will depend critically on the structure design and experimental arrangement. We assume a polarized incident beam, with the polarization vector lying in the plane of incidence. (Any polarization component perpendicular to the plane of incidence will lie in the QW plane, and so not couple to the interwell transition.) All of the incident beams can couple into the sample when the beam is incident at Brewster's angle ( $\theta \approx 75^{\circ}$  for a dielectric constant of 13.5), but less than 7% of the incident power per unit area will then have the appropriate polarization to excite interwell transitions. The coupling efficiency can be increased by a factor of order 5 by integrating a  $45^{\circ}$ prism onto the sample surface with the prism refractive index equal to that of the DQW sample,<sup>13</sup> to give a coupling efficiency of over 33%. The power incident per unit area can also be increased through use of an appropriate mirror, with a THz beam of order 1 cm<sup>-2</sup>, say, focused onto a 0.03  $\times 0.03$  cm<sup>-2</sup> structure (see, e.g., Ref. 14). Thus, it should be possible to observe a photoinduced response from a source of order 10 mW if a photinduced peak can be resolved with an amplitude of a few percent of the dark current.

#### **IV. CONCLUSION**

In summary, we have described the photon-assisted tunneling processes between independently contacted quantum wells. Taking into account both the nonscreened random potential and the scattering due to the short-range potential we have compared the dark and photon-assisted contributions to the current. The tail of the dark current is determined by the homogeneous scattering processes while the photon-assisted peak is limited by the homogeneous and inhomogeneous mechanisms. This fact determines the sensitivity which may be achieved by the phototransistor structure under consideration. Numerical estimates of the THz photon-assisted interwell current demonstrate the possibility to use an independently contacted DQW structure as a tunable far-IR detector, with the spectral characteristics easily varied by applying a transverse gate voltage to change the peak absorption frequency. The detailed characteristics of such a device (noise factors, contact phenomena, angle resolution, etc.) require further consideration and numerical modeling, particularly in view of the recent demonstration of stimulated THz emission in a quantum cascade structure.<sup>15</sup>

We finally discuss briefly the key assumptions made in the calculations presented here. The description of the tunnelcoupled levels is based on a tight-binding approach, which is suitable for independently contacted QW's with a thick barrier. Both the quasiclassical consideration of large-scale inhomogeneties and the phenomenologically introduced homogeneous broadening in the Green's functions (A4) are generally accepted approaches. A simplified consideration of the nonscreened potentials is performed in Appendix B in analogy with Ref. 8 where a quantitative description for the DQW conductance was performed. We have also used a local approximation for the large-scale inhomogeneities, neglecting the drift contributions, as discussed in Refs. 8 and 10. We consider the single-particle resonant response of the DQW structure, neglecting Coulomb effects due to the depolarization field and exchange contributions. Although the effects of these two contributions will partially cancel each other, a measurable shift of the transition energy is possible, but the amplitude of the absorption should not change noticeably (see Ref. 16 where the linear response was considered). Finally, we consider the low-energy region, where  $\Delta$  and  $\hbar \omega$  are less than the optical phonon energy, and the low-temperature case, so that elastic scattering and large-scale potential variations are mainly responsible for broadening the resonant transition peaks.

The results presented here describe the factors determining the line shape of the photon-assisted tunneling current in an independently contacted DQW structure, including a comparison of the photon-assisted and dark currents for the lowpump case. Further experimental analysis of this photonassistent tunneling current is now timely, to evaluate the prospect of using an independently contacted DQW structure as a tunable photodetector in the THz spectral region.

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#### APPENDIX A: GREEN'S FUNCTION

Below we evaluate the single-particle Green's function that appears in Eqs. (11) and (12) for the *j*th QW in the Wigner representation. After averaging over the short-range potential we obtain the set of equations for the retarded Green's function in the coordinate representation:

$$(\tilde{h}_{j} - \varepsilon - i\gamma_{j})G_{j\varepsilon}^{R}(\mathbf{x}, \mathbf{x}') = \delta(\mathbf{x} - \mathbf{x}'),$$
$$(\tilde{h}_{j}' - \varepsilon - i\gamma_{j})G_{j\varepsilon}^{R}(\mathbf{x}, \mathbf{x}') = \delta(\mathbf{x} - \mathbf{x}').$$
(A1)

