

**Statistics of charge fluctuations in chaotic cavities**

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We consider the zero-frequency fluctuations of the charge inside a mesoscopic conductor in the large capacitance limit. In analogy to current counting statistics, we derive the characteristic function of charge fluctuations in terms of the scattering matrix of the conductor. Using random matrix theory, we evaluate the characteristic function semianalytically for chaotic cavities. Our result is universal in the sense that it describes not only the fluctuations of the charge but of any observable quantity inside the cavity. We discuss equilibrium and nonequilibrium fluctuations and extend our theory to the case of contacts with arbitrary transparency. Finally, we investigate the suppression of fluctuations in the small capacitance limit due to charge screening.

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The full counting statistics of current fluctuations in mesoscopic conductors have attracted the attention of many theoretical works during the last decade. In a pioneering paper, Levitov *et al.* explain the universality of current statistics. Coherent charge transfer through a two-terminal conductor can be seen as a probabilistic process governed by a set of transmission probabilities.<sup>1</sup> In the following, several methods have been developed to obtain the full counting statistics of mesoscopic conductors. In the original work, the authors used a fully quantum-mechanical approach and described the conductor by its unitary scattering matrix.<sup>2-4</sup> Nazarov provided a description in terms of Keldysh-Green's functions that can be applied conveniently to conductors with a large number of channels.<sup>5</sup> De Jong characterized double barrier structures successfully by a fully classical method<sup>6</sup> based on the exclusion principle. Nagaev proposed a semiclassical diagrammatic scheme to obtain higher cumulants in a systematic manner.<sup>7,8</sup> Recently, Pilgram *et al.* expressed the full counting statistics in terms of a saddle point solution to a semiclassical stochastic path integral.<sup>9</sup>

In parallel, different works addressed the statistics of quantities related to current, such as the phases in superconducting devices<sup>10,11</sup> or the momentum transfer from electrons to a bent conductor.<sup>12</sup> The interpretation of the statistics of these quantities is less intuitive, in general, these cannot be characterized by transmission probabilities only. Momentum transfer, for example, is not quantized in contrast to charge transfer.<sup>13</sup>

In this paper, we report on a quantity of fundamental interest in physics: the fluctuations of charge inside a mesoscopic conductor. As soon as parts of a mesoscopic system are only coupled capacitively (i.e., metallic gates close to a mesoscopic conductor), charge fluctuations play an important role. Decoherence for instance is described as an average over a fluctuating potential,<sup>14,15</sup> which then in turn is coupled electrostatically to the charge. Charge correlations also contribute to nonequilibrium decoherence when a measurement process is taking place.<sup>16,17</sup> These are as well at the origin of Coulomb drag effects.<sup>18</sup> Such models often assume Gaussian fluctuations, which is justified in the weak-coupling limit. When coupling becomes stronger the non-Gaussian fluctuations should be taken into account. Methods to calculate higher cumulants are, therefore, of great use.

In some sense, charge fluctuations are diametrically opposed to the fluctuations of current. For a current measurement, we may arbitrarily choose the cross section of the conductor. For a charge measurement, we are forced to define exactly a volume of charge we are interested in.<sup>19</sup> The choice of this volume will necessarily influence the statistics of charge fluctuations. At this point, it becomes clear that statistics of charge noise are, in general, non-universal in contrast to current noise. There is, however, a class of conductors in which we expect universality to be preserved: Disordered systems and in our case chaotic cavities. In this case, the charge of interest can be characterized by one single parameter, the dwell time of the electrons inside the volume of the cavity.<sup>20</sup> Furthermore, the Hamiltonian of a chaotic cavity is random, no basis nor any operator is preferred. Therefore, the fluctuations of any other internal operator, such as the dipolar moment, will obey the same universal statistics as the charge operator.

In this paper, we present a method based on random matrix theory<sup>21</sup> to calculate the cumulants of charge noise in chaotic cavities analytically order by order. For the full generating function, we show semianalytical results that we confirmed numerically. We emphasize that the example of the chaotic cavity in the many channel limit can be treated as well in the semiclassical framework developed in Ref. 9. As we checked, the semiclassical results completely agree with the calculation presented here.

In Sec. I, we derive the generating function of charge fluctuations in terms of the scattering matrix. A similar expression for the second cumulant has been derived long before.<sup>22</sup> Results are, in particular, available for the geometry considered in this publication: the chaotic cavity.<sup>23</sup> The charge-charge correlation function has also been found at finite frequencies.<sup>24</sup> These works all emphasize on the importance of charge screening, interaction effects have to be taken into account. The charge in the cavity responds to the fluctuations of the internal electrostatic potential that is conjugated with the charge.

We explain in detail the evaluation of the characteristic function for an open cavity in equilibrium in Sec. II. We present results for the nonequilibrium case and contacts with arbitrary transparency in Sec. III. We briefly discuss how to include interaction effects into our calculation scheme. As an

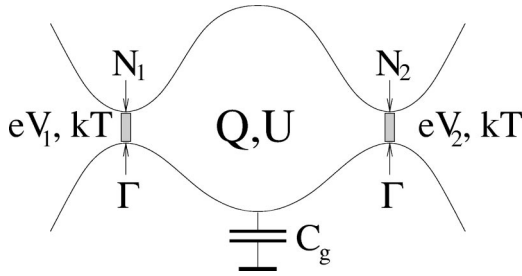


FIG. 1. Example of a mesoscopic conductor considered in this paper: the chaotic cavity. We calculate the statistics of the charge  $Q$  inside the cavity.  $U$  is the electrostatic potential of the cavity,  $V_1, V_2$  are the applied voltages,  $N_1, N_2$  are the number of channels in the contacts,  $\Gamma$  is the contact transparency,  $kT$  is the temperature, and  $C_g$  is the geometrical capacitance of the cavity.

application, we finally estimate the contribution of the non-Gaussian fluctuations to dephasing of electrons passing the chaotic cavity.

### I. GENERATING FUNCTION

We consider a mesoscopic conductor as shown in Fig. 1. The conductor is described by a Hamiltonian  $\hat{\mathbf{H}}_0$ . The electrostatic potential  $U$  on the conductor can be varied. The full Hamiltonian is, therefore,  $\hat{\mathbf{H}} = \hat{\mathbf{H}}_0 + U\hat{\mathbf{Q}}$ , where  $\hat{\mathbf{Q}}$  is the charge operator. In analogy to current statistics,<sup>4</sup> we define the following characteristic function (we set  $\hbar = 1$ ):

$$\begin{aligned} \chi_t(U) &= \langle e^{it[\hat{\mathbf{H}}_0 + (U/2)\hat{\mathbf{Q}}]} e^{-it[\hat{\mathbf{H}}_0 - (U/2)\hat{\mathbf{Q}}]} \rangle \\ &= \left\langle \hat{\mathbf{T}}_{\mathbf{K}} \exp i \frac{U}{2} \int_0^t \int_0^t dt' (\pm) \hat{\mathbf{Q}}(t') \right\rangle. \end{aligned} \quad (1)$$

The first line of Eq. (1) in the Heisenberg picture will serve as a starting point for our calculation. The second line in the interaction picture contains the Keldysh time ordering operator  $\hat{\mathbf{T}}_{\mathbf{K}}$  and an integral  $\int \int$  along the Keldysh contour from zero to  $t$  and back again. The  $\pm$  sign is positive on the upper Keldysh contour and negative on the lower contour. The interaction representation allows us to verify that the derivative  $C_n = (-i)^n \partial^n \ln \chi(U) / \partial U^n |_{U=0}$  generates the  $n$ th cumulant of charge fluctuations.

The Fourier transform of this characteristic function is the probability distribution of charge integrated over time  $\int_0^t dt' Q(t')$ , a quantity without direct physical meaning. Divided by the elementary charge  $e$ , it can be understood as the time spent by all electrons in the cavity after time  $t$ . Alternatively, it can be divided by a geometrical capacitance  $C_g$  to obtain the probability distribution of the phase, which is the potential integrated over time  $t$ . In this paper, we mainly concentrate on the characteristic function itself that generates the moments of zero-frequency charge fluctuations.

Using the procedure of Refs. 2,4, the time evolution operators defined in Eq. (1) can be expressed by the unitary scattering matrix of the mesoscopic conductor and we obtain

$$\ln \chi_t = \frac{t}{2\pi} \int dE \text{Tr} \ln \left[ \mathbf{1} - \mathbf{n}_{\mathbf{E}} + \mathbf{n}_{\mathbf{E}} \mathbf{S}^\dagger \left( E, -\frac{U}{2} \right) \mathbf{S} \left( E, +\frac{U}{2} \right) \right]. \quad (2)$$