Here the Hamiltonian  $\tilde{h}_j = \hat{p}^2/(2m) + w_{j\mathbf{x}}$  includes the largescale potential  $w_{j\mathbf{x}}$  and  $\tilde{h}'_j$  acts on  $\mathbf{x}'$ . After the change of coordinates  $(\mathbf{x}+\mathbf{x}')/2 \rightarrow \mathbf{x}$  and  $(\mathbf{x}-\mathbf{x}') \rightarrow \Delta \mathbf{x}$  we introduce the Wigner representation as follows:

$$G_{j\varepsilon}^{R}(\mathbf{p},\mathbf{x}) = \int d\Delta \mathbf{x} \exp\left(-\frac{i}{\hbar}\mathbf{p}\cdot\Delta\mathbf{x}\right) G_{j\varepsilon}^{R}\left(\mathbf{x}+\frac{\Delta\mathbf{x}}{2},\mathbf{x}-\frac{\Delta\mathbf{x}}{2}\right),$$
$$G_{j\varepsilon}^{R}\left(\mathbf{x}+\frac{\Delta\mathbf{x}}{2},\mathbf{x}-\frac{\Delta\mathbf{x}}{2}\right) = \int \frac{d\mathbf{p}}{(2\pi\hbar)^{2}} \exp\left(\frac{i}{\hbar}\mathbf{p}\cdot\Delta\mathbf{x}\right) G_{j\varepsilon}^{R}(\mathbf{p},\mathbf{x}).$$
(A2)

Adding and subtracting Eqs. (A1) after the Wigner transformation according to Eqs. (A2) we obtain

$$\left(-\frac{\mathbf{p}}{m}\cdot\nabla_{x}+\nabla_{x}w_{j\mathbf{x}}\cdot\nabla_{p}\right)G_{j\varepsilon}^{R}(\mathbf{p},\mathbf{x})=0,$$

$$\left(-\frac{\hbar^{2}\nabla_{x}^{2}}{8m}+\varepsilon_{p}+w_{j\mathbf{x}}-\varepsilon-i\gamma_{j}\right)G_{j\varepsilon}^{R}(\mathbf{p},\mathbf{x})=1,$$
(A3)

where  $(w_{j\mathbf{x}}+w_{j\mathbf{x}'})/2$  is replaced by  $w_{j\mathbf{x}}$ . Under the condition  $(\hbar/l_c)^2/8m \ll \varepsilon$  we neglect the contribution  $\propto \nabla_x^2$  in the second equation of Eqs. (A3) so that the Green's function then takes the form

$$G_{j\varepsilon}^{R}(\mathbf{p},\mathbf{x}) = \left(\frac{p^{2}}{2m} + w_{j\mathbf{x}} - \varepsilon - i\gamma_{j}\right)^{-1}.$$
 (A4)

This expression is also satisfied by the first equation of Eqs. (A3) due to the condition on the total energy  $[(\mathbf{p} \cdot \nabla_x)/m + (\nabla_x w_{j\mathbf{x}} \cdot \nabla_p)](\varepsilon_p + w_{j\mathbf{x}}) = 0$ . Representing the denominator in Eq. (A4) as an integral over time we finally obtain Eq. (12).

#### APPENDIX B: INHOMOGENEOUS BROADENING

We consider here the inhomogeneous broadening energy  $\Gamma$  which appears in Eq. (15) due to unscreened large-scale variations in the *u*- and *l*-energy levels. These variations are caused by such factors as heterointerface roughness and non-uniform modulation doping. Following the considerations in Refs. 8 and 10 we write the nonscreened potentials as<sup>17</sup>

$$w_{u\mathbf{x}} = -w_{l\mathbf{x}} = \frac{a_B}{a_B + 2Z} \frac{\delta \varepsilon_{u\mathbf{x}} - \delta \varepsilon_{l\mathbf{x}}}{2}, \qquad (B1)$$

where  $a_B$  is the Bohr radius and the in-plane variations of the u and l levels due to QW width fluctuations,  $d_{jx}=d_j + \delta d_{jx}$ , are given by

$$\delta \varepsilon_{j\mathbf{x}} = 2\varepsilon_j \delta d_{j\mathbf{x}} / d_j. \tag{B2}$$

We assume the fluctuations in the two QW's to be random, uncorrelated, and of equal magnitude. We then obtain that the broadening due to the heterointerface roughness is given by

$$\Gamma_r = \frac{2\sqrt{2}a_B}{a_B + 2Z} \frac{\overline{\delta d}}{d} \varepsilon_1, \tag{B3}$$

where  $\overline{\delta d}$  is the averaged variation in well width and  $\varepsilon_1$  is the confinement energy of the lowest level.