The energy integral reaches from the bottom of the band to infinity. The matrix  $\mathbf{S}(E, U)$  is the energy and potential dependent scattering matrix that links incoming and outgoing current amplitudes. The matrix  $\mathbf{n}_{\mathbf{E}}$  is diagonal and contains the Fermi occupation factors  $(\mathbf{n}_{\mathbf{E}})_{ii} = n_i(E) = [1 + \exp((E - eV_i)/kT)]^{-1}$  of channel  $i$  on its diagonal. In practice, it is useful to subtract the background charge. We consider separately the characteristic function at zero temperature and voltage that can be expressed by the eigenvalues of the scattering matrix  $e^{i\phi_n}$ :

$$\ln \chi_0(U) = i \frac{t}{2\pi} \int_0^0 dE \left\{ \sum_n (\phi_n(U/2) - \phi_n(-U/2)) \right\}. \quad (3)$$

It is obvious from Eq. (3) that all even cumulants vanish at zero temperature and voltage. The odd cumulants except the first vanish after disorder averaging,<sup>25</sup> because  $\langle \partial^k \phi_n / \partial U^k \rangle = 0$  for  $k > 1$ . In the presence of disorder, subtracting  $\ln \chi_0(U)$  from Eq. (2) is thus equivalent to subtracting the background charge. We arrive at the following symmetric expression for the characteristic function:

$$\begin{aligned} \ln \chi &= \frac{t}{2\pi} \int_{-\infty}^0 dE \text{Tr} \ln \left[ \mathbf{n}_{\mathbf{E}} + (\mathbf{1} - \mathbf{n}_{\mathbf{E}}) \mathbf{S}^\dagger \left( E, \frac{U}{2} \right) \mathbf{S} \left( E, -\frac{U}{2} \right) \right] \\ &+ \frac{t}{2\pi} \int_0^{\infty} dE \text{Tr} \ln \left[ (\mathbf{1} - \mathbf{n}_{\mathbf{E}}) + \mathbf{n}_{\mathbf{E}} \mathbf{S}^\dagger \left( E, -\frac{U}{2} \right) \mathbf{S} \left( E, \frac{U}{2} \right) \right]. \end{aligned} \quad (4)$$

At equilibrium, one can easily check that Eq. (4) generates only even cumulants. The characteristic function then only depends on the eigenvalues  $e^{i\phi_n}$  of the scattering matrix and is symmetric with respect to  $U \rightarrow -U$ .

### II. AVERAGE FOR A CHAOTIC CAVITY

So far we have been very general. Equation (4) can be applied to any mesoscopic conductor, as soon as its scattering matrix is known.<sup>26</sup> We now investigate more closely the generic example of a many-mode chaotic cavity (we neglect weak localization corrections). In this case, the average of products of at most four energy (potential) dependent scattering matrices is known.<sup>20,27</sup> It remains, nevertheless, a non-trivial problem to calculate the average of the logarithms in Eq. (4), which contain infinitely high powers of scattering matrices. We solve this problem in two steps. First, we express the characteristic function through a Green's function. We then calculate the average of this Green's function using a method developed in Ref. 21.

We define the following Green's functions:

$$\mathbf{F}^{eq}(z, U) = \left\langle \frac{1}{z - \mathbf{A}^{eq}(U)} \right\rangle, \quad \mathbf{F}^{neq}(z, U) = \left\langle \frac{1}{z - \mathbf{A}^{neq}(U)} \right\rangle. \quad (5)$$

The brackets denote the disorder average. The matrices  $\mathbf{A}$  abbreviate products of scattering matrices

$$\mathbf{A}^{eq} = \mathbf{S}^\dagger(-U/2)\mathbf{S}(U/2),$$

$$\mathbf{A}^{neq} = \mathbf{S}_{11}^\dagger(-U/2)\mathbf{S}_{11}(U/2) + \mathbf{S}_{21}^\dagger(-U/2)\mathbf{S}_{21}(U/2). \quad (6)$$

The dimension of the full scattering matrix is  $N \times N$ , its block  $\mathbf{S}_{11}$  (the reflection matrix) measures  $N_1 \times N_1$ . We introduce the temperature  $kT$ , the applied voltage  $V = V_1 - V_2 > 0$ , and the integration time  $t$ . The characteristic functions at equilibrium ( $kT \gg eV$ ) and for transport ( $kT \ll eV$ ) can then be written as integrals over the variable  $z$  of the Green's functions:

$$\chi^{eq}(U) = \frac{tkT}{2\pi} \int_1^\infty dz \frac{\ln(z)}{1+z} [2N + (1+z)\text{Tr}(\mathbf{F}^{eq}(-z, U) + \mathbf{F}^{eq}(-z, -U))], \quad (7)$$

and

$$\chi^{neq}(U) = \frac{teV}{2\pi} \int_0^\infty \frac{dz}{1+z} [N_1 + (1+z)\text{Tr} \mathbf{F}^{neq}(-z, U)]. \quad (8)$$

To outline the principles of the calculations, we first specially discuss the simplest case of an open cavity at equilibrium. As Brouwer and Büttiker,<sup>20</sup> we introduce the potential dependence of the scattering matrix via a virtual stub described by a  $M \times M$  reflection matrix

$$\mathbf{r}_s(U) = -\mathbf{e}^{ieU\Phi}, \quad \phi = \text{Tr} \Phi. \quad (9)$$

The trace  $\phi$  is linked to the mean level density by  $\phi = 2\pi dn/(edU) = 2\pi N_F$  and to the dwell time by  $\phi = G\tau$  where  $G$  is the total dimensionless conductance into the cavity and  $e$  is the elementary charge. The stub is assumed to be very large compared to the exits of the cavity; then the precise structure of the matrix  $\Phi$  becomes unimportant for the calculation. The total scattering matrix of the chaotic cavity may be expressed as the geometrical sum

$$\mathbf{S}(U) = \mathbf{U}_{aa} + \mathbf{U}_{ab}(\mathbf{1} - \mathbf{r}_s(U)\mathbf{U}_{bb})^{-1}\mathbf{r}_s(U)\mathbf{U}_{ba}. \quad (10)$$

For an open chaotic cavity, the  $(N+M) \times (N+M)$  matrix  $\mathbf{U}$  is distributed according to the circular ensemble of unitary matrices. It is divided into blocks  $\mathbf{U}_{aa}$  of size  $N \times N$  and  $\mathbf{U}_{bb}$  of size  $M \times M$ . The average over this ensemble can be carried out by means of a diagrammatic technique.<sup>21</sup> The technical details of this average are presented in appendix A. The trace of the Green's function turns out to be

$$\text{Tr} \mathbf{F}^{eq} = N \frac{1}{z - y(z, U)}, \quad (11)$$

where  $y$  is the root of a cubic Dyson equation

$$z - z(1 - ieU\tau)y - (1 + ieU\tau)y^2 + y^3 = 0. \quad (12)$$

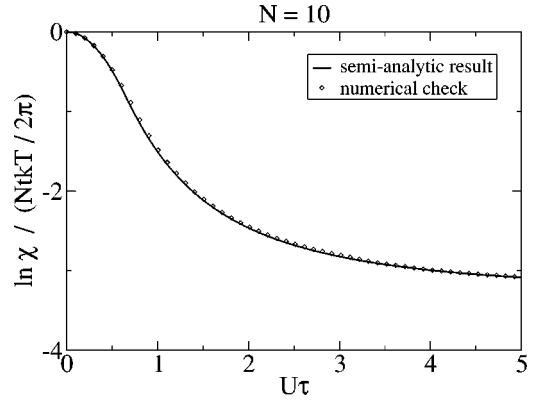


FIG. 2. Comparison of semianalytical and numerical results for the logarithm of the characteristic function of equilibrium charge fluctuations in an open cavity. We choose  $M = 160$  for the size of the numerical random Hamiltonian describing the cavity.

Unfortunately, the analytic solution of this polynomial equation is rather cumbersome. But we have two ways to continue. If we are only interested in the first few cumulants of charge fluctuations, we may expand the root  $y$  up to a certain order in the potential  $U$ :

$$y = y_0 + y_1 U + y_2 U^2 + y_3 U^3 + \dots \quad (13)$$

and then solve the cubic equation order by order (As a side product of our calculation, we can obtain in the same way the average  $\text{Tr} \langle (\mathbf{S}^\dagger(-U/2)\mathbf{S}(U/2))^n \rangle$  by expanding  $y$  in powers of  $1/z$ ). We then obtain the following expansion of the characteristic function

$$\ln \chi^{eq}(U) = N_1 \frac{tkT}{2\pi} \left\{ - (eU\tau)^2 + \frac{1}{12} (eU\tau)^4 - \frac{1}{45} (eU\tau)^6 + \dots \right\}. \quad (14)$$

The equilibrium fluctuations of the charge in the open cavity are non-Gaussian. In the following section, we will study how this result depends on the back-reflection probability  $1 - \Gamma$  of the leads.

If we are interested in the full characteristic function, we can find the root of Eq. (12) numerically and then carry out the integration in Eq. (7). The result is plotted in Fig. 2. The diamonds indicate a numerical check that was performed by numerically averaging over a large set of scattering matrices. This fully numerical solution is very time consuming compared to finding roots of the Dyson equation. The large  $U$  limit of the characteristic function can be determined analytically to be  $\ln \chi(U \rightarrow \infty) = -\pi N k T / 12$ . The characteristic function  $\chi$  is, therefore, properly behaved for large times  $t \gg (NkT)^{-1}$ . As discussed in Ref. 4, this is the range of validity of Eq. (2). Out of equilibrium, the applied voltage plays the role of the temperature  $t \gg (NeV)^{-1}$ .