We also consider the inhomogeneous broadening energy due to nonuniform modulation doping. According to Ref. 18, the Fourier component of the nonscreened potential due to variations in the donor atom concentration for the *j*th QW is written as

$$w_{j\mathbf{q}} = -\frac{\delta n_{j\mathbf{q}}}{\rho_{2D}} e^{-qa},\tag{B4}$$

where *a* is the distance between the donor sheet and the two-dimensional electrons. Neglecting the influence of the adjacent QW and considering the case of a correlation lengths longer than *a*, we obtain  $w_{jx} \approx -\delta n_{jx}/\rho_{2D}$ . As a result, the broadening energy due to nonuniform doping takes the form

$$\Gamma_{d} = \sqrt{\langle w_{u\mathbf{x}}^{2} \rangle + \langle w_{l\mathbf{x}}^{2} \rangle} = \frac{\sqrt{\delta N_{u}^{2} + \delta N_{l}^{2}}}{\rho_{2D}}, \qquad (B5)$$

where  $\overline{\delta N_j}$  is the averaged variation of the donor concentration in the *j*th QW.

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- <sup>1</sup>A. Wacker, Phys. Rep. **357**, 1 (2002); M. Helm, Semicond. Sci. Technol. **10**, 567 (1995).
- <sup>2</sup>J. Ohe and K. Yakubo, J. Phys. Soc. Jpn. **69**, 3650 (2000); M. Asada and N. Sashinaka, Jpn. J. Appl. Phys., Part 1 **40**, 5394 (2001).
- <sup>3</sup>Y. Dakhnovskii, R. Bavli, and H. Metiu, Phys. Rev. B 53, 4657 (1996); R. Aguado and G. Platero, *ibid.* 55, 12 860 (1997); T. Fromherz, *ibid.* 56, 4772 (1997).
- <sup>4</sup>C. Niu and D. L. Lin, Phys. Rev. B 62, 4578 (2000).
- <sup>5</sup> A. P. Jauho and N. S. Wingreen, Phys. Rev. B 58, 9619 (1998); K. C. Lin and D. S. Chuu, *ibid.* 64, 235320 (2001); H. Qin, F. Simmel, R. H. Blick, J. P. Kotthaus, W. Wegscheider, and M. Bichler, *ibid.* 63, 035320 (2001).
- <sup>6</sup>S. K. Lyo and J. A. Simmons, Physica B **314**, 417 (2002).
- <sup>7</sup>J. A. Simmons, M. A. Blount, J. S. Moon, S. K. Lyo, W. E. Baca, J. R. Wendt, J. L. Reno, and M. J. Hafich, J. Appl. Phys. 84, 5626 (1998).
- <sup>8</sup>F. T. Vasko, O. G. Balev, and N. Studart, Phys. Rev. B **62**, 12 940 (2000).
- <sup>9</sup>S. Q. Murphy, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **52**, 14 825 (1995).

- <sup>10</sup>F. T. Vasko, JETP **93**, 1270 (2001); F. T. Vasko, P. Aceituno, and A. Hernandez-Cabrera, Phys. Rev. B **66**, 125303 (2002).
- <sup>11</sup>L. Zheng and A. H. MacDonald, Phys. Rev. B **47**, 10 619 (1993).
- <sup>12</sup>O. E. Raichev and F. T. Vasko, J. Phys. C 9, 1547 (1997); 8, 1041 (1996); Superlattices Microstruct. 15, 133 (1994).
- <sup>13</sup> M. B. Johnston, D. M. Whittaker, A. Corchia, A. G. Davies, and E. H. Linfield (unpublished); M. B. Johnston, D. M. Whittaker, A. Dowd, A. G. Davies, E. H. Linfield, X. Li, and D. A. Ritchie, Opt. Lett. **27**, 1935 (2002).
- <sup>14</sup>H. G. Roskos, M. C. Nuss, J. Shah, K. Leo, D. A. B. Miller, A. M. Fox, S. Schmitt-Rink, and K. Kohler, Phys. Rev. Lett. 68, 2216 (1992).
- <sup>15</sup>R. Kohler, A. Tredicucci, F. Beltram, H. E. Beere, E. H. Linfield, A. G. Davies, D. A. Ritchie, R. C. Iotti, and F. Rossi, Nature (London) **417**, 156 (2002).
- <sup>16</sup>D. E. Nikonov, A. Imamoglu, L. V. Butov, and H. Schmidt, Phys. Rev. Lett. **79**, 4633 (1997). O. E. Raichev and F. T. Vasko, Phys. Rev. B **60**, 7776 (1999); F. T. Vasko and E. P. O'Reilly, Phys. Rev. B (to be published).
- <sup>17</sup>Here we took into account the factor  $a_B/(a_B+2Z)$  omitted in Ref. 8.
- <sup>18</sup>F. T. Vasko and G.-Q. Hai, Phys. Rev. B **65**, 045314 (2002); Solid State Commun. **122**, 89 (2002).