### III. RESULTS

The generalization of our approach to cavities with back reflection at the contacts is not completely straightforward.

On the one hand, the scattering matrix as given in Eq. (10) must be extended and its average  $\langle S(U) \rangle$  is no longer zero. This complication modifies the averaging procedure for the Green's functions. We discuss the crucial points in Appendix B. On the other hand, the calculation becomes lengthy and requires some algebraic software.

We use the parameters  $N=N_1+N_2$  for the total number of channels and  $\lambda=(N_1-N_2)/N$  to describe the asymmetry of the cavity (see Fig. 1). The transmission probability to pass one of the contacts of the cavity is denoted by  $\Gamma$ . For simplicity, we choose it to be the same in both contacts.

For the cumulants of fluctuations at equilibrium, we obtain

$$\begin{aligned} C_2^{eq} &= N\Gamma \frac{kTt}{\pi} (e\tau)^2, \\ C_4^{eq} &= N\Gamma \frac{kTt}{\pi} (e\tau)^4 (2-\Gamma), \\ C_6^{eq} &= N\Gamma \frac{kTt}{\pi} (e\tau)^6 (24-24\Gamma+8\Gamma^2). \end{aligned} \quad (15)$$

The dwell time is given by  $\tau$  and  $e$  is the unity charge. This result is universal in the sense that it describes also the fluctuations of any other operator inside the cavity. One has simply to replace the constant  $(e\tau)$  by other units. It is interesting to note that the higher cumulants increase with decreasing transparency of the leads. We observe the same behavior for nonequilibrium charge fluctuations and for current fluctuations in chaotic cavities<sup>8</sup>. Towards the limit of tunneling contacts, the non-Gaussian fluctuations get stronger.

In the transport regime, the characteristic function can be written in the following way:

$$\ln \chi^{neq} = \frac{NteV}{2\pi} \sum_n K_n(\Gamma) L_n(\lambda) (ieU\tau)^n. \quad (16)$$

The functions  $K_n$  and  $L_n$  describe the dependence of the cumulants on transparency of the contacts and asymmetry of the cavity. The first few coefficients are given by

$$\begin{aligned} K_1 &= \Gamma, & K_2 &= \Gamma(2-\Gamma), & K_3 &= \Gamma(3-3\Gamma+\Gamma^2), \\ L_1 &= \frac{1+\lambda}{2}, & L_2 &= \frac{1-\lambda^2}{8}, & L_3 &= -\frac{\lambda(1-\lambda^2)}{12}. \end{aligned} \quad (17)$$

These factors are plotted in Fig. 3 as a function of transparency and asymmetry. The higher the cumulant, the slower the factors  $K_n$  diminish towards the tunneling limit. The plot indicates a nonanalytic behavior for  $\Gamma \rightarrow 0$  and  $n \rightarrow \infty$ . A similar effect occurs in extremely asymmetric cavities at  $|\lambda| \approx 1$ ; Non-Gaussian fluctuations are enhanced.

Up to now we considered a completely noninteracting system ( $C_g \gg e^2 N_F$ ). We now relax the condition  $C_g \gg e^2 N_F$  by including the effect of charge screening. In the many-channel limit considered here, Coulomb blockade effects are unimportant. Interaction effects can then be treated by including a self-consistent response of the electrostatic

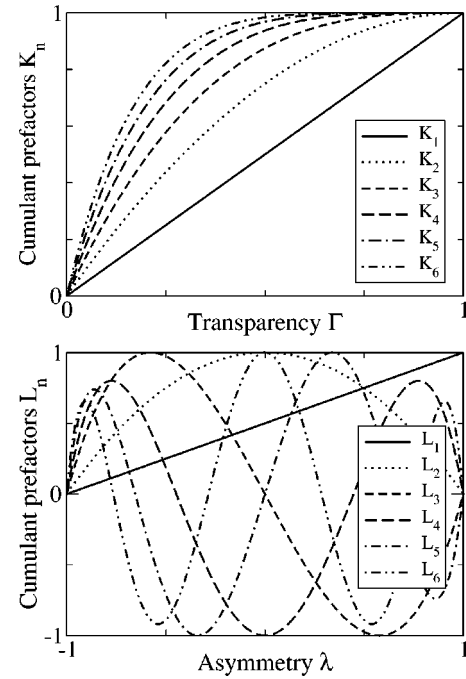


FIG. 3. Parameter dependence of nonequilibrium cumulants of charge noise. Upper panel: Each cumulant  $C_n$  depends on transmission  $\Gamma$ . The prefactors of low cumulants tend to zero in the tunneling limit, whereas high cumulants stay important. Lower panel: Each cumulant depends on the asymmetry  $\lambda=(N_1-N_2)/(N_1+N_2)$ . Cumulants higher than two may change their signs.

potential  $U$  (the bottom of the band) to the charge fluctuations.<sup>23</sup> We introduce a fluctuating electrostatic potential  $U=U_0+\delta U(t)$  that is coupled capacitively to the fluctuations of the total screened charge on the cavity  $\delta Q_{scr} = C_g \delta U$ . We then have to include the response of charge cumulants  $C_n$  to fluctuations of the potential  $\delta U$ , the so-called cascade corrections.<sup>7,8</sup> In the case of cavities this procedure is almost trivial, because only the first cumulant of charge  $C_1$ , the mean charge, depends on the electrostatic potential, see appendix C. It turns out that the screened cumulants differ from the unscreened cumulants by a universal prefactor (in the linear bias regime),

$$C_n^{scr} = \left(1 + \frac{e^2 N_F}{C_g}\right)^{-n} C_n. \quad (18)$$

An identical prefactor has been obtained in Ref. 23 for the second cumulant. This simple result relies on two facts. All cumulants higher than one do not depend on the electrostatic potential, because correlators of scattering matrices  $\langle (S^\dagger(U_1)S(U_2))^n \rangle$  only depend on the difference  $U_1-U_2$  after disorder averaging. Furthermore, the density of states  $N_F$  depends, in principle, on the particular scattering matrix. Only in the many channel limit, we can consider  $N_F$  and  $C_n$  as being independent when averaging over disorder. In general, a relation as simple as Eq. (18) cannot be expected.

#### IV. APPLICATION: PHASE FLUCTUATIONS

In this section, we estimate the contribution of the non-Gaussian fluctuations to the dephasing of electrons passing a

chaotic cavity. This allows us to formulate a validity condition for the Gaussian approximation. Models for dephasing in chaotic cavities have been considered for instance in Refs. 29,28. Dephasing rates in cavities have been measured by Huibers *et al.*<sup>30</sup> and Hackens *et al.*<sup>31</sup> Dephasing due to charge fluctuations has been treated in the scattering formalism in Refs. 15,32,33.

We approximate the dynamical phase picked up by an electron in the cavity by

$$\phi \simeq e \int_0^\tau dt' U(t') = \frac{e}{C_g} \int_0^\tau dt' Q^{scr}(t'). \quad (19)$$

The screened charge is denoted by  $Q^{scr}$ , its statistics is in the long-time limit given by Eq. (18). We consider the phase  $\phi$  to be a classically fluctuating field<sup>15</sup> (with non-Gaussian fluctuations) and average the phase over equilibrium charge fluctuations,

$$\langle e^{i\phi} \rangle \simeq \exp \left\{ -\frac{1}{2} \frac{e^2}{C_g^2} C_2^{scr} + \frac{1}{24} \frac{e^4}{C_g^4} C_4^{scr} - \dots \right\}. \quad (20)$$

Here, we assumed additionally that fluctuations on the time scale of  $\tau$  behave as fluctuations in the long-time limit. This is justified at high temperatures  $kT \gg \tau^{-1}$  if the frequency dependence of the response function  $\partial \langle Q \rangle / \partial U$  (see Appendix C) may be neglected. We combine Eqs. (15) and (18), and insert the cumulants into Eq. (20). A comparison of second and fourth cumulants gives the following condition for the validity of the Gaussian approximation

$$\frac{1}{G^2} \frac{C_\mu^2}{C_g^2} \frac{2-\Gamma}{12} \ll 1, \quad (21)$$

where we introduced the electrochemical capacitance  $C_\mu^{-1} = C_g^{-1} + (e^2 N_F)^{-1}$  and the dimensionless conductance  $G = \Gamma N$ . Note that  $C_\mu < C_g$ , therefore condition (21) always holds in the large conductance limit considered in this paper. Nevertheless, Eq. (21) indicates that the non-Gaussian corrections become important in the limit of few channels. There are strongest in the charge neutral limit  $C_\mu = C_g$  and increase in the tunneling limit  $\Gamma \rightarrow 0$ .

## V. CONCLUSIONS

In this paper, we described a way to calculate higher cumulants of charge fluctuations inside a mesoscopic conductor in the long-time limit. We first derived a general generating functional for such fluctuations in terms of the scattering matrix of a noninteracting mesoscopic system. As a specific example we considered a chaotic cavity and showed how to average the generating functional over disorder using the random matrix theory. We then studied the dependence of the generating functional on parameters as the asymmetry or transparency of the leads attached to the cavity. We find that higher cumulants contribute most to fluctuations in the tunneling limit. Finally we calculated the suppression of fluctuations in the presence of screening. Although higher cumulants of charge fluctuations are probably not directly

measurable, their knowledge is of great practical importance. It allows for instance to study decoherence effects due to fluctuations beyond the Gaussian theory.

## ACKNOWLEDGMENTS

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## APPENDIX A: DISORDER AVERAGE OF GREEN'S FUNCTIONS

In this appendix, we discuss how to average the potential dependent Green's functions (5) in the equilibrium case. In Ref. 21, Brouwer and Beenakker present a procedure to calculate the distribution of transmission eigenvalues of chaotic cavities from a disorder averaged Green's function. We adapt this procedure to our purposes. In a first step, the series in Eq. (10) is reexpressed in a compact formulation by introducing the following set of matrices:

$$\begin{aligned} \tilde{\mathbf{S}} &= \begin{pmatrix} \mathbf{S}(U/2) & 0 \\ 0 & \mathbf{S}^\dagger(-U/2) \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}, \\ \tilde{\mathbf{F}}(z) &= \begin{pmatrix} 0 & \mathbf{F}'(z) \\ \mathbf{F}(z) & 0 \end{pmatrix}, \quad \mathbf{T}' = \begin{pmatrix} \mathbf{t} & 0 & 0 & 0 \\ 0 & 0 & \mathbf{t}^\dagger & 0 \end{pmatrix}, \quad (A1) \\ \mathbf{T} = \mathbf{T}'^T, \quad \mathbf{R}' &= \begin{pmatrix} \mathbf{r}' & 0 & 0 & 0 \\ 0 & \mathbf{r}_s(U/2) & 0 & 0 \\ 0 & 0 & (\mathbf{r}')^\dagger & 0 \\ 0 & 0 & 0 & \mathbf{r}_s^\dagger(-U/2) \end{pmatrix}. \end{aligned}$$

The transmission and reflection matrices  $\mathbf{t}, \mathbf{r}$ , and  $\mathbf{r}'$  describe the back reflection in the contacts. Without barriers, we have  $\mathbf{t} = \mathbf{1}$  and  $\mathbf{r} = \mathbf{r}' = \mathbf{0}$ . The fluctuating part of the scattering matrix then is

$$\delta \tilde{\mathbf{S}} = \mathbf{T}' (1 - \tilde{\mathbf{U}} \mathbf{R}')^{-1} \tilde{\mathbf{U}} \mathbf{T}, \quad \tilde{\mathbf{U}} = \begin{pmatrix} \mathbf{U} & 0 \\ 0 & \mathbf{U}^\dagger \end{pmatrix}, \quad (A2)$$

where  $\mathbf{U}$  is the uniformly distributed unitary random matrix. The matrix Green's function  $\tilde{\mathbf{F}}$  that contains the desired Green's function (5) as off-diagonal element can be written as difference  $\tilde{\mathbf{F}} = \tilde{\mathbf{F}}^+ - \tilde{\mathbf{F}}^-$ . Both components

$$\tilde{\mathbf{F}}^\pm = \frac{1}{2\sqrt{z}} \mathbf{C} \frac{1}{\sqrt{z} \mp \tilde{\mathbf{S}} \mathbf{C}} \quad (A3)$$

can be averaged separately. According to Brouwer and Beenakker,<sup>21</sup> they obey a Dyson equation

$$\tilde{\mathbf{F}}^\pm = \mathbf{X}^\pm (1 + \Sigma^\pm(\tilde{\mathbf{F}}^\pm) \tilde{\mathbf{F}}^\pm) \quad (A4)$$

with a self-energy matrix  $\Sigma^\pm = \Sigma^\pm(\tilde{\mathbf{F}}^\pm)$  depending on the Green's function  $\tilde{\mathbf{F}}^\pm$ . The matrix  $\mathbf{X}^\pm$  is defined as  $\mathbf{X} = \mathbf{R}' + \mathbf{T} \mathbf{C} \mathbf{T}' z^{-1/2}$ . The equation for the self-energy may now be expanded up to first order in  $1/M$ , where  $M$  is the dimension

of the virtual potential dependent stub (9). After impurity averaging and some algebra, we obtain that the self-energy is of the form

$$\Sigma^\pm = \pm \frac{y}{\sqrt{z}} \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \quad (\text{A5})$$

and fulfills Eq. (12).

### APPENDIX B: EXTENSION TO CAVITIES WITH BARRIERS

In the presence of back reflection at the contacts, we have to modify the summation carried out by Brouwer and Beenakker.<sup>21</sup> Now the scattering matrix of the leads is given by  $\mathbf{t} = i\sqrt{\Gamma}$  and  $\mathbf{r} = \mathbf{r}' = \sqrt{1-\Gamma}$ . The additional complication due to barriers arises from the fact that the mean of the scattering matrix  $\bar{\mathbf{S}}$  is no longer zero. The needed form of the matrix  $\mathbf{C}$  is given by

$$\mathbf{C} = \begin{pmatrix} 0 & \mathbf{C}_1 \\ \mathbf{C}_2 & 0 \end{pmatrix}. \quad (\text{B1})$$

In order to obtain the density of transmission eigenvalues as in Ref. 21, one chooses

$$\mathbf{C}_1 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{C}_2 = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{1} \end{pmatrix} \quad (\text{B2})$$

and exploits the fact that  $\mathbf{C}_1, \mathbf{C}_2, \bar{\mathbf{S}}$  commute and  $\mathbf{C}_1\mathbf{C}_2 = 0$ . These conditions do not hold in our case. The matrix  $\mathbf{C}$  contains more entries. In equilibrium, for instance, we need  $\mathbf{C}_1 = \mathbf{C}_2 = \mathbf{1}$ . We overcome this problem by replacing  $\mathbf{C}$  through a matrix  $\mathbf{C}'$  that encounters the back reflection in the leads

$$\mathbf{C}' = \mathbf{C}(\mathbf{1} - \mathbf{R}\mathbf{C}z^{-1/2})^{-1}, \quad \mathbf{R} = \begin{pmatrix} \mathbf{r} & 0 \\ 0 & \mathbf{r}^\dagger \end{pmatrix}. \quad (\text{B3})$$

One can easily verify that this substitution generalizes the Green's function introduced in Eq. (A3) to the case of barriers at the contacts. The rest of the calculation follows the lines of Appendix A.

### APPENDIX C: SCREENED CHARGE FLUCTUATIONS

In this appendix, we explain the derivation of Eq. (18) using the cascade principle (for an introduction to cascade corrections, we refer the reader to Refs. 7–9). Charge and potential fluctuations of the cavity are linked via the capacitance  $C_g$ :

$$\delta Q_{scr} = C_g \delta U, \quad \delta Q_{scr} = \delta Q + \frac{\partial \langle Q \rangle}{\partial U} \delta U. \quad (\text{C1})$$

The charge fluctuations are composed of bare fluctuations at constant potential  $\delta Q$  and the linear response  $\partial \langle Q \rangle / \partial U = -e^2 N_F$  to  $\delta U$  (the screening charge). The fluctuations of  $\delta Q$  are known from the noninteracting problem [see Eqs. (15) and (16)]. Solving Eq. (C1) for the total charge fluctuations  $\delta Q_{scr}$ , the third cumulant without cascade corrections may be written as

$$\langle (\delta Q_{scr})^3 \rangle = \left( 1 + \frac{e^2 N_F}{C_g} \right)^{-3} \langle (\delta Q)^3 \rangle. \quad (\text{C2})$$

The two possible cascade corrections are given by<sup>7</sup>

$$3 \frac{\partial \langle (\delta Q_{scr})^2 \rangle}{\partial U} \langle \delta U \delta Q_{scr} \rangle, \quad 3 \frac{\partial^2 \langle \delta Q_{scr} \rangle}{\partial U^2} \langle \delta U \delta Q_{scr} \rangle^2. \quad (\text{C3})$$

But these corrections are zero, because  $\langle (\delta Q_{scr})^2 \rangle$  does not depend on  $U$  and  $\langle \delta Q_{scr} \rangle$  depends linearly on  $U$ . Cascade corrections to all higher cumulants vanish in the same way. Equation (18) is therefore valid for arbitrary  $n$ .

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<sup>19</sup>In principle, this volume can be of microscopic dimensions. In practice, we are often interested in charge fluctuations associated with a capacitance, and the volume is defined by the geometry of the conductor.  
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- <sup>25</sup>The first derivative  $\partial\phi_n/\partial U$  is proportional to the density of states  $N_F$ , which does not depend on the potential in a disordered system (up to small corrections of the order  $eU/E_F$  where  $E_F$  is the Fermi energy).
- <sup>26</sup>To achieve a Gauge invariant theory, which depends only on potential differences, a self-consistent treatment of the bottom of the band in the energy integral of Eq. (2) is necessary. We return to this point in Sec. III where we discuss how to include interaction effects.
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